Plastic Working of Metallic Materials
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Module No # 01
Lecture No # 02
Uniaxial Tension Test Analysis

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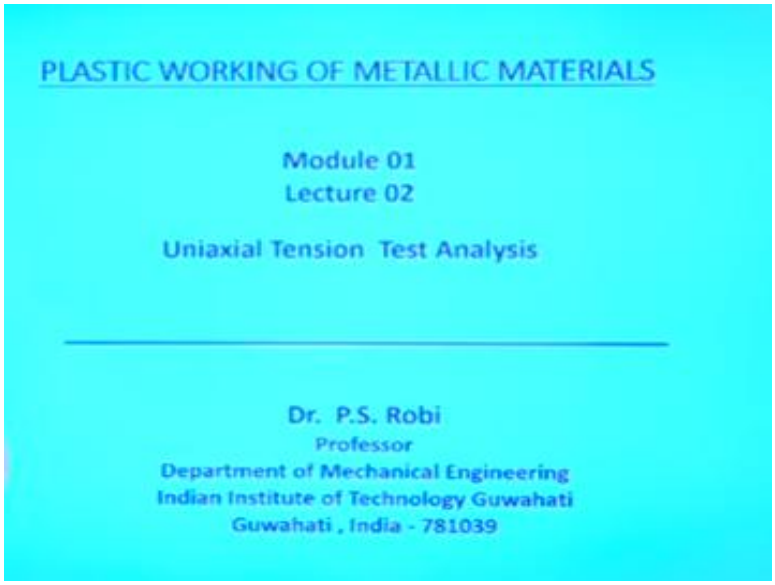

Yeah welcome to this second lecture of module 1 will be it will be a continuation of your last lecture number 1. So here today we will be discussing about the uniaxial tension test analysis.

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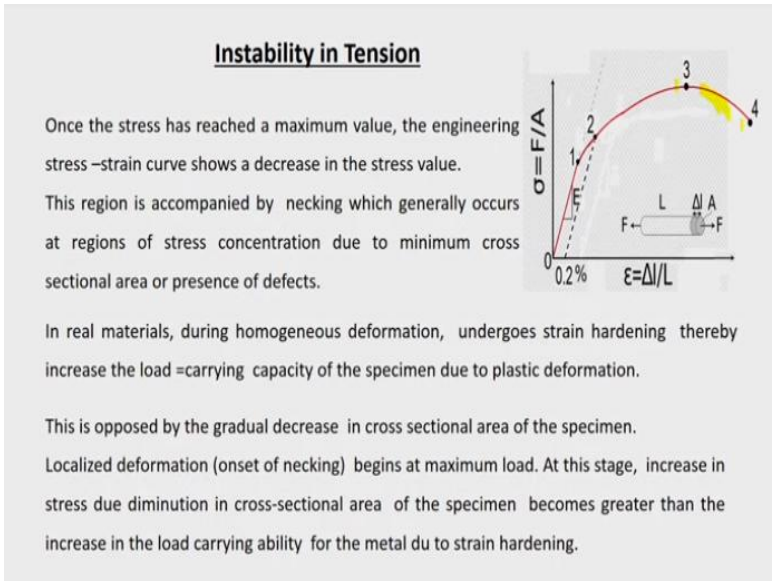

Though we explained about the flow stress flow curve and the instability in tension which I did not complete so we find at in the stress strain curve true stress true sorry engineering stress engineering strain curve the load during the uniform deformation the load reaches a maximum value here and then further that you will find that the load is decreasing or the stress engineering stress keeps on decreasing and so these it has a maximum value.

Many times it is required to find out the maximum value which generally corresponds to which is nothing but it is ultimate tensile strength of the material. So this in real material during the homogenous deformation it undergoes strain hardening there by increasing the load carrying capacity of the specimen due to plastic deformation and but this is opposed by the gradual decrease in cross sectional area of the specimen and you will find that after it reaches this point 3 the ultimate tensile strength or the maximum load condition.

A localized deformation takes place that is the onset of necking takes place at this maximum load and then further loading you will find that further deformation you will find the load decreases continuously till the fracture point is reached that is corresponding to point number 4 here. So during this time you will find that so though from this curve it appears that okay this stresses are decreasing but in actual true stress true strength are it will be continuously increasing.

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The condition of instability ,

i.e., the onset of necking, is dP = 0

$P = \sigma A$ (1)

i. e., $dP = \sigma dA + A d\sigma = 0$ (2)

From constant volume relationship,

$\frac{dL}{dL} = -\frac{dA}{dA} = d\varepsilon$ (3)

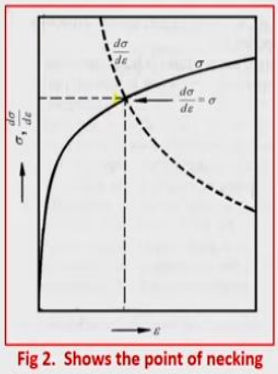

Fig 2. Shows the point of necking

Substituting in the condition for instability, using Eq. (2) and (3),

$-\frac{dA}{A} = \frac{d\sigma}{\sigma} = d\varepsilon \Rightarrow \boxed{\frac{d\sigma}{d\varepsilon} = \sigma}$ (4)

So these are the condition we said so this is the strain hardening rate it decreases and may be the true stress true strain curve now it increases and at some point it reaches these two are equal. So

the condition we get this also we discussed the onset of necking is the case in the true stress in the engineering stress engineering strain curve where the change in the load is 0 because it is initially increasing then reaches a maximum point and then decreases maximum condition is there.

Then dP = 0 and the external load P is given by σ into A and we will find that these dP in the differential form we can write like this. So it will give you this relationship dL / sorry it should be dL / L = - dA / A = $d\varepsilon$ and from this we can get this relationship we derived last day $d\sigma$ / $d\varepsilon$ is equal to σ that is the conditions for the maximum load.

$$P = \sigma A$$

$$dP = \sigma dA + A d\sigma = 0$$

Constant volume relationship,

$$\frac{dL}{dL} = \frac{-dA}{dA} = d\varepsilon$$

Substituting in the condition for instability,

$$-\frac{dA}{A} = \frac{d\sigma}{\sigma} = d\varepsilon$$

$$\frac{d\sigma}{d\varepsilon} = \sigma$$

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From the true stress true strain curve, the point of necking at maximum load can be obtained by finding the point on the curve where the sub-tangent is unity, or the point at which there of strain hardening is equal to the stress.

