Plastic Working of Metallic Materials
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Lecture - 14
Analysis of Forging

Today we will be discussing about the analysis of forging when your forging analysis what is the pressure distribution across in open die forging and what is the mean pressure and from that how we can find out the forging load? So for the forging analysis will be done,

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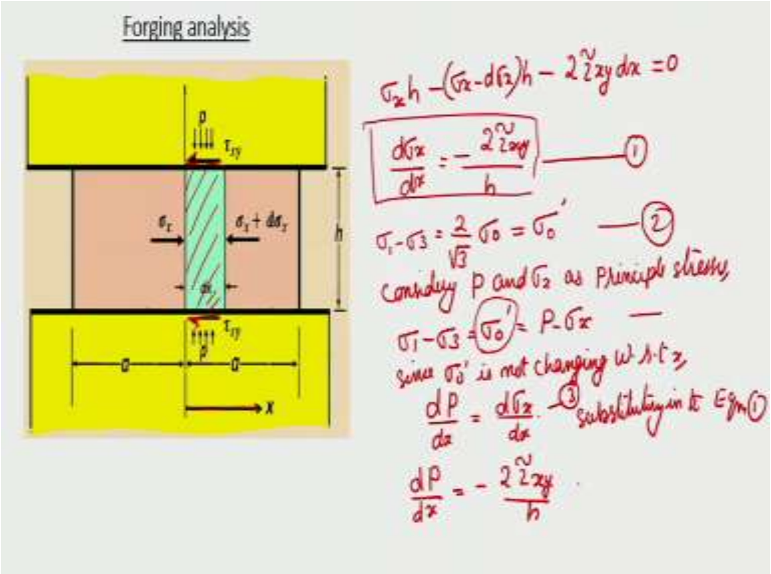

So let us considered that a case of a plane strain deformation when your front for the material open die forging so because this part which is on this color region this is a work place material and it is kept on bottom die with a stationary and then to the top die your applying a pressure and allowing the ramp to move downwards. So in this way your allowing the ramp to move downwards.

So when as the ramp move downwards, when the ramp is moving downwards the height X keeps on changing, it keeps on decreasing and here we are assuming that constant volume relationship is there during plastic deformation when you consider the constant volume relationship. So as the height keeps on decreasing there will be a lateral flow are the materials in this direction or perpendicular to the ramp moment.

So what will happen is that, when this is trying to move towards the side this leads to the development of frictional force at this interface region, when the material is trying to move in this direction there will be frictional force in this. So at the die work phase, interface region there will be a frictional forces which is represented we can say that this τ_{xy} is the frictional force which is there.

So this frictional force will be opposing the movement of the lateral direction, so that means the frictional force will be directly towards the centre line, so here in this direction it is, so here also you will find, on both the surface this frictional force are coming into the picture. So that the surface sphere results in frictional force and the presences of this frication causes an imbalance of forces on the element.

This particular if you take a small element in this the presences of a friction you will find that there is an imbalance of forces in the X direction. If your taking this as a X direction, along the X direction. So here we can assume that this is σ_x in this direction and $\sigma_x + d\sigma_x$ in this direction condition. So you will find there is going to be a difference in stresses in the element.

So let us assume that the material deforms under plain strain condition, the variation of barling, so for the time being for this analysis let us assume the baraling is not the and this work place is a plain of uniform thickness h. So these are the or basic assumption, one is plain strain condition so material is flight of uniform thickness and there is no barling during the deformation.

