Plastic Working of Metallic Materials
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Lecture - 21
Strain Rate in the Deformation Zone

Today, we will be discussing about the strain rate during rolling. Generally, see at room temperature any plastic deformation; it is not having much effect on the strain rate. Only very marginal effect only will be there, which can be peacefully neglected but when you go to higher temperature, the material you will see that there is strain rate sensitivity okay. So, it is highly dependent upon the strain rate at which you are deforming.

And when the material is passing through this roll through the roll gap or through the deformation zone, when it is undergoing the deformation, you will find that there is, the strain rate continuously changes and it may reach to a very high value also. So, this point we have to consider because the flow stress of the workpiece, it is dependent upon its temperature, strain and strain rate.

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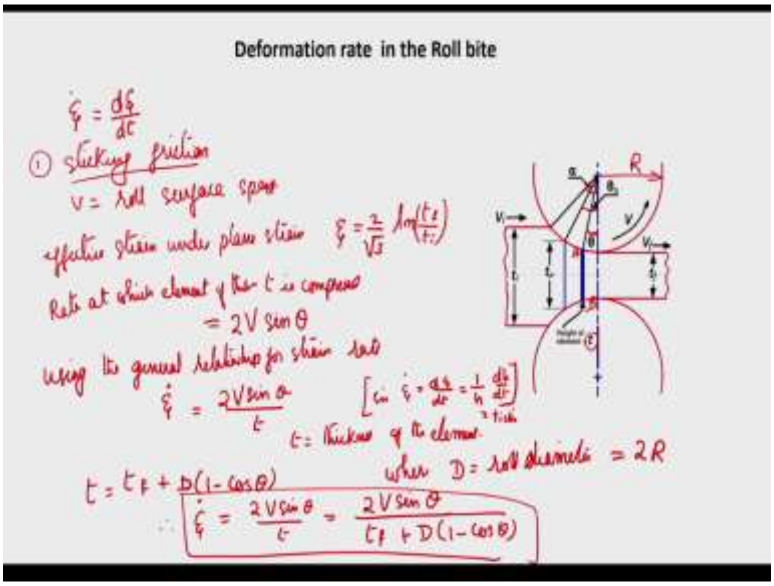

And the flow stress since it depends upon the rate at which deformation takes place; we can say that this strain rate is nothing but the d epsilon/dt. So, how this is going to affect or how this is going to change or affect, say based on this we have to find out when the material is moving from this inlet to the rolls to the outlet of the roll, at each region the strain rate changes and for all practical purpose strain rate is better for calculation purpose okay for simplicity and other thing.

$$\dot{\epsilon} = \frac{d\epsilon}{dt}$$

But still let us come out at arriving at some expression for the strain rate at each and every point during the rolling process. So, we can approach this considering one is the sticking friction, let us assume the first is the sticking friction, assuming sticking friction and we know that the surface velocity of the roll V, V is the surface speed and if you are considering that from a thickness of t_i it is being rolled to a thickness of t_f using roll, using rolls of say let us say diameter or R okay or radius R okay.

So, we can say that the effective strain in plane strain, so that is effective strain under plane strain condition, so that we can say that that is equal to say 2/root 3 into log tf/ti okay. So, that is the strain which you are going to get it, the effective strain. Now, suppose you consider a height of h, suppose this is height h of height t an element during the rolling process, the rate at which this element, this element is compressed.

$$effective\ strain\ under\ plane\ strain\ \ \epsilon = \frac{2}{\sqrt{3}} \ln\left(\frac{t_f}{t_i}\right)$$

Let us say this is A and B, the rate at which element of thickness t is compressed that is given by 2V sin theta because you take the vertical component of your velocity, velocity component and then so that is what we are taking and since from the both the sides it is there, so we will find that it is 2V sin theta and the rate at which element is deformed okay. So, using the general relationship for strain rate, we can say that strain rate is equal to 2V sin theta/t.