Using the engineering strain , we can express the necking criterion as

$$\frac{d\sigma}{d\varepsilon} = \frac{d\sigma}{de}\frac{de}{d\varepsilon} = \frac{d\sigma}{de}\frac{dL/L_0}{dL/L} = \frac{d\sigma}{de}\frac{L}{L_0} = \frac{d\sigma}{de}(1+e) = \sigma$$

$$\boxed{\frac{d\sigma}{de} = \frac{\sigma}{1+e}} \quad \text{.... (5)}$$

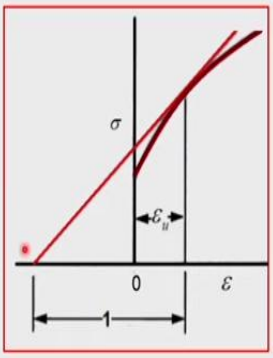

Fig. 3 Representation of necking criterion

Now, let us look at how to determine this value from a stress strain curve so the true stress true strain curve the point of necking at maximum load can be obtained by finding where the sub

tangent is unity. So because from the previous curve, from this derivation, we can get this value or the point at which the strain hardening is equal to stress that is what we have to find out so that the stress whatever it be the condition is coming and by equation that using the engineering strain we can express the necking criteria as d sigma / $d\varepsilon$ that is equal to σ.

We can write it is equal to $d\sigma$ / de into de / $d\varepsilon$ because this is the condition where you are going to find out in terms of engineering strain okay. So the previous case was for the case of true strain okay so in this case so $d\sigma$ / de into de / $d\varepsilon$ that is equal to this we can write in this form and finally we will get that $d\sigma$ / de = 1 + into 1 + e =σ or we can say $d\sigma$ / de equal to σ / 1 + e that can be represented by, you have to do little bit of iteration and find out at what point this is equal to 1 okay.

So and then draw a sub-tangent okay so such that know this condition is satisfied so this is slightly difficult a tedious job because exact point you do not know where it is? It all depends on how you shift this side or this side, okay so that becomes very difficult.

$$\frac{d\sigma}{d\varepsilon} = \frac{d\sigma}{de}\frac{de}{d\varepsilon} = \frac{d\sigma}{de}\frac{dL/L_0}{dL/L} = \frac{d\sigma}{de}\frac{L}{L_0} = \frac{d\sigma}{de}(1+e) = \sigma$$

$$\frac{d\sigma}{de} = \frac{\sigma}{1+e}$$

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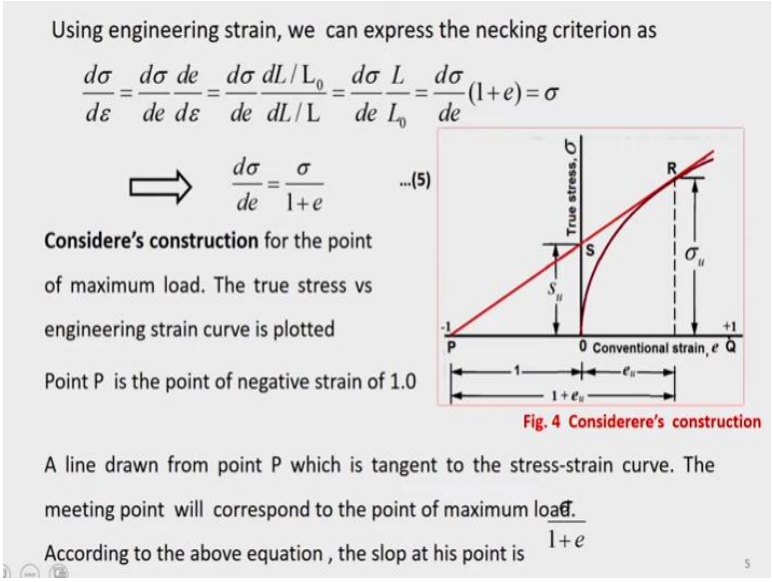

So that is why in another case we can just find out it is $d\sigma$ / de, if you just find out as per this relationship $d\sigma$ / $d\varepsilon$ is equal to this one $d\sigma$ / de into de / d epsilon so that we can write it as $d\sigma$ / de into de = dL / L_0 $d\varepsilon$ = dL / L so that we will get it as L / L_0 into $d\sigma$ / de. So that is equal to you get this equation or from this it implies that $d\sigma$ / de is equal to σ / 1 + e so that is very easy compared to the previous case and this is called as the considere's construction for the point of maximum load.

So what we do is that from the 0 point 0 strain you take a strain equal to – 1 in the opposite direction, in the negative direction of the strain axis and with from that point you draw a line which is tangent to your true stress engineering strain curve. So you are plotting that true stress versus engineering strain curve, so from a strain value of – 1 you just draw a tangent to this curve wherever it is meeting.

So what will be your the stress at maximum value, so that see that is nothing but σ u / say here it is ε u you are getting so 1 + ε u so that is the condition we are getting $d\sigma$ / ε, so that is the slope which is equal to σ / 1 + e which is engineering strain okay. So that is that way this drawing those this curve will be very easy to find out and this is called as the considere's construction for the point of maximum load.

$$\frac{d\sigma}{d\varepsilon} = \frac{d\sigma}{de}\frac{de}{d\varepsilon} = \frac{d\sigma}{de}\frac{dL/L_0}{dL/L} = \frac{d\sigma}{de}\frac{L}{L_0} = \frac{d\sigma}{de}(1+e) = \sigma$$

$$\frac{d\sigma}{de} = \frac{\sigma}{1+e}$$

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For computational purpose, it is necessary to represent the experimentally determined true stress- true plastic strain curves (flow curve) by empirical equations. Some of these are:

Hollowmon equation	$\sigma = A\varepsilon^n$
Modified Ludwik power law	$\sigma = \sigma_y + A\varepsilon^n$
Swift	$\sigma = C(m+\varepsilon)^n$
Voce	$\sigma = C(1 - me^{-n\varepsilon})$
Ramberg-Osgood	$\varepsilon = \frac{\sigma}{E}\left\{1 + \alpha\left(\frac{\sigma}{\sigma_0}\right)^{m-1}\right\}$

Whenever you wanted to study the plastic deformation for computational purpose because conducting an experiment initially you know it takes lot of time and it also involves lot of money also you need very complicated machines and other things so all these things comes into picture, so it is many times for deformation studies always you go for some computational work, so that your amount of experimental can be reduced okay.

So for that purpose many times it is necessary to represent the experimentally determined flow curve okay. So you conduct a tensile test and get your flow curve that is, the true stress versus true strain curve. So if you have that data for the particular material normally what is done is that in the fully annealed condition where the material is soft the people carry out that experiment, okay it do a tensile testing.

And because during plastic deformation there is going to be say work hardening. So you do not know when you are getting a material from the market these most of the raw materials are subjected to plastic deformation some are subjected to hot working condition, some are cold working condition say so in the cold working condition you may find that depending upon the amount of the strain or amount of plastic deformation we have given.