So if you take the equilibrium forces on this element along the X direction, then we can write this as a σ_x and if you just take for unit which is perpendicular to this plain of this diagram or the screen if you say unit bit is taken, that means width is assumed to be unity then we can say sigma x * h – sigma x – d sigma x * h – there are 2 surface are coming so 2 tau xy * dx = 0 this is the equilibrium condition of forces in the x direction.

$$\sigma_x h - (\sigma_x - d\sigma_x)h - 2\tau_{xy}dx = 0$$

When you do like that and if your just do a small mathematical manipulation and then we can get this equation in this form that is d sigma x by dx = -2 tau xy by h so your getting this relationship. If you consider the Von Mises criteria for a plain strain condition which we have discussed earlier at several conditions, that is sigma 1 – sigma 3 = 2 by root 3 sigma 0 where sigma 0 is the union axial strength of the material that we can write it, this is the notation which you can use it that.

$$\frac{d\sigma_x}{dx} = -\frac{2\tau_{xy}}{h}$$

$$\sigma_1 - \sigma_3 = \frac{2}{\sqrt{3}}\sigma_0 = \sigma_0^{'}$$

Here sigma 0 dash and so considering this pressure P and σ_x as the positive compression principle stresses. Considering P and σ_x as principle stresses we get that sigma, sigma X and sigma 3 = P if you give that, substitute that we will end with this relationship that is sigma 1 – sigma 3 = sigma 0 dash = P – sigma x from this you getting. Now if you look at the $\sigma_0^{'}$ this is not changing with respective x.

$$\sigma_1 - \sigma_3 = \sigma_0^{'} = P - \sigma_x$$

So we can write this form dp by dx is = d sigma x by dx so from that if your substitute in this d sigma by dx in the relationship so substituting may be I write this as two and this as equation 3 putting into equation 1, we can write it as dp by dx = - 2 tau xy divided by h this we can write it as equation number 4. So if the sheering stress is related to the normal pressure by coulombs law of friction.

$$\frac{dP}{dx} = \frac{d\sigma_x}{dx}$$

$$\frac{dP}{dx} = -\frac{2\tau_{xy}}{h}$$

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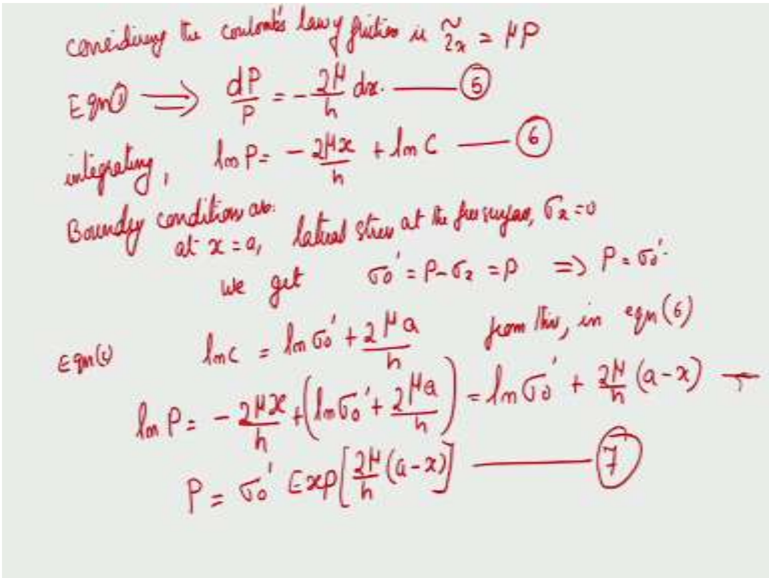

So that means considering the coulombs law of friction that is $\tau_x = \mu P$ where mu is the coefficient where is the coefficient of friction and P is the normal pressure which is shown in the figure the equation number 4 equal becomes dP by P is = - 2 mu by h * dx, so we are getting this relationship tends this form. So now if you try integrate this equation we will get log P = -2 mu x by h + constant of integration so let us give this equation number 6.

$$\frac{dP}{P} = -\frac{2\mu}{h} dx$$

Now we wanted to find what is the value of C or log C, so you apply the boundary conditions, so boundary condition are, see if you look at the boundary condition at x = a from the centre at x=a then you will find that sigma x = 0 at the free surface. So when the x=a the sigma x = 0, so from the Von Mises criteria which we wanted we can write it as at boundary conditions are at x = a, the lateral stress because it at the free surface, lateral stress at the free surface.

