Plastic Working of Metallic Materials
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Module 2
Lecture - 4
Slip Line Field Theory

So we will be continuing with the slip line field theory.

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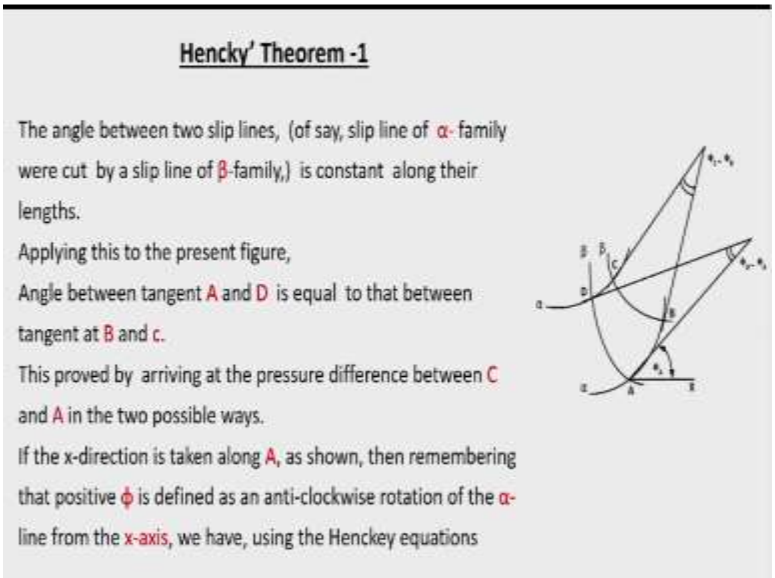

Last day, we were discussing about the Hencky’s theorem 1 where the angle between two slip lines, say, of say slip line of alpha family when it is cut by a slip line of beta family is constant along their lengths and we also derived this part, the proof of that has been derived.

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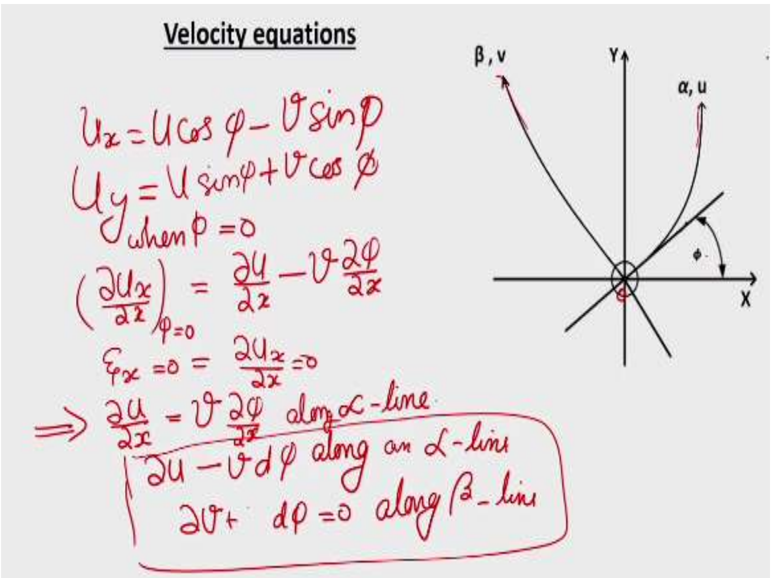

Now we will come to the second part of that, that is the velocity equations, which people just consider. So let us assume that u and v are the component velocity of a particle at a point O along a pair of alpha and beta lines. So, this is an alpha line and this is a beta line and then component velocities are u and v along these lines. Alpha line you will find that it is inclined at 5 degree to the x axis of a pair of orthogonal Cartesian coordinate axis which passes through O here.