So the it is yield strength will be high when it is in the especially when it is in the work hardening region or homogeneous deformation case. So you do not know whether how much amount of strain has been given to the material so that what will be yield strength, you do not know about it and not only that in deformation studies if you are getting if there is a prior history of deformation, then you your all your calculation may get distorted based on the data which you are going to get it.

Because sometimes it might have been subjected to a high value that further deformation the material may fail because there is a limit for the plastic deformation which is possible. So all these information is required. So in the normal case what people do is the you take the material, may be you might got it from the market or locally purchased it you do a fully annealing condition, so that the material becomes soft there is no effect of strain hardening, the dislocation density is inside the materials are minimum and so you can conduct the material is very soft in that case you carry out a tensile test.

So you get the exact representation of the stress versus strain plot from the data you are going to get it. And now and this data will be used for further plastic deformation studies for analysis purpose so in that case you wanted this data to be represented in some form of a empirical equations okay. So this flow curve and large number of people have come forward with the different types of equations for fitting the curve okay.

So it is basically a curve fitting technique. So one is the Hollowmon equation where it gives that the stress sigma is equal to A ε raised to n where n is the work hardening exponent and A is called as the strength coefficient sigma is the true stress and this is true strain. Now this was modified later by Ludwik the same power law he introduced that in yield strength in the initial case and other thing and then.

$$Hollowmon\ equation \quad \sigma = A\varepsilon^n$$

$$Modified\ Ludwik\ power\ law \quad \sigma = \sigma y + A\varepsilon^n$$

So these strain here refers to the plastic strain, not the total strain it refers to the plastic and so this can be represented by this swift as another form of equation so that is σ equal to C into m + ε, all this case ε is nothing but the plastics strain and we know that the total strength consist of elastic strain and plastic. And elastic strain because it is very small the considere's is around 0.2% so it can be neglected because the plastic deformation, in case of plastic deformation the strain is very large.

So that we can as a safely ignore the elastic deformation so here this Cm and then are constant n is again the work hardening exponent okay C and m are constant. Similarly Voce also come out with another equation and then another say Ramberg-Osgood equation generally this step of equation used for fraction studies and other things and this is also good form of representing the stress strain curve which is given by strain is equal to σ / e where e is the young's modulus value of the material + α into σ / σ_0 raised to m – 1.

$$Swift \quad \sigma = C(m + \varepsilon)^n$$

$$Voce \quad \sigma = C(1 - me^{-n\varepsilon})$$

$$Ramberg - Osgood \quad \varepsilon = \frac{\sigma}{E}\left\{1 + \alpha\left(\frac{\sigma}{\sigma_0}\right)^{m-1}\right\}$$

So this alpha and m and sigma 0 these are all constant for the material now out of all you will find not like this a larger number of forms of empirical equations are available in the literature okay.

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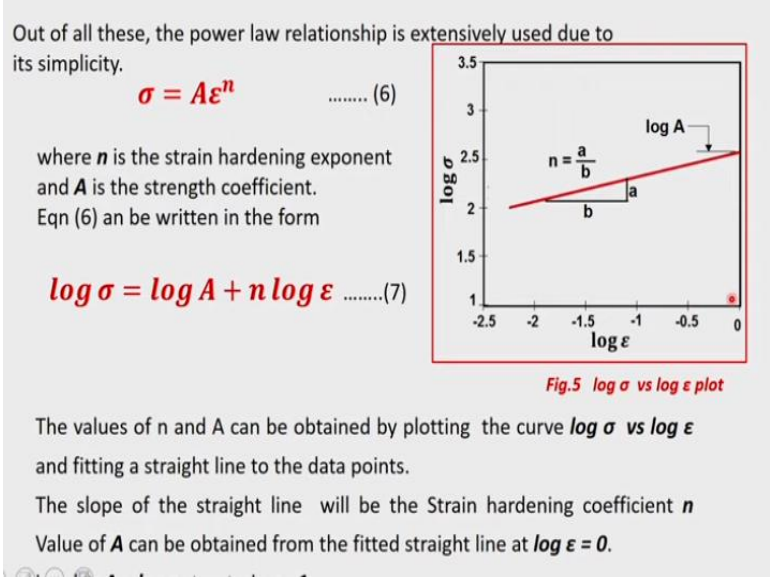

But out of all these the most convenient one is expression is that of the power law equation given in this form, σ is equal to A ε raise to n because of it is simplicity and the way it can be used for any computational purpose, this become a very handy relationship okay, it becomes a very convenient form of convenient relationship for all this analysis purpose.

So where n is the strain hardening exponent and A is the strength coefficient of the material. So it is basically the strength this strain hardening exponent which decides the material flow of the material the which decides the material flow during the plastic deformation. A since it is a constant that is the initial yield stress normally it will be the or maximum value. So this equation if you take the how to determine A and you take the logarithm so that it can be represented log σ equal to log A + n log ε, where epsilon is the plastic strain and this is of the form of a straight line equation for a straight line y = mx + c.

So if you take a to get to obtain the value of n and A what you have to do is that you take the log σ versus log ε plot from your tensile test data, you find out this log ε versus log sorry log σ versus log ε and you will find that it almost fix into straight line equation, there will be some

scatter but you can get a it may very good accuracy you can get it as a straight line equation and the slope of this line.

Because from this relationship slope of the line will give you the n and the strength of the coefficient or strain hardening coefficient strength coefficient can be obtained from the fitted straight line say where log ε is equal to 0 okay. So at log ε is equal to 0 what is the value of this σ so that value from that we can find out the value of A so log A is equal to log σ at log ε is equal to 0, okay so from that we can find out the value of A and n so that is.

$$\sigma = A\varepsilon^n$$

$$\log \sigma = \log A + n \log \varepsilon$$

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Values of strain hardening exponent , $0 < n < 1$

when n = 0 the material is perfectly plastic solid

when n = 1, the material is perfectly elastic.

For most materials, ***n*** generally varies between 0.1 to 0.5.

From the definition of ***n***,

$$n = \frac{d(\log \sigma)}{d(\log \varepsilon)} = \frac{d(\ln \sigma)}{d(\ln \varepsilon)} = \frac{\varepsilon}{\sigma}\frac{d\sigma}{d\varepsilon} \quad \text{or}$$

$$\frac{d\sigma}{d\varepsilon} = n\frac{\sigma}{\varepsilon} \quad \text{.....(8)}$$

So it is a very simple form of determining this constants, so the value of strain hardening exponent generally is between 0 and 1. So 0 means it will be elastic, then 0 means it will be a constant value, say perfect elastic material will be like, this okay it will remain constant. Now if it is when n = 1 the material is perfectly elastic which is in this region so which are getting it so here it is not deforming plastically okay.

