Plastic Working of Metallic Materials
Prof. Dr. P. S. Robi
Department of Mechanical Engineering
Indian Institute of Technology – Guwahati

Module 2
Lecture - 6
Plasticity Theory (plasticity equations)

Today, we will come to this lecture 6 of module 2. We will be discussing about the plasticity equations. When you wanted to go further, we wanted these certain terms, terminology which somehow was not discussed earlier. So if you look at, we have already discussed in the case that the flow rule like when the stresses at a given point in a material reaches a certain level, the plastic deformation or flow of the material starts okay. So that is the condition of yield criteria as per Tresca or von Mises criteria.

So that means, we have that the stress is reaching a certain value, then as per Tresca criteria, maximum shear stress is found and then for von Mises criteria also we have discussed about the equations under what condition the plastic deformation will take place okay. So many times for analysis purpose, it is required to arrive at certain relationships between the applied stress and the velocity field, either velocity or the strain field or the strain rate fields. So, this relationship between the stress and the strain or stress and the strain rate or stress and the velocity, these relationships we wanted for analysis purpose.

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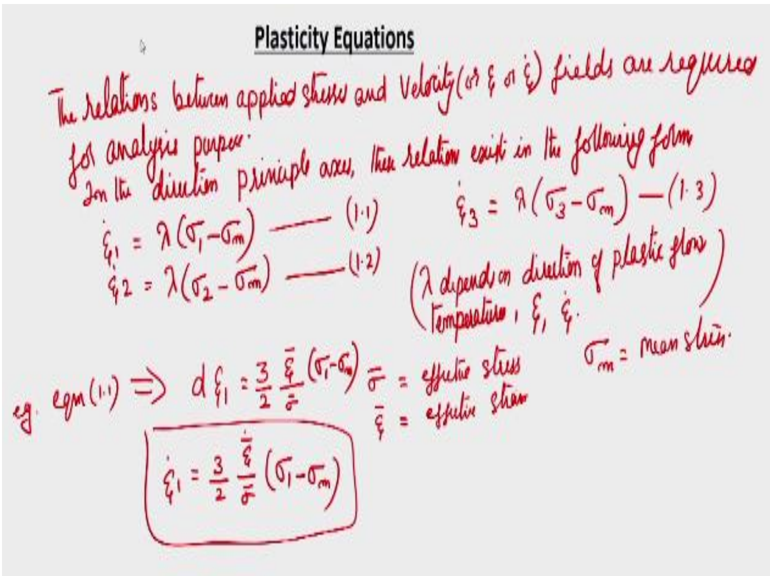

So the relations between applied stress and velocity or strain or strain rate fields are required for analysis. So in the direction of the principal axis, these relations exist in the following forms, that is, if you just look at it epsilon dot 1 = lambda into sigma 1 – your mean stress or hydrostatic stress, if I just put it as 1.1 and epsilon dot 2, the strain rate along this direction, lambda into sigma 2 – sigma m that is your mean stress and epsilon dot 3 = lambda into sigma 3 – sigma m, this is 1.3 okay and these equations are called as the plasticity equations.

$$\dot{\epsilon_1} = \lambda(\sigma_1 - \sigma_m)$$
$$\dot{\epsilon_2} = \lambda(\sigma_2 - \sigma_m)$$
$$\dot{\epsilon_3} = \lambda(\sigma_3 - \sigma_m)$$

The variable λ, this λ depends upon on direction of plastic flow, the temperature at which you are deforming, the strain, and the strain rate. So this is the thing. This depends upon these parameters, and we can write this equation, the flow equation in a different way. So taking in to consider the strain increments also we can write. For example like say equation 1.1, we can write in this form as d epsilon 1 = 3 by 2 into epsilon bar by sigma bar.

$$d\epsilon_1 = \frac{3}{2}\frac{\overline{\epsilon}}{\overline{\sigma}}(\sigma_1 - \sigma_m)$$

In that format also we can write where this ϵ bar and $\bar{\sigma}$ they are called as the see sigma bar is the effective stress or equivalent stress or this is called as the effective strain, and from this we can write epsilon dot 1 = 3 by 2 into epsilon dot effective strain rate by this one, we can arrive at this relationship also sigma 1 – sigma m, here I did not write the sigma 1 – sigma m where sigma m is the mean stress hydrostatic stress. So we can write it in this form, now why because this we have to discuss in the subsequent part today itself.

$$\dot{\epsilon_1} = \frac{3}{2}\frac{\overline{\epsilon}}{\overline{\sigma}}(\sigma_1 - \sigma_m)$$

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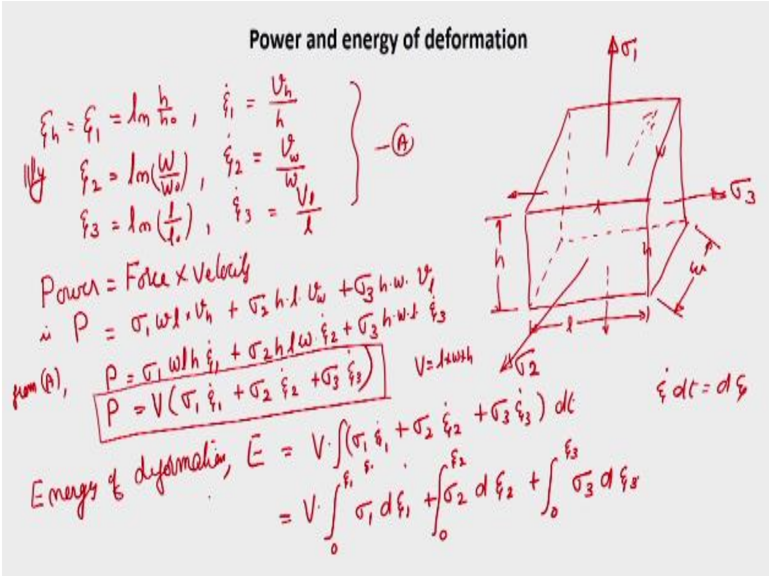

So let us come to this power and energy for deformation, this part we have not discussed because during the plastic working, we have to always know what is the power for the deformation and what is the energy for the deformation. So, this is very much important from a practical point of view. The power is very important actually. So in this if you consider a homogeneous deformation of a rectangular block, so maybe the rectangular block was like this with these dimensions and other things and the axis.