The component of velocities of the particles of this u along the ux and uy parallel to this ox and oy, we can write it as Ux = U cos phi – V sin phi and Uy = U sin phi + V cos phi. Now taking the x direction at a point a if you just rotate it and make this alpha line, say in line with parallel to the ox, that means when phi = 0, you take the derivative of this, you will find that dou Ux by dou x at phi = 0 which can write it as if you take a partial derivative dou U by dou x – V into say sin phi at phi tending towards 0 is always a phi, so V into dou phi by dou x okay.

$$U_x = U\ Cos\ \phi - V\ Sin\ \phi$$

$$U_y = U\ Sin\ \phi + V\ Cos\ \phi$$

$$When\ \phi = 0$$

$$\left(\frac{\partial U_x}{\partial x}\right)_{\phi=0} = \frac{\partial U}{\partial x} - V\frac{\partial \phi}{\partial x}$$

And you know that along slip line, any slip line, expansion and contraction is almost 0 that is our basic thing which we discussed in the previous class itself. So, that means if you say that epsilon x = 0 so that is equal to dou Ux by dou x = 0. So this implies that dou U by dou x = V x dou phi by dou x along alpha line or we can also write that dou U = V dou phi along an alpha line. In a similar way no, you take the partial derivative of Uy and with respect to dou y, with respect to y, you can also arrive at dou V + V dou phi = 0 along a beta line. This we can write like this. So these two things, these two relationships we will get it.

$$\epsilon_x = 0 = \frac{\partial U_x}{\partial x} = 0$$

$$\frac{\partial U}{\partial x} = V\frac{\partial \phi}{\partial x}\ along\ \alpha -\ line$$

$$\partial U - V\ d\phi\ along\ an\ \alpha -\ line$$

$$\partial V + d\phi = 0\ along\ \beta - line$$

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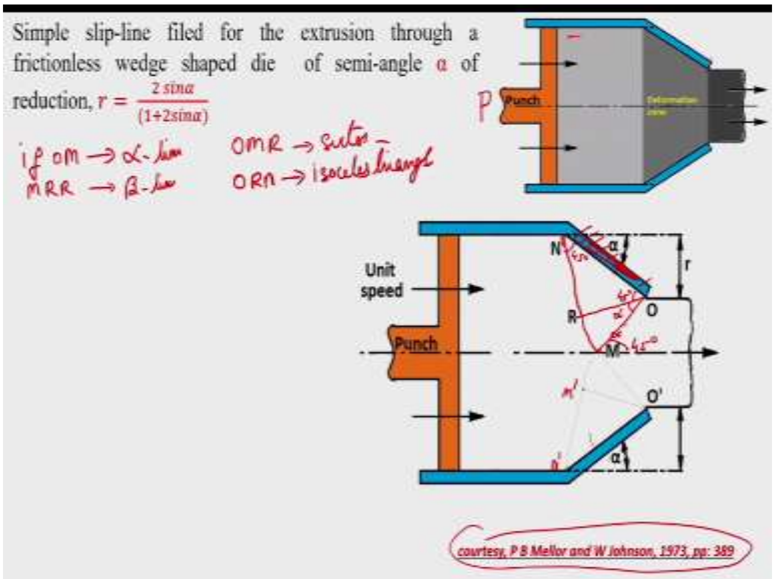

Now, let us take some typical example and find out how this slip line field theory can be applied for determining the load deformation. I will just take the case of an extrusion okay. Simple slip line field theory, it is a very simplest case. There are complicated slip line field theory which we will not be discussing. The simple slip line field for the extrusion through a frictionless wedge shaped die of semi-angle alpha of reduction r. This was taken from this, this book on Mellor and Johnson which was published in 1973, page number 389.

So this is the example which has been taken from that book itself, but let just take this and so that you understand the basic principle of that. So the top figure, it shows that, the blue colour shows the cylinder and the punch and say punch is moving and her your billet is there and then it is just moving towards the right at some velocity. So, this is the converging part of your die and the metal is deforming in that region, so here, this region, the metal is deforming in that region.

As the punch moves forward, initially the material up to this part is considered as rigid, but once it since the punch is moving, plastic deformation will take place and it will just move along this deformation zone, this is the deformation zone, and then finally when it comes out, you will find that okay there is, the metal is again rigid. So in this region, the deformation region, the material is considered as plastic, so that is what. The basic schematic of the process is shown here.

So, you wanted to know what is the stress which is applied here at the punch side so that the metal is able to deform and the metal extrudes out of the die okay. So, let us just take for that no, the conditions are that the semi angle is alpha, don't get confused with alpha line okay, but here that though it is interchangeably used here, we are using the semi die angle as alpha itself here and the reduction which is there is r = 2 sin alpha by 1+2 sin alpha is the relationship which has been produced. So, let us find out the slip line field.

$$r = \frac{2sin\alpha}{1 + 2sin\alpha}$$

The slip line field in this case because here is you will find that this is the die which is there, other part, the left of this part is container, but main die part is in this region only, here, this is the die part. So metal will be deforming inside this region. So die is assumed to be perfectly smooth and you will find that the slip line field it can just consist of an isosceles triangle ORN. So if I just draw like this ORN, here though I have drawn it, it is not clear.

