Plastic Working of Metallic Materials
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Module No # 08
Lecture No # 25
Drawing of Rods, Wires and Tubes (Contd.)

Yeah, we will continue with our lecture 1 from where we stopped last day. So, last day we were discussing about the draw stress for homogeneous deformation for a non-work hardening material as well as the draw stress required for the homogeneous deformation for a non-work hardening material with and without friction and we ignore the redundant deformation in that case also.

Because this redundant deformation is something, which we will discuss later, which is coming, earlier also we have discussed.

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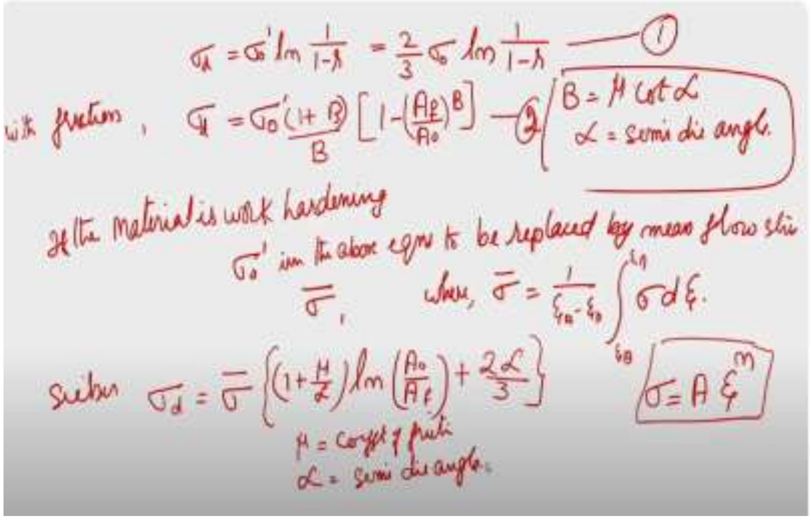

So, in the draw stress, we arrived at was sigma d = sigma dash log 1/1 – r. So, that is equal to 2/3 into your uniaxial yield strength sigma 0 log 1/1 – r. So, if you consider friction, so with the friction, if you are considering with the friction, sigma D we can write it as sigma 0 dash into 1 + B/B into 1 – A final, the cross-section area, final cross-sectional area by initial cross-sectional area raise to B where B = mu cot alpha and alpha is the semi die angle.

$$\sigma_d = \sigma_0' \ln \frac{1}{1-r} = \frac{2}{3} \sigma_0 \ln \frac{1}{1-r}$$

$$with\ friction, \sigma = \sigma_0' \frac{(1+B)}{B}\left[1-\left(\frac{A_f}{A_0}\right)^B\right] \qquad B = \mu \cot\alpha \quad \alpha = semi\ die\ angle$$

This is what we have arrived last day and you will find that these are not the 2 expressions. This we can say 1 and this is equal to 2 where several expressions have been developed for the draw stress required and other things okay and if the material is work hardening because this is just for a non-work hardening material. If the material work hardening, then the sigma 0. If the material is work hardening, the $\sigma_0{}'$ in the above expression to be replaced by the mean flow stress $\bar{\sigma}$, which you can get it.

We have discussed sigma bar is equal to 1/epsilon B - epsilon A into say where sigma bar is equal to 1/epsilon A - epsilon B into sigma from epsilon B to epsilon A sigma d epsilon. So, these are the strain under which we have to study okay. So, this is a very, using the average the flow stress is a very convenient form of using that if you know because it is much more easy because most of the time we have this expression sigma is equal to A epsilon raise to n.

$$\bar{\sigma} = \frac{1}{\epsilon_A - \epsilon_B}\int_{\epsilon_B}^{\epsilon_A} \sigma d\epsilon$$

$$\sigma = A\epsilon^n$$

So, from this know we can derive at it very easily. So, that is one very convenient form of using the average flow stress. If you have the flow curve of the material, that is the true stress versus true strain curve in the plastic region of the material, then this expression and replacing this stress component by the average flow stresses will be the most convenient thing and say there are many other expressions.

If you are considering this friction as well as the redundant deformation, a large number of works has been carried out and with different materials, different die angles, different frictional conditions and you will find that there is a wide range of this expression because material properties are coming into picture. So, it becomes very difficult to have a uniform relationship for that.

But in spite of that, the draw stress is necessary for the deformation for drawing, the stress necessary at the exit for the wire drawing considering say friction and redundant deformation, which has put forward by Siebel, which is of this form sigma D is equal to say sigma bar into this is a widely accepted relationship 1 plus mu by alpha into log A0 that is the initial cross-sectional area divided by the final cross-sectional area plus 2 alpha/3 where mu is the Coulomb’s, mu is the coefficient of friction and alpha is the similar die angle.

$$\sigma_d = \bar{\sigma}\{\left(1+\frac{\mu}{\alpha}\right)\ln\left(\frac{A_0}{A_f}\right) + \frac{2\alpha}{3}\} \qquad \mu = coefficient\ of\ friction, \qquad \alpha = semi\ die\ angle$$

So, that way we can find out this is one of the expression, one of the relationship which has been widely accepted but whatever be the type of relationship, when you are considering the friction and redundant deformation, the flow stress can be represented in a general way.

