Plastic Working of Metallic Materials
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Lecture 03
Analysis of Rolling (contd…)

So we will continue with the last class where we did not finish the complete derivation.

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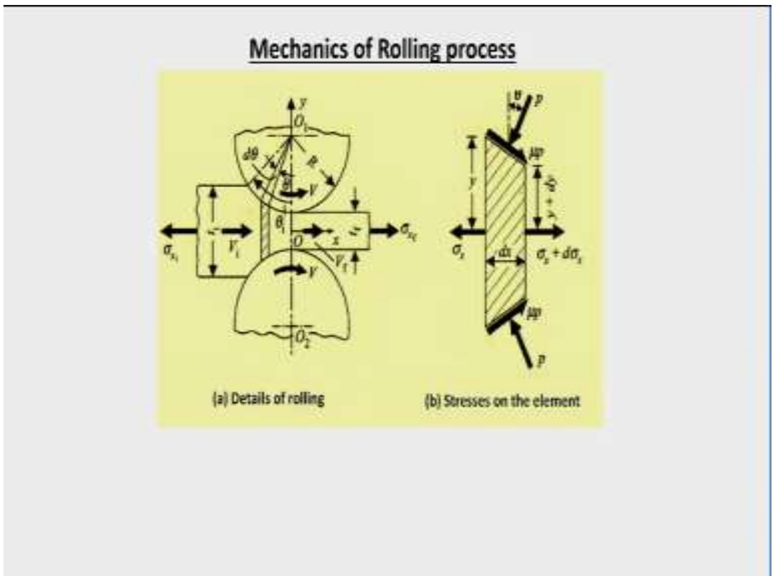

So we came across say like we were discussing with the trying to find out the pressure acting on the rolls. Okay, during the rolling operation. So here we reached these 2 equation C - and C + that equation 8 and 9.

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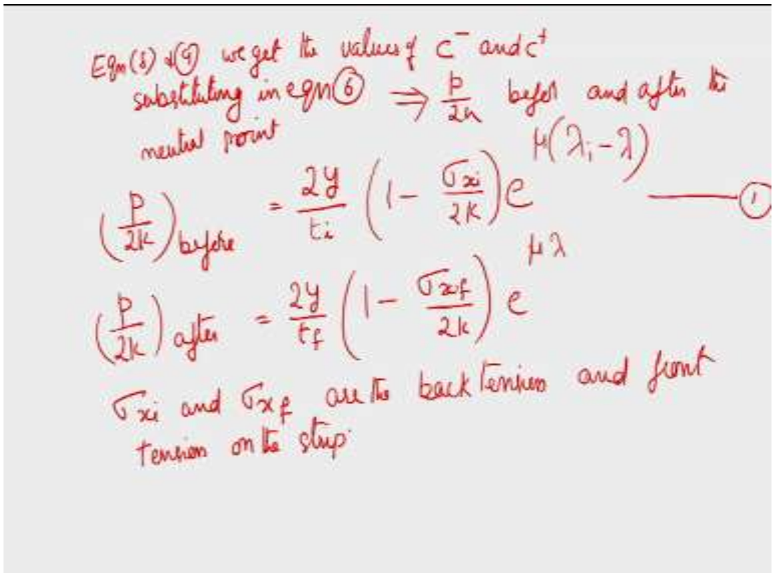

And 9 we get the values of C - and C +. One is before the neutral point and the other is after the neutral point. Okay. So if you substitute this substituting these values in equation 6 we will get a 2 equation for the dimensionless roll pressure. That is P by 2k P by 2k before and after the neutral point. So that we can get it as P by 2 k before the neutral point is equal to 2y and 2y is the width of the strip into 1 minus Sigma xi by 2k.

