

Plastic Working of Metallic Materials
Dr. P. S. Robi
Department of Mechanical Engineering
Indian Institute of Technology – Guwahati

Lecture - 26
Analysis of Wire Drawing

So we will be continuing with the analysis of the wire drawing operation the detailed derivations the last class we have discussed and you will find that there are a large number of expressions which has come up because though the process of wire drawing is very simple but the analysis of wire drawing process is very complicated.

Because there are many factors which are coming into picture when you are doing the analysis mainly the friction at the workpiece by interface the redundant deformation which takes place due to the converging die and the die angle due to which internal shearing of the material take place because there is going to be a discontinuity in the flow field. So due to that redundant deformation also takes place and all these factors it affects the power requirement for the drawing operation.

Redundant deformation it results in non-homogeneous deformation of the drawn wire and this results in steep variation in the structure and properties across the cross section of the drawn wire.

(Refer Slide Time: 01:41)

Wire drawing analysis

- Analysis of wire drawing process is complicated.
- Friction and redundant deformation increases the total power requirement for drawing operating
- Redundant deformation results in non-homogeneous deformation resulting in steep variation in the structure and properties across the cross section of drawn wire.
- Increase in die angle decreases the redundant deformation
- Increase die angle decreases the energy for overcoming friction
- Due to the above opposing tend, optimization of die angle is required to keep the Total power requirement a minimum

So across the section itself you will find that the properties are different and if you look at the die angle and increase in the die angle decreases the redundant deformation die angle if it is increasing if it is increasing, then decrease sorry increase in the die angle increases the redundant deformation and increasing the die angle decreases the energy for overcoming friction.

Because the die angle is increased the total contact area becomes less so that in the frictional area frictional contact area the interface contact here becomes less and thereby the frictional energy becomes rigorous. So these are 2 opposing phenomena which is taking place either by because of the die angle. So one is increasing the power requirement and other is decreasing the power requirement.

So in that case you need to arrive at an optimum die angle and this has been shown in the previous lecture also how it affects the total power requirement. So you need to have an optimum die angle for a particular reduction depending upon the deformation okay, so that the total power requirement is kept a minimum and you will find that say in the expressions which are coming a large number of expressions are there has been developed by various researchers to arrive at the draw stress.

The stress necessary for the drawing and each has its own effect but here in this case we will not be discussing all of them but we will be discussing a few cases of them which is found suitable

by large number of people because there is no expression which takes care of all matters. So which have been widely accepted those expressions we will discuss we will not go into those derivation.

The basics of derivation you have already discussed but the one which is being accepted by industries and for application purpose we will discuss and how we go ahead with it. So in the wire drawing stresses.

(Refer Slide Time: 04:00)

Draw stress
 $\epsilon_w = \ln\left(\frac{l_f}{l_0}\right) = \ln\left(\frac{A_0}{A_f}\right)$ — (1)
 Fractional reduction in cross sectional area, $r = \frac{A_0 - A_f}{A_0}$ — (2)
 $\epsilon_w = \ln\left(\frac{1}{1-r}\right)$ — (3)
 $\sigma_d = \frac{F}{A_f} = \beta \bar{\sigma} \ln\left(\frac{1}{1-r}\right)$
 $\beta = \frac{\text{total work done for drawing}}{\text{energy for homogeneous deformation}}$ | $\bar{\sigma} = \text{average flow stress over the strain interval}$
 $1 < \beta < 3$. | for $\beta=1$, $\sigma_d = \bar{\sigma} \ln\left(\frac{1}{1-r}\right)$
 β can be expressed in terms of an efficiency factor $\eta = \frac{1}{\beta}$

For the draw stress if you wanted to find out it is nothing but the you have to calculate the stress a strain for the wire drawing operation that is epsilon w it is based on the change in it is based on the initial and final length because you are just extending the rod or a wire. So due to that there is a reduction in the cross-section area there is an increase in the length which initially in the fundamentals of material marketing we have discussed so this during wire drawing operation epsilon w is we can write it as log say your lf/10.