$$Rate\ at\ which\ element\ of\ thickness\ t\ is\ compressed = 2Vsin\ \theta$$

So, that is since say if it is this one d epsilon/dt that is equal to 1/h into say dh/dt. So, in that case where h is the thickness. So, we can write like this where t is the thickness or t is the thickness of the element and from the geometry now we can always get that t is equal to from the exit side, from the exit side if you just do that, we can get which we have used it earlier tf + d into 1 - cos theta okay.

$$Using\ the\ general\ relationship\ for\ strain\ rate\ \ \dot{\epsilon} = \frac{2Vsin\theta}{t}$$

$$since, \dot{\epsilon} = \frac{d\epsilon}{dt} = \frac{1}{h}\frac{dh}{dt}$$

$$t = t_f + D(1 - cos\theta)$$

So, where D is the roll diameter and that is equal to 2R, 2 radius where r is the radius. So, therefore we can write that epsilon dot is equal to 2V sin theta/t is equal to 2V sin theta/tf where tf is the thickness of the strip at the outlet plus d into 1 minus cos theta. So, this is the relationship for your strain rate. Now, let us for easy calculation because at every point we are finding out the strain rate and then doing an analysis a very cumbersome job.

$$\dot{\epsilon} = \frac{2Vsin\theta}{t} = \frac{2Vsin\theta}{t_f + D(1 - cos\theta)}$$

So, rather what we will do is that so and you see that this strain rate is now dependent upon what is this theta because here this is the theta where the element makes with the center to center distance between the rolls, their end joining the center to center distance between the roll and you should also say that this alpha is the angle of bite okay.

And maybe at some point know you will find that the neutral point is there corresponding to this thickness is tn where the angle subtended at the center between the exit and the neutral point is theta n. So, these are our general normal terms which we are using it. So, let us now find out the mean strain rate okay.

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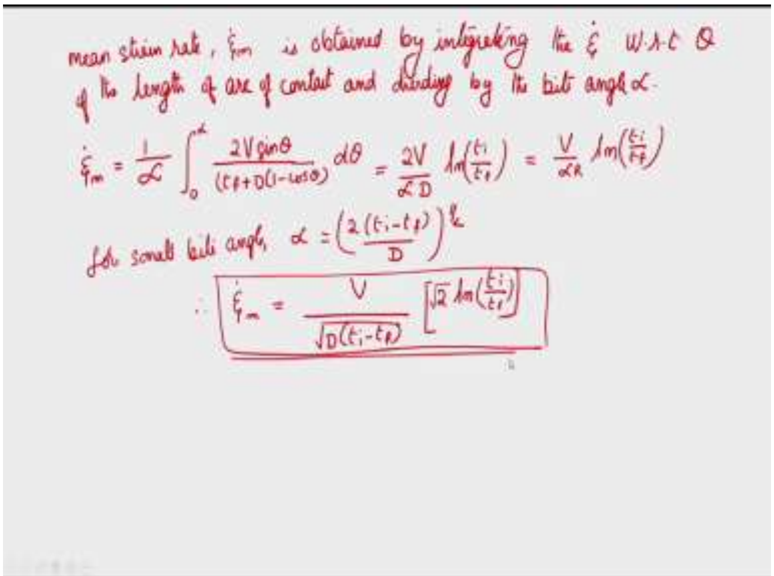

The mean strain rate or rate of deformation, so that is mean strain rate because you will find that at the entry the strain rate is high compared to at the exit okay. So, there is a variation in the strain rate and in that case know that is why which is very simple to calculate based on the mean strain rate. So, that is $\dot{\epsilon}_m$ and this is obtained by integrating the strain rate is obtained by integrating the strain rate with respect to theta over the length of arc of contact and dividing by the bite angle alpha.