$$\ln P = -\frac{2\mu x}{h} + \ln C$$

That is sigma x= 0, so from this so we get that is sigma 0 dash = P – sigma x = P or that is P = sigma 0 dash and then if you substitute into equation number 6 so we can write it as log c = log sigma 0 dash + 2 mu x by h. So if you take from this we can arrive at the equation that P = in equation number 6 if you substitute the equation number 6, if you substitute the value of log C, log P = -2 mu x by h + log C that is log + 2 mu x by h.

$$\sigma_0' = P - \sigma_x = P$$

$$\ln C = \ln \sigma_0' + \frac{2\mu a}{h}$$

$$\ln P = -\frac{2\mu x}{h} + (\ln \sigma_0' + \frac{2\mu a}{h}) \;= \ln \sigma_0' + \frac{2\mu}{h}(a - x)$$

So that we can write it as log sigma 0 dash + 2 mu by h here it should be 2 mu a a by h. So we can say that 2 mu * a – x, so this we can or from that P is = sigma 0 dash * exponential to mu by h * a – x so we are getting it question number 7. Now this equation we can expand it what is inside the inside the exponential term we can expanded it because the value of mu is very less it is always less than 1 normal it will be less than .5 normal cases.

$$P = \sigma_0' \, Exp\left[\frac{2\mu}{h}(a - x)\right]$$

So sometimes it may vary from anywhere from a very low value to normally for deformation conditions sliding friction when your considering it will come to maximum .3 or .2. Ok For analysis purpose you can use any case but normal case it is very less so we can write the exponential term we can expand it

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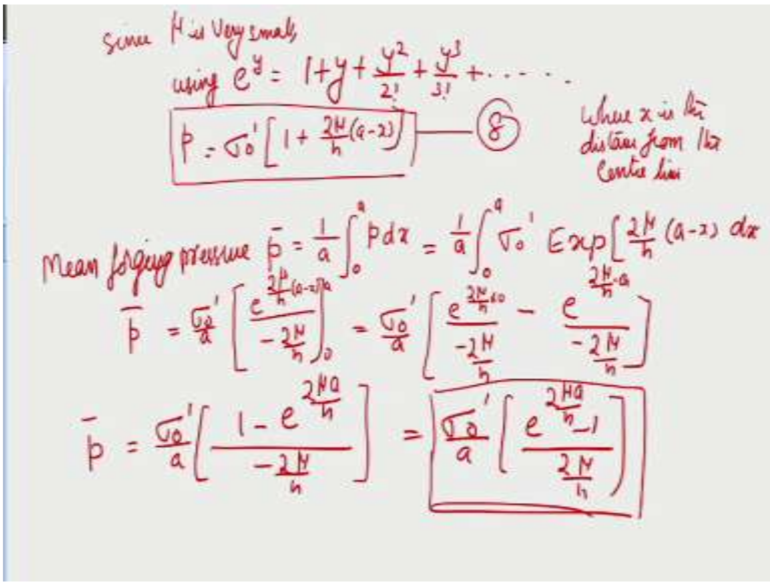

So since mu is very small the expansion we can write using e raised to y is = 1 + y + y square by 2 factorial + y cube by 3 factorial so + infinite term. When you substitute that we can write P is = sigma 0 dash * 1 + 2 mu by h * a – x in this form we can write. So this is the relationship for the variation of pressure on your work piece with respect to the distance from the central line where exceed the distance from the centre line where x is the distance from the central line.

$$e^y = 1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \cdots$$

$$p = \sigma_0^{'}\left[1 + \frac{2\mu}{h}(a - x)\right]$$

Now for analysis per person becomes much easier so we can also find out what is a pressure distribution the stress distribution or at the die from the central line that variation we are getting it but for calculation purpose when you want to find out the forging load it is always a simple method is by calculating the mean forging pressure and multiplier with this other ways to keep on integrating from the centre to surface.