So in terms of cross sectional area that we can write it as log A0/af this 0 stands for the initial original and f for the final the suffix okay. So which will be following throughout this vector and instead of strain the term which is extensively used is the fractional reduction in area the fractional reductional in cross sectional area, area r is defined as A0-Af/A0 okay. So this is

equation number 1 this is equation number 2 from this we can directly correlate between your strain and the frictional reduction as $\epsilon_w = \ln \frac{1}{1-r}$.

$$\text{Draw stress, } \epsilon_w = \ln \left(\frac{l_f}{l_0} \right) = \ln \left(\frac{A_0}{A_f} \right)$$

$$\text{Fractional reduction in cross sectional area, } r = \frac{A_0 - A_f}{A_0}$$

$$\epsilon_w = \ln \left(\frac{1}{1-r} \right)$$

So in terms of a frictional reduction in area we can write it and the drawing stress σ_d can be obtained from the work done by the force F the total pulling force F by the final cross sectional area A_f . So that we can write it as in this form a beta into say your average flow stress over the strain which is taking place into $\ln \frac{1}{1-r}$. Okay where beta sorry where $\bar{\sigma}$ is the average flow stress over the strain interval and beta is a factor whose value lies between 1 and 3 if beta = 1 so for beta for beta = 1.

$$\sigma_d = \frac{F}{A_f} = \beta \bar{\sigma} \ln \left(\frac{1}{1-r} \right) \quad 1 < \beta < 3$$

You will see that $\sigma_d = \bar{\sigma} \ln \frac{1}{1-r}$ which is nothing but $\ln \frac{1}{1-r}$ which is nothing but the stress necessary for homogeneous deformation or the stress necessary for homogeneous session but since this redundant work and what is the frictional energy is coming into picture. So that is why this beta for a simplicity in the simple form we have introduced this beta. So that will give an idea about what is the beta is the ratio of the total work to the homogenous.

$$\text{for } \beta = 1, \quad \sigma_d = \bar{\sigma} \ln \left(\frac{1}{1-r} \right)$$

So that means beta is nothing but the ratio of total work done for drawing to the energy for homogeneous deformation for the same production and this factor beta we can also express in the form of an efficiency factor. Such that now beta is equal to or an efficiency factor this instead of that beta can be expressed in terms of an efficiency factor η which is nothing but $1/\beta$.

$$\beta = \frac{\text{total work done for drawing}}{\text{energy for homogeneous deformation}}$$


$$\beta \text{ can be expressed in terms of an efficiency factor } \eta = \frac{1}{\beta}$$

(Refer Slide Time: 09:17)

η consists of (i) η_ϕ → efficiency due to the redundant work loss
(ii) η_f → efficiency due to frictional losses.

$\eta = \eta_\phi + \eta_f$

Drawing stress involving friction and redundant work



$\sigma_d = \frac{F}{A_f} = \bar{\sigma} \left\{ \left(1 + \frac{\mu}{\alpha}\right) \ln\left(\frac{A_0}{A_f}\right) + \frac{2\alpha}{3} \right\}$ — (5)

$\mu = \text{Coeff of friction}$
 $\alpha = \text{semi cone angle}$
 $B = \mu \cot \alpha$

$\sigma_d = \bar{\sigma} (1+B) \phi_1 \ln\left(\frac{A_0}{A_f}\right)$ — (6)

$\sigma_d = \bar{\sigma} \left(\frac{1+B}{B}\right) \phi_1 \left(1 - \left(\frac{A_f}{A_0}\right)^B\right)$ — (7)

(1+B) ϕ_1 is similar to β in eqn (4)
 $(1+B) = \frac{1}{\beta}$

$\phi_1 = \frac{1}{\eta_\phi}$

So it is just a representing in a different way and this beta the efficiency factor eta consists of 2 factors consist of 1 and efficiency due to due to the redundant workload and second eta f the efficiency due to frictional losses. So that means beta equal to this phi + f, so if you just looked at that is from this as a basic these are the most simplest form of representation of the drawing provided.

$$\eta = \eta_\phi + \eta_f$$

We know the efficiency factor or if you know that beta value any of these things if you know you can find out what will because in the expression the stress necessary for homogeneous deformation is there other things are based on this you are multiplying with the efficiency factor okay.