See this is how you are getting, therefore now we can write that epsilon dot m is equal to dividing 1/alpha into 0 to alpha 2V sin theta/tf + D into 1 - cos theta into d theta. So, that if you do that integration and putting these limits from 0 to n applying that, we can arrive at this relationship that is 2V/alpha D into log ti/tf, so that way we can get it and that is equal to V/alpha log ti/tf.

$$\dot{\epsilon}_m = \frac{1}{\alpha}\int_0^{\alpha} \frac{2V sin\theta}{t_f + D(1 - cos\theta)} d\theta = \frac{2V}{\alpha D} \ln\left(\frac{t_i}{t_f}\right) = \frac{V}{\alpha R} \ln\left(\frac{t_i}{t_f}\right)$$

Say for small bite angle, if you just say this is the larger case for a small bite angle, small bite angle will come and this radius of the roll is large okay, so for small bite angle we can write alpha = 2 into ti – tf/D raised to 1/2 or ti – tf/R, so that also we can write. So, therefore now we can if you substitute that now we can get this relationship epsilon dot m is equal to say V/root of D into ti – tf into 2 log ti/tf.

$$for\ small\ bite\ angle, \alpha = \left(\frac{2(t_i - t_f)}{D}\right)^{\frac{1}{2}}$$

$$\dot{\epsilon}_m = \frac{V}{\sqrt{D(t_i - t_f)}} [\sqrt{2} \ln (\frac{t_i}{t_f})$$

So, this relationship we can get it okay and do some, so this is the relationship we are getting. So, mean strain rate by this relationship we can get the mean strain rate if you are assuming the sticking friction. Now, let us come to the case. This sticking friction is much more prone at a higher temperature okay, especially for hot rolling. This is applicable for hot rolling but when the temperature is very high; it will be a sticking friction.

But if it is a slightly lower temperature is there and if your lower surfaces are very smooth and other things, then you may end up with a sliding friction also okay.

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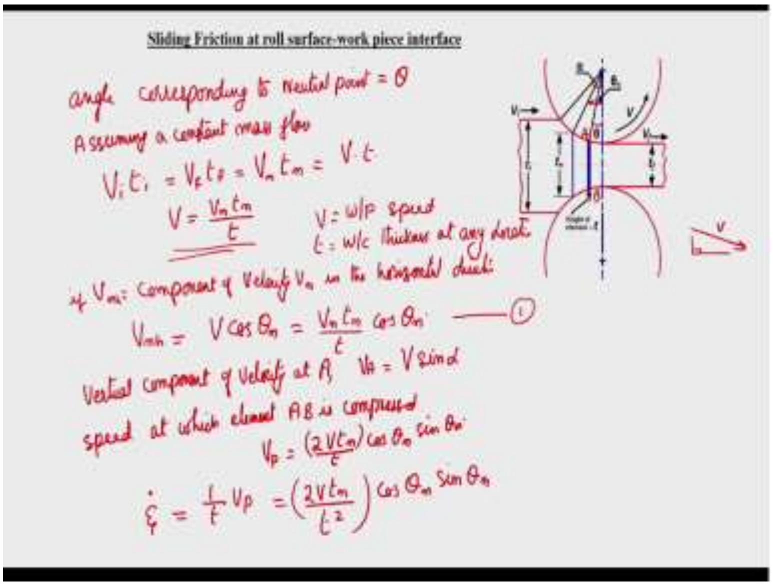

So, let us consider the case for sliding friction. So, strain rate during rolling considering sliding friction okay here itself. So, in this let us just consider the neutral point. So, the angle corresponding to neutral point is equal to θ. That means the theta is taken from the center to center, the line joining the center to center between the roll, so that we are taking, that is what this θ n okay.

The speciality of the neutral point as we have discussed earlier is that the speed of the workpiece and the surface speed of the roll they are saying at the this one and if you assume that constant flow, constant volume flow, mass flow, assuming a constant mass flow that means from the entry what was that the volume was the material which was entering at any section know it remains the same, the rate of mass flow is same.

So, that means we can say at the inlet know the velocity is Vi into ti so that is equal to Vf the velocity at the exit finish okay into tf that is equal to the velocity at the neutral point which is equal to your surface velocity of the roll in the tn that is equal to V into t at any point okay. So, that is what we are going to get it but at this point what is this, at this section what is that thickness, so at any for the element.

$$V_i t_i = V_f t_f = V_n t_n = V.t$$

$$V = \frac{V_n t_n}{t}$$

So, therefore now from this constant mass flow rate assuming, we can write say V of the element is equal to Vn into tn divided by your thickness t the instantaneous value of the thickness t and where V is the, remember we are not considering this as the surface speed, we are considering the velocity of the workpiece okay, mass flow rate V is the workpiece velocity or workpiece speed we can say, workpiece speed.

