Plastic Working of Metallic Materials
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Lecture – 19
Rolling of Metals

We will now come to the analysis of rolling. So, in this why we need analysis of this rolling process the mechanics of rolling we have to look at it there are various things which one has to look at when you are prosecuted deforming the material, you wanted to know what is the capacity of your rolling mill? What are the forces which are generated because the method is passing through between 2 rolls is similar to what you can see in like in our roadside know the making the sugarcane juice that is also like the sugarcanes are passing through this 2 rolls.

And then it gets juice between and the other person takes it from the output. And again bends it and put it into inlet and other things all those things are there. Where he is not adjusting the rod gap rather he increases the thickness and other things that is the thing, but a stage may come where the rod may not be able to rotate because the pressure requirement is very large and but an industrial purpose when you are going to do the rolling operation of metals, the forces which are going to be developed are very high.

The stresses which are going to be develop are very high because here compressive process in the roll gap or in the deformation zone that will have the reaction of that will result in the roll separating force. The roll separating force may come. So, if there is the roll separating force, if it is very high, then what will happen is that your dimension of the work board after it comes out will not be what you wanted it may be less or it may be a more okay?

So, these problems will come, second the pressure or the forces are very large the roll deflection may take place reflection on the along the radius also the deflection along the length of the roll also can take place. So, you may find that the thickness of the role is not uniform. So, all this matters comes into the picture. But more important that what is the role separating force which is going to come? What is the torque which is required for the motor for this rolling operation, this is these are the most important thing.

Then other characteristics comes with what is the friction effect which is coming whether you want an external force to allow the metal to move into the roll gap or whether the roll itself can draw the material out. So, all these things can be analyzed so, far that we have to look at the mechanics of crawling operation. So, this is the typical case of the rolling with which we are finding it out.

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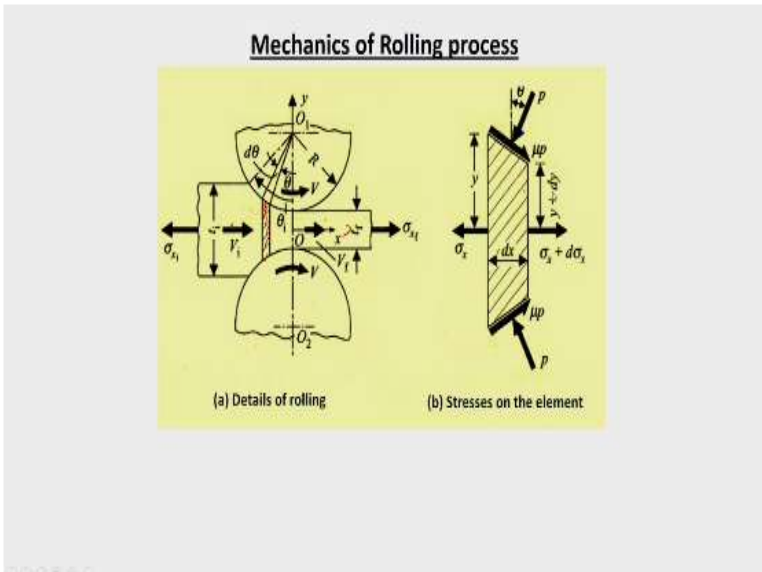

So, these are 2 counter rotating rolls. So, one is here one is here and the metal is entering into this roll gap okay. So, before proceeding further we have to assume certain things the first assumption the rolls are straight and rigid cylinder. So, it is rigid we are assuming that the rolls are rigid it is not going to deflect any type of deflection we are ignoring it in our assumption in this when you are discussing about this mechanics.

Now, without the strip which you are going to roll the width that means which is perpendicular to this screen is much higher than the thickness of the sheet okay. So, it is much higher than the thickness that means you have the when that condition there would not be any widening okay. So, sensitivity is very large compared to the thickness there would not be widening and you end up with the conditions of plane stress.

So, plain strain deformation we can assume in our analysis the coefficient of friction μ is very is low and it remains constant when the metal moves from here to here till the, to the from the inlet to the exit. The coefficient of friction is a constant while in actual case as it may not be, but this is one assumption we are taking and the yield strength of the material remains constant we are assuming that okay.