But if you are assuming the mean forging pressure so this mean forging pressure can be calculated, mean if you are considering only on one side forging pressure P bar so that will be 1 by a because it is from 0 to a it is varying and we are taking only one side 0 to a P dx so this is small p. So P dx so that if you just write the expansion form so 1 by a into integral from 0 to a writing the equation 7, so if you use the equation 7 so you will find that it is sigma not exponential to 2 mu by h * a – x.

So it is = sigma not dash * 2 mu by h *a – x it should be under exponential term * exponential or e raised to 2 mu by h * a - x dx. So that that will be like if you just do that sigma not dash if you take it outside, so we can write it in this form e raised to 2 mu by h -2 mu by h from 0 to a. So that will be sigma 0 dash by a * e raised 2 mu by h * 0 by -2 mu by h – e raised to 2 mu by h * a by -2 mu by h.

$$Mean\ forging\ pressure = \bar{p} = \frac{1}{a}\int_0^a \sigma_0^{'}\ Exp\left[\frac{2\mu}{h}(a - x)dx\right]$$

$$= \frac{\sigma_0^{'}}{a}\left[\frac{e^{\frac{2\mu}{h}(a-x)}}{-\frac{2\mu}{h}}\right]_0^a = \frac{\sigma_0^{'}}{a}\left[\frac{e^{\frac{2\mu}{h}\times 0}}{-\frac{2\mu}{h}} - \frac{e^{\frac{2\mu}{h}a}}{-\frac{2\mu}{h}}\right]$$

So that we will get it as P bar = sigma not dash by a * if you just simplification you will get it as 1 – e raised to 2 mu a by h -2 mu by h or that is = sigma 0 dash by a * e raised to 2 mu a by h -1 by 2 mu by h, so this is a mean forging pressure we are getting it for this condition so we forging pressure distribution.

$$\bar{p} = \frac{\sigma_0^{'}}{a}\left[\frac{1 - e^{\frac{2\mu a}{h}}}{-\frac{2\mu}{h}}\right] = \frac{\sigma_0^{'}}{a}\left[\frac{e^{\frac{2\mu a}{h}} - 1}{-\frac{2\mu}{h}}\right]$$

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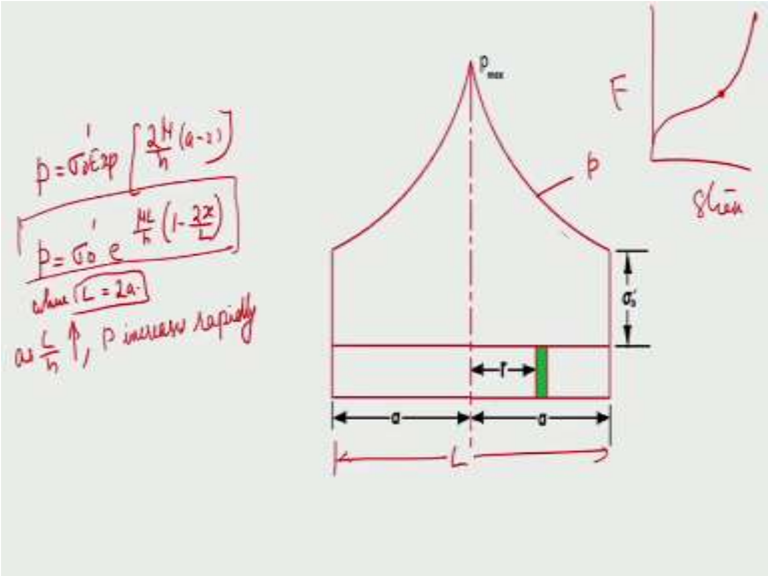

And if you look at the variation of the pressure distribution you will find that in this case the equation which we got it as P is = sigma not dash * exponential say 2 mu by h into a - x the variation is like this. This is a variation of P so at the centre you have a maximum pressure when $x = 0$ you will have the maximum pressure and towards the free surface you will find it is a sigma 0 which is the assumption which is we check also we are getting sigma 0 dash.