So this is an isosceles triangle with here it is 45 degree and here also 45 degree and this is the die angle alpha which is shown and then this is one part of that and another part is there, this is a sector okay, which is this will be equal to your alpha okay. So, this part if I just take, these are the straight line and this part is there, we can assume that this is one slip line field and this is the next slip line field. So we can assume that this is an alpha line, if this is an alpha line, this MRN.

So if you consider, if OM is an alpha line, then MRN is a beta line because this is at 90 degree to that and OMR is a sector, OMR is a sector, ORN is an isosceles triangle. So OM is the axis at 45 degree and OMR is the sector of an angle alpha. So now if you just consider this, we choose OM, if you choose OM so that it makes 45 degree with the axis okay. If this is 45 degree with the axis, then you can say that if you take along the axis no, this is a symmetric case okay.

So, a symmetry can be obtained for the slip line about the axis in the total force to the right of OM is 0. So we can say that the metal enters region and comes out of this region, so the path which is to the right of OM is rigid, the path which is left of NRM is also rigid and the path which is inside this NOM that is plastic, similarly if I just mark this an N dash and M dash, so O dash N dash M dash and M no, see this part also is plastic. So towards the left it is rigid and towards the right it is also rigid, so inside you have a plastic region.

So, slip line about the axis and the total force to the right of OM is 0 because of this part, that is what, especially the path which comes out, the forces are 0 on this. Therefore, as per the convention, if OM is an alpha line and MRN is a beta line in that case because the direction of the algebraically greatest principal stress in this particular case if you look because along this it is K, this is K, this is K, these are perpendicular to that, so if you just resolve that, you will find that the algebraically largest principal stress in this case it is 0, which is parallel to x axis okay. So this is the slip line field theory which comes.

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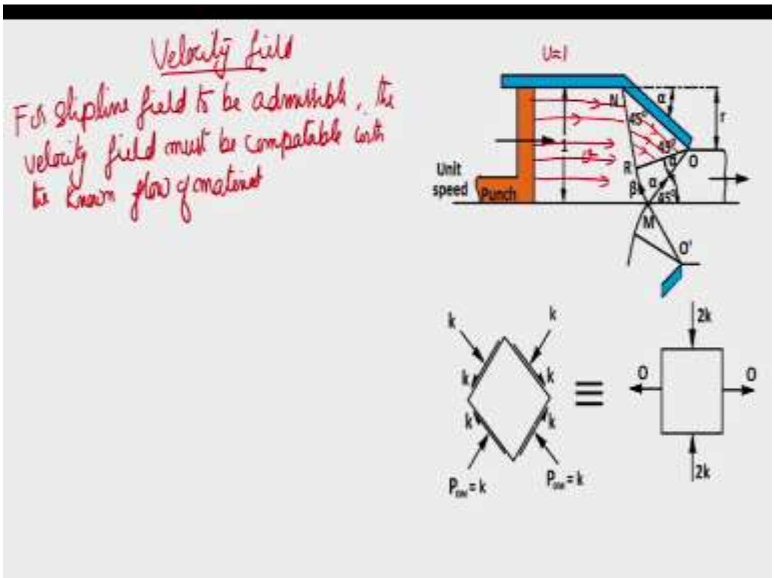

Now, let us look at that, what are the conditions? So you have to have a velocity field inside this. So for the slip line field to be admissible, the velocity field must be compatible with the known flow of the material, this is very important in this case. So, I will write that. For the slip line field to be admissible, the velocity field must be compatible with the known flow of material. So if you look at that what I have discussed, before this metal, see this is the region where material is solid okay, so this is your worker's material.

So, when the punch is pushing this up, up to say this region NRM, the material is considered as rigid, but the moment it crosses NRM, then the metal flows plastically because all the plastic deformation is taking place inside this region only. So initially, up to this NRM, you will find the material flow is in this direction, so all these things are moving in this direction okay. This is the direction of say may be velocity, but once it enters this, once it crosses this, there will be a change in the direction of metal flow and it will be parallel to your die axis.