So let us just a rectangular block is being deformed, so where this is your length l and this is your width and maybe this is your height and you are applying stressors, maybe a generalized σ_1 here. These are your principal stresses, σ_3 here in this direction and maybe in this direction σ_2 okay. So if you just complete this even in this direction also, it is there. So a rectangular block of size l by w by h is there and it is compressed along the x-direction.

So if you can say that the strain epsilon h at any instant we can say as epsilon 1 = log h by h0 and maybe if you consider the strain rate along this direction is equal to here Vh by h. Instantaneous value of the velocity by the instantaneous height, so that is what your strain rate is, we have discussed earlier. Similarly, we can write epsilon 2 = log, 2 is equal to w okay, so w by w0 and this strain rate along this direction tau = your V into V of w by w, and epsilon 3 = log l by lo or epsilon dot 3 = your Vl along the direction l by l. So these 3 equations are there.

$$\epsilon_h = \epsilon_1 = \ln\frac{h}{h_0}, \quad \dot{\epsilon_1} = \frac{v_h}{h}$$

$$\epsilon_2 = \ln\left(\frac{W}{W_0}\right), \quad \dot{\epsilon_2} = \frac{v_W}{w}$$

$$\epsilon_3 = \ln\left(\frac{l}{l_0}\right), \quad \dot{\epsilon_3} = \frac{v_l}{l}$$

So from this, under this stresses, the material has deformed by a small amount, so we can just write by a small strain increment in all the direction if it is there, so what is the power? So the power you know it that is force x velocity. So power P = force into velocity, so that is P = if you look at all these directions, sigma 1 into this is w, this is l okay and this is h okay. So P1 into w into l into Vh + sigma 2 into h into l into Vw + sigma 3 into h into w into Vl.

$$P = \sigma_1 wl \times v_h + \sigma_2 h \cdot l \cdot v_w + \sigma_3 h \cdot w \cdot v_l$$

So from the above relationship, from A, we can write it as P = sigma 1 wlh epsilon dot 1 because Vh = h into epsilon dot 1 + sigma 2 into h into l into w into epsilon dot 2 + sigma 3 into h into w into l into epsilon dot 3 or that we can write just w into l into h = the volume V. So that we can write it as it is equal to V into sigma 1 epsilon dot 1 + sigma 2 epsilon dot 2 + sigma 3 epsilon dot 3. So this is the power which we can get it if you know the strain rate and velocity and the stresses okay, the principal stresses if you know.

$$P = \sigma_1 wlh\,\dot{\epsilon_1} + \sigma_2 hlw\,\dot{\epsilon_2} + \sigma_3 h \cdot w \cdot l \cdot \dot{\epsilon_3}$$

$$P = V(\sigma_1\dot{\epsilon}_1 + \sigma_2\dot{\epsilon}_2 + \sigma_3\dot{\epsilon}_3)$$

Now if you look at the energy of deformation E that is equal to you have to just integrate it okay, so that is equal to V into integral sigma 1 epsilon dot 1 + sigma 2 epsilon dot 2 + sigma 3 epsilon dot 3 the whole into integrating with respect to time okay, because since epsilon dot dt, epsilon dot dt = d epsilon. So, we can write this as volume into integral from 0 to epsilon 1 sigma 1 d epsilon 1 integral from 0 to epsilon 2 sigma 2 into d epsilon 2, the strain increments you are writing, + integral from 0 to epsilon 3 sigma 3 d epsilon 3 okay. So this way, we can write out the total energy of deformation okay.

$$Energy\ of\ deformation, E = V\int (\sigma_1\dot{\epsilon}_1 + \sigma_2\dot{\epsilon}_2 + \sigma_3\dot{\epsilon}_3)dt \qquad \dot{\epsilon}dt = d\epsilon$$

$$= V\int_0^{\epsilon_1} \sigma_1 d\epsilon_1 + \int_0^{\epsilon_2} \sigma_2 d\epsilon_2 + \int_0^{\epsilon_3} \sigma_3 d\epsilon_3$$

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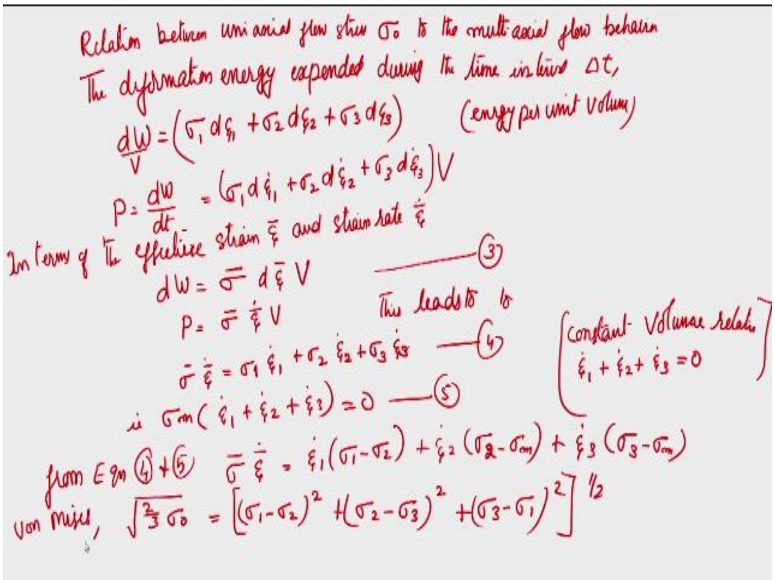

See many times it is necessary to relate the uniaxial flow stress σ_0 to the multi-axial material flow behavior and considering the element and the principal directions, the deformation of the dw expended during the time interval dT. So relation between uniaxial flow stress sigma 0 to the multi-axial flow behavior and considering the element and the principal direction, the deformation energy ΔW, the deformation energy during a small interval time, the deformation energy expended during the time interval dt or delta t is dw = sigma 1 d epsilon 1 + sigma 2 d epsilon 2 + sigma 3 d epsilon 3.