$$p = \sigma_0^{'}\, Exp\left[\frac{2\mu}{h}(a - x)\right]$$

So that is what we are getting it here and so you will find that it is not a straight line but exponential function it is there will be coming down with respect to any distance radial distance from the centre this is a variation of that. So now we can also find out that this relationship can also be, this we can write this in terms of length so suppose this you take this as $L = 2a$ from that so we can also write it as this form P = sigma 0 dash * e raised to mu L by h * 1 – 2x by L, where $L = 2a$.

$$p = \sigma_0^{'} e^{\frac{\mu L}{h}\left(1 - \frac{2x}{L}\right)}$$

So if you look at this relationship this particular relation with respective to your L and h, as L by h increases the resistant compressive forces deformation P it increases rapidly. So this point is used as an advantage in impression die forging. So there we have already discussed when die is clausing, as the die clauses the flash is formed. Now once the flash card forming you will find that in that curve which found the stress keeps on increasing.

That is this is a strake and this is the force so we found like this, so onces the this is a point where the flash starts forming and the pressure required for the deformation keeps on increasing and when the thickness say with L by x if h is very small compared to the L then you will find that your forces are going to increase like anything. So with the thickness of the the flash decreasing the forces necessary will be on the stresses necessary for the deformation will be very high.

That is why you will find that it is much more safer because the material to deform and the form the flash and then fall in the flash cutter, the pressure will be higher, that is what ever forces your applying that will be completely used inside the die cavity and then it will be so high inside the die cavity the pressure or stress will be very high so that complete filling of the die cavity takes place.

And still when the pressure increases this flash, through the flash it will curve out and then fall in the flash cutter. So that were the formation of the flash is very important and the flash thickness decreases the pressure buildup will be very high for the resistance to deformation will be very high in the flash but that high stress will be transmitted into the work piece material inside the die cavity.

Now this case we were considering only the, we are considering the case of sliding friction where the coulombs law of friction was applicable and the material we assume that material is just a sliding. So because of the lateral force though there is a frictional force which is coming the material sliding and moving we have and that the interface there is a relative motion but in almost all the cases of forging this may not be true.

Because if you look at the centre the pressure is very high but it is coming down, so there will every chances there is a sticking which is coming, it will be sticking to the die. This effect of sticking and sliding friction we have discussed in our effect of friction earlier lecture. So let us consider do the analysis considering sticking friction, so the interface here so the only thing initial equation which we came across was 2 let us a sticking friction.

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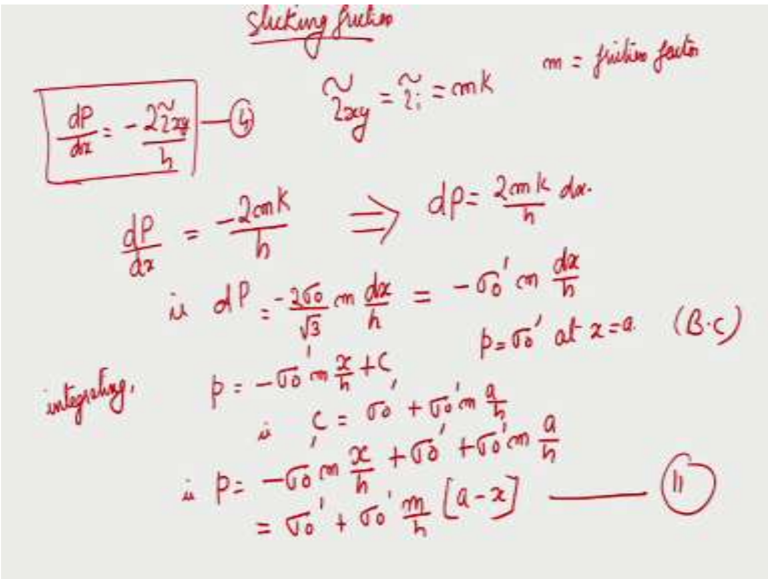