So in all these cases, you will feel that there is parallel to die axis and let us assume that velocity V is equal to unity, initial velocity okay is unity and then whatever be the speed, with that you can always multiply. So now, we have to draw a velocity diagram for the metal, for the particle which are moving, first at the rigid area, then at the plastically deformed region, there itself you find two regions are there okay.

The moment a particle just crosses this boundary NR okay, you will find that its direction is altered the way I have shown, is altered suddenly to proceed parallel to the die face oil. So this results, this change in the direction results in a tangential velocity discontinuity. So that is what we can see there.

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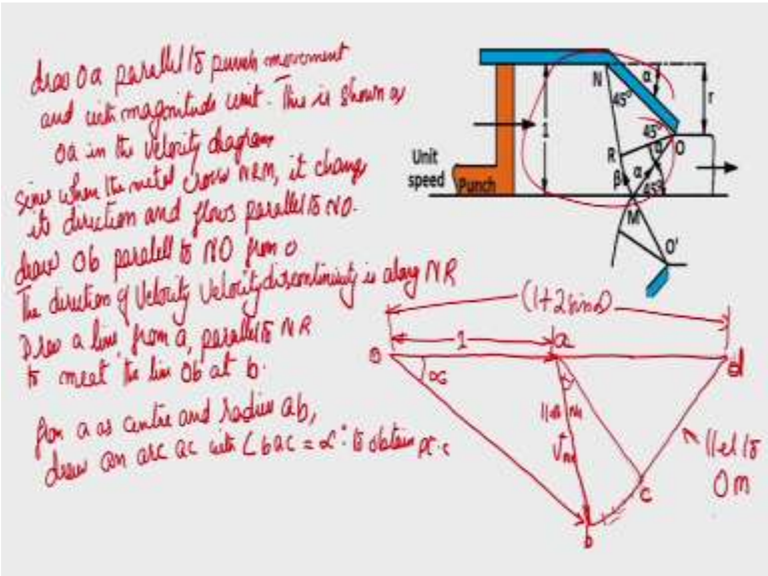

So if you try to draw the velocity diagram in this case, how you do, say because here it is not clear, I have just, I tried to draw it, but anyway since it is not clear, I will draw it. Say from O, this is your initial point, you draw that. We found that the punch velocity is unity, so you just draw a line parallel to the punch of flow direction ant it reaches at a, so that is your initial punch velocity. This represents the movement of the particle just ahead of the punch till it enters the deformation zone NRM.

Now when it crosses this, when it crosses this NR, there is a change in the direction, so this results in a tangential velocity discontinuity, so to draw that you have to draw the line parallel to NR okay. So from O, you draw a line at an angle α, so which represents this direction NO, so this is your α because it is flowing in this direction and then from a you draw this velocity discontinuity, so that this means and you get the velocity triangle oab okay. So the triangle oab is completed from its magnitude of oa and ob can be determined. So how is that?

Draw oa parallel to punch velocity direction and magnitude unity. This is represented as oa in the velocity diagram. Since when the metal crosses NRM, it changes its direction and flows parallel to NO. So, draw ob parallel to NO from o. The direction of velocity discontinuity is along NR. So, draw a line from a parallel to NR to meet the line ob at b and you get this velocity triangle oab okay. So this velocity discontinuity, this is parallel, this is parallel to NR, so we can say that V star NR okay and this is b.

Now, you will find that ac is at 45 degree to oa produced. We can draw this ac because ORM is a sector, so you will find that okay this is also the velocity diagram for that. So, what we can do is that you can just draw this as α because here the angle subtended is α, so here from a draw a line at α degree and with a radius ab you draw bc. From a as center and radius ab draw an arc ac with angle bac is equal to α degree to obtain point c, so you are getting this ac.

So, whatever is happening inside this, say the moment the metal crosses this region, you will find that okay this is your velocity, so at any point, the velocity at may be here, here, here, everywhere you can find it out like this at this point okay and see this is also your beta line. So, you will find that and now if you just draw that after crossing over OM, the metal may be flowing like this and then okay anywhere it is coming and then it will cross this OM, once it crosses the OM, again the material is rigid and the direction of flow is in the initial case.

Because there is a reduction which has been specified in this problem, the velocity at the exit will be higher than the velocity at the inlet, so that we can find out this point d, you just extend this, this oa you extend it and this beta slip line is also at 45 degree to the M. So, here also it is at 45 degree to that. So, draw this parallel to OM, so parallel to OM so that it will meet at this point d and the entire velocity diagram we can get it okay.