There is no work hardening but we can take that way assuming a average flow stress and doing our calculation okay. So, as long as the reduction is not very large, then this will be a reasonable assumption for our analysis. But still if you if you are considering the work hardening phenomenon that also can be concerned without any problem in our analysis that can be done, but for simplicity we are assuming the yield strength remains constant. Now, certain things are there, you are allowing the billet to enter in the roll gap.

The billet is moving into velocity Vi and the billet is having an initial thickness Ti as it is entering into the into the roll gap with the velocity Vi the rolls are the rotating the counter rotating rolls they are moving with a circumferential velocity of V and this V is higher than Vi and after the material gets deformed after that is rolled, it comes out with the velocity which is Vf. So, you will find that the velocity keeps on changing from inlet to the outlet the velocity gives on.

This is because the moment it comes in contact with the roll and a small amount of deformation is taking place as the thickness reduces there is no increasing in width that is our assumption prevention condition there is no widening. So, it is only whatever height is reduced correspondingly the length of the billet or the strips keep on increasing okay. So, from here it was having a velocity Vi when the reduction comes the metal has to move with a slightly higher velocity.

So, as with the more and more deformation taking place along the deformation zone or along the roll gap, the more and more the velocity keeps on increasing, velocity of work piece keeps on increasing. So, finally, when it comes out, you will find that the velocity is much higher than the inlet velocity. So, outlet velocity of the strip is much higher than that of the inlet velocity but all these things are happening when rolls are moving at the constant velocity of V that is that.

So, this V, so when initially the velocity is less and we found that it keeps on increasing as it moves through that, but at some point in the roll gap, you will find that the velocity of the strip is equal to the surface velocity of the roll okay. So, that is condition. So, they are the roll velocity and the strip velocity is same and that point you call it as the neutral point okay. And then beyond the neutral point the velocity of the strip will be higher than the velocity of the road.

So, but before the neutral point before means from exit from entry to the neutral point, the velocity of the roll will be higher than the velocity of the strip whereas after from the neutral point to the exit side the velocity of the billet will be higher than the velocity of the roll. Now when that is happening and we are assuming that there is friction the coefficient of friction is less, but friction or forces are coming into the picture. So, because the metal is compressed and there is a friction which is there and the friction will always try to oppose this moment of that.

So, in that case you will find from the inlet side of the roll to the neutral point you will find that the direction of directional force μp is something and from the neutral point to the exit of the roll the real friction forces reverses. So, these things are there. So, in this what we are showing here the fictional force which is coming here is region before the neutral point when I said before the neutral point in Rolling analysis because our analysis when we start we are starting from the exit and moving towards the inlet in analysis that is done.

So, what comes is that from the exit to the neutral point, you will find that the direction of the coefficient of friction the frictional force will be as shown here, but if it is from the neutral point to the inlet side it will be the reverse direction. Why? Here you will find the velocity of the roll is higher than the velocity of the strip. So, naturally the velocity frictional force will be in this direction from the inlet side to the neutral point because there the velocity of the roll is higher.

But whereas on the neutral point to here you will find the strip velocity is higher than the velocity of the roll. So, in that case this will have opposite direction that means it will be in this direction okay. So, that is the so, this point one has to be very clear the direction of the frictional forces reverses when it crosses the neutral point. Now, in this case, though, it may not be require, but when you are doing this analysis.

We will assume there is back tension at the inlet side and there is a front tension at the exit side. Back tension represented by σ_{xi} and the front side it is σ_{xf} and the for our coordinate system which you are going to consider the origin of the coordinate system will be at the center of the line center to center line turning the roll center. So, that is it will be here and our X direction is taken towards the right. So, these are coordination with respect to that we will be discussing the things.

Now, for our analysis, let us take an elemental strip of this and its enlarging view shown it here. What are the shows? So, let us look at the free body diagram of the strip. So, because it is continuously being deformed so, your σ_x is around this direction and because there is a difference in the axial stresses, so, here you are getting outside of on the strip you are having $\sigma_x + d\sigma_x$ and then at the roll metal interface you are having this friction force μp.