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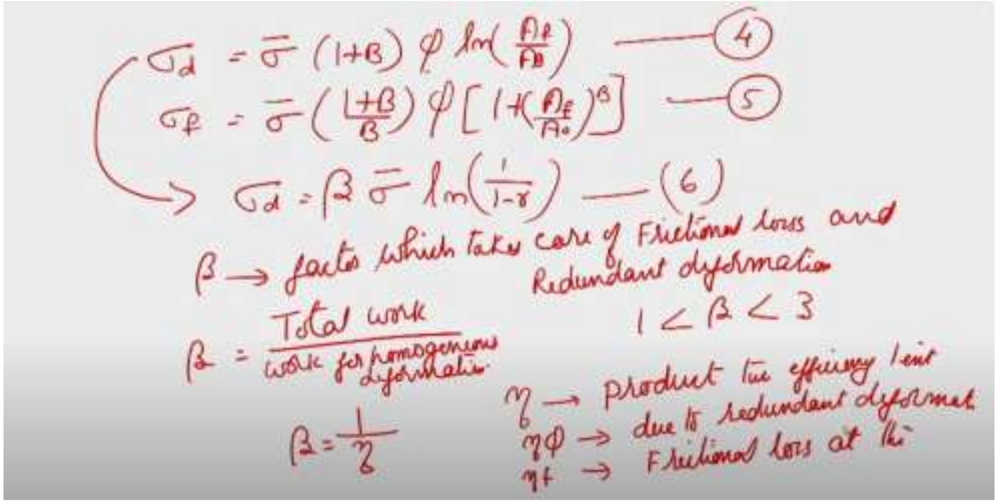

As sigma bar into 1 + B into phi where phi is your friction factor into your strain log Af/A0. This is a general expression which we can represent it considering friction and redundant deformation. So, it is equation number 4 and another expression is that say these are the 2 expressions, which we can get. One is 1+ B/B, so because we are introducing this term phi into 1 plus, this is the relationship which earlier we found without considering the redundant deformation.

$$\sigma_d = \bar{\sigma}(1+B)\phi \ln\left(\frac{A_f}{A_0}\right)$$

So, in that also we can write but only thing is that you are just considering this phi term to take into account of the redundant deformation, that factor is called as a redundant work factor phi okay. So, this expression we can just write in a general form as d is equal to say beta because if you eliminate all other things into average stress into your strain okay that is log 1/1 - r okay.

$$\sigma_f = \bar{\sigma}\left(\frac{1+B}{B}\right)\phi\left[1+\left(\frac{A_f}{A_0}\right)^B\right]$$

$$\sigma_d = \beta\,\bar{\sigma}\ln\left(\frac{1}{1-r}\right)$$

So, we can write in this form where beta is the factor, beta is a factor which takes care of your frictional loss and redundant deformation. So, that is why this beta is introduced and generally the value of beta is between 1 and 3, so where beta is nothing but the ratio of the total work including friction and redundant deformation to the work for homogeneous deformation.

$$1 < \beta < 3$$

$$\beta = \frac{Total\ work}{work\ for\ homogeneous\ deformation}$$

And this beta also, in some people they express it as an efficiency factor 1/eta where this eta is a product of 2 efficiency terms. One is say eta phi due to redundant deformation and this f due to frictional loss at the die workpiece interface okay. So, that is how this beta is termed okay.

$$\beta = \frac{1}{\eta}$$

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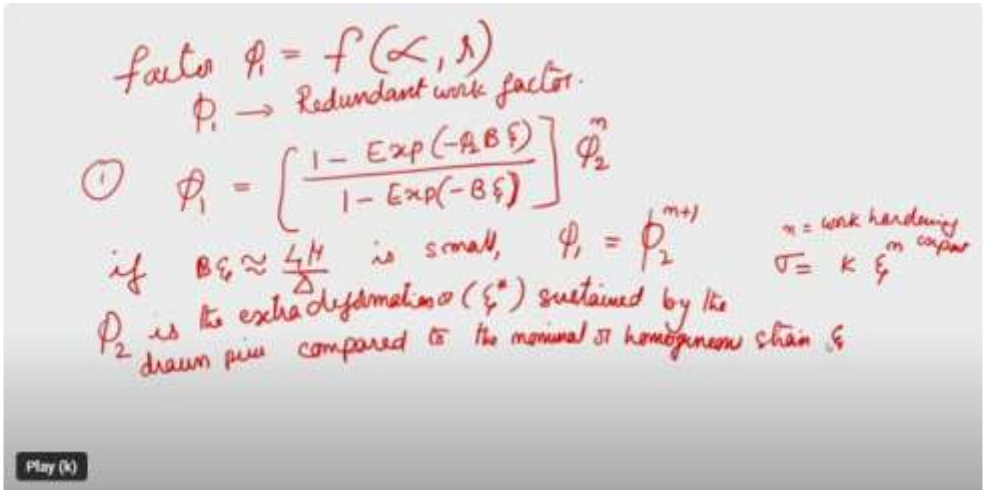

The factor phi, it is a function of your alpha and r where alpha is a die angle and the r is your reduction. So, that is so we can call it as phi 1 which is nothing but phi 1 is called as the redundant work factor or that corresponds to the, so from this we can find it out the enhanced strain corresponding to the yield strength of the metal, which has been homogeneously deform to a strain of epsilon okay.

$$factor\ \phi_1 = f(\alpha, r)$$

$$\phi_1 = Redundant\ work\ factor$$

So, that is what from our earlier discussion, we have discussed it. So, that means and phi can be estimated from the displacement of a flow curve, after flow curve after tensile testing of the drawn wire, so you have the wire, so before the drawing operation, you conduct a tensile testing in an annealed condition and the same material you just do the drawing operation and that wire again you test it after the drawing for after some particular strain.

And then you will find that the yield strength has increased in the drawn wire, so you just displace it towards the right, so that it matches with that and from that when it matches with the actual flow curve, what is that strain, that is the epsilon sharp. So, from that we have discussed this part earlier. So, how to calculate this redundant deformation that is another important because it is not that easy.

What is this redundant deformation? When the metal undergoes deformation through the deformation zone for any material, so we are just now since we are discussing about the wire drawing operation in a conical die, the material is deforming. We mentioned that if it is a homogeneous deformation, the square grid will be distorted to a rectangular grid but depending upon the die angle and the friction conditions, you will find that okay it is not a homogeneous deformation which is taking place.

If the die angle is very large, then actually what happened is after certain angle, die angle there will be an internal shearing of the material, so too much of extra work has to be done. So, it is very difficult to arrive at or derive an expression for this redundant work or redundant deformation. So, only way is that we have to carry out some experiment and then find out empirical relationships okay.