$$\frac{dW}{V} = (\sigma_1 d\epsilon_1 + \sigma_2 d\epsilon_2 + \sigma_3 d\epsilon_3)$$

So we can write that dw by V also per unit volume okay. So the deformation power P = dw by dt. So that will come to sigma 1 d epsilon dot 1 + sigma 2 d epsilon dot 2 + sigma 3 d epsilon dot epsilon 3, so this if you write it in terms of effective strain rate, strain and strain rate we can define. The effective strain and the strain rate in terms of effective strain epsilon bar and strain rate epsilon bar dot, we can define thus dw = sigma bar into d epsilon bar into V. So let us write this as equation number 3 and so we can write P = say sigma bar into epsilon bar dot into V.

$$P = \frac{dW}{dt} = (\sigma_1 d\dot{\epsilon}_1 + \sigma_2 d\dot{\epsilon}_2 + \sigma_3 d\dot{\epsilon}_3)V$$

$$In\ terms\ of\ the\ effective\ strain\ \bar{\epsilon}\ and\ strain\ rate\ \dot{\bar{\epsilon}}$$

$$dW = \bar{\sigma} d\bar{\epsilon} V$$

$$P = \bar{\sigma}\dot{\bar{\epsilon}}V$$

$$\bar{\sigma}\dot{\bar{\epsilon}} = \sigma_1\dot{\epsilon}_1 + \sigma_2\dot{\epsilon}_2 + \sigma_3\dot{\epsilon}_3$$

$$constant\ volume\ relation \quad \dot{\epsilon}_1 + \dot{\epsilon}_2 + \dot{\epsilon}_3 = 0$$

So this leads to epsilon dot bar = sigma 1 epsilon dot 1 + sigma 2 epsilon dot 2 + sigma 3 epsilon dot 3. If you apply the constant volume relationship, constant volume relation is some of the strains are 0 and similarly it is the sum of the strain rates are also 0. So that or sigma m the hydrostatic stress into epsilon dot 1 + epsilon dot 2 + epsilon dot 3 = 0, let it 5. So from this 4 and 5, we can get this relationship sigma bar epsilon dot bar = epsilon dot 1 into from the plasticity relationship okay.

So that is as what we discussed in the morning itself, sigma 1 – sigma 2 + epsilon dot 2 into sigma 2 – sigma m + epsilon dot 3 into sigma 3 – sigma m, earlier we have discussed this relationship. So that will lead to say cubed root of sorry square root of 2 by 3 sigma 0 = sigma 1 – sigma 2, see this is the von Mises criteria + sigma 2 – sigma 3 square + sigma 3 – sigma 1 square the whole raised to 1 by 2, this is the von Mises criteria.

$$\sigma_m(\dot{\epsilon}_1 + \dot{\epsilon}_2 + \dot{\epsilon}_3) = 0$$

$$\bar{\sigma}\dot{\bar{\epsilon}} = \dot{\epsilon}_1(\sigma_1 - \sigma_2) + \dot{\epsilon}_2(\sigma_2 - \sigma_m) + \dot{\epsilon}_3(\sigma_3 - \sigma_m)$$

$$\sqrt{\frac{2}{3}}\sigma_0 = [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{\frac{1}{2}}$$

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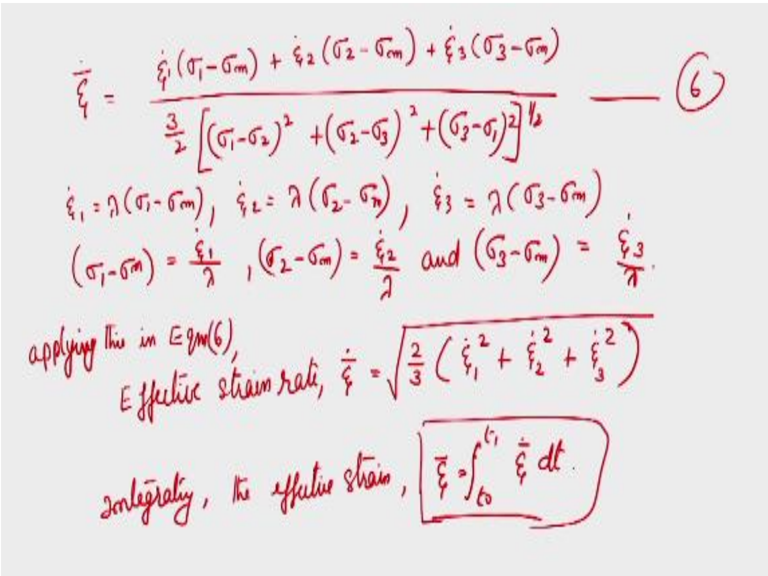

So from this relationship, we can write it as epsilon dot bar = epsilon dot 1 into sigma 1 – sigma m epsilon dot 2 into sigma 2 sigma m where sigma m is the hydrostatic stress + sigma 3 + epsilon dot 3 into sigma 3 sigma m divided by 3 by 2 into sigma 1 – sigma 2 square + sigma

2– sigma 3 square + sigma 3 – sigma 1 square the whole raised to 1 by 2. This is equation number 6. We also arrived at that sigma dot 1 = lambda into sigma 1 – sigma m and epsilon dot 2 = lambda into sigma 2 – sigma and the first slide itself, no sorry, I think it was first slide no, ha yeah, right right, and epsilon dot 3 = lambda into sigma 3 – sigma m.

$$\dot{\bar{\epsilon}} = \frac{\dot{\epsilon_1}(\sigma_1 - \sigma_m) + \dot{\epsilon_2}(\sigma_2 - \sigma_m) + \dot{\epsilon_3}(\sigma_3 - \sigma_m)}{\frac{3}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{\frac{1}{2}}}$$

$$\dot{\epsilon_1} = \lambda(\sigma_1 - \sigma_m), \quad \dot{\epsilon_2} = \lambda(\sigma_2 - \sigma_m), \quad \dot{\epsilon_3} = \lambda(\sigma_3 - \sigma_m)$$