These are the extreme condition which generally, you will not find it in real material the normal value for the n for real material lies between 0.1 to 0.5 okay. So this is the extreme condition limit but in real material the maximum value you may find is 0.5, may be exceptional cases may be there which may go beyond that but otherwise the value of n lies between 0.1 to 0.5. So from

that definition of n, n from our previous equation n = d log σ / d log ε that is equal to you can write in this form log σ / d log ε that is equal to epsilon / σ into d σ / d ε. And so from this relationship we can get that d σ / d ε = n into σ / ε.

$$n = \frac{d(\log \sigma)}{d(\log \varepsilon)} = \frac{d(\ln \sigma)}{d(\ln \varepsilon)} = \frac{\varepsilon}{\sigma}\frac{d\sigma}{d\varepsilon}$$

$$\frac{d\sigma}{d\varepsilon} = n\frac{\sigma}{\varepsilon}$$

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Measures of ductility

While dealing with plastic working of metals, ductility is important since it:

a) is the extent to which a metal can be deformed without fracture during metal working operation

b) indicate the ability of metal to flow plastically before facture. Provide warning before failure

c) act as an indicator of change in impurity level or processing conditions

The conventional measurement of ductility is from tension test. These are obtained from

- Engineering strain at fracture e_f and
- Reduction in area at fracture q

These are obtained by joining together the tensile fracture specimen and measuring the gauge length L_f and cross sectional area A_f after fracture.

Now let us come to the measures of ductility because for plastic deformation what is important is to how much extend the material you can deform, so that is what is required. So you may have to conduct this tensile test at various conditions and ductility the knowledge about ductility is very important because all engineering materials for application purpose specially for structural purpose it requires some minimum amount of ductility because you will not use a material which is brittle, okay.

Because that brittle material will fail with without giving a warning whereas if it a ductile material before failure you will get an indication that the material is going to fail, so for this case even for when you going to have a plastically deformation material also you should know to what extend the material can deform. So that is why we wanted that ductility, the value of the ductility for that material and form a tensile test, the ductility generally is to what extent it is elongating before failure.

So this is the extent which a metal can be deformed without fracture during metal working operation and it indicates the ability of the metal to flow plastically before fracture providing a warning before the failure, so these advantage are there and it also act as a indicator for change in the impurity level of processing condition something which may are important for the metallurgist.

But as a mechanical engineering these are the two things we two advantages we can have if you know about the ductility. So about the material ductility how to measure it the conventional measurement of ductility of any material it is by using a tensile test or a compression test. For deformation studies, people generally use compression testing because it is of much more easy, specially when you are carrying out the experiment at higher temperature.

But otherwise at room temperature the data you can have it is from a tensile test, these are obtained from the data of the ductility values can be obtained from a engineering strain at a fraction that is the ef total engineering strain and the reduction the area of fraction because as we know the deformation which is taking place it reaches a maximum till that time. The deformation is homogenous and after that the necking starts place and that the failure strain what is the minimum cross sectional area.

So if you have an idea about their maximum load which was supporting at the time of failure and the cross sectional area at that time of failure then you can know what was the stress also which was there. So this gives the reduction in the cross sectional area because initially you had cross sectional area which was A naught and now you have cross sectional area A from that we can find out what is the reduction in the area, cross sectional area to fracture.

So any of this criteria can be used both are having its own advantage and its own disadvantages but remember the engineering strain at fracture ef and the reduction area of fracture q these are being determined from the after fracture of tensile sample, you join together and across a non-distance how much it deforms that from that you will get this ef. And what was the initial cross sectional area and what is the final cross sectional at fracture from that we can find out the reduction in area.

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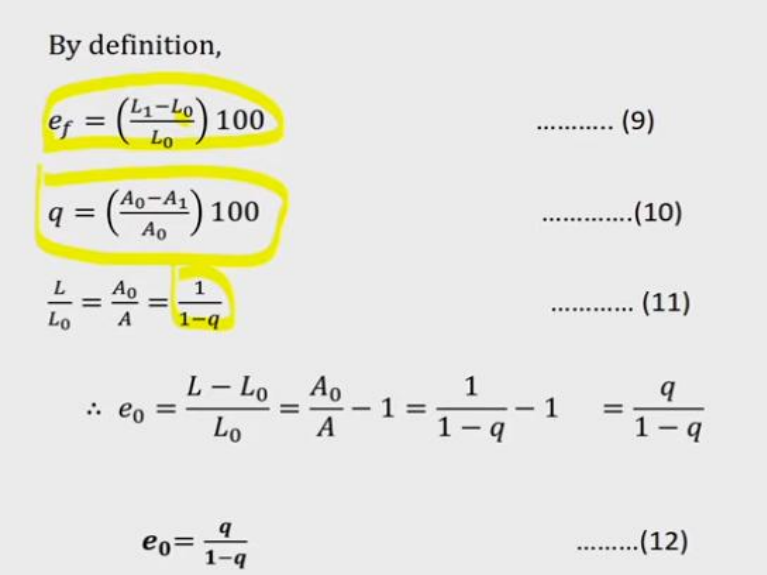

So by definition the fracture strain = L1 – L_0 / L_0into 100 so into should be there and the reductional area is A_0 / A1 / A_0into 100. So from these relationship, we can find out that L / L_0 = 1 / 1 – q, where q is the reduction in area cross sectional area. So the failure strain you can find it out = q / 1 – q, so you are getting the relationship between these engineering strain and reduction area by this relationship.

$$e_f = \left(\frac{L_1 - L_0}{L_0}\right) 100$$

$$q = \left(\frac{A_0 - A_1}{A_0}\right) 100$$

$$\frac{L}{L_0} = \frac{A_0}{A} = \frac{1}{1-q}$$

$$e_0 = \frac{L - L_0}{L_0} = \frac{A_0}{A} - 1 = \frac{1}{1-q} - 1 = \frac{q}{1-q}$$

$$e_0 = \frac{q}{1-q}$$

But before that I should like to tell you about one thing many times the percentage elongation which is given by this relationship e_f, when you quoting that value you have to mention across what gauge length it as to be there either you may tell it was across a 12.5 mm or half inch length or a 1 inch length or maybe 2 inch length. The problem is that necking can take place at any point within that gauge length or may be sometimes outside the gauge length, then you are not considering it okay.

So when you are keeping it 2 inches specimen a gauge length, failure will be taking place necking will be at small area only, if it is a 1 inch gauge length then you will find the total length is reduced. So if it is a 12 inch gauge length is section if you are taking then you get a much big better accuracy. But depending upon the gauge length your strain value you may get at different value, so you have to always specify what was the percentage elongation across the gauge length, you have to specify across may be 25mm or 1 inch or half inch or 2 inches gauge length you have to tell.

Whereas that is not required when you are going for your reduction cross sectional area because you are going to get the final cross sectional area and at that time what was the stress you can get it. So that is one thing so how much the reduction area was there you are calculating, so this may give a better accuracy but the difficulty is conducting the test also okay. So in any case you are going to join together and measure it.