Where k is the shear stress xi Sigma xi is the back tension, which is being applied on the Strip as per our error diagram itself e raised to or exponential mu into Lambda i minus Lambda. So this is equation number 10 and we can say P by 2k. After the neutral point is equal to 2y by t before is t i. This is t f into 1 minus Sigma x f by 2k into exponential mu into Lambda. So Sigma xi and sigma xf are the back tension and the front tension on the strip.

$$\left(\frac{P}{2K}\right)_{before} = \frac{2y}{t_i}\left(1-\frac{\sigma_{xi}}{2K}\right)e^{\mu(\lambda_i-\lambda)}$$

$$\left(\frac{P}{2K}\right)_{after} = \frac{2y}{t_f}\left(1-\frac{\sigma_{xf}}{2K}\right)e^{\mu\lambda}$$

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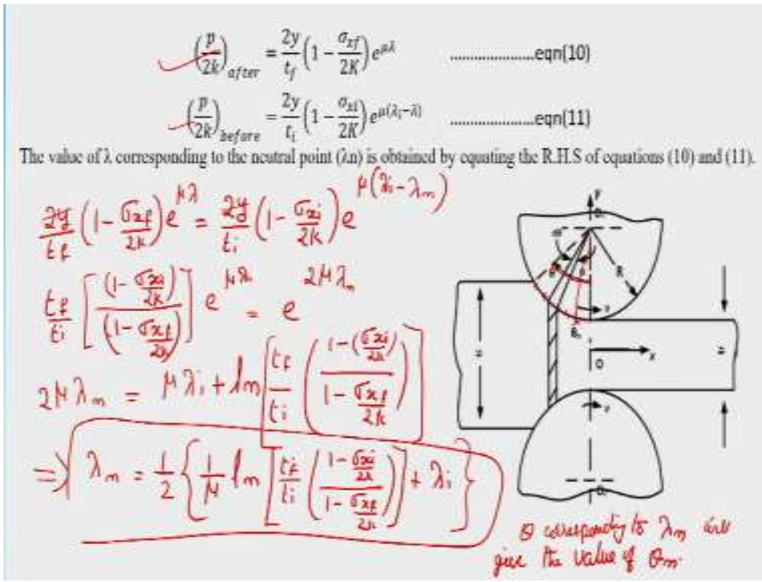

So once you get this, so that means what it represents is that at any point on this, so these are the 2 equations which are getting, so you have to take your θ from this direction. Okay, that means you are taking from the exit side. So $\theta_1, \theta_2, \theta_3, \theta_4$ like that we can just take it and correspondingly you find out the value of λ_i okay. So this is before means from here to here which we are doing it giving different value of theta and giving it in the equation for θ. Now, you will get it to that earlier.

We have written about the equation of λ and λ if you give the value of theta i for corresponding to any theta value you can get it at that particular point. So if you just take it and suppose this is the neutral point suppose. This is the neutral point, which you are getting it. We can always find out from here at any point for each value of θ. What is the pressure, which is acting the dimensionless value of P by 2k we can find out? So we can write it as P 1 here P 2 here and the P 3 here like that.

We can go now the thing is that because of the change in the frictional force. You will find that occur. There is a variation from the so you will find the friction hill now you wanted to find out the value of theta n so that means that will be a common for both this cases. If you substitute this value corresponding to that θ which is equal to theta and so you have to substitute for λ n and in the λ equation there is a term which is equal to θ.

So, which is a function of θ so for each value of θ you can find out the λ and then calculate these values? So in that case if you wanted at the neutral point the dimensionless value, whether you use it by this equation, which is after the neutral point or whether you use by this equation before the neutral point because these are common, so you will find that that is a peak maximum value which you will be getting so that value corresponding to this θ n you can substitute by getting the λ n.

Once you get the λ you substitute that back calculate it and find the value of θ so that will be corresponding to theta n. So that how to get it. This theta n is buy equating equation 10 and 11. Okay. So that means if you write it as P by 2k after and P by 2k before if you equate it so you can write the 2y by t f into 1 - Sigma xf by 2k into e raised to mu lambda is equal to 2y by t i into 1 - Sigma xi by 2k into e raised to mu lambda i minus lambda. Okay.