So now let us look at the drawing stress involving friction and redundant work okay what are the things, you will find that several expressions have come up because earlier our derivation was simple with the friction redundant work did not come and we also discussed about trying a wire drawing operation how we can find out the redundant work from a stress simple stress-strain curve of a fully annealed wire and also a material which has been drawn to this particular reduction and from that if you shift it and superimpose it how much it comes.

So epsilon star that we have found due to the drawing operation from that we can find out the redundant deformation but it is always it may not be true because it will not take care of the variation in the strain across the cross section. So that is one major problem which is coming and it also takes most of this case takes only for a single drawing operation multiple drawing operation it becomes very complicated it is not behaving in a proper way.

So you will find several expressions are has been proposed to account for the friction and redundant work and the geometry of the die. These three factors are interrelated and that drawing stress necessary depends upon all these three factors. One of the expression which has been widely accepted is by Siebel. So where the draw stress sigma d which is nothing but the total force by the final cross sectional area they have expressed in terms of your mean flow stress or average flow stress in this way it is = $1 + \mu/\alpha$ into $\log A_0/A_f$.

So this is $1/1 - r$ okay + $2\alpha/3$ so this relationship people have arrived at. So this we can write it as expression number 4 in this you will see that μ is the coefficient of friction alpha is the semi die angle if it is a converging die like this the material is being drawn like this and it is coming out. So this is that 2α or we can say this angle is alpha okay this is 2α .

$$\sigma_d = \frac{F}{A_f} = \bar{\sigma} \left\{ \left(1 + \frac{\mu}{\alpha} \right) \ln \left(\frac{A_0}{A_f} \right) + \frac{2\alpha}{3} \right\}$$

So semi die angle is alpha and in this this last term it takes care of the redundant deformation $\frac{2\alpha}{3}$ it takes care of the redundant deformation okay but you have to be very careful in using this expression you see that it is coming all other things are there. So if there is a small difference in the inaccuracy in your average flow stress this is going to what is called magnify in a big way in the total draw stress.

So one has to be very careful that okay so this equation this equation number 5 it deals with the redundant work by adding an extra term to the basic equation. Basic equation is for your homogenous deformation that is this part okay. So this extra terms are being added and there is some merit in dealing with the redundant work by using a multiplying factor on the basic equation for draw stress in the presence of friction okay.

So 2 mostly used equations are another 2 equations are $\sigma_d = \bar{\sigma}$ into which we have discussed in the previous class i hope into ϕ_1 into $\log A_0/A_f$ you will find that these more or less all these things are of the similar way and another expression is there $\sigma_d = \bar{\sigma}$ into $1 + B/B$ into ϕ_1 log ϕ_1 into these are all expressing in different way that $1 - A_f/A_0$ to the power B.