Our differential equation which we found was dP by dx = -2 tau xy by h this is what are equation number 4 we have written that. Now for sticking friction the interface air stress with the friction factor M, so we can write it as tau xy which is sheer stress at the interface so that is tau i we can written it as mk where m is the friction factor and you know that for perfect sticking m = 1for perfect sliding m=0 that is what our basic assumption we have taken.

$$\frac{dP}{dx} = -\frac{2\tau_{xy}}{h} \qquad \tau_{xy} = \tau_i = mk$$

So from this if you assume that tau xy is = mk if you substitute into this equation you find that dP by dx = -2 mk by h. So from this we can arrive at dP = tau mk by h * dx and we know that the value of k = so from that sigma not so from that we can just write that it is dP by dx so we can write it as is equal to substituting for 2k, we can say that 2 sigma 0 by root 3 * m * dx by h so that also is = - sigma 0 dash * m * dx by h we are getting.

$$\frac{dP}{dx} = -\frac{2mk}{h} \qquad dP = \frac{2mk}{h}dx$$

$$dP = -\frac{2\sigma_0}{\sqrt{3}}m\frac{dx}{h} = -\sigma_0^{'}m\frac{dx}{h}$$

So now integrating this becomes very simple straight forward, we will get it as P= - sigma not m * x by h + C because it is a straight line equation that is why your taking C. But earlier also we found that P = sigma not we got it the same at x = a. So from that, that is the boundary condition, that is from that we can write it as C = sigma not + sigma not * m * a by

h. If you substitute that in this equation so we will write that is P = -sigma not dash * m * x by h + sigma not dash + sigma not dash * m * a by h.

$$integrating, P = -\sigma_0^{'} m \frac{x}{h} + C \qquad P = \sigma_0^{'} \; at \; x = a$$

$$C = \sigma_0^{'} + \sigma_0^{'} m \frac{a}{h}$$

$$P = -\sigma_0^{'} m \frac{x}{h} + \sigma_0^{'} + \sigma_0^{'} m \frac{a}{h}$$

$$= \sigma_0^{'} + \sigma_0^{'} \frac{m}{h} [a - x]$$

So that is P you will get it as sigma not dash + sigma not dash * m by h * a – x so you get this as equation number 11. This pressure distribution is linear with respect with the distance from the central line,

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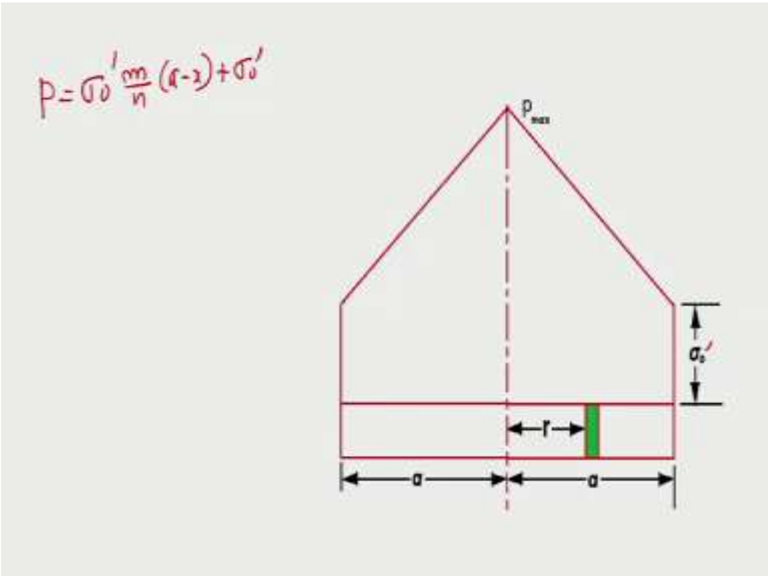

See it will be like, this is the it is linear so from the central line you will find because the equation is that so if you look at if P is = sigma not dash * m by h * a- x + sigma 0 dash. So this is at this x = a your getting this is 0 and you will get it as sigma 0 dash here and then x = 0 you will get the maximum value P max so that it was we are getting it. So when that, that means there is a limit, since the pressure distribution here is linear we can see that in actual case there will be which is neither be perfectly be sliding.

$$P = \sigma_0^{'} \frac{m}{h} (a - x) + \sigma_0^{'}$$

So it is perfectly sliding then $m = 0$ and where coulombs law of friction will be applicable and if it is perfectly sticking m = 1 then we can get this relationship with $m = 1$ we can get the value of k for perfect condition. But in actual case it will be an intermediate up to some distances from the centre see if you calculate the value of P_{max} for using sliding friction and the sticking friction then you will find that it is like this relationship will be there.