Once you are right that you just do manipulation then 2y and 2y will get cancelled and then you can write it as in terms of t f by t i into 1 minus Sigma xi by 2k, where k is the shear modulus Shear real strength uni axial Shear real strength by 1 minus Sigma xf by 2k into e raised to mu lambda i is equal to 2 mu lambda. So this lambda should be equal to your lambda n for the since we are going to find out this lambda and that is corresponding to your theta n.

$$\frac{2y}{t_f}\left(1-\frac{\sigma_{xf}}{2K}\right)e^{\mu\lambda} = \frac{2y}{t_i}\left(1-\frac{\sigma_{xi}}{2K}\right)e^{\mu(\lambda_i-\lambda_n)}$$

So that is for neutral point if you are doing it so you can say that 2 mu Lambda n is equal to mu lambda i + log t f by t i into 1 - Sigma xi by 2k / - Sigma x f by 2k. So from that we can get it as say maybe this you divided by lambda on this side know so you can write it as lambda n is equal to 1 by 2 into 1 by mu t i by t f by t i into 1 - Sigma xi by 2k / 1 - Sigma x f by 2k plus lambda i this. So once you get this value of lambda n that is corresponding to your theta n here. Okay so corresponding to this theta n.

$$\frac{t_f}{t_i}\left[\frac{\left(1-\frac{\sigma_{xi}}{2K}\right)}{\left(1-\frac{\sigma_{xf}}{2K}\right)}\right]e^{\mu\lambda_1} = e^{2\mu\lambda_n}$$

$$2\mu\lambda_n = \mu\lambda_i + \ln\left[\frac{t_f}{t_i}\left(\frac{\left(1-\left(\frac{\sigma_{xi}}{2K}\right)\right)}{\left(1-\left(\frac{\sigma_{xf}}{2K}\right)\right)}\right)\right]$$

So now in that the earlier had that relationship for theta n lambda in terms of theta which was given by which equation to us so because it was in the previous slide, let me just see that so lambda is equal to if you write like this lambda is equal to say 2 root R/t f into tan inverse. Okay. So, something you are getting like that. Okay. So, the equation for the lambda that relationship if you substitute and then corresponding to that will be the theta value Okay? So the theta corresponding to lambda n will give the value of θ n.

$$\lambda_n = \frac{1}{2}\left\{\frac{1}{\mu}\ln\left[\frac{t_f}{t_i}\left(\frac{1-\frac{\sigma_{xi}}{2K}}{1-\frac{\sigma_{xf}}{2K}}\right)\right] + \lambda_i\right\}$$

So, that way you can find out the thing, but only thing you have to remember is that always know we are taking this θ in the anti-clockwise direction from the center field Center Line of the center the center line of the roll. So, from here you are taking this θ so always in our calculation θ should be from this way from the center to the center line and it was the anti-clockwise direction. You have to take it. Okay.

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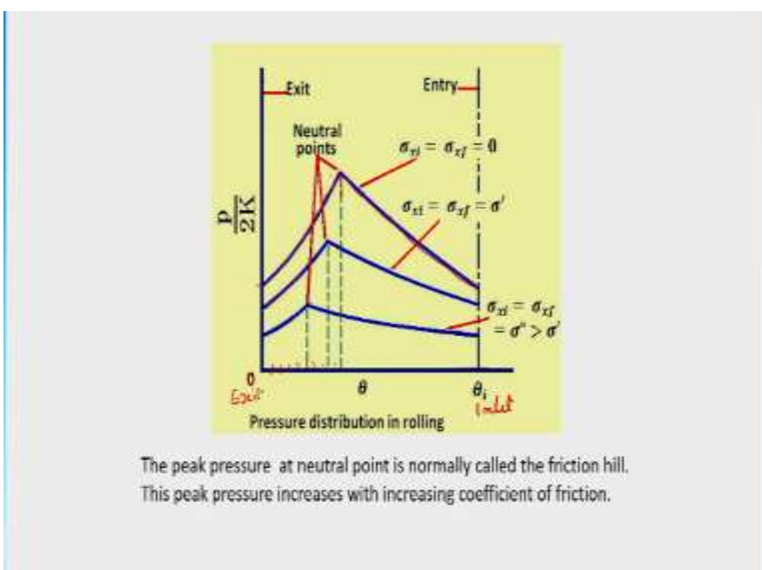

So, now once we have reached let us see what is this is the P by 2k value corresponding to θ value so when you say that this θ is equal to 0 is the exit side. Okay. This is the exit side and we are going to the when in our analysis part you are going to take it from the exit towards the inlet side actually. Okay. So when θ is equal to 0 you have some value as per our relationship we are getting.

We are so for θ is equal to any particular value we can find out and then it comes out and you will find that with theta increasing the non dimensional pressure on the roll pressure keeps on increasing reaches a maximum at the neutral point. And then with the further towards the exit side when it come it will keep on decreasing like this continuously and it will at the exit the inlet side. This is the inlet side.