So using the plasticity equation, we can write it as sigma 1 – sigma m = lambda dot 1 sorry epsilon dot 1 by lambda and sigma 2 – sigma m = epsilon dot 2 by lambda and sigma 3 – sigma m = epsilon dot 3 by lambda. So using this equations, applying this in equation number 6, we get the effective strain rate. So effective strain rate epsilon bar dot = if you substitute you will get as 2 by 3 into epsilon dot 1 square + epsilon dot 2 square + epsilon dot 3 square, and if we integrate it, the effective strain can be obtained from this = integral from t2 to t1 dt.

$$(\sigma_1 - \sigma_m) = \frac{\dot{\epsilon_1}}{\lambda}, \quad (\sigma_2 - \sigma_m) = \frac{\dot{\epsilon_2}}{\lambda} \; and \; (\sigma_3 - \sigma_m) = \frac{\dot{\epsilon_3}}{\lambda}$$

$$Effective\ strain\ rate, \dot{\bar{\epsilon}} = \sqrt{\frac{2}{3}(\dot{\epsilon}_1^2 + \dot{\epsilon}_2^2 + \dot{\epsilon}_3^2)}$$

$$Integrating, the\ effective\ strain, \dot{\bar{\epsilon}} = \int_{t_0}^{t_1} \dot{\bar{\epsilon}}\, dt$$

See this is how we can calculate the effective strain and effective strain rate for a plasticity problems okay. Now with this in background, let us do one or two problem, today we can do that.

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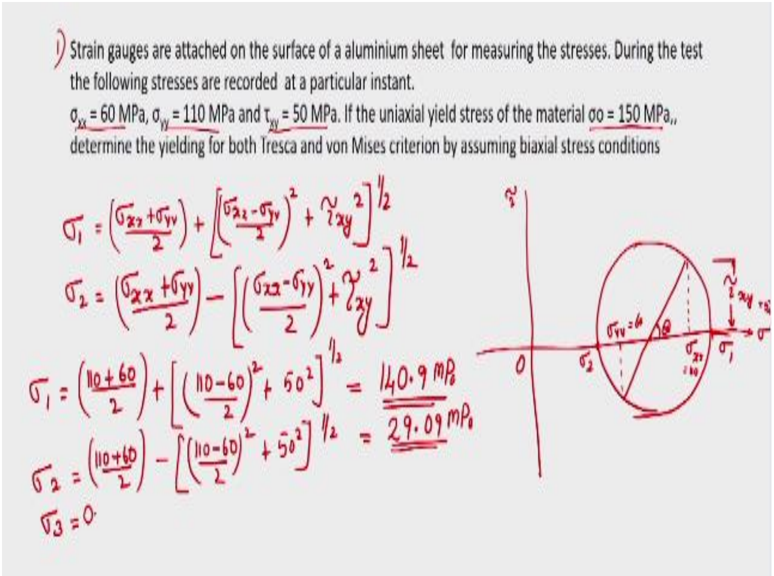

So the question number 1 is since we have not discussed about it and the assignments are given, but I thought it is always better to do some problem in this relationship. So the question number 1 is the strain gauges are attached on the surface of aluminium of an aluminium sheet for measuring the stresses. So there was an aluminium sheet, so its thickness is very less, you can assume that it to be plane stress condition, stresses are along a plane only, third direction there is no stresses okay, so that we can assume because it is very thin.

During the test, the following stresses are recorded at a particular instant, any instant, you found this these are the stresses the values of which are obtained from the strain gauges in the particular directions okay. So one is you found that the sigma xx = 90 okay, sigma yy = 110 megapascal and tau xy = 50 megapascal. So these three things we got it. If the uniaxial yield stress of the material sigma 0 = 150 megapascal, determine the yielding for both the Tresca and von Mises criteria by assuming biaxial stress conditions.

So if the uniaxial yield strength of the material sigma 0 is 150 megapascal, determine the yielding for both the Tresca as well as for von Mises criteria assuming this one, assuming that it is a biaxial stress condition because when it is a thin aluminium sheet, we can always consider it as a biaxial stress condition okay, so no triaxial because along the third axis the stresses can be neglected that is what the basic assumption we are going to take it.

Now this problem can be done by 2 methods, one is by a simple method is by the drawing the drawing the Mohr circle and determining the values and another is by the mathematical analytical method using the stress invariants which you have discussed in the earlier classes, the straight of stress, there we have discussed about the stress invariants, by using that we can do it. So it is like, first let us discuss about how to solve this by von Mises criteria though you have, sorry by Mohr circle determination though you have studied that in the classes, at least for revision, this will be of helpful for you.

So in a biaxial state of stress if I just draw the circle like this, so this is your sigma 1 principal stress, this is your sigma 2, and if I draw a state of stress at any instant say with this as theta, so if I just drop this, this largest one give you sigma xx and if I drop the projection, this will give you sigma yy and this will give you your tau xy okay. This is the general and this is your 0, this is your tau, and this is your sigma okay, in this direction you can write it and her it may not be good here we let write okay.

So applying this, this is theta, applying the Mohr circle from the mathematics part of that, we can write it as sigma 1 = sigma xx + sigma yy divided by 2, that is your half part here which is coming + square root of sigma xx – sigma yy by 2 the whole square, so that means this radius square okay, + tau xy square is 1 by 2. This is the mathematical relationship, if you look at this, we can always find out what is sigma 1 and similarly sigma 2 will be the same thing, only here it is negative, sigma xx + sigma yy by 2 – sigma xx – sigma yy by 2 the whole square + tau xy square whole raised to 1 by 2.

$$\sigma_1 = \left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right) + \left[\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2\right]^{\frac{1}{2}}$$

$$\sigma_2 = \left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right) + \left[\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2\right]^{\frac{1}{2}}$$

Now, we get that this value is sigma xx = 110 = 110 megapascal and this is equal to 60 megapascal okay and this value is equal to 50 okay. So if you just substitute, so we can find it out sigma 1 = 110 + 60 divided by 2 + 110 – 60 divided by 2 the whole square + 50 square the whole square, so that we will get, you will arrive at 140.9 megapascal. Sigma 1 = 140.9 megapascal and sigma 2 = since it is negative, so you will find 110+ 60 by 2 – 110 – 60 by 2 the whole square + 50 square raised 1 by 2, that you will get it as 29.09 megapascal.

