

Plastic Working of Metallic Materials
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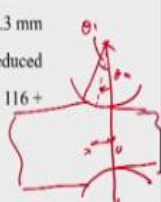
Lecture – 23
Sample Problem in Rolling

So, today we will come to this last part of this module that is a rolling, I will just today; today's lecture will be based on a problem which will be trying to do, so that all those attending this course will have a feel for this how to carry out rolling analysis or how to do a problem in this with related to rolling.

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Problem.1:
 High purity aluminum strip of 1500 mm width is cold rolled from 4 mm to 3.3 mm thickness using 500 mm diameter rolls. The material was previously been reduced to a plane strain of 0.6. The flow curve is represented by the relationship $\sigma = 116 + 39.6 \epsilon$. If $\mu = 0.06$ and the roll speed is 3 rad/sec.

(1) determine the roll separating force and total power required for rolling.
 (2) State whether the work piece can be drawn in to the rolls unaided?



$w = 1500 \text{ mm}$, $t_i = 4 \text{ mm}$, $t_f = 3.3 \text{ mm}$, $R = \frac{500}{2} = 250 \text{ mm}$
 $\epsilon_0 = 0.6$, $\sigma = 116 + 39.6 \times (\epsilon)$, $\mu = 0.06$, $\omega = 3 \text{ rad/sec}$
 \rightarrow bite angle, $\theta_i = \sqrt{\frac{t_i - t_f}{R}} = \sqrt{\frac{0.7}{250}} = 0.052915 \text{ rad}$
 $\theta_m = \sqrt{\frac{t_f}{R}} \tan \left[\frac{\mu}{2} \sqrt{\frac{t_i}{R}} \right]$

So, the question is like this; high purity aluminium strip of 15,000 mm width is cold rolled from 4 mm to 3.3 mm thickness using 500 mm diameter rolls. The material was previously been reduced to a plane strain of 0.6, the flow stress is represented by the relationship $\sigma = 116 + 39.6 \epsilon$ that is the strain; true strain. If $\mu = 0.06$ and the roll speed is 3 radians per second, determined the roll separating force and total power required for the rolling.

This is what we have to the question; second is state whether the work piece can be drawn into the rolls unaided, so these are the two things which we will have to solve but the major thing will be it takes a little bit of time in solving the first part that they are all separating force and total power required for rolling in this case, we wanted to find out the roll pressure from the inlet to the exit side along the roll diameter.

So, what are the normal pressure which is acting on it and then you have to integrate that so, one is from the inlet to the neutral point and the other is neutral point to the exit point, we have 2 equations which we have arrived at and so now considering that now we have to find out these 2 different phase; one is the frictional force is directed towards the exit side whereas, in the second case since the velocity is higher than the velocity of the roll, the frictional force is directed towards the from the exit towards the neutral point.

So, this change in the direction is there in the frictional forces there, so you get 2 equations; one from the exit to the neutral point, one equation for the non-dimensional pressure and the second case is from the neutral point to the exit, so these 2 you will have to do. So, let us now just try to solve this problem, so the data which are given is the width $w = 15,000$ mm so, t_i ; the inlet thickness $t_i = 4$ mm, the outlet thickness after rolling $t_f = 3.3$ mm.

The roll radius = $500/2 = 250$ mm, the material was initially reduced to a plane strain, so you can say $\epsilon_0 = 0.6$ and the flow curve is given by $116 + 39.6 \ln \epsilon$ that is the strain, the frictional force coefficient of friction is $\mu = 0.06$ and they roll speed $v = 3$, not b ; ω , 3 radians per second, so these are the data which has been given. Now, let us find out with various factors okay.

So, when you come to that we have 2 equations okay, first; the first thing which we have to do is that find out the neutral point that is very important, so one is so, we have to find out the bite angle, so that is from the inlet to the exit, bite angle is the contact angle, the angle subtended at the arc of contact between the billet and the work piece, so that bite angle $\theta_i = \sqrt{t_i - t_f}$ divided by R which we have discussed in our previous in the lecture class itself.