$$\sigma_d = \bar{\sigma}(1 + B)\phi_1 \ln\left(\frac{A_0}{A_f}\right)$$

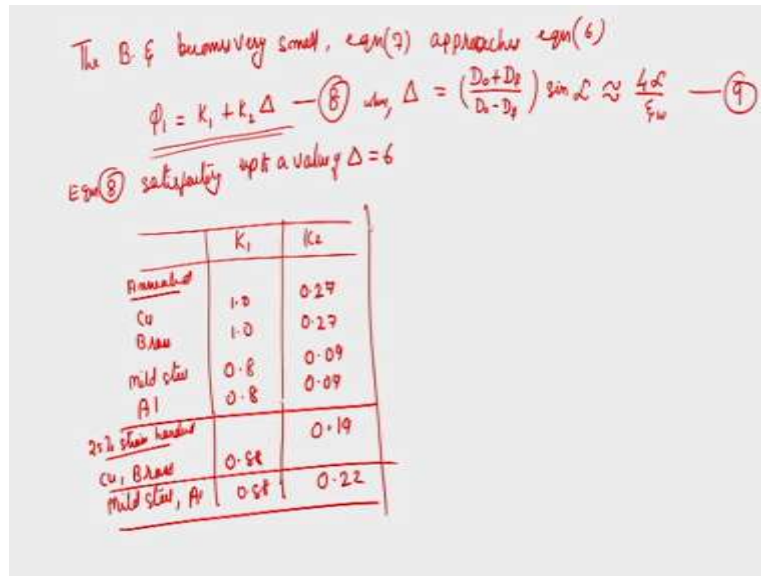
$$\sigma_d = \bar{\sigma}\left(\frac{1 + B}{B}\right)\phi_1\left(1 - \left(\frac{A_f}{A_0}\right)^B\right)$$

So this is expression 7 where $B = \mu \cot \alpha$ μ is the coefficient of friction and α is the die angle okay semi die angle. So the in equation 6 the term $1 + B$ into ϕ_1 has the same meaning as your beta here as i say in equation number 4. So it is almost the same okay and so we can say that is similar to or same meaning to beta in equation 4 or we can say that $1 + \beta$ $1 + B$ we can say is equal to 1 by this f frictional force because b is nothing but it is a function of your coefficient of friction okay. Whereas this ϕ_1 is nothing but 1 by this efficiency due to redundant work loss.

$$B = \mu \cot \alpha$$

$$(1 + B) = \frac{1}{\eta_f} \quad \phi_1 = \frac{1}{\eta_\phi}$$

(Refer Slide Time: 17:50)



And you will find that when the product when b into epsilon becomes very small what was that equation number equation 7 approaches equation 6 almost to equation 6 not completely and we should know this what is this phi 1. So phi 1 is related to the geometry of drawing that means your deformations on geometry. So that is a linear function of the what you call does the that parameter delta that means $\phi_1 = k_1 + k_2 \Delta$ where this parameter delta which we have discussed I think it may be in the lecture number 3 or 2 or 3.

$$\phi_1 = k_1 + k_2 \Delta$$

I do not remember exactly that deformation I think it is lecture 3 so in this. So that can be related for your wire drawing operation as the $d_0 + d_f$ the final diameter by d_0 minus final diameter into sin alpha which you can say it is almost equivalent to $4\alpha/w/\epsilon_w$ and this this equation is a linear equation with respect to this parameter delta the deformation zone parameter delta and that is satisfied it is a linear relationship up to say it is satisfactory up to say $\Delta = 6$ up to a value of $\Delta = 6$ equation 8 is satisfied. But beyond that if the delta value goes beyond that if you recollect that 1 curve 1 graph.

$$\Delta = \left(\frac{D_0 + D_f}{D_0 - D_f} \right) \sin \alpha \approx \frac{4\alpha}{\epsilon_w}$$

If have shown it which was taken from writer book that with the up to very high value also there but beyond that what happens as the linearity is lost resolve we will end up with a lot of errors in