$$\sigma_1 = \left(\frac{110 + 60}{2}\right) + \left[\left(\frac{110 - 60}{2}\right)^2 + 50^2\right]^{\frac{1}{2}} = 140.9\, mPa$$

$$\sigma_2 = \left(\frac{110 + 60}{2}\right) - \left[\left(\frac{110 - 60}{2}\right)^2 + 50^2\right]^{\frac{1}{2}} = 29.09\ mPa$$

$$\sigma_3 = 0$$

So you know that sigma 3 = 0 because it is a biaxial state of stress, so sigma 3 = 0 that we know that. So now let us look at as per the Tresca criteria, the material will start plastically deforming or yielding when the shear stress, the maximum shear stress is the half of the largest and the smallest principal stresses okay. So in this case it is equal to, now let us check us as per the von Mises criteria, the yielding can take place under this condition.

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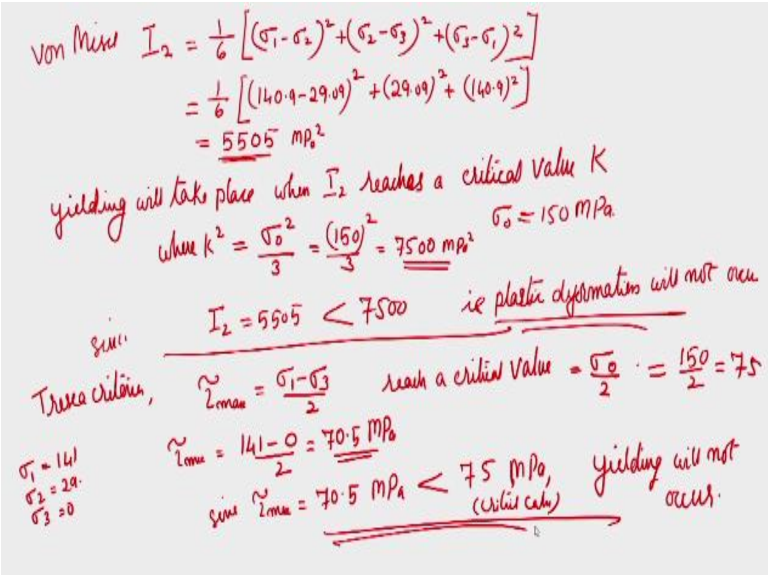

So when you look at it as von Mises criteria, it is that your second invariant I2 when is cross a certain level, plastic deformation take place, so that is given by say plastic deformation will take place when it crosses a certain value, that is 1 by 6 into your sigma 1 – sigma 2 square + sigma 2 – sigma 3 square + sigma 3 – sigma 1 square. So, this is the condition. So if you just find that value, that is equal to 1 by 6 into 112 square because sigma 1 = 140.9 – sigma 2 = 29.09 square + 29.09 square, you can take 29 itself, + 140.9, 141 also we can do it, it is not going to make much of a difference.

So, this value comes to something around of 5505 megapascal square we can say. Second invariant comes to this value.

$$I_2 = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

$$= \frac{1}{6}[(140.9 - 29.09)^2 + (29.09)^2 + (140.9)^2]$$

$$= 5505\, mPa^2$$

So like this, the yielding criteria, that means yielding will occur or plastic deformation will occur or it will take place as per the von Mises criteria when I2 reaches a critical value K where K square = sigma not uniaxial yield strength of the material, that is sigma0 by 3. So since sigma0 is equal to, in our problem sigma0 = 150 megapascal, this is equal to 150 square by 3.

$$k_2 = \frac{\sigma_0^2}{3} = \frac{(150)^2}{3} = 7500\, mPa^2$$

This comes to 7500 mps, so this is energy okay, distortion energy this is what. So yielding will take place when the distortion energy reaches the value of 7500 megapascal square, but since based on our principal stresses, the value of I2 is only 5505 since I2 = 5505, since this second invariant in our case it is 5505 which is much less than this critical value of 7500 megapascal square, you will find that that is yielding will not take place, that is plastic deformation will not take place, deformation will not occur.

$$since, I_2 = 5505 < 7500\ , that\ is\ plastic\ deformation\ will\ not\ occur$$

Now, let us just consider that Tresca criteria. The Tresca criteria states that the tau max when it reaches a critical value, that is tau max decided by sigma 1 – sigma 3 by 2, it reaches a critical value that is equal to sigma 0, uniaxial yield stress by 2 okay, so that is equal to, so let us see that. This tau max is equal to or sigma 1 = what is the value of 141 we can write and sigma 2 = 29, Sigma 3 = 0, that is 141-0 divided by 2 which is 70.5 megapascal and this critical value is sigma 0 is, how much is sigma 0, 150, 150 divided by 2 that is equal to 75.

So since tau max = 70.5 megapascal which is less than critical value 75. So under both the conditions, yielding will not take place, so this is what. So in this case what you have to find out, you find out the principal stresses and then you apply the von Mises criteria and yield criteria. The second method is by finding out the stress invariants okay, so that method also we will just discuss.