So, there are 2, 3 methods by which this redundant deformation can be found out okay. So, one is we can find out the redundant work factor. So, that is phi by people have carried out this expression and then arrived at this relationship that is 1 - phi 1 = 1 - exponential - phi 2 B epsilon/1 - exponential - B epsilon into phi 2 to the power n. So, I will come to that what is phi 2.

So, if the product this beta and epsilon, so if beta B into epsilon, it will be approximately equal to 4 mu/delta where delta is deformations on geometry which we have refreshed earlier, we have come out across, I will discuss immediately also and if this is sufficiently small, then phi 1 will be equal to we can approximate just phi 2 to the power n + 1 where n is the work hardening exponential obtained for the material which is nothing but K epsilon raise to n, so n is equal to work hardening exponent.

$$\phi_1 = \left[\frac{1 - Exp(-\phi_2 B\epsilon)}{1 - Exp(-B\epsilon)}\right]\phi_2^n$$

$$if\ B\epsilon \approx \frac{4\mu}{\Delta}\ is\ small, \phi_1 = \phi_2^{n+1} \qquad \sigma = k\epsilon^n$$

So, this is what happens. Now, one should be able to distinguish very clearly between phi 1 and phi 2. Many books know sometimes it is confusing but one should be able to know what is the difference between this phi 1 and phi 2. So, phi 2 is the redundant deformation whereas phi 1 is the redundant work factor. The redundant deformation this phi 2 is the extra deformation.

See for example phi 2, I am writing it clear, is the extra deformation or maybe say your this strain which is the equivalent strain sustained by the drawn piece compared to the nominal or the homogenous strain epsilon.

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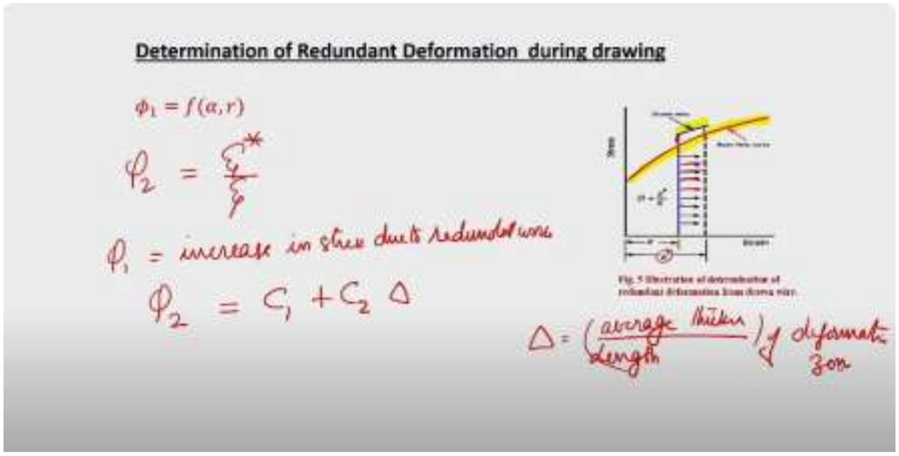

So, just like if you are doing this, so for example this is your basic flow curve. Suppose this is your basic flow curve of the annealed material. Now, the annealed material is subjected to say drawing operation and after a drawing with a reduction such that corresponding to a strain epsilon, then you take the tensile testing of that, you conduct a tensile testing and you will find that your this flow curve has, the yield strength has increased by some amount okay.

So, this is the thing, so but if you have just shifting this curve, if you are shifting this curve towards the right, if you are displacing towards the right and so that now this comes and matches with this, then you will find that this is the total work done due to redundant deformation. So, but that corresponds to a strain, enhanced strain of a epsilon star though your actual strain, actual reduction during the drawing operation or strain but you will find that now this is an enhanced strain.

So, that is the equivalent strain which we are considering to that. So, from this, phi 2 can be defined as epsilon star/epsilon this. So, that is the, it is an extra deformation which is taking place okay. Whereas this phi 1 is the stress due to the redundant work, the increase in stress due to redundant work. So, that is the difference. So, here you have an increase in the stress here okay, so that is what is phi whereas this is the strain which is coming.

$$\phi_1 = f(\alpha, r)$$

$$\phi_2 = \frac{\epsilon^*}{\epsilon}$$

Now, you will find that if you find out, try to find out this the relationship between these deformations on geometry and the redundant deformation, you can always find it correlate in these forms, phi 2 = C1 + C2 into delta. So, that is what where delta is your geometrical factor, which is coming okay, it is the average thickness to the length of maybe of deformation zone, so that we have discussed earlier.

$$\phi_2 = C_1 + C_2\Delta$$

$$\Delta = \left(\frac{average\ thickness}{length}\right) of\ deformation\ zone$$

Now, another method is you are also finding out the stress-strain curve or the wire drawn under different conditions and these are compared to your stress-strain curve or the fully annealed wire okay. So, the same case which you have done. The flow stress of drawn wire is always higher than the annealed wire. So, that is what we have seen in the previous diagram. The strain is in simple tension of the drawn wire is thus equivalent to your epsilon star.

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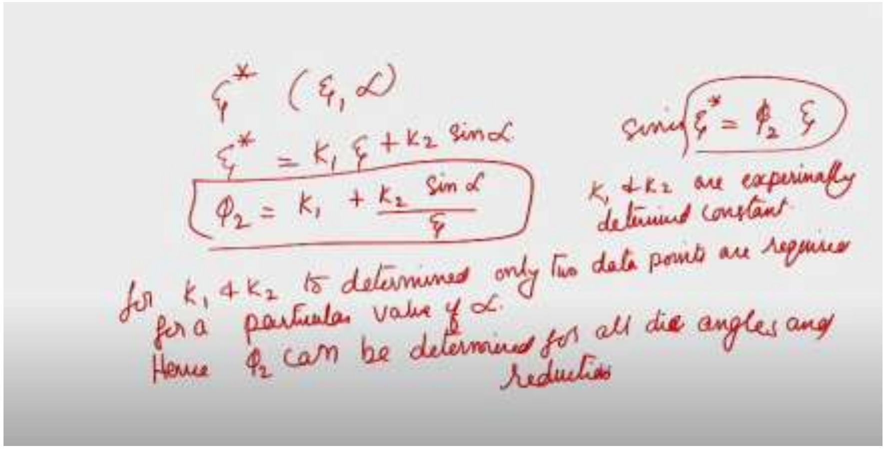