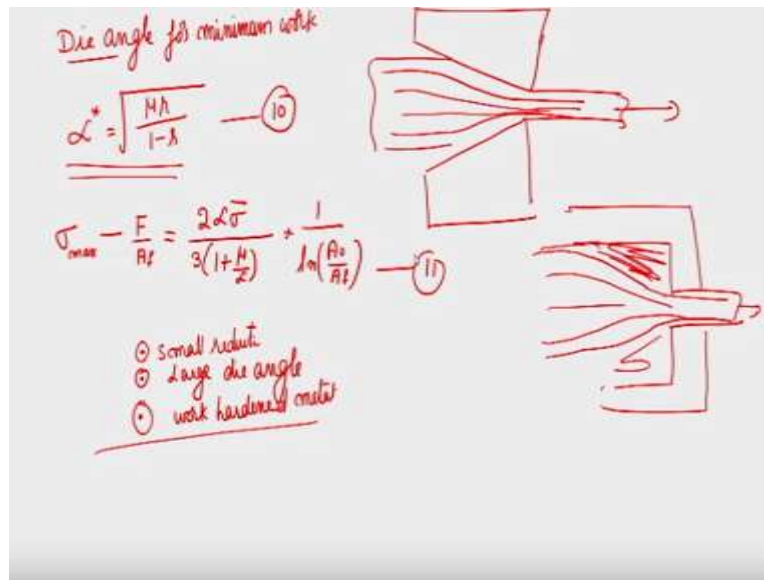
their prediction and you will find that lot of work has been done some values of k_1 and k_2 . So maybe we can write it as in a table form we can write say k_1 and k_2 in the annealed condition if you look for materials say for example copper brass this is 1 and the 0.27 and brass is also same value 0.27 and mild steel it is 0.8 and 0.09 and aluminium you will find that it is 0.8 and 0.09.

From experimentally determine the values of k_1 and k_2 and if it takes at 20 after 25% strain hardening. So copper and the brass it is 0.88 and this is 0.19 whereas for aluminium it is 0.88 and this is 0.22 k_2 is 0.22. So that way we can find it from the literature different values and then we can find you see that there is not much variation but it is satisfactory by using this equations and by using this equation 6 and 7 any of this equation on the using this mean yield stress whether this is a correct value where the predictions are going to be right.

So people did investigators carried out extensive work and then they found that if you are using this mean flow stress in these equations if you are using the mean flow stress a σ bar over the strain following the Hollomon relationship that means σ is equal to $A \epsilon^k$ raised or k epsilon raised whatever you in whichever form you write and if you are satisfied with that and taking the average stress value you found that using the average stress value the draw stress which was obtained was an underestimation.

However, the error which comes through by using of the underestimated by using the main flow stress is already 10%. So when it just 10% is almost acceptable okay my investigator so you do not have to spend it too much of energy and the time on having a very accurate prediction of the draw stress required because your machine also has a limitation. So that is what their conclusion said.

(Refer Slide Time: 24:00)



Now we also discussed that the minimum die angle for minimum work see there are 2 opposite end if the die angle is small the work and for a particular constant reduction if the die angle is small then your work phase die work phase interface area is large. So that will result in a very high frictional force. So frictional force will increase with the reduction in the die angle but if the I have drawn earlier also if you look at the metal flow condition it will just come take a turn like this and go.

So you are in general shearing becomes so with a low die angle the velocity discontinuity will be reduced. So that means in general shearing will be reduced that will result in the less amount of redundant work for when you are using low die angle. But with the with the die angle increasing see for example if you take a 90 degree die $2\alpha = 90^\circ$ and you are trying to pull the material like this.

The metal which will be flowing will be like a dead material so on will come and then there is an internal shearing which is taking place okay. So here you will find a dead metal zone. So large amount of energy will be spent for this redundant work with the die angle but the thing is that the frictional force will be reduced considerably. So these 2 opposing trends come so when you look at it you have to have a minimum die angle and several empirical relationships and derived relationships were also proposed one by WES range.

That the minimum die angle alpha star is equal to in this lecture I am not going into the derivation of all these things so that these details can be used for practicing engineers so mu into your reduction/1 - r where r is the reduction in cross sectional area. This is a very simple expression and this.

$$\alpha^* = \sqrt{\frac{\mu r}{1 - r}}$$

So simple but it can be used to have a first range state estimator of the drawing stress that is the biggest advantage of using this relationship and you may see that from this expression this alpha star that the minimum die angle is it dependent only on your mu and reduction okay and so that is the die angle will depend only on the coefficient of friction and reduction it will not be depending upon the precise work hardening behaviour of the material. What is the work hardening exponent?

It is not interested it is not interested in what is your strength the coefficient of that or the constants in the Hollomon relationship which is not dependent that means it is not dependent upon the deformation behaviour of the material itself or flow stress of the material it completely depends upon the coefficient of friction at the die work piece interface and the reduction which is required.