So, there is a compressive force which is coming which is normal to the strip. So, at an angle with that you are getting the pressure p. So, this is and the width of your element is dx and the initial thickness t_i we are taking is the if you take that that is we are equating it into as x equal to 2y. So, t_i is equal to 2y. So, we are taking similarly here t_f is equal to equal to twice y +dy. So, that is the thing. So, this is a symbol element on accelerated view when you are taking it this is what.

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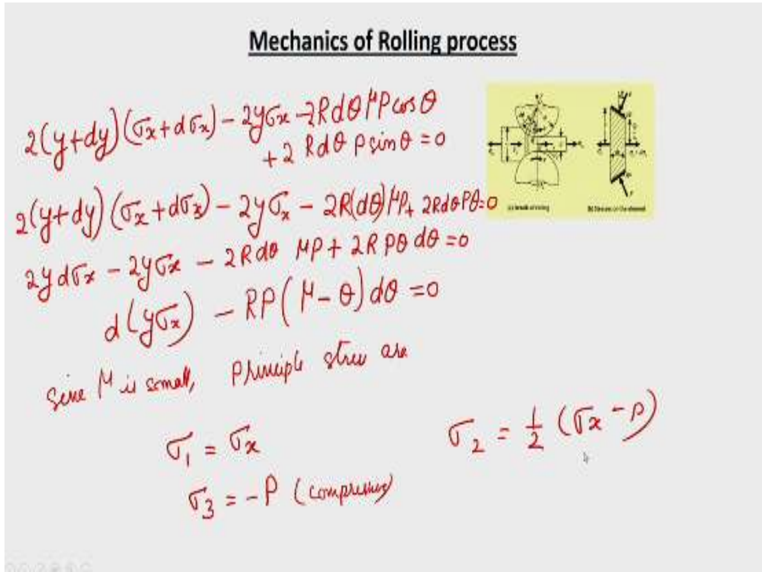

So, let us know go to the equilibrium forces on the element in the x direction and just draw it here for that knows this part will be clear. So, when you look at the equilibrium of forces in the x direction you can write this as a 2 into y + dy into sigma x + d sigma x – 2 y into sigma x – R d theta mu p because if you resolve this along this direction for this P as well as mu p if you resolve it mu P cos theta + because 2 surfaces are there that is why this is coming us 2 2 R + 2R d theta p sin theta is equal to 0.

$$2(y+dy)(\sigma_x+d\sigma_x)-2y\sigma_x-Rd\theta\mu Pcos\theta+2Rd\theta Psin\theta=0$$

So, we are taking this along the x axis direction and if you just look at it since θ is very small. So, when θ is very small we can see sin theta tending to cos theta and cos theta tending to 1. So, we can write it in this form. So 2 into y + dy into sigma x + d sigma x – 2y into sigma x -2R d theta mu P d theta is one mu P into theta into 1 + 2R d theta P into theta is equal to 0. So, if you are just to neglect this higher order terms, this can be simplified into 2 and eliminate these things.

What are common things and the thing 2y into d sigma x – 2y into sigma x – 2R d theta mu P + 2R P theta d theta is equal to 0. So, with the rearranging, we can write it in the differential forms as d into y sigma x – R into P into mu – theta into d theta that is equal to 0. Remember, the direction of the mu is what we have found here is for the case from the before the neutral point that means mainly in the exit side this direction for the exit side that you should have it in mind okay. So, that should not be confused. And since friction forces assumed to be small.

$$2(y+dy)(\sigma_x+d\sigma_x)-2y\sigma_x-2R(d\theta)\mu P+2Rd\theta P\theta=0$$
$$2yd\sigma_x-2y\sigma_x-2Rd\theta\mu P+2RP\theta d\theta=0$$
$$d(y\sigma_x)-RP(\mu-\theta)d\theta=0$$

The principal stresses in the element can be taken as σ_x. So, since mu is very small, because you will find that in this mu the value of mu will be somewhere around less than point 2 point 3 in the everything. So, since mu is very less is small. The principle stress are sigma 1 is equal to your sigma x and sigma 3 is equal to - p here - sign comes because it is compressive and you can also write that sigma in that case sigma 2 will be under plane strength and condition it will be a half into sigma x - P. So, that also will come.