A particle on the axis because since this is a symmetry, I have shown only the half of that path, the remaining half which was shown in the previous slide has been removed, so because of the symmetry, the same thing is there, may be up to here we can see that end okay. A particle on the axis you will find that it is subjected to two successive jumps, initially the first one is ac and secondly one of equal direction of magnitude MO, in this direction MO, which is also at 45 degree to oa which is produced, oa to produce, and derived from the lower half of extrusion.

So, this is equal to your cd, these two, so two jumps are there, and the value of do is found to be, so if you take this as 1, this do will be equal to $1 + 2\ sin\ \alpha$, then only you will find that reduction. The material after emerging out from MO, from this part, is again rigid and this velocity of any particles in the physical plane inside this region which is represented by this lower figure, this is called as a hodogram.

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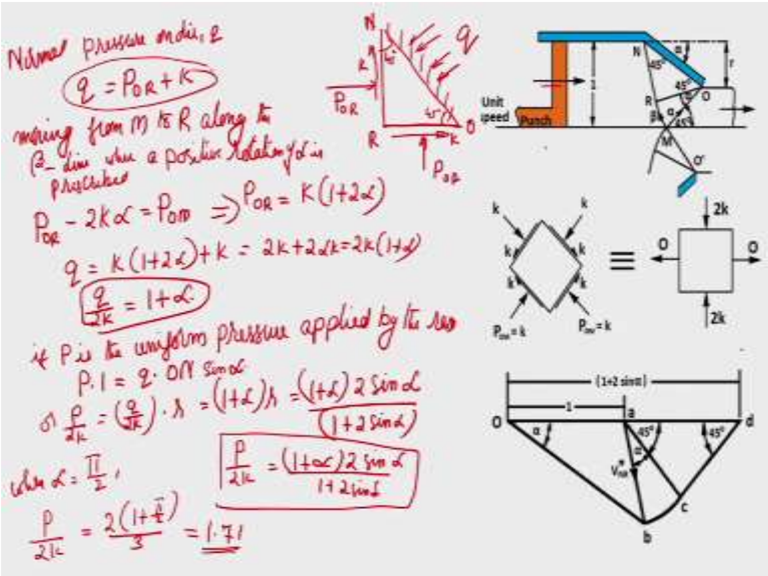

Now we have to find out, calculate the forces which are there. The normal pressure on the die is q. So like if this is the die angle, this is 45 degree and you can say that this is your pressure P_{OR} and this is also pressure P_{OR}. Because it has slip line, you have this shear stress which is K and here also the shear stress is K. So, this is R, this is N, and this is O. So, this is your die surface. So, the normal pressure, normal pressure on the die surface is like this okay, which is coming. So this is q, so that q is equal to, okay normal pressure q.

So you can write that $q = P_{OR} + K$, so that you can get it here okay. So P_{OR} is the hydrostatic pressure normal to the plane OR okay, normal to this plane OR is hydrostatic pressure, that is this P_{OR}. In this case, it is normal to this line, so that is what, so you can get it when you just find out, equated this, you will get this P_{OR}, so now if you are using the Hencky equation to find the P_{OR}, we can say that P_{OM} = K that is coming.

So in this if you just write, convert it into this one, transforming to this one, you will find that along this axial direction it is 0 that is the principle of maximum stress is 0 along this x axis and here you get 2K. So if P_{OM}= K and that moving from M to R along the beta line MRN and you take a positive rotation. So moving from M to R along the beta line where a positive rotation of alpha is prescribed. So we can write it as P_{OR}– 2 K alpha = P_{OM} or that is it implies that P_{OR}= K x 1+2 alpha, we can get this relation.

$$P_{OR} - 2K\alpha = P_{OM}$$

$$P_{OR} = K(1 + 2\alpha)$$

If you substitute this P_{OR} in this relationship here, so you can get the equation for that say like q = K into 1+2 alpha + K, so this is equal to 2K + 2 alpha = 2 alpha K, that is 2K into 1+ alpha you are getting or you can say that q by 2 K nondimensional pressure = 1 + alpha you are getting and when you look at the uniform pressure applied by the ram if it is P, if P is the uniform pressure applied by the ram, then P into 1 = q into ON sin alpha or we can say P by 2 K = q by 2 K into r which is nothing but the reduction okay or that is equal to q by 2 K we got it as 1+ alpha into r, and r = 1 + alpha into 2 sin alpha by 1+2 sin alpha.