So, we can say that that is equal to $0.7/250$ under root, so that comes to 0.052915, see this I have just done it this way we can say its radian, bite angle we have to write it now, we have to find out the neutral point, so neutral point is the relationship is θ_n , the angle of the neutral point this is θ_n is from the exit to the inlet, so that is what so, if you just consider this is the roll, this is our reference point okay, between the centre to centre between the rolls.

$$\text{bite angle, } \theta_i = \sqrt{\frac{t_i - t_f}{R}} = \sqrt{\frac{0.7}{250}} = 0.052915 \text{ rad}$$

And if this is the; so this is θ_i so, this is the theta m, so this is our θ , okay so we have to take into the opposite direction, okay that is what we were just doing okay, so in this case this is the θ_m , so this neutral point you have to find out, so that is given by; that also we have written the t_f/R into $\tan \lambda_m$ divided by 2 into t_f/R , this is the value of theta m, so in this we had to find out what is lambda m, for finding out the lambda, we have to find out what is lambda i.

$$\theta_n = \sqrt{\frac{t_f}{R}} \tan \left[\frac{\lambda_m}{2} \sqrt{\frac{t_f}{R}} \right]$$

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$\lambda_i = 2 \sqrt{\frac{R}{t_f}} \tan^{-1} \left(\sqrt{\frac{R}{t_f}} \theta_i \right) = 2 \sqrt{\frac{250}{3.3}} \tan^{-1} \left(\sqrt{\frac{250}{3.3}} \times 0.52915 \right) = 7.5133$
 $\lambda_m = \frac{1}{2} \left[\frac{1}{H} \lambda_m \left(\frac{t_f}{t_i} \left(\frac{1 - \frac{v_{2i}}{v_{1i}}}{1 - \frac{v_{2i}}{v_{1i}}} \right) + \lambda_i \right) \right]$ $v_{2f} = v_{2i} = 0$
 $\Rightarrow \lambda_m = \frac{1}{2} \left[\frac{1}{H} \lambda_m \left(\frac{t_f}{t_i} \right) + \lambda_i \right] = \frac{1}{2} \left[0.06 \lambda_m \left(\frac{3.3}{4} \right) + 7.5133 \right] = 2.153548$
 $\theta_m = \sqrt{\frac{t_f}{R}} \tan \left[\frac{\lambda_m}{2} \sqrt{\frac{t_f}{R}} \right] = 0.014286 \text{ rad}$
 $\theta_i = 0.05295, \theta_m = 0.014286$
 $P_{\text{exit}} = 2K \left(\frac{t_f}{t_i} \right) e^{M \lambda} \quad M(\lambda_i - \lambda)$
 $P_{\text{inlet}} = 2K \left(\frac{t_f}{t_i} \right) e^{-M \lambda}$
 $\Delta \theta_{\text{byp}} = \frac{\theta_i - \theta_m}{8} = \frac{0.05295 - 0.014286}{8} = 0.004629 \text{ rad}$
 $\Delta \theta_{\text{inlet}} = \frac{\theta_m}{8} = \frac{0.014286}{8} = 0.0017857 \text{ rad}$

So, the thing is that; the next thing is that lambda i that is with respect to your theta i we can find out that is lambda i = 2 into root of R/ t_f into tan inverse root of R/ t_f into theta i, this is a value of lambda i which we will be getting that = 2 into root of 250 divided by t_f is 3.3 into tan inverse root of 250/3.3 into 0.52915, so that value we will be getting as 7.5133, so that is the value of lambda i we are getting.

$$\lambda_i = 2 \sqrt{\frac{R}{t_f}} \tan^{-1} \left(\sqrt{\frac{R}{t_f}} \theta_i \right) = 2 \sqrt{\frac{250}{3.3}} \tan^{-1} \left(\sqrt{\frac{250}{3.3}} \times 0.52915 \right) = 7.5133$$

Now, once you get the lambda i then, we can find out the value of lambda m because these are all we have come across and discussed about that what is lambda i, lambda m, m is that lambda corresponding to a neutral point, lambda i is the value of lambda corresponding to your inlet point, these are all with respect to the references from the exit side, okay, centre to centre, between the roll, so that is the line, okay that as I have shown in this drawing.