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \quad reach\ a\ critical\ value = \frac{\sigma_0}{2} = \frac{150}{2} = 75$$

$$\tau_{max} = \frac{141 - 0}{2} = 70.5\, mPa$$

$$since\ \tau_{max} = 70.5\, mPa\ < 75\, mPa, yielding\ will\ not\ occur$$

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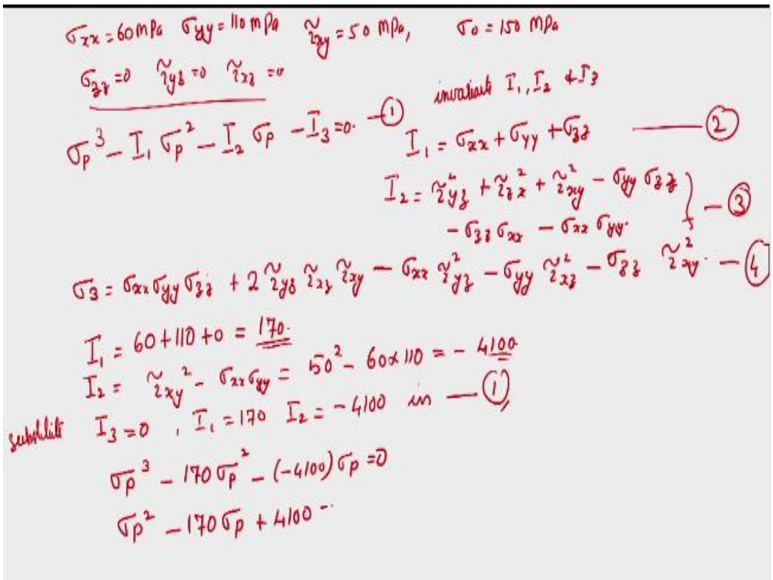

So the same problem of finding out the principal stresses by the invariants, tensor invariants no we can also do that, see because what are the things which are given for us is sigma xx = 60 megapascal, sigma yy =110, and tau xy = 50 and sigma 0 = 150 megapascal. These are the things which we have given. So since it is a biaxial stress mode, like we can assume that sigma zz = 0, tau yz = 0, tau xz = 0 okay. So the solution when you do it, earlier we have discussed I think in the first or second lecture, the principal stresses.

$$\sigma_{xx} = 60mPa \quad \sigma_{yy} = 110mPa \quad \tau_{xy} = 50mPa, \qquad \sigma_0 = 150mPa$$

$$\sigma_{zz} = 0 \quad \tau_{yz} = 0 \quad \tau_{xz} = 0$$

Sigma P is the principal stresses, we got a cubic equation sigma 3 cube – the first invariant I1 into sigma P square I2 the second invariant into sigma P – I3 = 0 okay. So if you look at that where I1, I2, I3 are the invariants okay, invariants I1, I2 and I3. So what are these I1, I2, I3? So let us say that I1 the first invariant which we have written earlier itself is equal to sigma xx + sigma yy + sigma zz.

$$\sigma_p^3 - I_1, \quad \sigma_p^2 - I_2\sigma_p - I_3 = 0$$

$$invariant\ I_1, I_2 and\ I_3$$

$$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$$

$$I_2 = \tau_{yz}^2 + \tau_{zx}^2 + \tau_{xy}^2 - \sigma_{yy}\sigma_{zz} - \sigma_{zz}\sigma_{xx} - \sigma_{xx}\sigma_{yy}$$

So sigma xx + sigma **yy** + sigma zz, sigma zz is 0 okay. So I2 = tau squared yz + tau zx + tau square xy – sigma yy sigma zz – sigma zz sigma xx – sigma xx sigma yy okay and similarly I3 = sigma xx sigma yy sigma zz + 2 into tau yz into tau xz into tau xy – sigma xx into tau square yz – sigma yy x tau square xz – sigma zz into tau squared xy, so this is. If I write this as equation number 1, this is equation number 2, this is equal to equation number 3 and this is equation number 4, these 4 equations. Now we have to solve it.

$$\sigma_3 = \sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\tau_{yz}\tau_{xz}\tau_{xy} - \sigma_{xx}\tau_{yz}^2 - \sigma_{yy}\tau_{xz}^2 - \sigma_{zz}\tau_{xy}^2$$

So let us find out what is the value of I1 sigma xx + sigma yy + sigma zz, sigma z = 0. So sigma xx = 60+ 110 + 0 = 170 and I2, I2 = only two terms are coming, so that is your sigma xx sigma yy and tau xy square, all others are 0 because these terms are 0, so you will get it as 50 square that is tau xy square minus so here if I just write like the tay xy square – sigma xx sigma yy, so that is equal to 50 square – 60 into 110 = –4100, so that is the I2 you are getting; and I3, I3 = 0 because all the terms are 0 because here multiplication by 0 is coming in all the terms it is 0, so I3 = 0.

$$I_1 = 60 + 110 + 0 = 170$$

$$I_2 = \tau_{xy}^2 - \sigma_{xx}\sigma_{yy} = 50^2 - 60 \times 110 = -4100$$

$$I_3 = 0, I_1 = 170 \quad I_2 = -4100$$

$$\sigma_p^3 - 170\sigma_p^2 - (-4100)\sigma_p = 0$$

So if you substitute this, that is I1 = 170 and I2 = – 4100 in equation 1, you will get that is a sigma P cubed – I1 that is 170 into sigma P square minus – I2 = – 4100 into sigma P = 0. So that is anyway sigma P is common, so we can write it as sigma P square – 170 sigma P, these are the principal stresses + 4100 = 0.

$$\sigma_p^2 - 170\sigma_p = 4100 = 0$$

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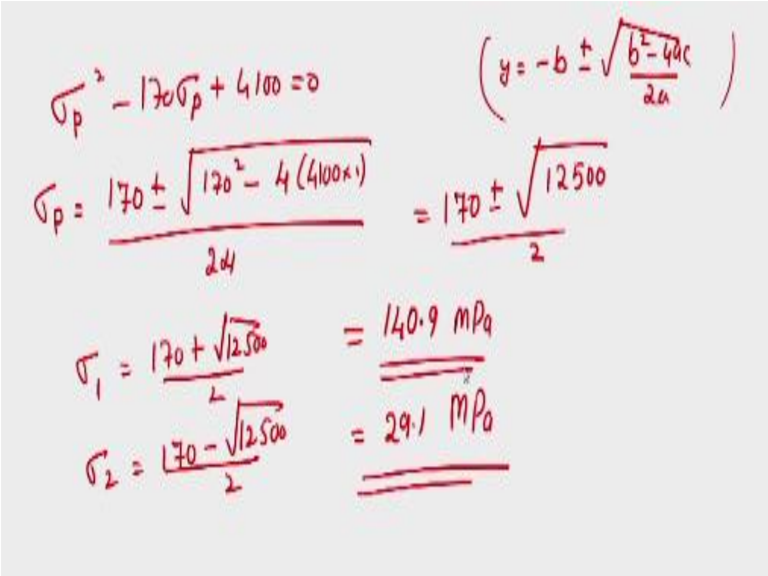

Binomial equation so that is sigma P square – 170 sigma P – 4100 = 0. So this if you solve it, we can write it solving for sigma P, sigma P = what is that, so y = –b plus or minus square root of b squared – 4acby 2a okay. So that if you look at it, it will be –170, that is 170 plus or minus root of 170 square – 4 into, minus this is plus, – 4100 into 4ac divided by 2 into 1. So that is equal to 170 plus or minus root of this will come to around 12500 divided by 2. So one of the solution is what is sigma 11= you can write it as 170 plus or minus root of 12500 divided by 2 and sigma 2 = 170 – root of 12500 by 2.