Suffix and other things corresponding to say i, f and then is corresponding to your inlet or out exit or neutral point or at any point here so that is what and t is the workpiece thickness at any location okay. So, at the neutral point, you will find that the component of velocity Vn in the horizontal direction okay, in the horizontal direction it is related to the velocity by the relationship say the component of velocity if Vn is the component of velocity Vn.

Vn corresponds to the velocity at the neutral point okay in the horizontal direction. So, that means Vnh = V cos theta n and you know that V is this so that is equal to Vn tn/t into cos theta n. Let us say that this is equation number 1. So, if the velocity component at A, this is the point A, let us say A and B. The vertical component of the velocity, the vertical component of velocity at A, VA = V sin alpha.

$$V_{nh} = V cos\theta_n = \frac{V_n t_n}{t} cos\theta_n$$

$$Vertical\ component\ of\ velocity\ at\ A, V_A = V sin\alpha$$

Because when you are taking this know, this velocity component is like this, so this is V, so you just take vertical or horizontal component, so that will be V sin alpha. The speed at which this element AB is compressed, so that is what we have to find out. Now, what is the speed at which element AB is compressed, the speed at which element AB is compressed. So, that we can say that Vp = 2V tn/t into cos theta n sin theta n, so that is what we are going to get it okay.

$$V_p = \left(\frac{2Vt_n}{t}\right) cos\theta_n sin\theta_n$$

So, this path now, this relationship when you are going to differentiate it that is what we are getting. So, therefore strain rate is equal to your strain rate epsilon dot = 1/t into Vp, so that is equal to 2V tn/t square into cos theta n sin theta n.

$$\dot{\epsilon} = \frac{1}{t} V_p = \left(\frac{2Vt_n}{t^2}\right) cos\theta_n sin\theta_n$$

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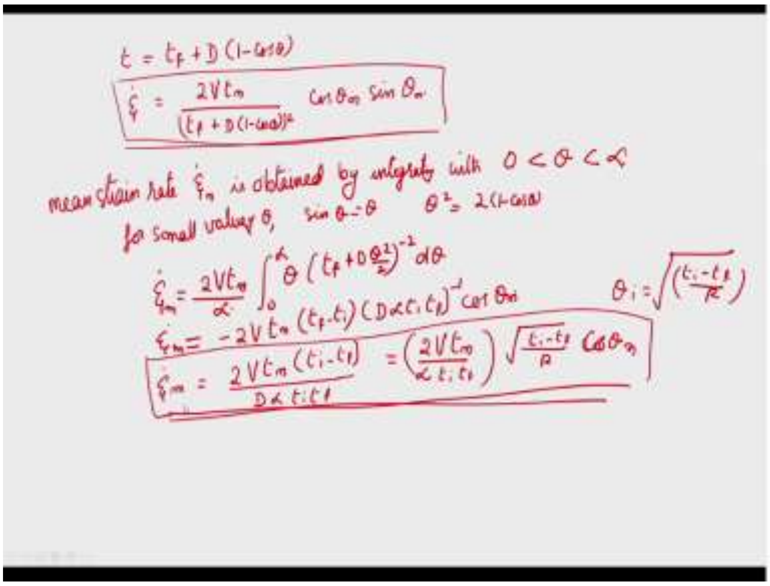

And from that relationship know if you look at it since t earlier we wrote it as t is equal to with respect to your exit side tf + D into 1 - cos theta. When you substitute in that, that value of t, this epsilon dot = 2V tn/tf + D into 1 - cos theta the whole square into cos theta n sin theta n. This is by considering the sliding friction okay. So, in this condition, the mean strain rate we can obtain by integrating within the limits say theta between 0 and theta n.

$$t = t_f + D(1 - cos\theta)$$

$$\dot{\epsilon} = \frac{2Vt_n}{\left(t_f + D(1 - cos\theta)\right)^2} cos\theta_n sin\theta_n$$

Theta n means it is sorry this one not theta n within the limits 0 to your bite angle okay. So, mean strain rate okay so that and if you just find that for small values okay is obtained by integrating within say 0 to theta between alpha okay. So, for small values of θ, we can substitute sin theta = theta and theta square = 2 into 1 - cos theta. So, this we can just put it, so that if you substitute that, this we will get it as 2V tn/alpha into 0 to alpha.