$$Since\ \mu\ is\ small, Principle\ stress\ are\quad \sigma_1=\sigma_x$$
$$\sigma_3=-P(compressive)\quad \sigma_2=\frac{1}{2}(\sigma_x-P)$$

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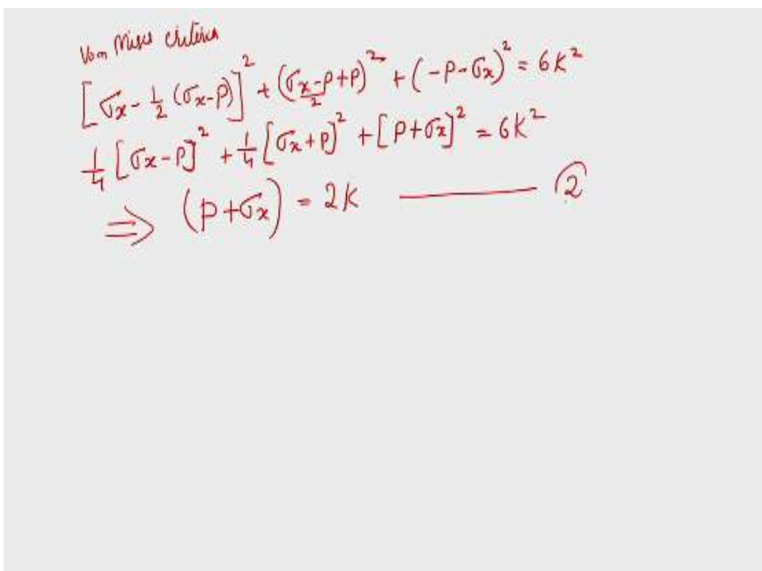

So, now, if you apply the Von Mises criteria to these applying Von Mises criteria you can write that sigma x - half into sigma x - P that is sigma 2 from our previous equation whole square that is sigma 1 – sigma 2 okay + sigma x - P + P that is sigma 2 + sigma 2 - sigma 3 that is what whole square + sigma 3 is - P - sigma 1 that is sigma x the whole square is equal to 6 k square where k is the shear yield strength uni axial strength of the material.

So from these if you do you can write it as it comes to 1 by 4 into sigma x – P the whole square + 1 by 4 into sigma x + P the whole square + P + sigma x the whole square is equal to 6 k square + 6 k square. So, that finally on simplification you will arrive at P + sigma x is equal to 2k this is the final. Applying the Von Mises equation you will get the equation 2 as this and if you look at the equation 1 as this.

$$Von\ Mises\ criteria$$

$$\left[\sigma_x - \frac{1}{2}(\sigma_x - P)\right]^2 + \left(\frac{\sigma_x - P}{2} + P\right)^2 + (-P - \sigma_x)^2 = 6K^2$$

$$\frac{1}{4}[\sigma_x - P]^2 + \frac{1}{4}[\sigma_x + P]^2 + [P + \sigma_x]^2 = 6K^2$$

$$(P + \sigma_x) = 2K$$

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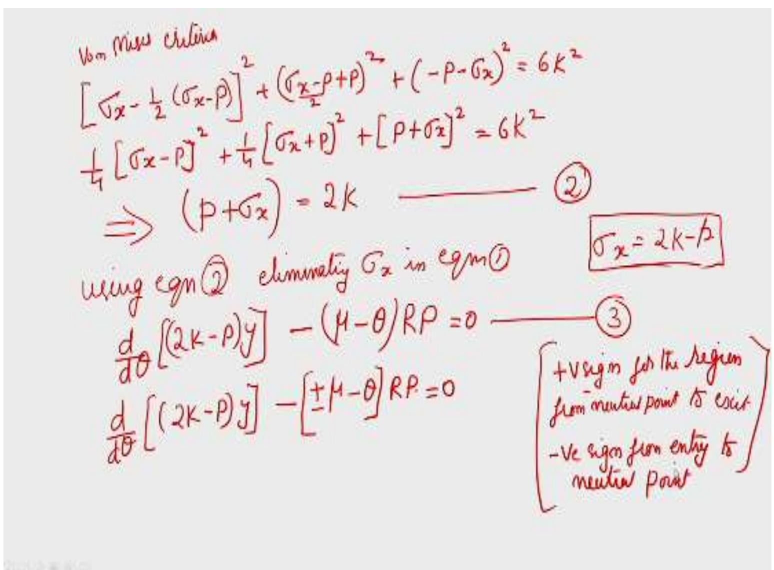