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sigma_p - 170 + 4100 = 0$$

$$\sigma_p = \frac{170 \pm \sqrt{170^2 - 4(4100X1)}}{2X1} = \frac{170 \pm \sqrt{12500}}{2}$$

$$\sigma_1 = \frac{170 + \sqrt{12500}}{2} = 140.9mPa$$

$$\sigma_1 = \frac{170 - \sqrt{12500}}{2} = 29.1mPa$$

So this comes to something 140.9 and this is 29.1 megapascal. So this is how you find out the principal stresses. So you can do by any of this method, either from Mohr circle representation or by solving this analytical equation you can find out. Once you find out this principal stresses, then you can check for whether the yielding will take place as per the Tresca criteria or as per von Mises criteria which is asking the question. So that we have already solved it okay, the remaining**,** the sigma 1 and sigma 2 we have got it, so you can solve it by any of this method.

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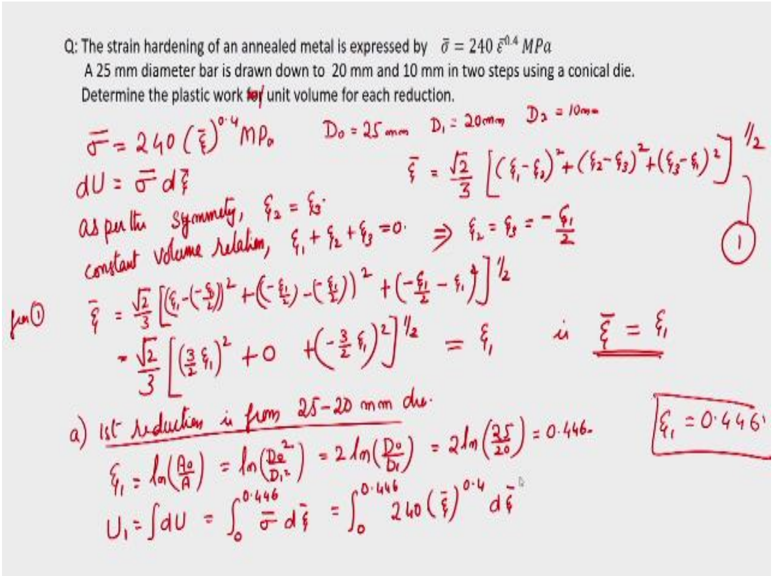

Let us now come to the second question. The strain hardening of an annealed metal, the strain hardening is expressed by this relationship sigma bar, it is the effective strain or average strain = 240 into epsilon bar 0.4 megapascal okay, there should be a bracket. So the given is that the sigma bar = 240 into epsilon bar, otherwise no the way I have typed it people may think that it is – 0.4. So we can write it as D0 = 25 mm, D1 = 20 mm and D2 = 10 mm. So that means a 25 mm diameter bar is drawn down to 20 mm diameter and after that subsequently it went to 10 mm.

$$\overline{\sigma} = 240(\overline{\epsilon})^{0.4} mPa \quad D_0 = 25mm \quad D_1 = 20mm \quad D_2 = 10mm$$

So these are the two steps. In the first step, it was reduced to 20 mm diameter and in the second step it was reduced to 10 mm and this was done using a conical die. Determine the plastic work plastic work per unit volume for each reduction, so that is the question. So we know that the plastic work du = sigma bar d epsilon bar okay, see this is the thing. Epsilon bar we can write it in this form, that is equal to epsilon 1 – epsilon 2 square + epsilon 2 – epsilon 3 square + epsilon 3 – epsilon 1 square whole raised to 1 by 2.

$$dU = \overline{\sigma} d\overline{\epsilon} \qquad \overline{\epsilon} = \frac{\sqrt{2}}{3}[(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2]^{\frac{1}{2}}$$

This is the effective stress okay and epsilon bar is your effective strain also, so that is what, maybe because the bar is in a cylindrical piece, the work piece is a cylindrical piece and you are also drawing it into a cylindrical piece, output also is a cylindrical piece, there is a symmetry. So as per the symmetry of the sample, we can write it as epsilon 2 = epsilon 3, and if you apply the constant volume relationship, that is epsilon 1 + epsilon 2 + epsilon 3 = 0. So that means epsilon 2 = epsilon 3, so that will come to minus of epsilon 1 by 2, this is how we will get from this relationship.

$$as\ per\ the\ symmetry, \epsilon_2 = \epsilon_3$$

$$constant\ volume\ relation, \epsilon_1 + \epsilon_2 + \epsilon_3 = 0 \qquad \epsilon_2 = \epsilon_3 = -\frac{\epsilon_1}{2}$$

So now, we can calculate what is the effective strain. So effective strain will be, if I just put this as equation number 1, from 1, effective strain = root 2 by 3, sorry here it is root 2 by 3, root 2 by 3 into epsilon 1 – epsilon 2. So that means epsilon 1 – epsilon 2 = –epsilon by 2 the whole square + –epsilon 1 by 2 the whole square –minus of epsilon 1, this one the whole square + epsilon 3 = –epsilon 1 by 2 – epsilon 1 the whole square raised to 1 by 2. So that is equal to root 2 by 3 into this is 3 by 2 epsilon 1 square + 0 and + again this is 3 by 2 epsilon 1 one okay.