$$q = K(1 + 2\alpha) + K = 2K + 2\alpha K = 2K(1 + \alpha)$$

$$\frac{q}{2K} = 1 + \alpha$$

$$if\ P\ is\ the\ uniform\ pressure\ applied\ by\ the\ ram$$

$$P.1 = q.ON\ sin\alpha \quad or$$

$$\frac{P}{2K} = \left(\frac{q}{2K}\right).r = (1 + \alpha)r = \frac{(1 + \alpha)2sin\alpha}{1 + 2sin\alpha}$$

So this is the relationship, that is P by 2 K = 1 + alpha into 2 sin alpha by 1+2 sin alpha. Now, if you just take the condition a special case when alpha is equal to pi by 2, you will find that P by 2 K you are getting it as 2 into 1 + pi by 2 by 3 because α is 90, so sin alpha is 1 and here also sin alpha is 1, so you will get it this which is equal to 1.71. So, from this we can find out what is the value of PK, so that means if alpha is 90 degree means there is a sharp change in the direction which is taking place, here alpha is a small value, whatever will be that is the extreme condition when it is coming.

$$\frac{P}{2K} = \frac{(1 + \alpha)2\sin\alpha}{1 + 2sin\alpha}$$

$$when\ \alpha = \frac{\pi}{2},$$

$$\frac{P}{2K} = \frac{2\left(1 + \frac{\pi}{2}\right)}{3} = 1.71$$

So you can find out if you know the value of your shear stress of the material, then you can directly find out this, this is the pressure, uniform pressure which is applied at this punch multiplied by its cross sectional area of the punch will give you the total load requirement for the deformation process.

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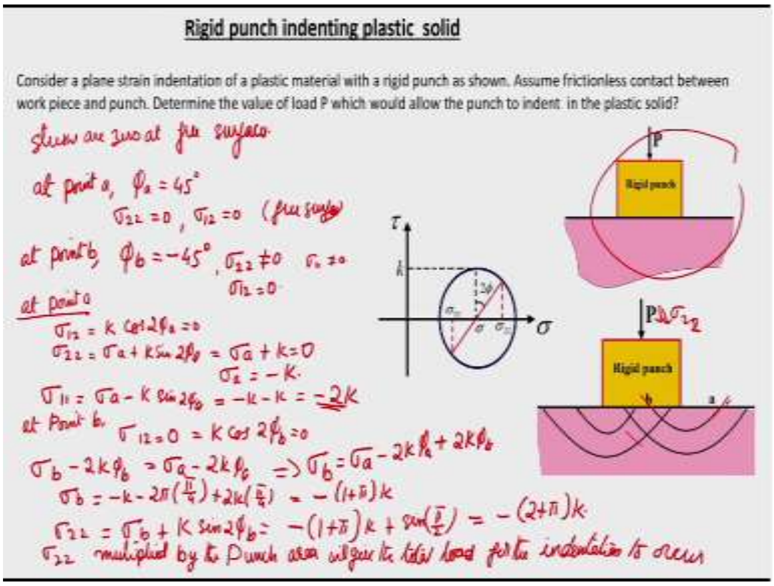

Now let us take another simple problem with the application of slip line field to finding out the in a rigid punch indenting a plastic solid. It is like a hardness testing type, but only the difference is that in this it is, the indentation is carried out with a rigid punch okay, flat punch, circular punch we can say. So, we are just assuming that this is also a case of plain strain indentation of a plastic material with a rigid punch which is shown in the top figure here.

We are also assuming that the frictionless interface between the rigid punch and the plastic material is taking place because there is no, we are ignoring that, friction which is coming. What is required, what is to be determined is the value of the load P which would allow the punch to indent in the plastic solid, so that is what we have to consider. So we can just construct a slip line field here as shown in this below this diagram okay. So here, this is an alpha line and this may be a beta line, so these are the two alpha lines which are there and beta lines are there.

So, if you take this, the assumptions are that at surfaces, the stresses are 0. So, stresses are 0 at free surfaces, so if this a to b is an alpha line, just consider, if you considering that a to b is an alpha line, the metal is trying to deform in this okay. So, now let us look at this condition, one is a, one is b okay. So because when you are trying to do that, at the free surface since the stresses are free, so you will see that this inclination of this alpha line is at 45 degree to the surface. Similarly, at this also it is 45 degree to the surface.