So, inlet side that is why theta is equals to θ inlet i, we are getting okay. So this is the angle of bulk total angle of bulk. So this from our previous equation, we were able to find out what is the value θ. Now you can play with this there is a back tension there is a front tension. Now if θ xi which is the inlet that is the back tension is equal to θ x f that is the front tension or if those are 0 then you get a value here, but you apply some value which is equal to your θ dash Sigma dash which is equal to your k value.

You are getting corresponding to k value know you are getting the θ dash that is 2 by root 3 into your uni axial yield strength. Okay, so that is what you are getting it so that if you give it you will find it is such a lower value. Okay, now if θ x i equal to θ f and it is still a larger know then you will find that the so when you are having a front tension and a back tension, you will find that you are non dimensional or the roll pressure keeps on decreasing.

This is one thing now if they are equal now at the if you are just having the front tension higher, then you will find that this is Shifting θ value is Shifting. So if you are increasing the front tension. And if you are decreasing the back tension, then you will find that the neutral point keeps on changing towards that side where your forces are there. Okay, so if you are having a front tension, then you will find that the neutral point is increasing.

Okay towards the shifting towards the right side. So this is what you will find so you can always apply the front tension and back tension and then see what is happening in the other case where both are applied. It is like a stretching and rolling operation. Okay, so that is why the roll pressure keeps on decreasing in that case. But if that is not there only a front tension is the neutral point will shift towards the exit side.

So, if you are having front tension, it will keep on shifting like this towards this side exit side. And if the back tension is higher, it will keep on shifting towards the inlet side. So, that is the and you are going to find that from inlet to the neutral point you are roll velocity is higher than that of the inlet. So you will find that there the frictional force are trying to the your frictional forces will be acting opposite to the roll direction.

Whereas after neutral point Know your roll velocity, the surface velocity of the roll is lower than that of the Billet. So, you will find that the frictional forces are aiding the deformation process. So, that way by adjusting this front tension and back tension. You can always reduce the total force requirement. Your roll separating force can be reduced you are torque on the roll can be reduced. So, all these things you will be able to do it by adjusting this.

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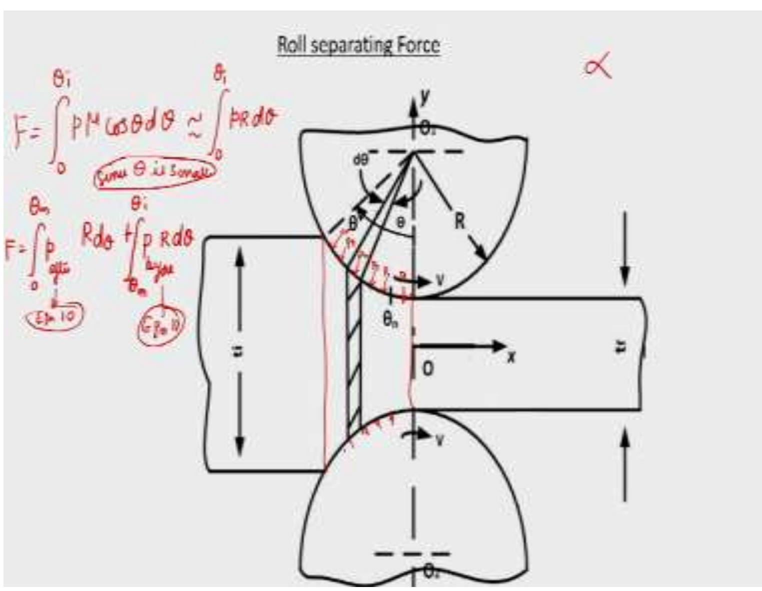

Now, let us see that we have to calculate the roll separating force so if you look at that, we know the roll pressure it is acting normal to that here. So, this is P 1 P 2 P 3 if I just put it from your so if I say P 1 P 2 P 3 P 4, which is continuously changing P 4 P 5 like that. So, certain things you have to look at it. If you look at that at the inlet the maximum width of the strip easier and as the rolling takes place when it moves to this place already some deformation has taken place.