$$\bar{\epsilon} = \frac{\sqrt{2}}{3}\left[\left(\epsilon_1 - \left(-\frac{\epsilon_1}{2}\right)\right)^2 + \left(\left(-\frac{\epsilon_1}{2}\right) - \left(\frac{-\epsilon_1}{2}\right)\right)^2 + \left(-\frac{\epsilon_1}{2} - \epsilon_1\right)^2\right]^{\frac{1}{2}}$$

$$= \frac{\sqrt{2}}{3}\left[\left(\frac{3}{2}\epsilon_1\right)^2 + 0 + \left(-\frac{3}{2}\epsilon_1\right)^2\right]^{\frac{1}{2}} = \epsilon_1 \quad is \quad \bar{\epsilon} = \epsilon_1$$

So that is minus of, so – 3 by 2 epsilon 1 the whole square raised to 1 by 2, so that you will get it as = epsilon 1 one, that is, effective strain epsilon = epsilon 1, not I epsilon 1 okay, so that is what we are getting. Now we can find out, see the question number if I say the first step a, first reduction, that is from 25 to 20 mm dia. So your epsilon 1 because it is touched, it is just wire drawing only, epsilon 1 you can write it as = log A0 by a**,** for the initial cross sectional area and the instantaneous value, instantaneous value initially was 25.

So this can be in terms of diameter, you can write it as A = pi by 4 d square. So that means, numerator and denominator pi by 4 will go off, so you can say that to D0 square by D1 square. So that is equal to 2 log D0 by D1. So that is equal to you will get 2 log 25 by 20 and that you will get it as 0.446 that is what you are getting. So the plastic work U1, so this is your first case, epsilon 1 = 0.446. The plastic work per unit volume = U1 = integral dU, so that is equal to integral from 0 to 0.446 sigma bar d epsilon bar, so what is directly you can write as 0.0 to 0.446 integral within this limit 240 into epsilon bar raised to 0.4 d epsilon bar.

$$\epsilon_1 = \ln\left(\frac{A_0}{A}\right) = \ln\left(\frac{D_0^2}{D_1}\right) = 2\ln\left(\frac{D_0}{D_1}\right) = 2\ln\left(\frac{25}{20}\right) = 0.446$$

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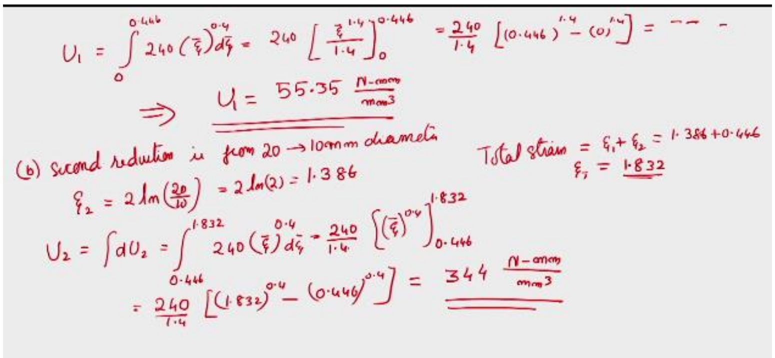

So that if you do it here, so U1, that is U1 = I integral 240 into epsilon bar raised to 0.4, this is from integral from 0 to 0.446 = 240 epsilon bar raised to 1.4 by 1.4 between this, 0.446 okay, or that we can say 240 by 1.4 into 0.446 raised to 1.4 – 0 raised to 1.4. So this comes to around 55.35 okay. This is equal to, so U1 = 55.35, unit will be Newton millimeter per millimeter cubed, this is the first one. Now b, the second reduction, that is from 20 to 10 mm diameter. So there, if you find out the strain, so that means epsilon 2 = 2 log 20 by 10, that is equal to 2 log 2 = 1.386.

$$U_1 = \int dU = \int_0^{0.446} \overline{\sigma} d\overline{\epsilon} = \int_0^{0.446} 240(\overline{\epsilon})^{0.4} d\overline{\epsilon}$$

$$U_1 = \int_0^{0.446} 240(\overline{\epsilon})^{0.4} d\overline{\epsilon} = 240\left[\frac{\overline{\epsilon}^{1.4}}{1.4}\right]_0^{0.446} = \frac{240}{1.4}\left[(0.466)^{1.4} - (0)^{1.4}\right]$$

$$U_1 = 55.35\frac{Nmm}{mm^3}$$

So the first reduction, the strain was 0.446 and with that because if it is a annealed material, you have to calculate from because you should know about the strain history, so in the strain history after the first step, already it has been strained to 0.446, now from 0.446, another strain is coming which is around 1.386. The total strain, so total strain will be epsilon 1 + epsilon 2 that is equal to 1.386 + 0.446, so that comes to 1.832 total strain.

So final strain you will find it is 1.832, you should not make this as 1.386. So now this U2, plastic energy work, so we can say du2 is nothing but integral from, you have to start with the previous one that is 0.446, integral from 0.446 to 1.832, this is the final strain, so which will come to say 240 into epsilon bar raised to 0.4, this is your flow stress equation, so that it will come to 240 by 1.4 into epsilon bar raised to 0.4 between the limits 0.446 to 1.832 d epsilon, here d epsilon is required, I somehow forgot it, no here d epsilon not required, here d epsilon, let me just see whether, yes d epsilon bar.

$$U_2 = \int dU_2 = \int_{0.446}^{1.832} 240(\overline{\epsilon})^{0.4} \, d\overline{\epsilon} = \frac{240}{1.4} [(\overline{\epsilon})^{0.4}]_{0.446}^{1.832} \, d\epsilon$$

$$= \frac{240}{1.4} [(1.832)^{0.4} - (0.446)^{0.4}] = 344 \frac{Nmm}{mm^3}$$

So this we can get it as 240 by 1.4 into 1.832 to the power 0.4 – 0.446 to the power 0.4, so that will come to 344 Newton per millimeter per millimeter cube. So if you just look at the value of the plastic work we has been done from 25 to 20, it is only very small amount compared to say 20 to 10 mm diameter that is a big difference. So it is almost coming to say around 6 times or something approximately. So these are the 2 cases we are finding out.