So, you will find that epsilon star is linearly related to the nominal stress as a function of nominal stress epsilon and die angle. So, that means this is linearly related that is K1 into epsilon w or epsilon, if it is wire only epsilon plus K2 into sin alpha where alpha is the die angle and since earlier we have written this is equal to epsilon to sorry phi 2 into epsilon. So, from this relationship, we can arrive at say like if you divide it by epsilon, then you will find this as phi 2 into epsilon and divided by epsilon.

$$\epsilon^*(\epsilon, \alpha)$$

Then, we can get it as phi 2 = K1 + K2 into sin alpha/epsilon where this K1 and K2 are experimentally determined constants. So, under different conditions, we can find out what are the value of K1 and K2 and phi 2 that we can find out if you know the value of alpha and then this. So, the advantage of this, the significance of this equation is that only 2 data points are required for one particular value of alpha.

$$\epsilon^* = k_1\epsilon + k_2 \sin\alpha \qquad \sin\epsilon^* = \phi_2\epsilon$$

So, if you just look at it, you will find because in this case you know, from this you can find out what is the value of phi 2 with different strain values and if you get 2 points you know you can easily get it and enable K1 and K2 you can just find it out and hence phi 2 can be calculated for so for determination for K1 and K2 to be determined only 2 data points are required for a particular value of alpha.

$$\phi_2 = k_1 + \frac{k_2 \sin \alpha}{\epsilon}$$

So, hence this phi 2 can be calculated, can be determined for all die angles and reduction. This is the biggest advantage with this because it is a straight line equation which is coming.

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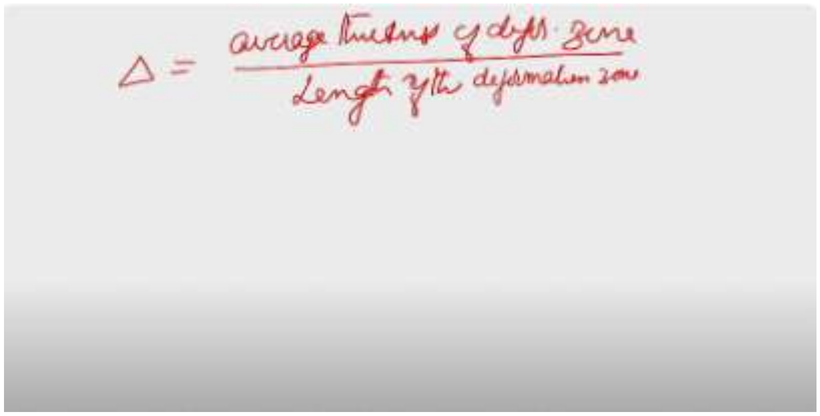

Now, coming to this delta, so delta as we have discussed earlier, delta is nothing but the average thickness ratio of the average thickness of the deformation zone to the length of the deformation zone okay.

$$\Delta = \frac{average\ thickness\ of\ the\ deformation\ zone}{length\ of\ the\ deformation\ zone}$$

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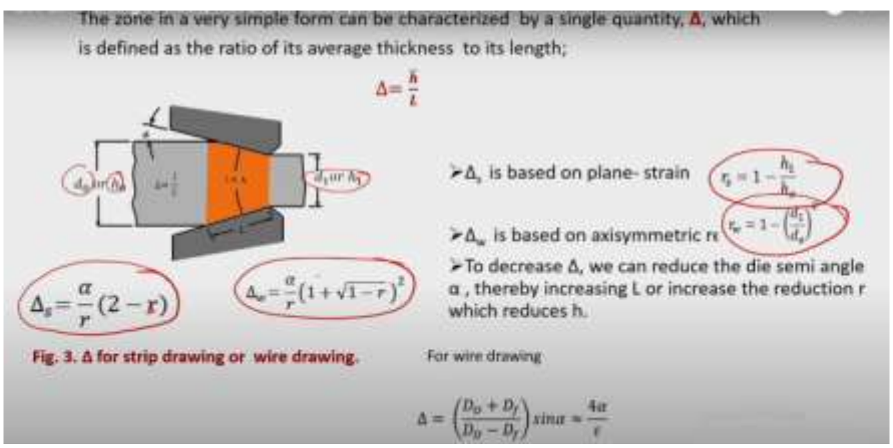

Now, see this is the thing, so if this is the wire drawing operation, you can have say maybe one is the plane strain condition, so you can find out whether it is diameter or height. So, if it is a strip drawing, then you consider this h0. If it is a wire drawing, you take this d0 and finally the corresponding value at the output will be h1 and d1 respectively. So, we can find out this sigma s by this relationship alpha/r into 2 – r okay.

$$\Delta_s = \frac{\alpha}{r}(2 - r)$$

So, that is based on the plane strain condition where r is equal to say we can say rs = 1 – h1/h0. For strip condition, it will be based on this height h at the beginning at the initial whereas if it is for wire drawing operation which is the diameter which is taking place, so you will consider that rw is equal to this one. So, this r will be rw here, here it will be rs, so rw is there.

$$r_s = 1 - \frac{h_1}{h_0}$$

In that condition, you are getting this relationship, the deformation zone geometry delta = alpha/r into 1 + under root 1 - r the whole square. So, for wire drawing operation, this is the general relationship which people use it and then you will find that in different books different ways it is being used okay, but from this you can also have a simplified form of delta is equal to 4 alpha by your strain. So, that is how you find out these deformations on geometry.