The material has undergone work hardening though in our assumption which we assumed that okay. This uniform yield strength is there but here in real case there will be a work hardening. So when work hardening is taking place your section thickness decreases. So that is why we made a reasonably good approximation that k into y where y is equal to this is y k into y remains constant. Okay. This is t_i so at any point. This is the y part in the y remains a constant.

So, that is a reasonable approximation. But now you have the normal pressures different at different places the total so, this if you take the vertical component and the horizontal component. The vertical component of this pressure p at any point, it will try to separate the roles. So, that is and these pressures are going to be very high. So, you just trying to separate the roll because you will have on here also. Okay, the normal pressures are there all these things are the so it the vertical component causes the role separation.

So you have to find out what is Roll separating force. Otherwise know when you are going to you are fixing certain things the material through the your structure should have that rigidity so that there is you are going to get a very accurate thickness. So, that is very important so, we had to calculate the Roll separating force for this case. So how to go for it the total Force trying to separate the roll can be obtained by integrating the vertical component of this.

So the total force we have to find out by integrating from θ is equal to 0 to θ is equal to your angle of bite. So, θ I we can say it. Okay, or normally it is better to use that angle of bite as alpha. That is the best thing. Otherwise, here you may get confused so across this contact length across the deformations on this is the deformation zone, which is coming. Okay. So, in that case, you just integrate this total force.

And if you assume the width the script is width is a unity and θ_i being very small because this contact angle of bite is always very less. It may be of the order of 2 degree 3 degree like that only so it is not very large. Okay, so unless it is very high reduction is there. Okay, so and you are having a smaller diameter rolls in that case is a different. Otherwise, you know for normal case we can assume that θ_i is very small. So when θ_i is very small the frictional force at the inter phase it is very small so that you can neglect it.

So, you consider only the vertical component of the force acting on that. So, the total Roll separating force f is equal to integral from 0 to theta i PR cos theta d theta since, theta is very small so that we can say that it is almost equal to 0 to integral from 0 2 theta i PR d theta since, theta is equal to small is small. So, that means now the thing is that there are two cases one is here one is here.

$$F = \int_0^{\theta_i} P\mu cos\theta d\theta \approx \int_0^{\theta_i} PRd\theta$$

So, you have different equation for that before the neutral point and after the neutral point. So, you have to write that f is equal to f is equal to integral 0 to theta i P after Rd theta plus integral from integral 0 to theta n from here to here, this is after. Okay, so that is plus integral theta n to theta i P before meant before the neutral point Rd theta where this expression for pay after and P before you have to take it from the previous relationship.

$$F = \int_0^{\theta_n} P_{after}\, Rd\theta + \int_{\theta_n}^{\theta_i} P_{before}\, Rd\theta$$

Which we have expressed it to hear the equation 10 and 11. Okay. So, that means you that is this is P after is equation. What was that after this equation 10? And this is equation 11. So that these two you have to use it. And then from that you have to normally what happens is that because this angle is very less. So you have to do at this one you can even use this your Simpsons rule or trapezoidal Rule and then integrated numerical methods that you the easiest to thing for that.

Otherwise, you have to plot the variation of the P by 2 k or P. Because multiplied by 2k will give you the value of P. So, the variation of P you can get it in a tabular form and then fit it into an equation and then okay, that is all for this and then for both the cases for whether P after or P before and then you have to sum up these two and then get the value of roll separating Force. Okay. This is a that is how it is generally being done.

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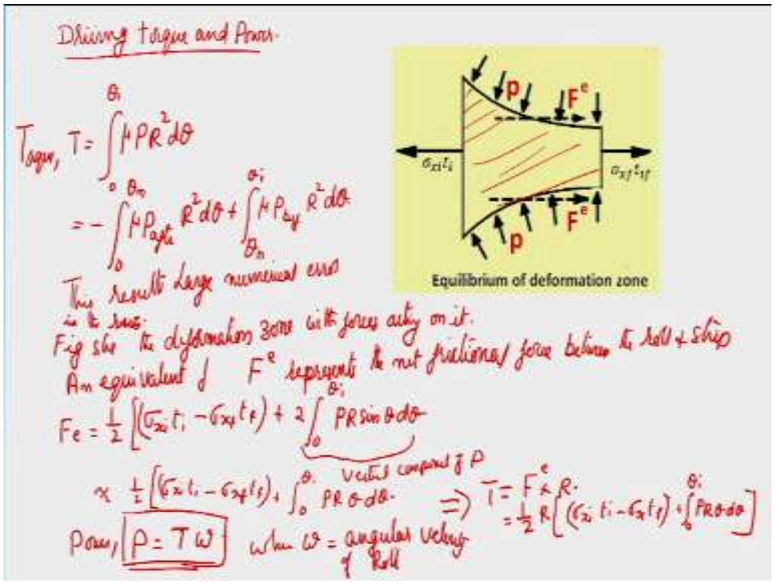