So that is equal to lambda m is equal to we can say 1/2 into 1/Mu, this is the equation we have written into a log t_f/t_i into $1 - \sigma_{xf}/2k$ divided by $1 - \sigma_{xi}/2k + \lambda_i$, in this equation since this σ_{xf} is the front tension and σ_{xi} is the back tension since in the problem it is not mentioned, we will assume that there is no front tension or back tension.

So that means this is $\sigma_{xf} = \sigma_{xi} = 0$ that is what we can assume that okay, in that case this will come to say 1/2 into or 1/ Mu into $\log t_f/t_i + \lambda_i$, so that is equal to 1/2 into 1/; your value of Mu was; $\mu = 0.06 \log 3.3/4 + \lambda_i = 7.5133$, so lambda m we are getting it as a 2.153548, this is the value of lambda n we are getting, once you get lambda n you can calculate here the position of the neutral point $\theta_n = \text{root of } t_f/R \text{ into } \tan \lambda_n / 2 \text{ into root of } t_f/R$.

$$\lambda_m = \frac{1}{2} \left[\frac{1}{\mu} \ln \left[\frac{t_f}{t_i} \left(\frac{1 - \frac{\sigma_{xf}}{2k}}{1 - \frac{\sigma_{xi}}{2k}} \right) + \lambda_i \right] \right] \quad \sigma_{xf} = \sigma_{xi} = 0$$

$$\lambda_m = \frac{1}{2} \left[\frac{1}{\mu} \ln \left(\frac{t_f}{t_i} \right) + \lambda_i \right] = \frac{1}{2} \left[\frac{1}{0.06} \ln \left(\frac{3.3}{4} \right) + 7.5133 \right] = 2.153548$$

So that comes to 0.014286 radians, so 2 things are there; $\theta_i = 0.05295$ and $\theta_n = 0.014286$, so this much we have got it, okay. So because now we wanted to do that see, we have to find out the total roll pressure if you wanted to find out, we have to integrate it but this equation for the two; the expression for this roll pressure is very complicated, so direct integration will be very, very cumbersome.

$$\theta_n = \sqrt{\frac{t_f}{R}} \tan \left[\frac{\lambda_m}{2} \sqrt{\frac{t_f}{R}} \right] = 0.014286 \text{ rad}$$

$$\theta_i = 0.05295, \quad \theta_n = 0.014286$$

So, we can just for by identifying the pressure at each and every point at equal interval from the inlet to the outlet and then by Simpson rule we can just integrate it, numerical integration we can do it, okay so that is the easiest rating, so it will be like this, if the roll is like this, this is your inlet and this is your exit maybe I am just drawing it here, so this is it.

So, this is your θ_n , so maybe at we can divide it this will maybe 8 sections and then at each every point now we can write 1, 2, 3, what are the roll pressure at this point you have to do and find the integration that is what, so that one is before, so the $P/2k$ before we have written as; so

we have got this bite angle as well as the position of the neutral point at which the velocity of the speed and the velocity of the billet are same, okay.

Now, from the inlet to the neutral point, we have to divide it into equal parts, so that now we can integrate the pressure equation from inlet to the neutral point as well as from the neutral point to the exit side by maybe the numerical integration we can do may be by Simpson rule, so it will be like this. If this is your roll and this is the inlet and this is the outlet, so this is your theta i and maybe this is your theta n.