And then divided by alpha u have to write into theta into tf plus D into theta square by 2 okay raise to minus 2 D theta. So, within this if you integrate, you will get this as say epsilon dot m mean flow rate is equal to minus 2V tn into tf minus ti, so that is what you will be getting into D alpha ti tf raise to minus 1 cos theta n and since theta i is equal to we can say that ti – tf/R, substituting that we can just get this equation.

$$\dot{\epsilon_m} = \frac{2Vt_n}{\alpha}\int_0^{\alpha} \theta \left(t_f + D\frac{\theta^2}{2}\right)^{-2} d\theta$$

$$\dot{\epsilon}_m = -2Vt_n\left(t_f - t_i\right)\left(D\alpha t_i t_f\right)^{-1} cos\theta_n$$

Epsilon dot m is equal to 2V tn into ti minus tf because this minus sign it will go by d alpha ti tf okay, from that we can just get this final value as 2V tn the thickness at the neutral point divided by your bite angle alpha into say ti and tf into root of ti minus tf by R which is your theta value theta i cos theta n. So, this is your mean flow strain. So, if you look at these two conditions for sticking friction and sliding friction, we can observe certain things.

$$\theta_i = \sqrt{\left(\frac{t_i - t_f}{R}\right)}$$

$$\dot{\epsilon_m} = \frac{2Vt_n\left(t_i - t_f\right)}{D\alpha t_i t_f} = \left(\frac{2Vt_n}{\alpha t_i t_f}\right)\sqrt{\frac{t_i - t_f}{R}} cos\theta_n$$

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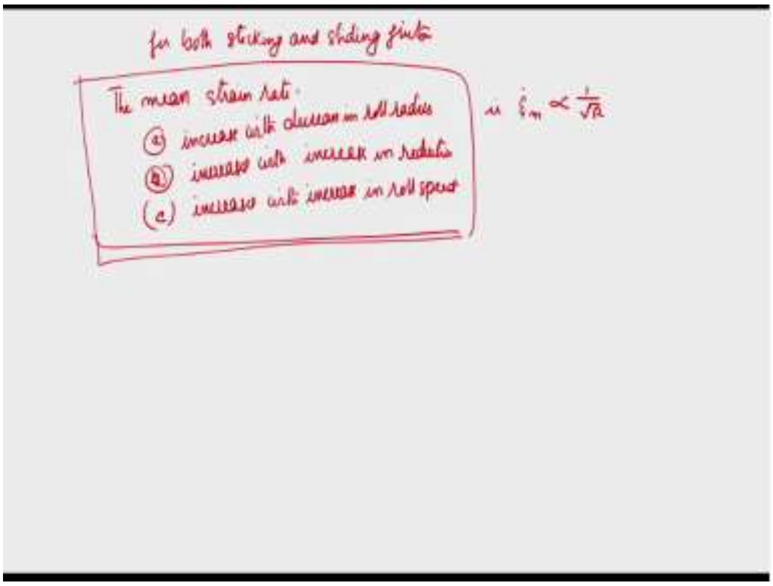

That is the mean strain rate for both sticking and sliding condition and sliding friction, the mean strain rate one, it increases with the decrease in roll radius. See for example say you will find that with roll radius know say it is square root of that is $\dot{\epsilon_m}$ is proportional to 1/root R we can say. $\dot{\epsilon_m}\,\alpha\,\frac{1}{\sqrt{R}}$ Second, it increases with increase in reduction. This is B and this is C that is it increases with the increase in roll speed.

That is what we are getting here. See, one is with the reduction increasing now strain rate is increasing with the increase in the reduction and with your bite angle, bite angle increasing what will happen is that this decreases okay and with the radius of the roll, it is increasing again it will decrease. So, we can write that with the bite angle has a direct relationship with this roll diameter okay.