And now next is more important is the driving torque which is required so, driving torque. So you need to know whether sufficient torque is there. Otherwise, the mill will start the basically the driving torque is required to overcome the torque exerted on the roll. Due to the interfacial friction force so, it is not that roll separating Force. You have the interfacial friction force. And then so the torque exerted on the roll has to be determined so driving torque.

We can find out this by the horizontal component of the thing. Okay and corresponding to unit width of the spring the torque T is equal to in this equation integral from 0 to theta i mu P R Square d theta, so that is equal to minus 0 to theta n mu P after R Square d theta plus integral from 0 to theta i theta n integral from theta n to theta i mu P before R Square d theta this is how you can get it.

$$Torque, T = \int_0^{\theta_i} \mu P R^2 d\theta$$

$$= -\int_0^{\theta_n} \mu P_{after}\, R^2 d\theta + \int_{\theta_n}^{\theta_i} \mu P_{before}\, R^2 d\theta$$

But actually the this is written in such a way that the because what happens is that after the neutral point you will find that okay, the direction of the frictional force is opposing it. So that will be reducing it whereas, before the neutral point at the exit side that will be increasing the torque requirement. So that is why the negative sign is coming here. Now the thing is that this equation is limited due to a numerical error since the result is difference between 2 nearly equal large numbers.

So, this is very inconvenient, this results in large numerical errors in the result. So, another way we can do is that you consider the horizontal equilibrium of the deformation zone if you took it take this is your deformation zone, which is like if you just consider the equilibrium the horizontal equilibrium of the deformation zone of the strip. So like so here now, this is the figure which has been given for the deformation zone where you have different pressures and the other thing and then equal force is acting equivalent force F^e is acting in this direction. Okay.

So, the reaction of F^e will be F^e which is in the opposite direction that has to be overcome by your driving torque. So F^e can be determined so, here F^e. So, here now if you consider the figure shows the deformation zone with the forces acting on it. So, the and the equivalent force F^e represents the net frictional force between the roll and strip Okay, so this we can find out.

The quantity of the value of F^e can be determined by 1 by 2 into your sigma x i into t i - sigma x f into t f plus on the two sides you are getting 0 to theta i P R sin theta d theta Okay, so this is the vertical component. So, this part is the vertical component of P. Okay, so this we can write it as approximately equal to 1/2 into your sigma x i t i -sigma x f t f + integral 0 to Theta i P R theta d theta. So, total power is equal to.

$$F^e = \frac{1}{2}[(\sigma_{xi}t_i - \sigma_{xf}t_f) + 2\int_0^{\theta_i} PRsin\theta d\theta$$

$$\approx \frac{1}{2}[(\sigma_{xi}t_i - \sigma_{xf}t_f) + \int_0^{\theta_i} PR\theta d\theta$$

So that means torque is equal to F^e into R where R is the roll diameter. So, that you can always find it out from that that is equal to that is equal to your half into R into so sigma x i t i - sigma x f t f + integral 0 to theta i P R theta d theta and then power P. The total power which requirement

P is equal to T into omega where omega is the angular velocity of the rolls Okay, so where omega is equal to angular velocity of rolls of roll. See this is how we can get it

$$T = F^e \times R$$

$$= \frac{1}{2} R[(\sigma_{xi} t_i - \sigma_{xf} t_f) + \int_0^{\theta_i} PR\theta d\theta$$

$$Power, P = T\omega$$

Now after this one thing we have to look into that. What is the there is a in some case you do not have to have a back tension or a front tension or you do not have to push it because if the frictional force is not high then it may be difficult to roll. The metal may not be able to go into that. So, you may have to apply a large pressure from the back side a tension you have to not tension force.