So but in between value you may find it very difficult to get for this whereas the actual value you may get it for change in length, reduction in area measurement during the test is very difficult that is the so both are having advantages as well as disadvantages.

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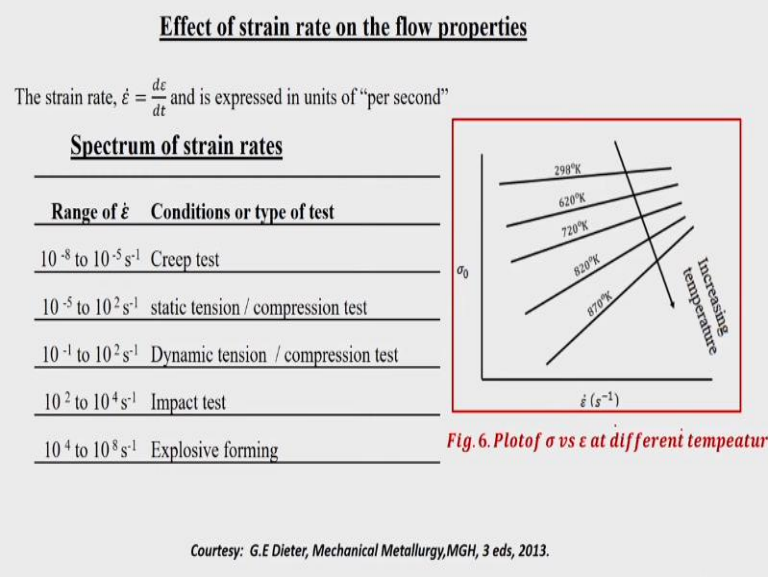

So the effect of strain rate and the flow properties of the material. So if you look at the various strain rates which are being used to for under different condition when the metal is subjected to straining. It is generally done at different strain rates, so the spectrum of strain rates are say like

we can say that when it is very slow strain rate to the power 10 to the power – 8 to 10 to the power – 5 per second, this is something like the Creep test.

$$10^{-8}\ to\ 10^{-5}\ s^{-1} Creep\ test$$

So in a creep deformation what is happening, that is the strain rate it is very small so like you are subjecting a material to high at an elevated temperature and stresses and keep it for there for long time even if it is below the yield strength of the material also after prolonged exposure you will find that some amount of deformation is taking place. Normal case, it is not going to affect much but special for those material which are subjected to higher temperature this is of more important.

Specially like if you look at the component in a gas turbine blade or may be the reformer tube in a chemical industry or a refinery. So these are subjected to high temperature and some amount of stress and normally know when you do the design of this components say for example reformer tube, it is a designed life is something around 12 years at that particular temperature and the pressure of application.

So but still you will find that by 12 years because beyond that, you will find that small amount of plastic deformation is taking place. So that is the case where it is the creep test that is so 10 to power – 8 to 10 the power – 5 per second, that is typical of a creep test and from 10 to the power – 5 to 10 to the power 2 it is a typical of a static tension or a compression test where you are just deforming the material between two jaws and one is the moveable head and another is a fixed head and then you are just pulling it.

So that normally will be at somewhat higher strain but it is not very high also so this 10 to the power 2 is sorry 10 to the power -1 it should be then 10 to the power -1 to 10 to the power 2 is the dynamic tension or compression test may be like your fatigue testing and other things when you commit or may be your slow impact test those type things are. Basically this range will be corresponding to your fatigue testing.

$$10^{2}\ to\ 10^{4} s^{-1}\ Impact\ test$$

Then you have the 10 to the power 2 to 10 to the power 4 the impact testing a Charpy V-notch or an Iso-testing or maybe you find that free following indentation impact test. So all those things

are coming into the picture so that will be under the impact testing then when you go for very high strength rate 10 to the power 4 to 10 to the power 8 per second it is something like explosive forming okay.

$$10^4 \; to \; 10^8 s^{-1} Explosive \; forming$$

Or so this are the strained spectrum of strained rate which one will be facing in normal engineering applications and if you look at the flow stress the dependency of flow stress on the strain rate and temperature it will flow like this. So you will find that with increase in strain rate the flow stress keep on increasing, that is the trend almost with increase in temperature in flow stress keeps on increasing and with a higher temperature the this slope will be much sharper compared to a lower temperature.

For example, room temperature you will find that the strain rate dependency on the flow stress is very small, so your strain rate sensitivity will be very low here. But at elevated temperature you find that the slope of these curves are changing. So when this slope is changing, you will find that at higher temperature strain rate is having a higher impact on the flow stress. So this is what I think increase in temperature the variation of the flow stress will be in this format.

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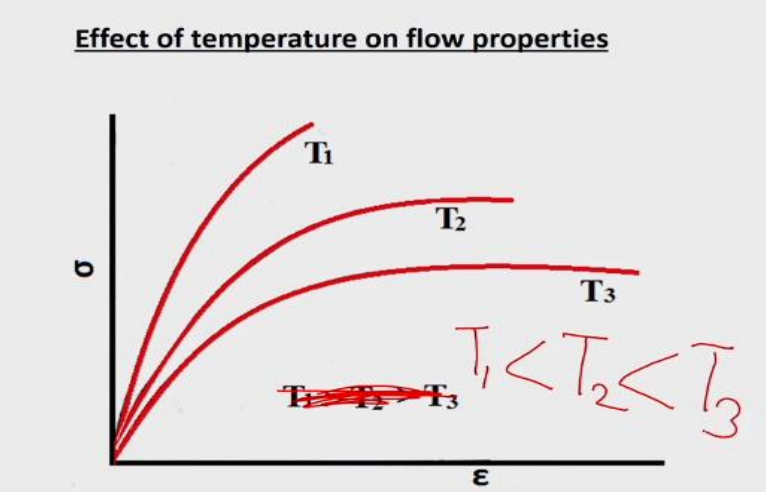

Figure showing the effect of temperature on flow curve

Now effect of temperature on the flow properties, you will see that with the stress and typical stress strain curve for a particular strain at a constant strain rate, if you look at it you will find that at higher temperature, okay, this should be the reverse actually. So it should be so at lower

temperature you will find that the stress is increasing for a particular strain and as the temperature and you have a very less amount of ductility because this strain the till failure is very less.

But at a higher when you are increasing the temperature any engineering material specially any metallic materials, it is a common sense also that if you heat it becomes much more soft okay and we get easily deform the material if it is heated otherwise you know one or two times you know you heat it know you bend it then its gets hardened and it become very brittle, so this is a very it is a very layman will understand these things.