$$r_w = 1 - \left(\frac{d_1}{d_0}\right)^2$$

$$\Delta_w = \frac{\alpha}{r}\left(1 + \sqrt{1-r}\right)^2$$

$$\Delta = \frac{4\alpha}{\epsilon}$$

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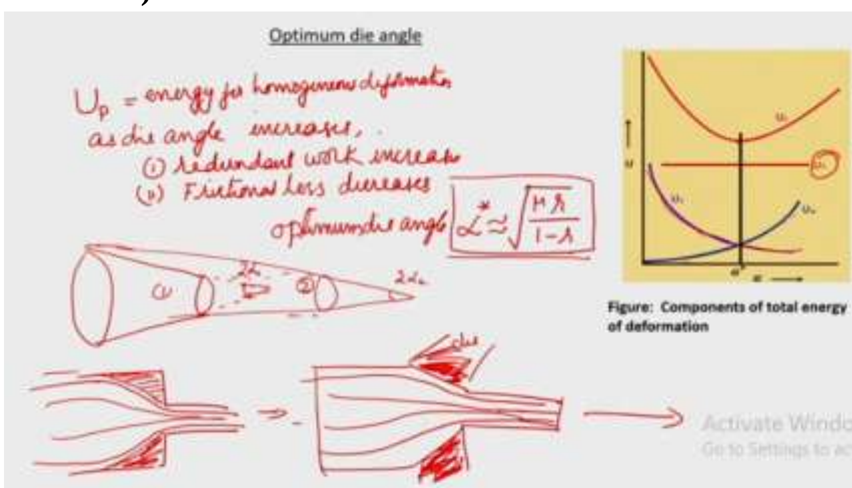

Now, when you consider this, there are 3 factors. One is the homogeneous deformation where there is no friction, there is no redundant deformation. If you just pull the wire, how it is happening and what is the energy required for that, so that is called as a work for plastic energy, we can say energy for homogeneous deformation, plastic energy we can say where there is no friction, there is no redundant deformation.

You just pull it and from that you find out for a strain how much is the energy required, so that is the U_p. Now, if you just look at it with a die angle, let us say a conical die angle is coming, so this with the die angle decreasing, 2 things are happening. See for example, I am just getting from say maybe diameter of this to a diameter of this, so this is the reduction which is taking place and I can say that okay this is going to be my die angle okay.

So, this is your 2α, this is the 2α angle. Now, if with alpha decreasing, when it is alpha decreasing suppose let us say we want the same diameter. So, this will be $2\alpha_2$, this is $2\alpha_1$ let us say. So, if these diameters are same, the output reduction is same but with a different die angle. With lower the die angle, you will find that the contact area between the die and the workpiece is higher for lower die angle.

Whereas the die angle increasing this distance, the deformation zone length keeps on decreasing, so when the deformation zone length keeps on decreasing, the total contact area between the die and the workpiece that decreases because it say as per this relationship, this figure say these 2 cases are there, this is 1 and this is 2 case, second case is coming. So, that means with the increase in the die angle the frictional loss keeps on decreasing.

Because the total contact area decreases but you just consider the case with the increase in the die angle what happens is that maybe the one is the die angle is like this, another is the die angle is like this okay. So, in this case, the metal has to come like this and deform like this, here it is that, so there is a sharp deformation which is taking place. So, I will just draw it in a better way.

The metal flow will be like this, it is going like this. So, this is the inlet and this is the outlet we are pulling like this. This is your die, so there is a sharp turn, change in the flow direction if the die angle is large, so whereas the diagonal was if it was small, then you will find that okay, you will see a very smooth flow of metal is there inside the die whereas this is your die. So, in this case with the die angle larger, the redundant deformation will be very high.

And if the die angle exceeds a certain value, then you may find that there is an internal shearing of the material which can take place and forming a dead material and so. Suppose my die was like this say where die angle 2 alpha is 90 degree, so what happened, when the metal is pulled, it will just deform like this and you will find that here there is a dead metal zone which is not going to flow at all depending upon your frictional conditions and the die angle.

So, beyond the certain increase in the die angle, this internal shearing of the material will take place, that means your redundant deformation has to be very high because there is an internal deformation and change in the direction of the material flow is taking place, so that will result in a very higher amount of energy which is required.

So, in that case, so with increase in the die angle, so as the die angle increases, so you will find that one is that as a die angle if you are plotting this energy required for this alpha versus alpha, your energy for plastic deformation remains constant okay. It has no relationship between your alpha okay, it remains constant but whereas with increase in the alpha you will find that the frictional energy loss increase in the alpha value, increase in the die angle it decreases.

Because why, the contact area between the workpiece and the die material that decreases, so your frictional energy decreases whereas the redundant deformation, the energy due to redundant deformation, with the work the deformation energy, the redundant work increases as the die angle increases. So, these 2 things are happening. So, one is as die angle increases one is the redundant work increases and second is the frictional loss decreases.

And these are the plots, so you will find that the work for plastic deformation or homogeneous deformation, it remains constant and this is the frictional energy with the die angle increasing and this is the redundant work which increases with the die angle okay. So, you will find that the total energy, which is required, will be a sum of all these things. So, that we will just keep on increasing, as keep on decreasing as die angle increases it reaches a minimum and then it again increases.

So, there is an optimum die angle for depending upon your frictional component and other things. So, you have to optimize the die angle and there are certain lot of work has been derived and people have arrived at certain relationship for the die angle. So, a simple expression I will tell you that for obtaining the optimum die angle okay. A simple alpha star is approximately equal to your root of mu into your reduction divided by 1 minus r okay.

$$\alpha^* \approx \sqrt{\frac{\mu r}{1-r}}$$

So, this is a very useful relationship for a first-hand approximation, estimator of the drawing stress, that way it is very useful. Only thing is over a large value there may be some inaccuracy or the accuracy may get reduced with a very large value of alpha okay. So, in this alpha star does not depend on the precise work hardening behavior of the material or on the type of the material that is the biggest advantage.

It already depends upon the frictional condition mu and the reduction. So, it is not dependent upon the material that is what this relationship has come out okay. This is from the literature you get it, so that means with this you can have a first-hand estimate of the die angle.