So, these are the main findings from this. So, when you are considering the hot extrusion sorry hot rolling, you have to consider the effect of strain rate on deformation okay. So, now let us come to the next part, few things which are required to be mentioned okay. That is a simplified analysis of rolling load we can just because otherwise a rolling load becomes very cumbersome.

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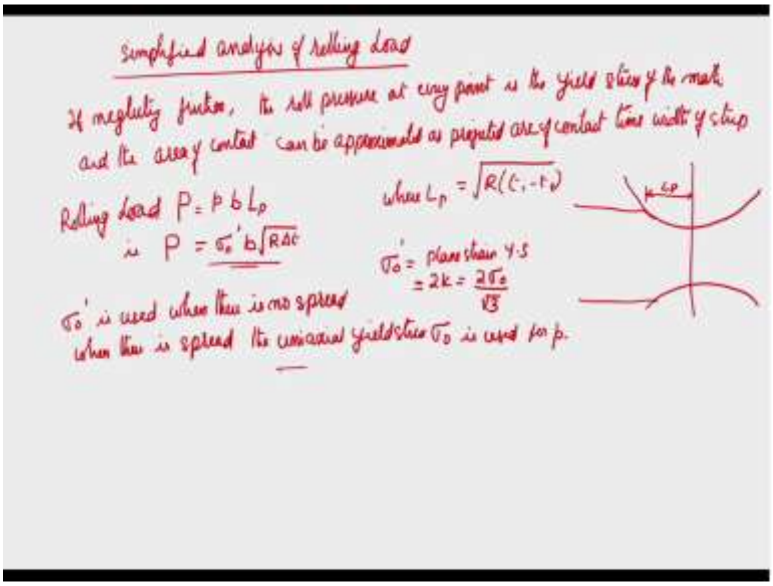

So, simplified analysis of rolling load, so that in a very quick way we can just look at it. So, basically the rolling load is the roll pressure times the area of the contact of the metal and the roll, the workpiece material and the roll. So, that is what it comes, so if you are able to know what is the roll pressure at each and every point that is what we have discussed earlier but if you just neglecting friction see mainly for thin sheets and other thing, it is always advisable to use smaller diameter roll okay.

In that case, the friction, total frictional force is not very high because their contact length is very less, so that way we can neglect it. So, if the friction is neglected, the pressure which is on the roll is basically the yield strength okay, yield stress of the material. So, if you neglect friction, the pressure roll pressure at every point is the yield stress of the material and the area of contact can be approximated as the projected arc of contact time width of strip.

See for example if this is your this one and this is the roll center and if this is the thing, for small radius know see this length and projected is this one Lp, so for larger radius know, you will find that Lp is almost equal to the arc of contact that is what the thing is that. So, in that case know, your roll pressure, roll load, rolling load P is equal to your pressure p into b into Lp where Lp can be approximated as say r into ti – tf.

$$Rolling\ Load\ P = P_b L_p \quad where\ L_p = \sqrt{R(t_i - t_f)}$$

$$P = \sigma_0' b \sqrt{R\Delta t}$$

We can just say where Lp = root of R into ti – tf that is what we can. So, this is equal to say your plane strain flow stress of the material into b into root, so we can say R into delta t that is what it is coming where the sigma 0 is the plane strain yield strength, which is equal to 2K is equal to 2 sigma 0/root 3 or sigma 0 is the uniaxial tensile yield strength of the material. So, this is a very quick way of finding out this one.

$$\sigma_0' = 2K = \frac{2\sigma_0}{\sqrt{3}}$$

And when you are using this sigma 0, you have to be very careful. σ_0' used when there is no spread. So, when there is spread, the uniaxial yield stress sigma 0 is used for p. So, this is a very simple way of finding out that okay. Maybe there will be a slight error and other things that is okay and then this is not considering much of the effect of friction also. This is basically the rolling load for the deformation purpose okay.

So, assuming that friction is neglected especially when you are using a smaller diameter roll and other things that is what is there. Now, let us come to your defects in the rolling part okay.