Now from using this equation 2 eliminating sigma x in 1. So, we can just write that question is d by d theta of see sigma x is equal to 2k –P is equal to 2k - P into y - mu - theta into RP is equal to 0. So, this is our equation number 3 remember this sign okay of mu now, when you said see like the direction of the frictional forces from the entry to the neutral point is one direction and once it crosses the neutral point you will find that the direction of the friction forces changes it reverses.

$$\frac{d}{d\theta}[(2K - P)y] - (\mu - \theta)RP = 0$$

So, to take care of this too across the from the inlet to the outlet we can write general form that you can add d by d theta of 2k – P into y we can write it as - of + and - mu - theta into RP. So, is equal to 0 so, remember that in this case positive sign for the region from neutral point to exit and negative sign from entry to neutral point so, this that is why this + or - you take any one of that depending upon which region it is. Now when you look at that as the matter is deforming from the inlet to the outlet it undergoes different version 2 things are happening.

$$\frac{d}{d\theta}[(2K-P)y]-[\pm\mu-\theta]RP=0$$

One the thickness keeps on reducing but when the material is being deformed there is a work hardening which is taking places okay. So, so, when the work hardening is taking place your value of the shear yield strength keeps on increasing. But total now we can assume which is almost a remaining constant because k into y if you just take it just we can have a reasonable assumption. We can assume that k into y remains constant because when k is increasing y is decreasing. So, it is a good assumption to assume that the product k into y remains constant.

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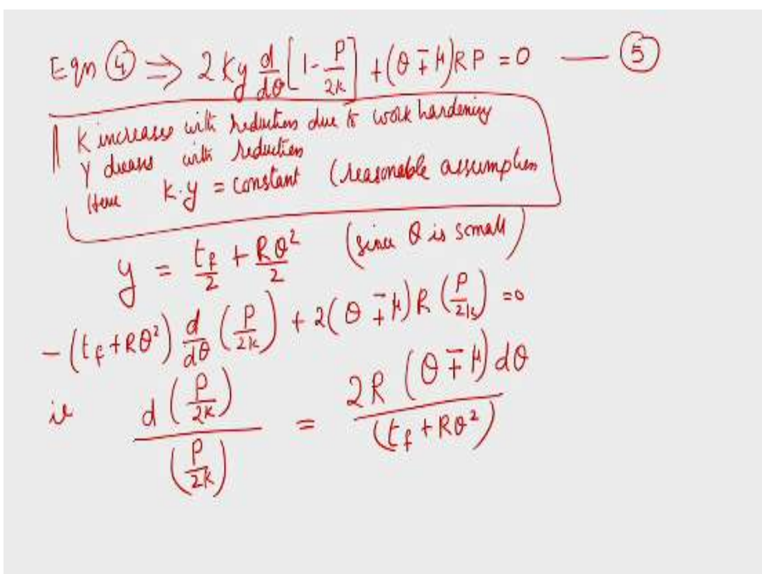

So, we can write this equation number 4 that case as to this is equation number 4. So, this equation number 4 can be written us 2ky into d by d theta 1 –P by 2k + theta - + this - sign we are changing into mu - or + mu into RP is equal to 0. So that we can write it as equation number 5. So that is what we can write in this form why because k increases with the reduction due to work hardening y decreases with reduction hence k into y is a constant which is a reasonable assumption. Okay now all these things this positive and negative sign one should not be I mean.