So if you take these two points, at point a phi a = 45 degree, that is the inclination of your alpha line is 45 degree and in that condition, you will find that sigma 22 which is your vertical line, sigma 22 = 0 and sigma 12 = 0 since it is free surface. Now if you take it at point two at point b, phi b = –45 degree compared to the initial 45 degree, here it is the appositive one -45 degree and you will find that sigma 22 is not equal to 0 and sigma 12 = 0, sigma 11 is also not equal to 0. So, this is what we are finding out.

$$at\ point\ a, \phi_a = 45°$$

$$\sigma_{22} = 0, \sigma_{12} = 0\ (free\ surface)$$

$$at\ point\ b, \phi_b = -45°, \quad \sigma_{22} \neq 0, \sigma_{12} = 0, \sigma_{11} \neq 0$$

Now, let us consider the condition at point a, so this point a. At point a, we can write sigma 12 = 0, that is K cos 2 phi a = 0 if you look at the Mohr circle diagram. If you are just referring to Mohr circle diagram, this is sigma 22, this is sigma 11 and this is the mean value sigma and this is 2 phi. So from this, we can write it as sigma 12, because at this point, it is equal to 0, similarly sigma 22 is equal to, you can say that sigma a + K cos 2 phi a, sigma a means here okay, 2 phi a, that is equal to sigma a + K because 2 phi a means 90 degree, sigma 22 since it is 0.

So, sigma + K = 0, that means sigma a = – K. similarly sigma 11 is equal to we can say from this equation, sigma 11 is sigma a – K sin 2 phi a, sorry this is not cos, this is sin. So that you will get it as sigma a is –K, –K that is equal to –2K, but at point b, sigma 12 = 0, that is equal to K cos 2 phi b = 0 when you come here okay. Now if you just look at it following the Hencky’s equation, you can write that sigma b – 2 K phi b = sigma a – 2K phi a, so that means sigma b = sigma a – 2K phi a + 2K phi b.

$$at\ point\ a$$

$$\sigma_{12} = Kcos2\phi_a = 0$$

$$\sigma_{22} = \sigma_a + Ksin2\phi_a = \sigma_a + K = 0$$
$$\sigma_a = -K$$
$$\sigma_{11} = \sigma_a - Ksin2\phi_a = -K - K = -2K$$
$$at\ point\ b,$$
$$\sigma_{12} = 0 = Kcos2\phi_b = 0$$
$$\sigma_b - 2K\phi_b = \sigma_a - 2K\phi_b$$
$$\sigma_b = \sigma_a - 2K\phi_a + 2K\phi_b$$

So that is sigma b = sigma a is –K, so –K–2K into phi by 4 that is 45 degree, phi a is 45 degree that is pi by 4 + 2K into – pi by 4, so that you will get it as– 1 + pi into K, sigma b value. So you can find out the stress at sigma 22. So sigma 22 = sigma b + K sin 2 phi b = – 1 + pi into K + sin pi by 2, that will come to –2 + pi into K and this sigma 22 is the pressure under the punch and which is the uniform pressure under the punch. So this is sigma 22 we are getting okay. So, that into the punch cross section area will give you the load necessary for indentation.

$$\sigma_b = -K - 2\pi\ \left(\frac{\pi}{4}\right) + 2K\ \left(\frac{\pi}{4}\right) = -(1+\pi)K$$
$$\sigma_{22} = \sigma_b + Ksin2\phi_b = -(1+\pi)K + \sin\left(\frac{\pi}{2}\right) = -(2+\pi)K$$

So this since the pressure under the punch is uniform, sigma 22 multiplied by the punch cross sectional area will give the total load for the indentation to occur. Now if you look at that these are 2 examples just to illustrate how this slip line field theory can be applied for determining the load required for the deformation. There are very advanced forms also with more accurate, this one you may find out discontinuities or several jumps are taking place, then central fan methods are also being used.

Depending upon each and every application, different types of slip line fields have been developed also. We will not be explaining all those things here in this course, but by this, you can just determine the load necessary for the deformation. So next, we will come to the bound theories. The next lecture, it will be the bound theories okay. Thank you very much.