So that means, with increase in the temperature you will find that the ductility is increasing and that increase in the ductility is very high but at the same time, the yield strength or the flow behavior of the material lower than that corresponding to at a higher temperature. So this is the general flow behavior at higher temperature, so with this. Now during this test we should have some idea like if the test is carried out at a high speed or high strain rate, at a high rate then you may be a get a different values specially that is of importance in the test are carried out at higher temperature, at elevated temperature.

So there is the total the flow stress with are going to get it depends upon the strain, the rate at which it is strain and the temperature at which the test has been carried out. So the flow stress is a function of stress of strain, strain rate and temperature. So you have to know about the effect of strain rate on the flow properties of the material. So if you are carrying out a different temperature's and now the strain rate it is expressed by this rate of strain that is $\dot{\varepsilon}$ dot is nothing but is equal to d ε / dt and this expressed in terms of per second.

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt}$$

So when you look at this spectrum of strain rate, say this data is taken from say Dieter the mechanical Metallurgy. So the range of strain rate, you will see that from 10 to the power – 8 to 10 to the power -5 per second then it will be something like which is equal to a creep test. Where the deformation rate is very small okay. So it may take several years to have a small amount of strain, so that corresponds to a creep testing or creep phenomena.

But if the strain rate is varying from 10 to power -1 to 10 to power 2 per second which corresponds to a static tension or a static compression stress which you always do it under a static machine, just loading it so that it is strained and measure the load versus the specimen gauge length extension. So that is the static correspond to a static tensile test condition or the static compression test condition.

Now if the strain rate varies from say if it is increasing it corresponds to dynamic tension compression test and with the further increase it corresponds to impact us and with a very high strain rate it is explosive forming. So there is this negative signs and these values have come there is a mistake is coming I am very sorry that I will correct it next day. But if we look at the when the experiments are carried out at different temperatures and at constant strain rates, we will come to that how it is conducted.

The flow stress values you will find that you are getting the curve like this with increase in temperature coming down, you will find at the flow curve keeps on decreasing and with the strain rate increasing, you will find the flow stress keeps on increase. So this is the case individual cases then carried out at different temperatures and so with increase in temperature flow stress value decreases, with increase in strain rate the flow stress value increases.

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Strain rate can be expressed in terms of either conventional linear strain or true strain.

For the case of extension of a cylindrical specimen with one end fixed and the other end is fixed to a movable cross head If the cross head is moving with a constant velocity v,

$$v = \frac{dL}{dt} \qquad \text{.....(13)}$$

Conventional strain rate $\dot{e}$,

$$\dot{e} = \frac{de}{dt} = \frac{d}{dt}\left(\frac{L-L_0}{L}\right) = \frac{1}{L_0} \cdot \frac{dL}{dt} = \frac{v}{L_0} \qquad \text{.....(14)}$$

i.e., $$\dot{e} = \frac{v}{L_0} \qquad \text{.....(15)}$$

i.e., the conventional strain rate $\dot{e}$ is proportional to the cross head velocity v.

In conventional testing machine it is possible to carry out tests at constant $\dot{e}$.

How to express or how to carry out at test by means of constant strain rate? So two types of strain rates are there. One is by using either the conventional linear strain or true and the second

case is considering the true strain, okay. So when you are taking the considering the conventional linear strain then we say that this experiment can be carried out under a tensile testing machine, in a tensile mode or in a compression mode okay.

So for that case of extension of a cylindrical specimen with one end fixed, most of this machines you will find it one end of the cross head is fixed and the other will be moving and the specimen is clammed into that so maybe the top one may move or bottom one may move, in any case the load is appeared and there will be load cell to measure, the load which is coming during the extension okay.

So if the cross head is moving with the, if the movable cross head is moving with a velocity v then we say that the v = dL / dt okay, where L is the change in length of the specimen, that is correspond to the how much it is moving. So conventional strain rate can be calculated by this relationship e dot = de / det that is nothing but d / dt of L – L_0 / L where L is the instantaneous value okay.

So that we can get it as 1 / L_0 into dL / dt L – L_0 / L_0 here okay, so that is = 1 / L_0into dL / dt that is equal to v / L_0. So in the conventional strain rate when you are looking at it, your considering only the initial gauge length and your velocity is constant, so when you are doing this experiment at a constant cross head velocity, your engineering conventional strain rate remains constant or engineering strain rate remains constant okay.

But your true strain is not constant the conventional strain rate is proportional to cross head velocity v in conventional testing machine it is possible to carry out at a test at a constant strain rate only, constant engineering strain or a conventional strain rate only. You cannot carry out test at a constant true strain testing.

$$v = \frac{dL}{dt}$$

$$\dot{e} = \frac{de}{dt} = \frac{d}{dt}\left(\frac{L - L_0}{L}\right) = \frac{1}{L_0} \cdot \frac{dL}{dt} = \frac{v}{L_0}$$

$$\dot{e} = \frac{v}{L_0}$$

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The true strain rate

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = \frac{d\left[ln\frac{L}{L_0}\right]}{dt} \qquad \Rightarrow \dot{\varepsilon} = \frac{1}{L}\cdot\frac{dL}{dt} = \frac{v}{L}$$

$$\Rightarrow \dot{\varepsilon} = \frac{v}{L} \qquad \text{.....(16)}$$

The relationship between true strain rate and conventional strain rate can be obtained by

$$\dot{\varepsilon} = \frac{v}{L} = \frac{L_0}{L}\cdot\frac{de}{dt} \qquad \text{.....(17)}$$

$$= \frac{1}{1+e}\cdot\frac{de}{dt} \qquad \left[\because v = L_0\frac{de}{dt} = L_0\dot{e}\right]$$

$$\Rightarrow \dot{\varepsilon} = \frac{\dot{e}}{1+e} \qquad \text{.....(18)}$$

So for true strain rate it is defined by say ε dot = d epsilon / dt that is by ε, we say d into log L / L_0/ dt which implies that epsilon dot can be obtained by this relationship with v / L. So if we compare with the previous equation, that is epsilon dot = v / L and e dot your conventional strain rate = v / L_0. So here for carrying out an experiment at a constant strain rate if it is tensile specimen, when the gauge length increases you will see that your strain rate has to your velocity of the cross head as to change.

So by conventional static machine this is not possible so when you wanted to carry out an experiment using an experiment with a true strain rate test constant true strain rate test, you need to have a machine with a feedback control system, so that as this specimen gauge a specimen length gauge length changes your velocity has to change. So that is what so the relationship between true strain rate and conventional strain rate also can be obtained by this relationship, that is ε dot = v / L that is equal to L/L_0 / de / dt.