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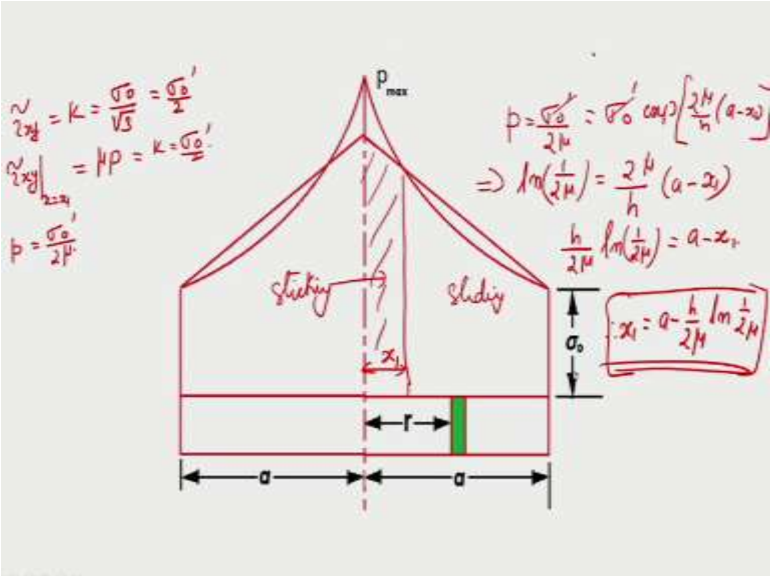

So from the centre to some distance if you just take some distance here from the centre after some distance you will find that the stress resistance is less here so the moments just reach here it will start deforming. So that means in this condition it is a sticking friction which will be prevailing from the centre to some distance. Now beyond that you find the lower stress is for the sliding friction.

So when the stress reaches the metal which starts sliding from that area so sliding over the material takes place. In actual case the application when the deformation is there from the centre to some distance there is so this part it is we can say sticking condition and here it is sliding condition. So when that so there is a limit upto which with the sliding friction can exist at the die work place surface.

So when that limit is reached the interfacial sheer of the work piece occurs at a value of the process which is = tau i is = K so that means tau i is = K where K is = sigma 0 by root 3. So in general we can write that tau i is = mk so that, so we say tau xy not tau i which is at the interface, tau xy is = K is = sigma 0 by root 3 is = sigma 0 by 2 when you located it and the distance from the central line is if you just consider this is x1 so that where is this 2 are equal.

So in that case we can write that tau xy at x is = x1 is = mu p is = K is = sigma 0 dash by 2, so that means P is = sigma 0 dash by 2 2 mu. Now if you use this for the sliding condition that is P is = sigma 0 dash by 2 mu the sliding condition if you write it as sigma 0 dash * exponential say 2 mu by h * a – x, where x = x1 what we are considering. So from this, this will get cancelled so we can write log 1 by 2 mu is = 2 mu by h * a - x 1.

$$\tau_{xy} = k = \frac{\sigma_0}{\sqrt{3}} = \frac{\sigma_0^{'}}{2}$$

$$\tau_{xy}|_{x=x_1} = \mu P = k = \frac{\sigma_0^{'}}{2}$$

$$P = \frac{\sigma_0^{'}}{2\mu} = \sigma_0^{'} \exp\left[\frac{2\mu}{h}(a - x_1)\right]$$

$$\ln\left(\frac{1}{2\mu}\right) = \frac{2\mu}{h}(a - x_1)$$

So that means we can write it is h = x by 2 mu * log 1 by 2 mu is = a - x1 so therefore x1 is = a - h by 2 * log 1 by 2 mu. So that way we can find out this the distance x1 at which the transition from sticking to siding condition exist. So that is one method of finding out actually. Now we can also just look at certain condition so in the striking friction when you look at it we can also find out what is the actual pressure should be like you have to integrate between from x is = 0 to x is = x1.