$$2Ky\frac{d}{d\theta}\left[1-\frac{P}{2K}\right]+(\theta\mp\mu)RP=0$$

Now, as theta is very small see whatever you do that reduction is there, but natural case when you look at the theta value is very small if there is especially when the diameter is very large you will find that theta is small okay. So, in that case, we can write that we can have an expression between the thickness and this one as t f by 2 + R theta square by 2. Since theta is small okay. So, now this equation number 5 if you expand it and substitute the value for y in this. So, we can write it like this say that is and remove all this other parts and the thing so.

$$y=\frac{t_f}{2}+\frac{R\theta^2}{2}(since\ \theta\ is\ small)$$

And on simplification, we can get it as - of t f + R theta square into because that is 2Ky you know into d by d theta of p by 2k + 2 into theta - + mu into R into P by 2k is equal to 0. So this is what you are getting or that is we can write it in this form in terms of v by 2k. So, that is d of P by 2k by P by 2K is equal to 2R in to theta - + mu into d theta divided by t f + R theta square rearranging it you will get this relationship this equation lets us integrate.

$$-(t_f+R\theta^2)\frac{d}{d\theta}\left(\frac{P}{2K}\right)+2(\theta\mp\mu)R\left(\frac{P}{2K}\right)=0$$

$$\frac{d\left(\frac{P}{2K}\right)}{\left(\frac{P}{2k}\right)}=\frac{2R(\theta\mp\mu)d\theta}{(t_f+R\theta^2)}$$

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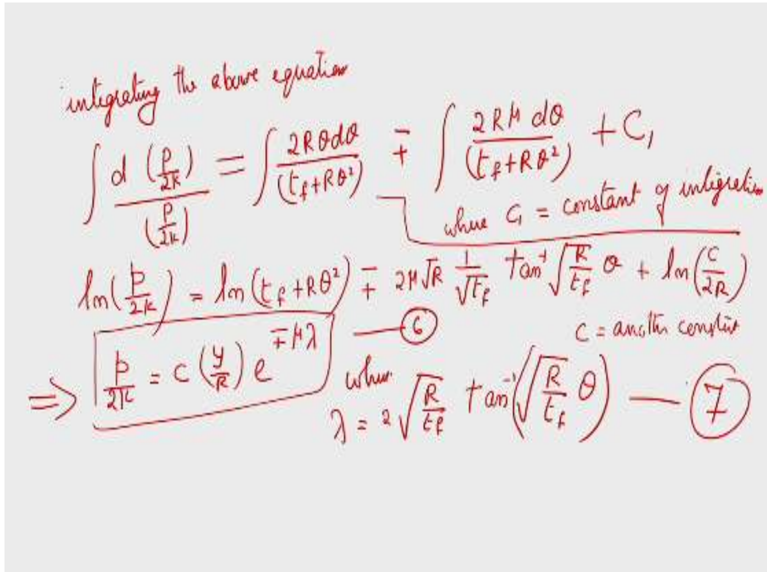

So, when you are integrating it we can write it as integrating the above equation you will get integral d of P by 2K by P by 2K is equal to integral of 2R theta d theta divided by t f + R theta square - or + integral 2R mu d theta divided by t f + R theta square this is equal + constant C1, where C 1 is a constant of integration where is equal to a constant of integration. So, this integrating we can write it as log P by 2K is equal to log t f R theta square a very straight integration - + 2 mu root of R in the 1 by root of t f tan inverse is coming that is why.

$$\int \frac{d\left(\frac{P}{2K}\right)}{\left(\frac{P}{2K}\right)} = \int \frac{2R\theta d\theta}{(t_f + R\theta^2)} \mp \int \frac{2R\mu d\theta}{\left(t_f + R\theta^2\right)} + C_1$$

$$\ln\left(\frac{P}{2K}\right) = \ln\left(t_f + R\theta^2\right) \mp 2\mu\sqrt{R}\frac{1}{\sqrt{t_f}}\tan^{-1}\sqrt{\frac{R}{t_f}}\theta + \ln\left(\frac{C}{2R}\right)$$

Tan inverse root of R by t f into theta + we can write log C by 2R. So C is another constant. So, this will lead to the equation if you take the anti logarithm we can write it as that means it can be written in this form P by 2K is the non dimensional pressure P by 2K is equal to C into y by R into e raised to - or + mu lambda, where lambda is equal to 2 root of R by t f tan inverse root of R by t f into theta.