So, we have to find out the pressure at these points and once you get that point, then it becomes very easy, so at the normal pressure you just take the component; the vertical component of that and integrate it that is what, so we have these 2 equation; P after P/2k the non-dimensional thing which you have arrived at it, so we can write that it is equal to t/tf into e raised to Mu lambda, this is for after that.

$$P_{after} = 2k \left(\frac{t}{t_f} \right) e^{\mu\lambda}$$

And P before = 2k into t/ti into e raised to Mu into lambda i - lambda, the value of lambda here is corresponding to that whatever this theta value is there, corresponding to that is the lambda values, in this equation we have to substitute corresponding to each theta value, so then that is how we will be calculating. So, now these are the 2 equations which we have to get it.

$$P_{after} = 2k \left(\frac{t}{t_i} \right) e^{\mu(\lambda_i - \lambda)}$$

One is from this is the neutral point, I will say neutral point, from the neutral point to the exit to the first equation and from the inlet that is a before the neutral point, so inlet to the neutral point is the second equation you are using, so one is A and this is B, so when you are using this we will do it by numerical integration, so let us divide it into equal parts. So, one is from theta i to neutral point we will divide it into 8 equal parts.

So that means, one is the delta theta; equivalent roll you take before the region before the neutral point is equal to theta i - theta n divided by 8, so that is = 0.05295 – 0.014286 divided by 8, we are getting it as 0.004829 radians similarly, the equal angle because these are not same

okay, so this is after the neutral point also, let us divide into; so that means from theta n to 0, this is the 0 value, okay.

$$\Delta\theta_{before} = \frac{\theta_i - \theta_n}{8} = \frac{0.05295 - 0.014286}{8} = 0.004829 \text{ rad}$$

So that is theta n/ 8, so that is equal to theta n = 0.014286/8, so we will get it as 0.0017857 radians, so here we will divide it into this value and at this region we will be divided in the angle to this, calculate and determine the value of theta for each case theta; sorry, lambda for each case and then we can find it out, okay. So that once you do that, we will just do all this thing in the form of a table.

$$\Delta\theta_{after} = \frac{\theta_n}{8} = \frac{0.014286}{8} = 0.0017857 \text{ rad}$$

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	1	2	3	4	5	6	7	8	9	10	11	
	$\theta(\text{rad})$	$t(\text{mm})$	t/R	$\sin^2(\theta/2)$	$\cos^2(\theta/2)$	k	$\cos(\theta/2)$	t/R	$P_n (N)$	$\cos(\theta/2)$	t/R	$P_n (N)$
ENTRY	0.0529	4	1	0.6	139.76	7.51	1.57	1.21	265.89	1	1	139.76
1	0.0481	3.678	0.9485	0.63	140.79	6.9			1.04	0.92	141.8	
2	0.0433	3.768	0.942	0.6598	142.13	6.29			1.08	0.94	144.3	
3	0.0384	3.669	0.9176	0.6843	143.48	5.62			1.12	0.96	147.14	
4	0.0336	3.582	0.8955	0.7103	144.13	4.95			1.17	0.9	150.6	
5	0.0288	3.502	0.8766	0.7385	144.99	4.29			1.21	0.88	154.4	
6	0.0239	3.443	0.8608	0.7699	145.69	3.58			1.27	0.86	158.8	
7	0.0191	3.391	0.8478	0.7859	146.3	2.87			1.32	0.85	163.9	
NP	0.0143	3.351	0.8378	0.772	146.77	2.15	1.11	1.02	169.6	1.38	0.84	169.6
1	0.0125	3.339	0.83425	0.7806	146.91	1.69	1.12	1.01	166.5			
2	0.0107	3.329	0.8328	0.7839	147.43	1.62	1.1	1.008	163.4			
3	0.0089	3.32	0.83	0.7864	147.14	1.35	1.08	1.006	160.5			
4	0.0071	3.313	0.82825	0.7885	147.23	1.08	1.07	1.003	157.7			
5	0.0054	3.307	0.82675	0.7902	147.29	0.81	1.05	1.002	154.9			
6	0.0036	3.303	0.82535	0.7914	147.34	0.54	1.03	1.0009	152.3			
7	0.0018	3.301	0.82505	0.7921	147.37	0.27	1.02	1.0003	149.8			
EXIT	0	3.3	0.825	0.7921	147.38	0	1	1	147.38			

$s = 0.4 + 1.6(i)$
 $F = 116 + 39.6(i)$
 $= 2.16$ mm.
 Column no
 $P_{yk} = (4) \times (10) \times (9)$
 $= P_c$
 $P_{yk} = (4) \times (6) \times (9)$
 Column no

See, I have just drawn this point, so in this may be from entry now, we have to start with this, so from exit if you are looking at it that is what, so your entry point will be 0.0529, okay and then that we will be adding up this 0.004528; 0.004829, interval we have to do it, so if you just keep on subtracting like this, find out the theta value for this corresponding to this, the T; thickness value for this.