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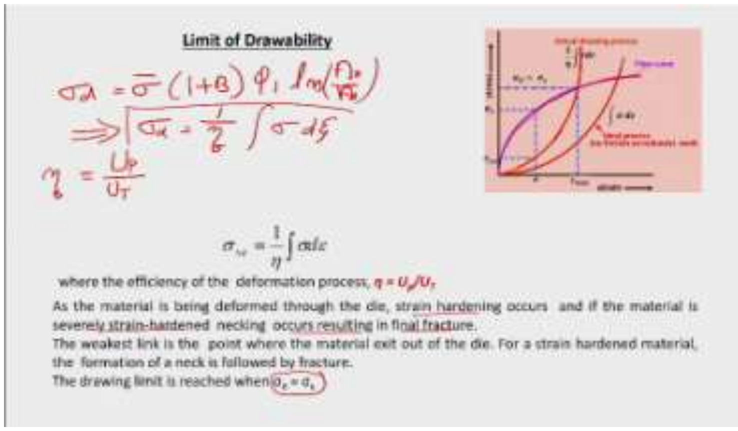

Now, we have found that there is an optimum die angle for this, whatever be happening we have also arrived at different relationship for the limit of drawability for the draw stress okay. The thing is that most of the real material say like there are different models, some are say may be perfectly elastic and perfectly plastic, then elastic plastic, elastic work hardening. So, most of the engineering materials for structural use is elastic and work hardening material.

That is what the case which comes, so with the each deformation with each amount of increase in the strain, plastic strain the yield strength keeps on decreasing and that is what we get the flow curve of that. So, if this is the typical flow curve of the material given by this curve and you will find that earlier also I am again writing that the draw stress sigma d = sigma bar into 1 + B into phi 1 into log A0/Af.

$$\sigma_d = \bar{\sigma}(1 + B)\phi_1 \ln\left(\frac{A_0}{A_f}\right)$$

Again and again I am writing this, this is a general expression. See this we can almost write it in this general form sigma d is equal to 1 by an efficiency factor eta into integral sigma d epsilon, this we can write okay. So, where this efficiency is nothing but your ratio of plastic work that means for a homogeneous deformation to your total work this is the thing. So, when you look at this, the flow curve is like this.

$$\sigma_d = \frac{1}{\eta}\int \sigma d\epsilon$$

But in an ideal process, this sigma d epsilon curve will come like this but because there is a friction and redundant deformation, the curve of the flow curve of the drawn wire will be following like this, so that we can write it in this form as this with a strain increasing. So, this will be that curve and now the thing is that as the material is being deformed through the die, strain hardening occurs.

$$\eta = \frac{U_P}{U_T}$$

And if the material is severely strain-hardened, the necking occurs resulting in final fracture but when you are pulling this material for the wire drawing operation, the weakest part or weakest link; it is a point where the material exit out of the die that is the weakest part, inside it is compressive stresses. The other part it is already having sufficient strength but basically at that part which is about to exceed that is the part which is the weakest material, weakest part of the link okay.

So, for a strain-hardened material, it is at that part a necking may take place when it reaches the limit of drawing okay and once the necking takes place and you are pulling it, the necking is followed by a sudden fracture. So, drawing limit is reached, when this condition is met. So, at that part of the exit, when the stress is equal to your drawing stress, so the stress due to your drawing operation is equal to your draw stress, then the fracture will take place.

Because beyond to that if you look at that, this is a drawing stress with a more strain, this will be higher than your flow curve okay, flow curve of the material. So, when it is higher than the flow curve of the material, naturally that part will just deform of and then necking will start. As long as, it is below the flow curve, it will not fail because once you cross that it just want to deform and necking will take place and it will fail at that point.

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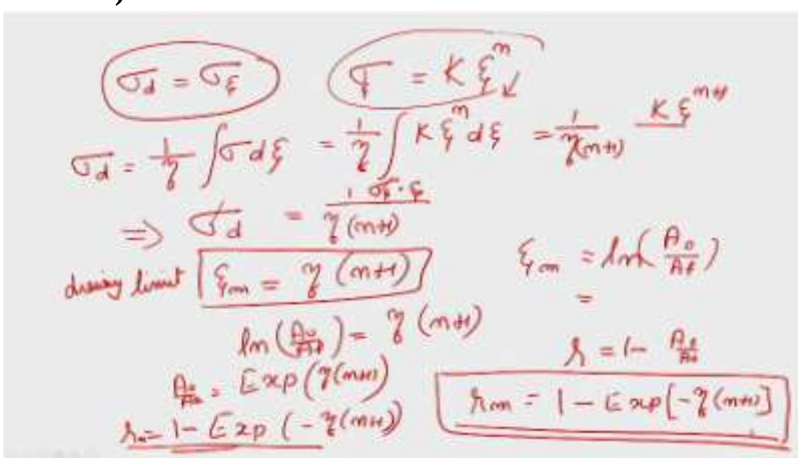

So, in that case, the drawing limit, what is the drawing limit, that is what we wanted to find where sigma d will be equal to sigma epsilon, sigma epsilon means from the flow curve what is that value, when these two equals the same, then you will have it. So, if we can write that sigma epsilon is given by the Hollomon relationship by K epsilon raise to n, so this is suffix is equal to K epsilon raise to n.

$$\sigma_d = \sigma_\epsilon \qquad \sigma_\epsilon = k\epsilon^n$$

So, we can write this relationship sigma d = 1/eta into 1 by this efficiency factor into sigma d epsilon. So, that will be equal to 1/K epsilon raise to n d epsilon from this equation you will get it. So, that is equal to 1/eta into n + 1 into K epsilon raise to n + 1 but K epsilon raise to n + 1 is equal to K epsilon raise to n that is this one into epsilon, so that is what. So, that means that will be equal to say we can say it sigma d = 1 by into n plus 1 into sigma into epsilon.

$$\sigma_d = \frac{1}{\eta}\int \sigma d\epsilon = \frac{1}{\eta}\int k\epsilon^n d\epsilon = \frac{1}{\eta(n+1)}k\epsilon^{n+1}$$

So, we can say sigma e epsilon oh sorry into epsilon and this is the condition for failure. So, that means sigma d, so this will get cancelled off, so you will get that the maximum limit from this the limit of drawing limit epsilon max is equal to so that means here we can say and this one will get cancelled, so is equal to eta into n + 1. So, that is the maximum limit for the drawing operation.