$$\frac{P}{2K} = C\left(\frac{y}{R}\right)e^{\mp\mu\lambda}$$

$$\lambda = 2\sqrt{\frac{R}{t_f}}\tan^{-1}\left(\sqrt{\frac{R}{t_f}}\theta\right)$$

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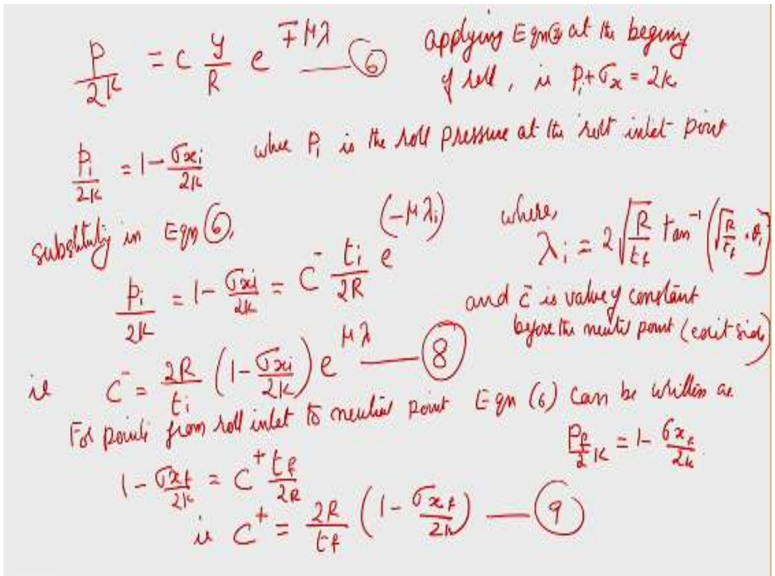

So, if you just come that P by 2K same equation I will write it P by 2K is equal to C into y by R e raise to - or + mu lambda. So, we can apply that at the beginning of the roll applying equation 2 at the beginning of the roll the change the sign change you should know that is P + sigma x is equal to 2k if you substitute here in that what we will be getting is P i inlet i refers to inlet so that we can write P i okay. So P i by to 2K P i by 2k this suffix i refers to inlet okay is equal to 1-sigma xi by 2K.

$$\frac{P}{2K} = C\left(\frac{y}{R}\right)e^{\mp\mu\lambda}$$

$$\frac{P_i}{2K} = 1 - \frac{\sigma_{xi}}{2K}$$

Where P i is the roll pressure at the roll inlet point so, if you use this into question number 6. This is a equation number 6. So, substituting in equation number 6 we can write it as P i by 2k is equal to 1-sigma xi by 2k is equal to C - let us say C minus because the inlet thing is that. So, that is equal to t i by 2R into e raise to - mu lambda i lambda i means especially at the inlet point that is why we are putting the lambda i okay.

$$\frac{P_i}{2K} = 1 - \frac{\sigma_{xi}}{2K} = C^{-}\frac{t_i}{2R}e^{(-\mu\lambda_i)}\ where, \qquad \lambda_i = 2\sqrt{\frac{R}{t_f}}\tan^{-1}\left(\sqrt{\frac{R}{t_f}}\theta_i\right)$$

So, here we can put e raise to exponential relationship where lambda i it is similar to our lambda i we can substitute it as let it by 2 root of R by t f into tan inverse its only difference in the theta that is only difference. If you put it as root of R by t f into theta i so that is what we are getting. So, this C value is the value of the constant before the neutral point is reached and C negative is the value of constant before neutral point.

One should remember that this theta i when we are taking this we should not forget theta i is taken because this is our this one this is directional feature theta is taken okay. So, it is not from inlet to the outlet, this is from the outlet to the inlet. So, that one should keep it in mind when you're taking the theta I value. So, that is why so, before the neutral points mean it is at the exit side.