So, we can just keep on writing that one is 4, then corresponding to this theta if you use that what is the thickness we can get it corresponding to that, see because any value you can get it corresponding to any theta you can just t_i , so from this say $t = R \theta^2$ by t_i , okay, so from this we can get it as at the value, so if you do that so, I will just write down these values

here so that it will be very simple; 3.878, I will suggest you yourself work it out, then 3.768, 3.669, 3.582, 3.507, 3.443, 3.391.

$$t = \frac{R\theta^2}{t_i}$$

This is the neutral point okay, neutral point as we look at the friction hill know, whether you use the equation for $P/2k$ before or $P/2k$ after that should be the same value because neutral point is common to both that and so here that is why I have highlighted that point 351, this is 3.339, 3.329, 3.32 because here after the neutral point you will find that the delta theta is very small, so the difference is very small in this okay.

Then 3.313, 3.313, 3.07, 3.303, 3.301 and 3.3 we are getting now, we can take out this, this is the t, t_i is 4, so that is this is equal to $\frac{t_i}{4}$, okay, so that you will get it as 0.9695. why I am taking this table is that it will be very convenient to find out this, do the problem in that and this is 0.942, 0.91725, 0.8955, 0.8955, 0.8766, 0.8608, 0.848, 0.8478, neutral point is 0.8378.

And then we will get 834, 0.83475, 0.83225, 0.83, 0.82825, 0.82675, 0.82575, 0.82525 and 0.825, so t/t_i we have got it. Now, the next problem comes is that strain now, strain how we will calculate, see at the entry we already have this 0.6, so in the question if you read it, it is mentioned that the material was previously been reduced to a plane strain of 0.06 because your the width is 15,000 mm and the thickness is this much okay, this problem is of a plane strain condition which I should have mentioned earlier.

So, the problem is of plane strain condition, so whatever we have derived was for a plane strain condition only, so this is satisfying that and but the material before rolling maybe they might have obtained it from the market and in that case know, it was given a deformation which is equivalent to a plane strain of 0.6, so at the compare to a; what do you call it as a fully annealed material.

So, our equation is there, this equation for $\sigma = 0.116 + 39.6 \epsilon$, this is the true strain so, already you have a true strain of 0.6, so you will have to always add to that, so your this 3; column 3, so strain = $0.6 + \ln\left(\frac{t_i}{t_f}\right)$ you have to take it, so that way when you consider it, so here at entry point it is already 0.6, so now you have to add that this $\ln\frac{t_i}{t_f}$ if you keep on adding, you will

get it as 0.631, 0.6598, 0.6863, 0.7103, 0.7315, 0.7499, 0.7651, 0.777, 0.7806, 0.7837, 0.7864, 0.7885, 0.7902, 0.7914, 0.7921 and 0.7924, so that values you have already got it, okay.

So, now the question is that you have to find out this what do you call it as the stress, okay the stress is given by this relationship $\sigma = 116 + 39.6 \text{ times } \epsilon$ in mega Pascal, okay so that is what the relationship we are having, so you have this ϵ is there, so this stress we can calculate by this stress, so that will come to 140, so here we are getting 139.76, 140.99, 142.13, 143.18, 144.13, 144.97, 145.69, 146.3, 146.77.

You will see that friction hill, it is increasing from 139 to 146 and now you can go down okay, so that is 146.91, this is the flow stress value, so that will continuously increase from entry to exit okay, so that means 147.03, 147.23, 147.29, 147.34, 147.37, 147.38, so this is the flow stress variation due to the strain which is there, okay so this much we have got it. Now, if you look at this entry to the neutral point is before the neutral point.