$$\sigma_d = \frac{\sigma_\epsilon . \epsilon}{\eta(n+1)}$$

$$drawing\ limit\ \epsilon_m = \eta(n+1)$$

And since this epsilon max is equal to log Ab/Af or maybe you can say this is Ao/Af starting and final, so that is the thing. So, we can write that that is equal to so log Ao/Af = eta into n + 1. So, we can write in this form E exponential and we also have the relationship r = 1 – Af/Ao, so that way we can write r = 1 – 1/e raise to eta into n + 1, r max so that is the limit of, so that means r max.

$$\epsilon_m = \ln\left(\frac{A_0}{A_f}\right)$$

$$\ln\left(\frac{A_0}{A_f}\right) = \eta(n+1)$$

$$\frac{A_0}{A_a} = Exp(\eta(n+1))$$

$$r = 1 - \frac{A_f}{A_0}$$

I will write it better way that the limit of this one is equal to 1 - exponential - eta into n + 1 okay. Now, when you are having a repeated reduction through a series of dies, the n will keep on decreasing because even though you are having repeated deformation also you know the strain keeps on decreasing and work hardening keeps on decreasing. This is the general nature of the material.

$$r = 1 - Exp(-\eta(n+1))$$

$$r_m = 1 - Exp[-\eta(n+1)]$$

So, if you look at the work hardened rate versus strain, you will find that it decreases. Initially, you have a very high work hardening rate but with the increase in the strain, it will just saturate off, so it will keep on decreasing and then it will remain constant. So, if you want a material which is not work hardening, the best way is that you do some large amount of plastic deformation.

Only thing is that ductility will be very poor but you will get that the material is not work hardening, so that is one way of carrying out experiments with the samples which are not work hardening or with the different work hardening rate if you wanted then these are the methods by which people do. So, when you are repeatedly doing reductions and other things what happens is that the n will decrease to 0 almost to 0 and allowable reduction keeps on decreasing in this case from this expression.

Now, the thing is that if you just look at the maximum strain, which is possible during the drop, if you make a comparison for the maximum strain in drawing with that of stretching, stretching means tensile testing.

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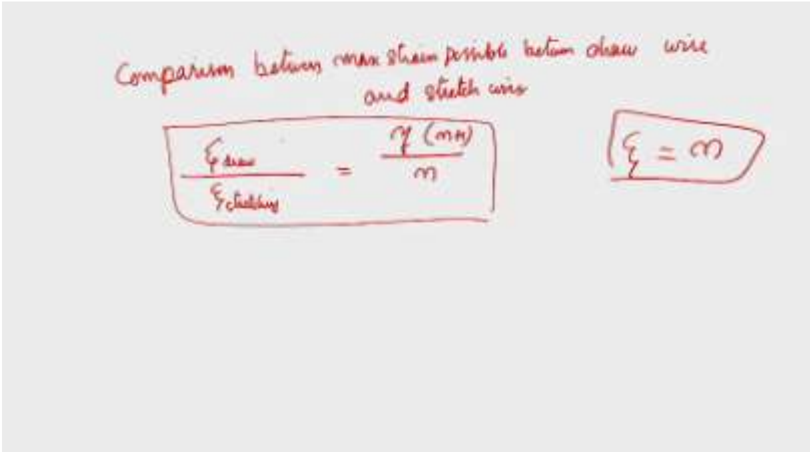

So, if you do a comparison between maximum strain possible between drawn wire and stretched wire, so you will find that the strain ratio of drawing by stretching is equal to 1 is this eta into n + 1 and another is n okay. So, why this is coming, because in a tensile pulled sample, you will find that the maximum necking will take place when strain is equal to your n, so your maximum strain was over the, the strain at maximum load at which that is where the instability sets in that, we have discussed in the earlier classes.

$$\frac{\epsilon_{draw}}{\epsilon_{stretc\,hing}} = \frac{\eta(n+1)}{n} \qquad \epsilon = n$$

At the instability where it starts, that means at the ultimate tensile strength, the value at that case your true strain is equal to that means is equal to your work hardening exponent for a material which is following the Hollomon relationship. So, if you just do like this, this is the relationship and then you will find that the stretching is the effect of work hardening exponent on this drawing and stretching, how it is influencing the, having a very high influence on the maximum strain, which is possible okay.

Now, if you look at that these are the limits which have been obtained. Now, we will just come to the analysis of tube drawing okay.

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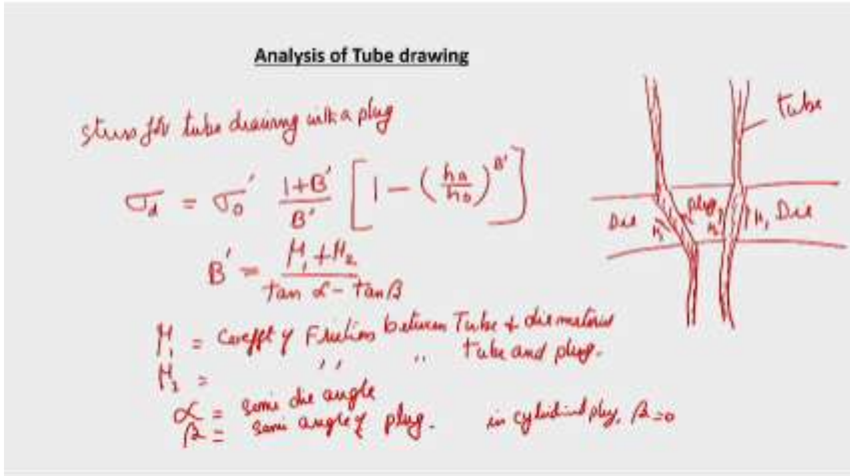

So, analysis of tube drawing is basically based on the plane strain conditions and with analogue with the draw stress for wire drawing, the stress for tube drawing with a plug. So, you can say the stress for I am not going into the derivation, detailed derivation and other thing for tube drawing with a plug because we have found that we have discussed the 4 types of tube drawing.