$$\frac{h}{2\mu}\ln\left(\frac{1}{2\mu}\right) = a - x_1$$

$$x_1 = a - \frac{h}{2\mu}\ln\frac{1}{2\mu}$$

Considering the striking friction and the pressure distribution and from this x is = x1 to x is = a for the sliding conditions. So we have to find out this how to determine this, so we know the pressure distribution but we can also find out,

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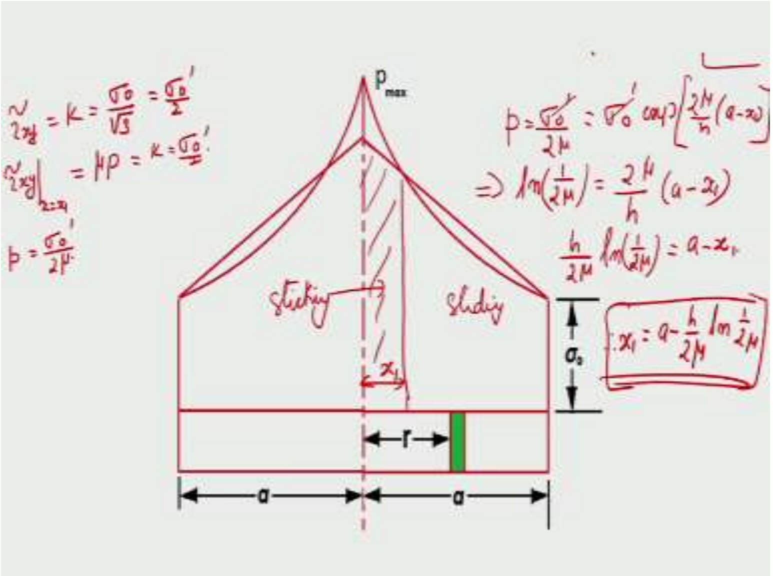

Like for special sky of a stitching friction for perfect striking condition know $m = 1$ for perfect striking condition for $m = 1$ and so we will get it P is = sigma not dash * a - x by x + 1 we can write this equation and the mean pressure P bar is = 1 by a * integral P dx that is = 1 by a * 0 to a sigma 0 dash * a - x by h + 1. So that is = sigma not dash by a * say a * x we can write, ax –x square by 2 + ah 0 to a.

$$P = \sigma_0' \left[\frac{a-x}{h} + 1\right]$$

$$mean\ pressure, \bar{p} = \frac{1}{a}\int_0^a Pdx = \frac{1}{a}\int_0^a \sigma_0' \left[\frac{a-x}{h} + 1\right]$$

$$= \frac{\sigma_0'}{ah}\left[ax - \frac{x^2}{2} + ah\right]_0^a$$

So that is P bar is = sigma not dash * a by 2h + 1 and you can just integrate from here to here using the mean stress and then from here to hear we can integrate using the earlier equation this equation for this and then find out the sum of all these things so that is what we can always get it. Now the thing is that once you get this you have to multiply without the specimen so that you get a total load this is what we are calculating as for unit load.

$$\bar{P} = \sigma_0' \left(\frac{a}{2h} + 1\right)$$

So you have to multiply with the total width of your total work place and then find out that the total rod which is required for this forging so open drive forging. The same thing we can also do it for the same relationship can be used for the deformation of the flash, so that also calculate so you find that the thickness comes when the thickness is very less the pressure total force required or the pressure required for the difference increase is very high that it why towards the end when the flash thickness keeps on decreasing the pressure will be very high. Thank you.