So, if you look at that we have to take this equation before 2k into 2k is say here, $2k = \text{your } \sigma$, what do you call it as a uniaxial flow stress sigma okay, so that is what; so that is what the 2k we are getting here, so this is equal to 2k is equal to you have to write it like this, this is equal to 2k because of the work hardening behaviour. So, then you are looking at this from entry to the neutral point that is P before this we have to take it.

So, it contains is your σ ; this is equal to your sigma and t/t_i and e raised to μ into λ into λ , so you have to find out the λ for each and every case okay, so that where λ if you calculate it as per the equation λ for each value of theta, for each increment of theta value corresponding to this theta value, you have to just find out the λ , substituting this λ is equal to whatever value is there, okay.

So that way we will get it as, so you have to substitute in this equation; this equation for θ value for each value of theta you substitute into this equation, so in that case what you will get it is this is 7.51, 6.9, 6.27, 5.62, 4.95, 4.27, 3.58, 2.87, and 2.15 at the neutral point, 1.89, 1.62, 1.35, 1.08, 0.81, 0.54, 0.27 and then here it will be $\lambda = 0$, because $\theta = 0$, then λ will become 0 in that equation.

So, once you do that for the region from entry inlet to the neutral point we have to take this value, so here in this equation λ_0 that is basically, it is λ theta we have to write what is that we used; λ_i , okay, this is lambda i, it should be $-\lambda$ that is as per our equation, so this will be 1 and here you will get 1.04, 1.08, 1.12, 1.17, 1.21, 1.27, 1.32 and 1.38, below that we do not need it.

Because in that case we need this equation only here; here only it will be required in the column number 6, so if we do not have to do, we can do that but it is of no use for us, okay maybe if you wanted to do that we can write it as 1.4, 1.42, 1.45, 1.47 it will be wasting our time only, so we do not have to do that okay. Now, the question is for the region from entry to the exit, so we have to use if you look at this P after, so P after we can write like here.

P after is equal to as per our equation $2k$ that is Sigma; sigma is in this one, column number 4 into t/t_f that is column number 1, sorry this is before, so 4 into 10 into; column number 4 into column number 10 into column number 9, this is that okay, these are column number; the values, so t/t_0 we have to note it down, so that will be one; 0.97, 0.94, 0.92, 0.9, 0.88, 0.86, 0.85, 0.84, below that may not be of important plus, so if you just substitute this here we will get this value.

So that is column number 11, the value will be you multiplied by this, so that is 141.8, 144.3, 147.14, 150.5, 154.4, 158.8, 163.9, 169.6 we are getting this, okay so, we got the roll pressure at an interval of say maybe this for each value of θ , we have got the roll pressure here, okay and the column number 11. Now, for the case for a neutral point to the exit, so that we have to calculate.

One is that the exponential μ lambda we have to calculate it, so that is if you do that; 1.11, 1.12, 1.1, 1.08, 1.07, 1.05, 1.03, 1.02, 1 we are getting this now, t/t_f we can get it because in our expression we have t/t_f , so that will be 1.02, 1.01, again 1.01, it will be 1.0; third digit we have to go that is what, so 1, 1, 1, 1, so there is a decimal place so, maybe to write it very accurately, I will just to tell that what is the value of t ; $t =$ say 3.329 divided by 3.3, t_f is; so at this point $t = 3.339$ divided by 3.3, 1.011; 3.329 divided by 3.3, 1.008, 3.32 divided by 3.3, 1.006.