Because de/ dt you can get and that is equal to 1 / 1 + e into de / dt okay. So this you can get it from this relationship so you will find that, ultimately because v = L_0 into de/ dt that is what we are getting. So ε dot = d dot / 1 / ε dot.

$$\dot{\varepsilon} = \frac{d\varepsilon}{dt} = \frac{d\left[\ln\frac{L}{L_0}\right]}{dt}$$

$$\dot{\varepsilon} = \frac{1}{L}\cdot\frac{dL}{dt} = \frac{v}{L}$$

$$\dot{\varepsilon} = \frac{v}{L} = \frac{L_0}{L} \cdot \frac{de}{dt}$$

$$= \frac{1}{1+e} \cdot \frac{de}{dt}$$

$$\dot{\varepsilon} = \frac{\dot{e}}{1+e}$$

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- From eqn. (17), the true strain rate will decrease as the specimen elongates when deformed at a constant cross head velocity
- A machine with open loop control is necessery to maintain a constant true strain rate during tests such that the cross head velocity increases as the speciman elongates.

The cross head velocity of the machine should vary according the expression

$$v = v(t) = \dot{\varepsilon}\, L_0 exp(\dot{\varepsilon} t) \qquad \text{.....(19)}$$

A general relationalship between σ and $\dot{\varepsilon}$ at constant strain is of the form

$$\sigma = C(\dot{\varepsilon})^m|_{\varepsilon,T} \qquad \text{.....(20)}$$

where m = strain rate sensitivity [obtained from a $\ln \sigma$ vs $\ln \dot{\varepsilon}$] plot

So from the pre as I was telling the true strain rate will decrease as a specimen elongates when deformed at a constant cross head velocity. A machine with an open loop control is necessary to maintain a constant true strength rate testing during the tests such that the cross head velocity increases as the specimen elongates. At initial stages there were equipment's where it was mechanical design so that for a constant you are able to get a constant form rate testing also.

But now a days you know with advanced electronic system and microprocessor system this with a opponent loop control machines are very easily available everywhere. So now a days people will follow that one. So since the for a constant strain rate testing if you wanted to carry out the cross head velocity will vary this by a simple derivation we can get this, of this form where velocity with respect to time is obtained by this constant velocity into L naught into exponential f naught t from that relationship we can always get with a time how the velocity should change.

So you can always allow your cross head displacement accordingly moving the general relationship between sigma the flow stress and strain rate, at constant strain is of the form sigma = c into epsilon dot raise to the power m at a constant strain and temperature. So where, m is called as the strain rate sensitivity okay, so this in similar to our previous equation for sigma versus, so you are Hollowmon relationship in the similar way you can find out but the experiment has simply done at a constant strain rate and constant strain and constant.

So at a constant value of strain and temperature if you take it and then from that, you know you can find out you do it at a different strain rate and then find out the value of sigma for a constant strain and temperature and from that value you can always find out. So you plot a log sigma versus log epsilon dot plot for a constant strain value and at a constant temperature and from that we can get it.

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finding out the corresponding flow stresses σ_1 and σ_2 at any constant strain. Another convenient method of determining the value of m is by carrying out a rate change test where the true strain rate is changed suddenly.

$$m = \left(\frac{\partial \ln \sigma}{\partial \ln \dot{\varepsilon}}\right)_{\varepsilon,T} = \frac{\dot{\varepsilon}}{\sigma}\left(\frac{\partial \sigma}{\partial \dot{\varepsilon}}\right)_{\varepsilon,T} = \frac{\Delta \log \sigma}{\Delta \log \dot{\varepsilon}}$$

$$= \frac{\log(\sigma_2 - \sigma_1)}{\log \dot{\varepsilon}_2 - \log \dot{\varepsilon}_1} = \frac{\log\left(\frac{\sigma_2}{\sigma_1}\right)}{\log\left(\frac{\dot{\varepsilon}_2}{\dot{\varepsilon}_1}\right)} \quad \ldots\ldots (21)$$

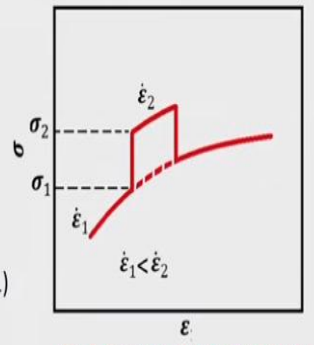

Fig. 7. Strain rate change test data plot

- The value of m of metals is quite zero (<0.1) at room temperature.
- m increases with temperature especially above $0.5\ T_m$.
- For hot working conditions, m valuses of 0.1 to 0.2 are common.

By m can be obtained by carrying out test at different true strength rates ε dot 1 and ε dot 2 and finding out the corresponding flow stress value σ 1 and σ 2 and at any constant strain and a constant temperature. Another convenient, because this takes lot of time specially when you are carrying out a test at very slow strain rate test when you are doing it one test itself will take lot of time.

So to save the time, people also carry out that in a single specimen you do a rate change test strain rate change test you do. You are initially doing an experiment at a constant strain rate

epsilon dot 1 with your feedback control machine and when it reaches a constant value of say may be some strain, then suddenly you jump the strain rate. Nowadays you know that is very it can be done very easily the machine with the microprocessor control so use from the ε dot 1 to another strain rate ε dot 2 you suddenly have a jump.

So when that is done you will find that when the strain rate is higher if strain rate ε dot 2 is higher than ε dot 1 you will find that the flow stress increases or if it is reduced flow stress will reduce. So and then you carry out the experiment for some or strain and then suddenly come back to the initial stage. So if you are carrying out the experiment at this strain rate it would have followed like this curve.

So now you know that ε 2 and ε sorry, σ 2 and σ 1 corresponding to ε dot 1 and ε dot 2 and epsilon dot 1 from this value you can always calculate even maybe for this strain we can find out in this case we can find out for the different strain value. And that we can get it by the basic definition m = del log σ / del log ε dot at a constant strain and temperature.

So that we can get it in this form ε dot by σ into d σ by d ε del sigma by del ε, so we can get in this differential form and from finally you can arrive that m is nothing but log σ 2 / σ 1 divided by log ε dot 2 / ε dot 1. So σ 2 is known σ 1 is known ε dot 1 and ε dot 2 is known so for a particular strain you get it so for 2 strains also one strain here one strain corresponding to here you can get it, so you can find out the value of the strain rate sensitivity of the material.

And you may please note that the value of m for metals its almost 0 it is much less than 0.1 at room temperature, when you are doing it at a room temperature it is not showing much influence on the strain rate, it remains almost constant but situation is complicated when you are doing specially at hot working condition at higher temperature when you are doing, it specially for hot working condition where the flow stresses will be lower in that condition you will find that you having a appreciable amount of value of m.