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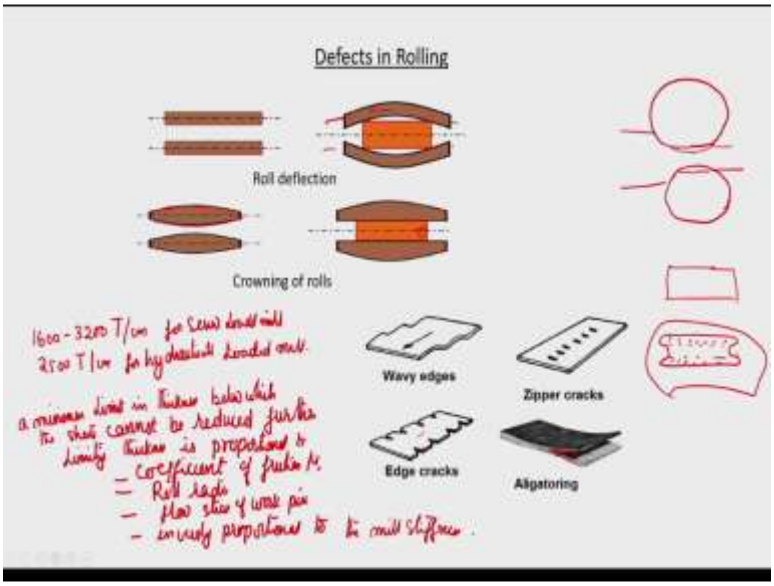

Defects can occur during the rolling as a result of the interaction of the plastically deforming workpiece with the elastically deforming rolls. This is one thing, so you can have this roll is, there is going to be elastic deformation depending upon specially for higher load know this is going to happen okay and then your material is plastically deforming, it also will get work-hardened all those things are going to happen.

So, what happens is that due to this non-uniformity will take place. Let us see what all this. See when there is a very high rolling force okay, then roll flattening will take place. So, instead of that here, if this is the in fact it should have been like this, but then you will find that okay this is what is happening. So, instead of being here, the roll gets flattened because of the elastic deformation.

Due to the pressure, the roll pressure, it gets elastically deformed and then what will happen is that your thickness is not maintained. So, whatever you wanted is that the thickness will always be more than that okay and that is one thing. Second thing is that because of the very high force you know, this roll will bend. See this is a roll, this is a roll, this is a roll and bending of the rolls also will take place especially when it is a longer roll.

So, due to this, this is basically due to the stiffness of the mill okay or the roll. The thickness of the exiting sheet will be greater than the roll gap which you have done at no load condition because before the starting you will be checking at the roll gap and other thing under no load condition and then only you are fixing. So, in that case when you do it because of the higher roll pressure, which is coming one is that it may slightly differ and there will be roll flattening which is taking place.

And at the same time know, there will be say bending also will take place. If bending is taking place, the thickness of the sheet will be not uniform so because you will find at the center the sheet thickness is higher than at the edge. So, there will be non-uniform thickness across the section, across the width. Due to roll flattening, the radius increases, apparent radius of the roll will be much higher.

And then okay in that case, the contact area will be larger, you will end up with large forces okay. So, you should know before fixing the roll gap, you should know a priori that what is going to be the deflection of the rolls. For that know, you should know that the stiffness of the mill. So, stiffness of the mill generally normal range is with 1600 to say 3200 tons per centimeter for screw loaded mill and maybe for 2500 roughly tons per centimeter for hydraulically rolling mill, loaded mill.

So, if you know that, then to obtain it, decide roll thickness you can do the adjustment. Now, there will also be elastic flattening of the roll, it will increase the roll pressure and it is a condition where roll eventually deform more easily than the workpiece. So, that is also elastic flattening will happen. Hence, there will be a minimum thickness below which the sheet cannot be reduced further.

So, it is not that okay you can reduce any amount and other thing, there is a minimum limit, a minimum limit in thickness below which the sheets cannot be reduced further and this limiting thickness is proportional to one your coefficient of friction, the roll radius, the flow stress of the workpiece and inversely proportional to the Young's modulus value okay, we can say the mill stiffness.