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Simpson's Rule for numerical integration $\int_a^b f(x)dx = \frac{h}{3} [y_0 + y_n + 4(y_1 + y_3 + \dots) + 2(y_2 + y_4 + y_6 + \dots)]$

Roll separating force/unit strip thickness, $F = R \left[\int_0^{\theta_n} P_{after} d\theta + \int_{\theta_n}^{\theta_i} P_{before} d\theta \right]$

$\int_0^{\theta_n} P_{after} d\theta = \frac{0.001786}{3} [(147.38 + 169.60) + 4(149.82 + 154.97 + 160.52 + 166.47) + 2(152.35 + 157.7 + 163.44)] = 2.2569 \text{ N/mm}$

$\int_{\theta_n}^{\theta_i} P_{before} d\theta = \frac{0.004629}{3} [(169.59 + 139.74) + 4(163.88 + 154.39 + 147.74 + 144.81) + 2(158.83 + 150.51 + 144.26)] = 5.868 \text{ N/mm}$

$\therefore F = R [2.2569 + 5.868] = 250 \times 8.1249 = 2031.21 \text{ N/mm}$

Total roll separating force = $1500 \times 2.0312 = 3046.82 \text{ KN}$

Torque/unit width = $T = R^2 N \left[\int_0^{\theta_i} P_{before} d\theta - \int_0^{\theta_n} P_{after} d\theta \right] = (250)^2 \times 0.06 [5.868 - 2.2569] = 13496.25 \text{ N-mm per cm width}$

Total driving Torque per each roll = $T \times 1500 = 13496.25 \times 1500 = 20244 \text{ KN-mm} = 202.44 \text{ N-m}$

Total power required = Driving Torque $\times 2 \times \omega$
 $= 202.44 \times 2 \times \dots = 31214 \text{ W} = 31.214 \text{ kW}$

So that is numerical integrations, Simpsons rule; Simpsons rule for numerical integration, I will write the equation that is integral from a to b $f(x) dx = h/3$; h is the interval at which you are taking into $y_0 + y_n$ the last value and the first value + 4 into your odd values $y_1 + y_3 + y_5 +$ and + 2 into the even values $y_2 + y_4 + y_6 +$; so this is the Simpson rule, so in our expression the roll separating force per unit strip thickness F as per our equation which we have derived earlier is equal to R into integral from 0 to theta n P after into d theta + integral from theta n into theta i P before into d theta.

Simpson's Rule for numerical integration

$$\int_a^b f(x)dx = \frac{h}{3} [y_0 + y_n + 4(y_1 + y_3 + \dots) + 2(y_2 + y_4 + y_6 + \dots)]$$

$$\text{Roll separating force per unit strip thickness, } F = R \left[\int_0^{\theta} P_{after} d\theta + \int_{\theta_n}^{\theta_i} P_{before} d\theta \right]$$

See, this is what we have to do so, let us just take it separately, so integral from theta to; 0 to theta n P after d theta is equal to; so in that case know we have taken this our h is different say, this h/3 is you have to write it h is our interval of the theta which we have taken that is equal to 0.001786 divided by 3 into 147.38 + 169.60 which is the last and the first point + 4 into 149.82 + 154.97 + 160.52.

If you take it the value which we have obtained, 166.47 + 2 into 152.35 + 157.7 + 163.44, this value we are getting, so that is equal to say 2.2569 similarly, the before theta n integral from theta n into theta i, so theta n into theta i is that integral values we have to write it, so that is

equal to P before d theta, this after should be here; is equal to here, the interval was different, we took 004829 as the interval of this; equal interval from your neutral point to your inlet value, okay.

$$\int_0^{\theta_n} P_{after} d\theta = \frac{0.001786}{3} [(147.38 + 169.60) + 4(149.82 + 154.97 + 160.52 + 166.47) + 2(152.35 + 157.7 + 163.44)] = 2.2569 \text{ N/mm}$$

So, that is that we will get it as; by 3 into; if you take it first value is 169.59 + next is 139.76 + 4 into 163.88 + 154.39 + 147.14 + 141.81 + 2 into 158.83 + 150.51 + 144.26, so that we are getting it as 2.0312, sorry this is equal to 5.868, so this is kilo Newton per millimetre; per millimeter width okay. So, therefore we will get the roll separating force F or therefore F = R into 2.2569 + 5.868 that is equal to 250 into 8.1249.