The m increases with the temperature especially above a homolog of temperature 0.5 that means 0.5 means the absolute scale when the temperature is more than half of its absolute temperature, you will find that the strain rate sensitivity comes into picture. So there the flow stress will heavily dependent upon the temperature at which dependent upon the strain rate with you are

doing okay. So for hot working condition m values of 0.1 to 0.2 are very common for most of the material.

$$m = \left(\frac{\partial\, in\, \sigma}{\partial\, in\, \dot{\varepsilon}}\right)_{\varepsilon T} = \frac{\dot{\varepsilon}}{\sigma}\left(\frac{\partial \sigma}{\partial \dot{\varepsilon}}\right)_{\varepsilon T}$$

$$= \frac{\Delta \log \sigma}{\Delta \log \dot{\varepsilon}}$$

$$= \frac{\log(\sigma_2 - \sigma_1)}{\log \varepsilon_2 - \log \varepsilon_1} = \frac{\log\left(\frac{\sigma_2}{\sigma_1}\right)}{\log\left(\frac{\dot{\varepsilon}_2}{\dot{\varepsilon}_1}\right)}$$

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High strain rate sensitivity is characteristics of superplastic metals & alloys. During superplastic deformation, the specimen undergoes vey high elongation viz., 100% to 1000% elongation . The material then deforms at a very low stress.

Conditions for superplastic deformation:

a) Very fine grain size (< 5 μm)

b) Two phases: one brittle and discontinuous phase and the other ductile and continuous phase

c) High m value (m > 0.5)

d) Very low $\dot{\varepsilon}$

Now high strain rate sensitivity, that is high value of m is characteristic of super plastic metal and alloys. So metals there are certain materials which can be deformed under super plastic condition that means the elongation which you can get is extremely very large, may be 100 or 1000 times of gauge length. So it is a typical example is that you take a glass rod heat it so in this case when there is a super plastic deformation the necking phenomena of necking is not taking place okay.

And at that condition at under certain conditions only you can carry out that, under that condition you will find that the flow stress is very less than one tenth of one fifteenth of the actual case at room temperature. So you may be able to deform even a ceramic material at a higher temperature

at a flow stress of even may be 10 or 12 mega Pascal. So that is the beauty of this super plastic deformation okay.

So there, under those certain conditions you can deform it to a very high amount of strain may be 100, 500 or 1000 or even 5000% depending upon the material, so the percentage elongation will be so high. And the material than deform under that conditions it a very low flow stress value. The main condition for super plastic deformations which people have highlighted but there are some exceptions also or that it should consist either grain size of the material should be very fine that means it should below 0.5 micrometer.

And it should consist of two phases, one should be a brittle phase and which is discontinuous and other is a ductile phase which is a continuous phase, okay. But in some cases this may not be necessary but in general metallic materials these are necessary okay. And the material should have a very high value of strain rate sensitivity index, that is the m value should be more than 0.5 and the strain rate should be very small, at a very high strain rate it may not work out.

So strain rate has to should be very small so in that case you know you can deform the material without any type of necking. So localized reduction in cross section will not be taking place and you can get some very particular shape inside a dye if you do it with a with an air pressure or hydrostatic pressure if you are doing it or with air pressure compressed air pressure at higher temperature if you do it the material will deform in a homogenous way and with a uniform thickness, we can get it in any complicated shapes we can be it can be obtained.

So that is the advantage even ceramic materials can be deformed by super, not all but certain ceramic materials are so can subjected to super plastic deformation.

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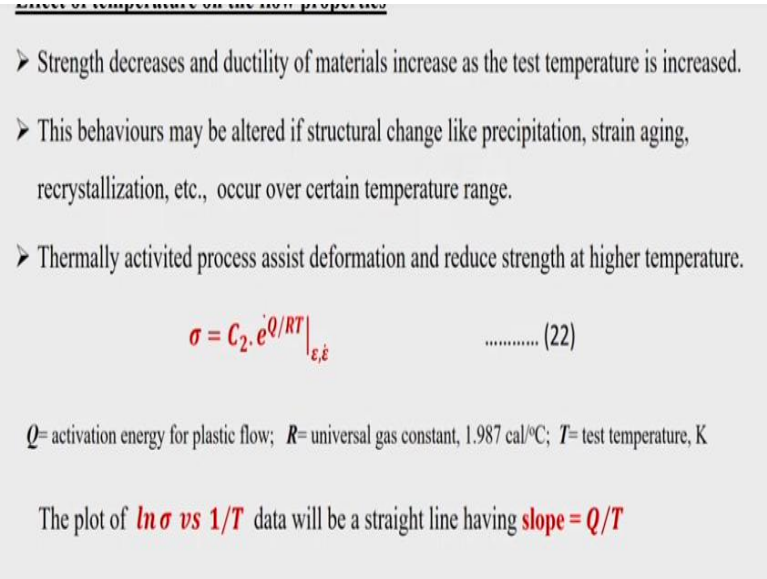

And if you look at the effect of temperature and flow properties the strength decreases and ductility of the material increases as the test temperature is increased. This behavior may be all altered, this is under the normal condition if there are structural changes taking place. Say like a precipitation phenomena in a precipitation hardenable alloy if very fine precipitates are nucleating and growing from the metrics from a super saturated matrix then this behavior may change.

Sometimes instead of decrease the strength, you may end up with increase in strength okay and like a strain aging if it is taking place so that means if it is done at a very low at a slow rate sometimes you now yield strength may again come back and other thing. So strain aging if it is also taking or maybe recrystallization taking place when the recrystallization taking place then okay ductility maybe you can have a very high amount of ductility especially at a lower temperature and other things compared to that.

So that means so all these metallurgical phenomena if it is taking place then okay there will be deviation from this case, this is the general trend sometimes, you know you may find that the phase transformation is taking place, then also you will find that okay it may not be following this trend but in the same domain where the structural changes are not taking place where it is a constant structure is obtained and there is no micro structural changes and other phenomena like recrystallization and other things are not taking place.

Then you will find that the strength it decreases and ductility of the material increases with increase in the temperature. So you can say that most of this thermally affected phenomena at specially when you got to higher temperature. The thermally activated phenomena that assist the deformation and reduces the strength at higher temperature. So that also comes into picture. One thing is that with increase in temperature okay it is softening is taking place another is the thermally assisted phenomena takes place okay.

So for so you have something called as the activation energy for the plastic deformation so you will find that based on that the yield strength or the flow stress can be represented in this from C2 to which is a constant into exponential – Q / RT say that is at a constant stress and strain rate. So you will find that the temperature dependent on the flow stress is there so this Q you can obtain by plotting from this different values you know you plotting the log sigma versus 1 / T, sometime people say it is 1000 / T also.

So in one whatever it be it is at the log sigma versus 1 / T data if you just plotted it will almost be a straight line and then you take the slope of the curve that will give you the value of Q / T. So from that depending upon the temperature you can calculate the activation energy for plastic flow for the material.

$$\sigma = C_2 \cdot e^{Q/RT} \big| \varepsilon, \dot{\varepsilon}$$

$$Slope = \frac{Q}{T}$$