You have to apply from the inlet side. Otherwise, it will not be drawn into the dye into the rolls in the roll Gap. So, for the metal to be a roll in where it has to be drawn into the roll gap minimum amount of friction is required at the roll metal interface. So, that is very important. Otherwise, you will have to have extra energy extra power has to be required to be pushed for pushing the material into that the normal case the roll should automatically draw the material into the roll Gap and then cause the deformation.

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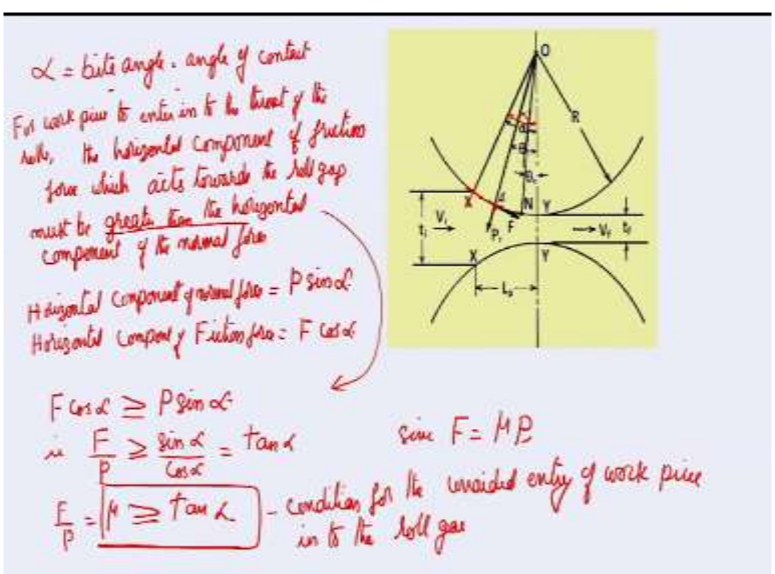

So, like if you just take that the angle between so, here that is what I telling in the bite angle bite angle means the angle subtended at the roll at the point of contact. So, this is the bite angle alpha. Okay, this is the alpha is bite angle that is equal to angle of contact between inlet and outlet. That is angle of contact that is bite angle if you say so, the centre line the entrance plain and this one these 2 these are the bite angle so, work piece enter into the throat of the roll. The horizontal component of frictional for work piece to be work piece enters into throat of the rolls.

The horizontal component of friction of frictional force which acts towards roll gap must be greater than the horizontal component of the normal force. If you just consider these part at any point. This is the normal pressure P acting and at this point there is going to the frictional force also acting if the take the horizontal component of this frictional force and this P normal component that is what so, for the metal to enter into the throat that means this point at this point.

What is required is that the force horizontal components of this frictional force and this should be matching. That means horizontal components frictional force should be greater than that is greater than horizontal components of the normal force. For that if you look at the horizontal component of the normal force that is equal to P or sin theta normal force. That is equal to we can P sin alpha is equal to P sin alpha and the horizontal component of frictional force is equal to F cos alpha.

$$Horizontal\ Component\ of\ normal\ force = Psin\alpha$$
$$Horizontal\ Component\ of\ Frictional\ force = Fcos\alpha$$

So, by these know what we required is that F cos alpha should be greater than or equal to P sin alpha or that is F by P is equal to sin alpha by cos alpha is equal to tan alpha. So, this should be F by P greater than or equal to tan alpha or F by P is equal to so, if you say that since F is equal to mu P F by P equal to mu. So, that should be greater than equal to your tan alpha. This is the condition for the unaided entry of work piece into the roll roll gap.

$$Fcos\alpha \geq Psin\alpha$$
$$\frac{F}{P} \geq \frac{sin\alpha}{cos\alpha} = \tan\alpha$$
$$since\ F = \mu P, \quad \frac{F}{P} = \mu \geq \tan\alpha$$

So, at least the minimum co-efficient of friction between the work piece and the roll should be greater than these tan alpha where, alpha is bite angel this is the necessary for is the friction force is less than that. Then the material will not drawn inside that unless you give an external force. So, that industry that will be very difficult you always wanted the material to be drawn that so need that minimum amount of frictional force otherwise you have to have external forces you have to provide. Thank you.