One is the drawing with the fixed plug, drawing with the floating plug and drawing with a mandrel, so these things we have discussed and sinking also, but all these drawing relationship which is almost similar, it is having an analogue with the case of which we have discussed earlier for drawing relationship. Only difference we can say is that the sigma drawing for a tube can be approximated as 1 is sigma 0 star which is 2/root 3 into your uniaxial yield strength of the material plus instead of 1/B we have to put 1/B dash.

B is replaced by B dash, that is the only difference, otherwise so 1 – ha/hb raise to B dash. There are 2 things, which are happening in a plug okay. In a plug, there are 2 interfaces coming, the interface between your die and the workpiece material, the tube material. Secondly, the interface between the tube material and the plug, these 2 are coming, so you have to compensate for this thing.

$$\sigma_d = \sigma_0^{'} \frac{1+B'}{B'}\left[1-\left(\frac{h_a}{h_b}\right)^{B'}\right]$$

So, in tube drawing, in this tube drawing know, you will find that this relationship, so B dash in this relationship is nothing but your mu 1 so 2 frictional constants are coming by tan alpha minus tan beta. These 2 things are coming. So, this is where you will find that mu 1 is equal to coefficient of friction between tube and die material and mu 2 is a coefficient of friction between your tube and the plug.

$$B^{'} = \frac{\mu_1 + \mu_2}{\tan\alpha - \tan\beta}$$

Alpha is the semi die angle or cone angle and beta is the semi angle of plug. If it is cylindrical plug, beta is equal to 0, so that will come. So, if now the thing is that see if you just look at this you have a plug maybe let us say plug is like this. So, this is the die, this is the plug and this is the die okay and this is your tube. So, in this case what happened, there is a friction here.

So, this is the mu 1 which is coming, so there is a friction here, which is mu 2, so this is mu 2 here, mu 1, so this is what. So, this mu 1 is the coefficient of friction between the tube and the die material and mu 2 is the coefficient of friction between the plug and the tube. So, these 2 things are coming and of course if you make a different plug angle and other things, you will have die angle is same, so that is the thing which comes.

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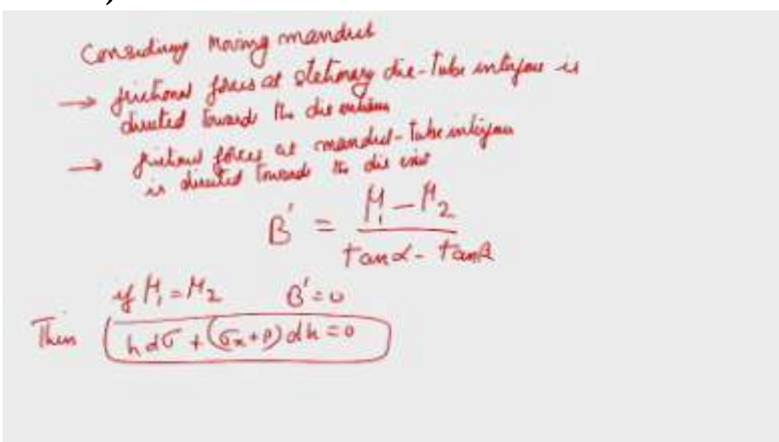

Now, if you are using that stress for a mandrel, a moving mandrel, the stress for a, when you are considering the frictional forces at the considering moving mandrel, there are 2 things which are happening. The frictional forces at stationary die-tube interface is directed towards the die entrance. Second, the frictional forces at mandrel-tube interface is directed towards the die exit.

This is because the mandrel is moving at the velocity equal to x velocity, exit velocity of the tube whereas at the inlet it is only very less, so you will find that with the when you are drawing the operation, the material which comes out is having a higher velocity, but the inside velocity is high but the diameter is very large and due to which there is an accumulation of material at that side.

And that is not going to affect the direction of the velocity, frictional force at the die-tube interface but at the mandrel-tube interface you will find that it is directed towards the exit. So, in that case know, you will find that the beta dash = mu 1 – mu 2 because change in the direction takes place by tan alpha - tan beta. See if you just look at the previous case, you will find that beta; relationship for beta dash is here.

$$\beta' = \frac{\mu_1 - \mu_2}{\tan\alpha - \tan\beta}$$

And that is mu 1 + mu 2 but the difference here it is mu 1 – mu 2 because these 2 are the opposite direction and since the velocity is higher than the velocity of the material confined in the die channel, so that is what is going to happen. The mandrel is moving at a velocity which is equal to exit velocity of the tube and it is higher compared to inlet velocity, so that is higher than the velocity of the material confined in the die chamber.

So, there is a forward frictional drag at the mandrel-tube interface. So, this tends to cancel the backward frictional drag between the die and the tube okay. So, that is one advantage, that way that is going to be very helpful because you need only less power for drawing with the mandrel if it is properly designed, then you will find that you need less power because the forward frictional drag at the mandrel-tube interface it cancels the backward frictional drag between the die and the tube.

And if you say that if mu 1 = mu 2 then beta dash = 0 and then the differential equation you will get it as hd sigma + sigma x + p dh = 0 which you can solve it in a very simple way and then do it, in the similar way we can do it actually okay. So, integrating we get the equation of ideal homogeneous deformation in this equation if you do it, so that is the condition, then simple idea of homogeneous deformation will come.

$$if\ \mu_1 = \mu_2 \qquad B' = 0$$
$$then, \qquad hd + (\sigma_x + p)dh = 0$$

So, that means if you are able to have this frictional coefficient same, then redundant deformation and that frictional and those factors will be get nullified. So, you will need only lower stress which is equal to your homogeneous deformation. It is also possible for the coefficient of friction on the mandrel that is at the side that is mu 2 to exit that at the die side, so by having a surface roughness and other things also, may be lubricating conditions if you do it properly, so that way it is.

In such case, B tend to be negative resulting in a less draw stress than that required by frictionless ideal deformation condition. So, you may need only lesser conditions also for that case okay.