So that is one advantage of that now having come across this after deformation if you know the deformation behaviour of the material then you can also have an idea about what is going to be the draw stress but you will find that after drawing operation there is going to be a longitudinal stress across the cross section if it is circular across the diameter if it is rectangular across the thickness there is going to be a variation the longitudinal stress because stresses are the longitudinal stress are developed because of the tensile stress.

The plastic deformation is taking place and due to which and it is plastic deformation taking place or longitudinal stresses will be there you will find at the centre a stress is higher than at the surface okay the longitudinal stresses. At the wire surface the stress is lower whereas at the

centre the stress is high okay and sometimes at the centre the stress which are developed is higher than your applied stress also applied drawing stress.

So that means if you look at the mean stress based on that if you compute you will find that at the centre it may be still higher than that. So the excess stress on the centre line above that required for drawing okay so we can say that is given by the difference between the centreline stress σ_{max} and the average draw stress. So f/A_f is people researchers have arrived at this value by 3 into $1 + \mu/\alpha$ into $1/\log A_0/A_f$ okay.

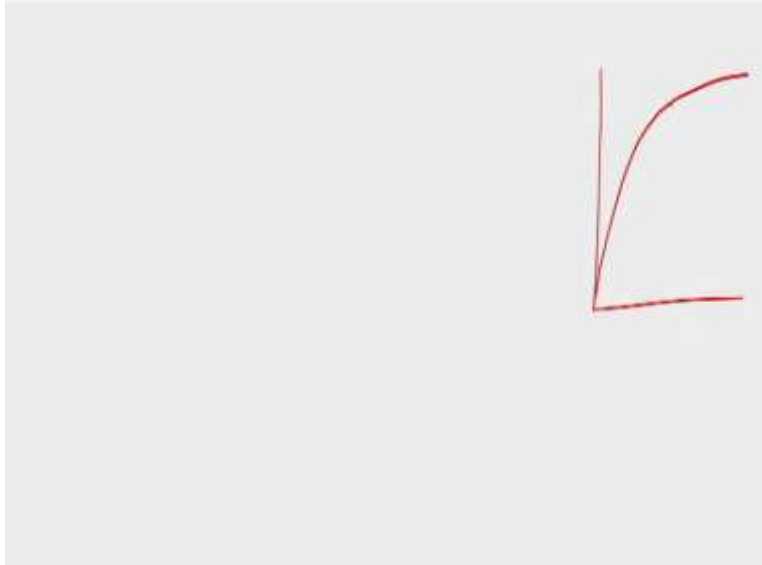
$$\sigma_{max} - \frac{F}{A_f} = \frac{2\alpha \bar{\sigma}}{3(1 + \frac{\mu}{\alpha})} \times \frac{1}{\ln\left(\frac{A_0}{A_f}\right)}$$

So this is that excess stress on the centre line above that required for drawing that is given by the difference between this the maximum centreline stress which is going to have and the average draw stress required there. See this differential stress across the cross section see that centre it is large the deformation is very large in tensile strain is taking place whereas at the outer surface the stress is very less.

So it will try to come back but the centre is trying to deform more what will happen is that at the centre of the sample during drawing you may end up with the chevron cracking. So cracking at the centre will take place in the presence of tensile stress if it is above certain limit it will result in centreline cracking as the shape of a chevron shape okay. So you know that what is chevron shape chevron that shape is there.

So that is that cracks may develop and the parameters by which this cracks can develop is one is a small reduction, second is larger die angle, third is the work hardened metal. So it depends upon all these 3 so this is equation number 11. So this is the general expressions and the minimum axial tensile stresses this comes as axial tensile stresses during the drawing operation.

(Refer Slide Time: 31:51)



That is fine we have arrived at this but how to determine the redundant work, this is very important okay. So we wanted to know how this redundant work can be estimated. So one method we have explained earlier was that you take a material in a fully annealed condition and find out what is its stress-strain flow curve and then when it is coming maybe after some amount you know you will find that you find out the stress-strain curve for the drawn wire and you will find that in the drawn wire for the same reduction the yield strength is high.