Now, based on this what are the normal defects which can occur? So, one is the waviness, the wavy edges are coming. See this wavy edges happens, see in this condition if you look know, edge is going to be strain the more. So, it is compressed more than at the center okay. So, you will find that the edge part there is an elongation compared to that at the centre of the sheet means along the width of the sheet mid width, mid width region compared to that the edge will have more strain.

So, that it will just deform like that and then what happens, from the center and the edge, it is not uniform, so width result in say a wavy appearance especially at the edge. So, that is not advisable okay. Then, another thing is that if this is going to be very large, so at the outside know material has been deformed much whereas at the center that much is not deformed. So, at the center portion, resultant force will be tensile in nature.

And if the deformation is very large, that will result in some defects, which is called as a zipper crack like a zip know it is continuously going on that, so this is that zipper cracks which are there. So, that can also form. So, in that case, the solution to avoid this is taking into the consideration of this one, you do that crowning or give a chamfer like this, you give a chamfer to roll, it is an exaggerated view which is shown, it will be only very small only.

You give a chamfer to the roll so that at the center though it is deforming and when you move towards the edge, you are going to have a uniform deformation. So, depending upon that we can do how much is this chamfering can be done so that ultimately what comes it will be a flat piece without any wavy structure okay and then there are also chances of cracking. Edge cracking can take place because as the workpiece it passes through the rolls.

All elements across the width experience some tendency to expand laterally. When it is passing through the rolls, so across the width there also it will have a tendency to expand laterally along the width direction okay, but then this is opposed by the transverse frictional stress and due to the frictional hill, if you look at the friction hill, which is high at the center compared to the edge, so you will find due to which what happened, the element at the center which spread at the center region.

If you take a small element at the center region, its spread will be much less than when you compare at the edge portion. At the edge, portion what happens is free to move, so that it can deform there, but whereas the center it is constrained for the elongation okay and then the thickness decrease at the center, it goes through a length increase and the thickness decrease because the thickness at the edge decreases much.

So, part of that goes to the spread and since there is a continuity between the edge and the center, you will find that it result in edge cracks okay. So, you have a strained tension at the surface here compared to at the center. So, here there is a longitudinal stresses are there, so tensile stresses are there at the edge compared to a centre. So, that may as after some time result in say edge cracking.

So, that is that, so center split also can take place. This also can take place and this edge cracking can also be caused by the inhomogeneous deformation in the thickness direction okay. When the rolling conditions are such that say suppose you have a thick space and then okay your strain is not very high, so what will happen is that earlier the surface only strains are there, reduction is not very high.

Then, what happens at the edge, the piece was like this and after some deformation, you will find that only at this region only the straining has taken place, at this region. So, that part you will see that okay it is deformed larger. So, you are getting this and but with the subsequent rolling passes what will happen is that this will also end up with splitting okay. The material is not compressed directly but is forced to elongate by the neighboring material close to this center okay.

So, this will lead to high secondary tensile stresses leading to the again this edge cracking, but if you are giving too high reduction, then barreling also may take place and the limit will come in the surface at the strain at the surface has reached a level where it is going to develop crack okay. So, that also can happen. So, then there is should be an optimum reduction which is possible based on your properties of the material and the working environment.

Now, another is this alligatoring part. So, this is also when this part goes to a very high extent, so this part when it goes to a very high extent, the stresses are again going to come okay so and this can happen due to curling of the sheet also say because if there is going to be a small if the material is not exactly at the center, if there is a curling of the thing, that can happen when either the surface velocity of the roll, if it is not uniform, then this can happen.

Because after sometime know, you will find that some displacement, the strains are very high, so it will just split in this way and you call it as alligatoring, say it is like an alligator opening its mouth, that type of a shape it is coming. So, basically it is splitting at the center, so that is what it resulting. So, these are some of the defects which are taking place during the rolling operation if you are not careful enough.

So, you may have to give some crowning of the rolls or the chamber has to be provided so that uniform width or thickness of the sheet is obtained from center to the edge spot and similarly you have to avoid these wavy edges, the zipper cracks can be avoided, all these things by proper use of the die and making sure that your inlet material is also uniform thickness and their all speeds are exactly the same. If there is a variation, it is going to split the sample, which is going to come out okay.