$$\int_{\theta_n}^{\theta_i} P_{before} d\theta = \frac{0.004829}{3} [(169.59 + 139.76) + 4(163.88 + 154.39 + 147.14 + 141.81) + 2(158.83 + 150.51 + 144.26)] = 5.868 \text{ N/mm}$$

So that means we are getting as 2031.21 newton per millimeter sorry, this is not given this is Newton; per millimeter width, this millimetre; per millimetre; per w value, okay so, therefore total roll separating force is equal to; because 2 rolls are there and that is equal to your width, 1500 into your 2.031 in kilo newton, if you write it that is equal to 3046.82 kilo Newton is your total roll separating force.

$$F = R[2.2569 + 5.868] = 250 \times 8.1249 = 2031.21 \text{ N/mm}$$

$$\text{Total roll separating force} = 1500 \times 2.0312 = 304682 \text{ kN}$$

Now, you have to find out the torque, so torque per unit width that is equal to $T = R^2 \mu$ into this also we have written last slide, theta n to theta i, say P before into d theta and minus because the frictional force is changing, so one part will be if you reduce the total torque okay, so this is eta n into P after d theta, so that will be $R = 250$ square into 0.06 into say 5.868 - 2.2569.

So that we will get it as that is equal to 13496.25 Newton meter; Newton millimeter per millimeter width okay, so that is the thing, Newton millimeter per millimeter width, so driving torque for each roll, therefore total driving torque; per roll is equal to this T into 1500 that is equal to 13496.25 into 1500, so you will get it as 202.44 kilo Newton meter, if you convert into meter you will get this sorry, this is kilo Newton millimetre.

$$\text{Torque per unit width} = T$$

$$= R^2 \mu \left[\int_{\theta_n}^{\theta_i} P_{before} d\theta - \int_0^{\theta_n} P_{after} d\theta \right] = (250)^2 \times 0.06 [5.868 - 2.2563]$$

$$= 13496.25 \text{ Nmm per mm width}$$

$$\text{Total driving torque per roll} = T \times 1500 = 13496.25 \times 1500 = 202.44 \text{ kNmm}$$

$$= 202.44 \text{ Nm}$$

So that is equal to 202.44 Newton meter, so now if you look at the total power requirement total power because two rolls are there, this is per roll we have found it, so we have to multiply it into 2 also, so into omega, so that is a driving torque into 2 into omega, so that is equal to 202.44 into 2 into omega is 3, in our problem which is 3, so that comes to 31214.6 watts that is equal to 31.214 kilowatts.

$$\text{Total power required} = \text{Driving torque} \times 2 \times \omega$$

$$= 202.44 \times 2 \times 3 = 31214.6 \text{ W} = 31.214 \text{ kW}$$

So now, because there is not much time I am going to the second part, so that is equal to μ is equal to $\tan \theta_i$ so that is a simple equation, so I am not going to do that because what I wanted was that if we just look at this, this will be how you do the rolling analysis and all the parts are there, in this problem if you look at it you have to determine the bite angle, you have to determine the neutral axis; neutral point, the θ_n , you have to find out the θ_i also we can find out.

And λ_i , λ_n all these things we can find out and for each point we can calculate using these two equation a and b, so from where $2k$ is equal to your flow stress σ we have taken and then you can do that so, at each point you find out the value of the roll pressure and then you find out the friction hill okay, the pressure variation across the roll from the entry to the exit, okay basically, when it is like this people normally do it the start with the exit as 0.

Because exit is the zero point of the theta, so you are taking it from the clockwise direction that is why so exit it is coming here and entering will be here, so and then plot it and then you integrate this to get the roll separating force. Once you get the roll separating force because then you can find out the torque but remember that the torque from the exit side that will always help them after the neutral point, if it is more, then that will be helping the in reducing the torque requirement that is one advantage with these.

So that way we can find out and then the total power consumption we can do now, the second part of the problem is state whether the work piece can be drawn into the rolls unaided, so that is equal to $\mu = \tan \theta$, if you just μ should be greater than or equal to $\tan \theta$, from that you can just to do it, okay and that is left for those attending this course, thank you very much. I hope that you will get familiar with this rolling by this problem.