Because of the internal shearing and other things. So you shifted towards the right displace it so that it matches with the stress-strain curve and then when it matches with that.

(Refer Slide Time: 32:46)

ξ^* corresponds to ξ

Estimation of Redundant work & redundant deformation

Method - I (for redundant work ϕ_1)

based on determination of redundant deformation ϕ_2 :

$$\phi_1 = \left\{ \frac{1 - e^{(-\frac{1}{2} B \xi_w)}}{1 - e^{(-B \xi_w)}} \right\} \phi_2 \quad (12)$$

if $B \xi_w \approx \left(\frac{\Delta \mu}{\Delta}\right)$ is very small, then $\phi_1 = \phi_2^m$

ϕ_2 is termed as "Redundant deformation".
This is the measure of extra deformation (or equivalent strain ξ^*) sustained by the 1st of wire compared to the nominal strain, ξ_w , (homogenous strain) in the wire drawing pass.

$$\xi^* = \phi_2 \xi_w \quad (14)$$

Method - II (for redundant work ϕ_1)

based on determination of redundant deformation ϕ_2 :

if $B \xi_w \approx \left(\frac{\Delta \mu}{\Delta}\right)$ is very small, then $\phi_1 = \phi_2^m$

ϕ_2 is termed as "Redundant deformation".
This is the measure of extra deformation (or equivalent strain ξ^*) sustained by the 1st of wire compared to the nominal strain, ξ_w , (homogenous strain) in the wire drawing pass.

$\xi^* = \phi_2 \xi_w$

$m = \text{work hardening exponent.}$

$\sigma = k \xi^m$

How much is the increase in the strain epsilon strain, ϵ^* okay corresponding to equivalent strain epsilon from that you find out what is the excess strain which just taken place. So that is one by and the ratio by this you will get the value of phi 1 that is what we discussed. But this will this is not a very accurate method also but in a very simple technique we can find out but there are other methods of finding out the redundant work is there.

So they are for finding out the value so estimation of redundant work and redundant deformation the method the first method 1. We have several methods I will just discuss 1 or 2 methods the method 1 is based on the determine and determination of the redundant deformation ϕ_2 , based on determination of redundant deformation ϕ_2 for redundant work phi 1. So this ϕ_2 we have to find out and then correlate with the ϕ_1 .

So phi = 1 - exponential - phi 2 B epsilon w divided by 1 - exponential - B epsilon w into phi 2 to the power n. This way we can find and you will find that this product 2 B into epsilon w is almost equal to say $4\mu/\Delta$. So if this is very small sufficiently small then phi 1 = phi 2 to the power n where n is the work hardening exponent. Exponent of the relation which is nothing but sigma is equal to say maybe k epsilon raised to n okay.

$$\phi_1 = \left\{ \frac{1 - e^{(-\phi_2 B \epsilon_w)}}{1 - e^{(-B \epsilon_w)}} \right\} \phi_2^n$$

if $B \epsilon_w \approx \left(\frac{4\mu}{\Delta}\right)$ is very small, then $\phi_1 = \phi_2^n \quad \sigma = k \epsilon^n$

So that is that work hardening exponent which is coming so you should understand the difference between ϕ_1 and ϕ_2 . So ϕ_2 has been termed redundant deformation it is not the redundant work it is a redundant deformation to distinguish it from ϕ_1 . So ϕ_2 I will write it ϕ_2 is termed as redundant deformation and this is a measurement of the extra strain as a result of redundant deformation at the work.

So equivalent strain we are reaching that if you refer back to that lecture maybe third lecture or second lecture I am not sure the basic fundamentals of plastic deformation if you look at it effect

of die angle and other things you have come across there you will find that redundant deformation is defined and I have very clearly mentioned that how to determine that ϵ^* okay.