So, from this we can find out the value of C - is equal to that is C - is equal to So, just 2R by t i into 1-sigma xi by 2k into e raise to because - will now become + mu lambda. Now, for this point beyond the neutral point that means from the inlet side to the neutral point there also you have a different C value so, that you have to calculate it. So, for that for point beyond the neutral point, for points from roll inlet to the neutral point equation 6 can be written as

Sigma xi is is your back tension what we are giving okay so, that is what we have. So, if you are applying it can be written as say like that, if you just say that P f is equal to P f by 2K is equal to 1- sigma xf by 2k where sigma xf is the front tension we are going to put in that case we can just where P f is the roll pressure at the exit so that means 1- sigma xf si equal to so that we can say 1- sigma xf by 2 K is equal to C + t f by 2R. This is the instead of t i we use t f here okay.

$$C^{-} = \frac{2R}{t_i}\left(1 - \frac{\sigma_{xi}}{2K}\right)e^{\mu\lambda}$$

$$\frac{P_f}{2K} = 1 - \frac{\sigma x_f}{2K}$$

$$1 - \frac{\sigma x_f}{2K} = C^+ \frac{t_f}{2R}$$

So, that is a thing. So, from that now, that is you will get C + is equal to 2R by t f into 1 - sigma xf by 2K. So, this is a equation number 9 equation number this is equation number 8. So, substituting this we are getting a so getting the value of the constant before and after the neutral point.

$$C^+ = \frac{2R}{t_f}\left(1 - \frac{\sigma x_f}{2K}\right)$$

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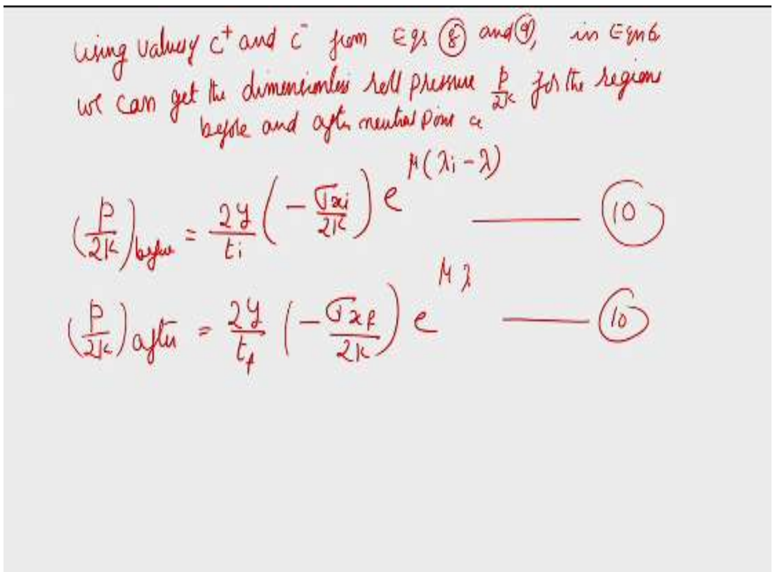

So, our generalized equation if you look at it we can just using this C - and C + we can write 2 equations. Using this values of C + and C - from equations 8 and 9 respectively. In equation we can obtain dimensionless roll pressure P by 2K for the regions before and after neutral point as P by 2K before is equal to 2y by t i into - sigma xi by 2K into e raise to mu lambda i - lambda equation number 10. And P by 2K after means after the neutral point is equal to 2y by t f into - sigma xf by 2K into e raise to mu lambda.

$$\left(\frac{P}{2K}\right)_{before} = \frac{2y}{t_i}\left(-\frac{\sigma_{xi}}{2K}\right) e^{\mu(\lambda_i - \lambda)}$$

$$\left(\frac{P}{2K}\right)_{after} = \frac{2y}{t_f}\left(-\frac{\sigma_{xf}}{2K}\right) e^{\mu\lambda}$$

So, you are getting this value of the dimensionless roll pressure. So once from μ calculated from multiplied by this 2k you get the value of the roll pressure at any point at any angle theta defined by your lambda. In the lambda know the theta value is coming so further particular value you can find out what is the value of the lambda and get it. Okay, so these are the 2 dimensionless equations which you are getting. And next we will also find out how to determine the value of the neutral point also so that will be in the next lecture. Thank you very much.