So and this is the measure of extra deformation or maybe equivalent strain sustained by the rod or wire compared to the nominal strain. What is nominal strain? Nominal strain is for the uniform homogeneous deformation. That is homogeneous in the wire drawing pass here we can say ϵ_w . So that means you can define this ϵ^* as nothing but ϕ_2 into ϵ_w from the stress-strain curve for a what you call it as a annealed material.

$$\epsilon^* = \phi_2 \epsilon_w$$

See if you just take this case it is like this is for the annealed material and then for you may find that after a strain of epsilon and take this you will find that it is just raised one. So if you just if you move this up to here it will match with this okay so this is called as the epsilon strain. So if you translate this into this region so you are getting this epsilon strain. So that is this relationship which you are getting here okay from this relationship. So that is what you should understand that this is the redundant deformation and not redundant work.

(Refer Slide Time: 40:49)

$\phi_2 = C_1 + C_2 \Delta$ — (15)
 $C_1 = 2.25(m)^{0.28 - 0.10} k$ — (16)
 $C_2 = 0.367(m)^{0.26 - 0.057} k$ — (17)

Method - 2
 If the σ - ϵ curves drawn at different conditions are compared to σ - ϵ curve of annealed wire, then the drawn wire will have higher strain for similar stress.

$\epsilon^* = K_1 \epsilon_w + K_2 \sin \alpha$ — (18)
 $\phi_2 = K_1 + K_2 \frac{\sin \alpha}{\epsilon_w}$ — (19)

ϵ^* vs ϵ_w plot will have slope as K_1 and intercept on ϵ_w axis.

So and this phi 2 can be related with your deformations on geometry okay as $c1 + c2$ into delta this is also a linear regression similar to the phi 1 also you are getting where phi is defined by equation earlier we have mentioned so the $d f - d_0$ $d_0 - d_f /$ that relationship where is that equation

number 9. So this relationship and it is very interesting to note that when you do a large number of when it was a large number of experiments are conducted with a wide work hardening characteristic on materials having wide work hardening characteristics.

$$\phi_2 = c_1 + c_2\Delta$$

This c_2 and c_1 they can be correlated by to its work hardening exponent as $c_1 = 2.25$ into your n raised to 0.28 into $k = -0.10$ and c_2 is equal to I will just not write it down 0.367 n raised to 0.76 k raised to -0.54 so this is 17. So with the wide variation in work hardening behaviour with n and dk where $\sigma = k \epsilon^n$ this is the flow stress relationship you will find that for a large number of materials when you do it the c_1 and c_2 is coming like this.

$$c_1 = 2.25(n)^{0.28}k^{-0.10}$$

$$c_2 = 0.367(n)^{0.76}k^{-0.054} \quad \sigma = k\epsilon^n$$

So that is a very interesting thing so that we can find out the c_1 and c_2 and then from that if you know the deformation zone geometry that parameter Δ from that we can calculate the ϕ_2 and then from that we can determine ϕ . This is one method of determining the deformation sort of redundant work. Another is method 2 in this what happened is that the stress-strain curves are drawn at different conditions.

You are obtaining the stress-strain curve for drawn wire maybe like you have drawn for different reduction and then they are compared with the stress-strain curve of a fully annealed field okay then the initial flow stress following any given strain in wire drawing is greater than the flow stress produced by straining homogeneously to the same strain in simple tension.

So that is what you will find out and this difference is due to the redundant deformation. So if the stress-strain curve drawn at different conditions are compared to the stress-strain curve of annealed material then what will happen the drawn wire will always have a higher stress for a particular strain then the drawn wire will have higher stress for a constant strain compared to your homogeneously differed deformation for a tensile pulled a sample what is the in that condition okay.

So this difference is due to the redundant deformation and the symbol the strain in simple tension corresponds to the yield stress of the drawn wire is thus an equivalent strain ϵ^* which we have discussed. So you will find that for a number of metals this ϵ^* can be correlated by the nominal strain by $\epsilon_w + k_2 \sin \alpha$ so if you use the relationship 14 okay this 14 if you use it then you can get that yourself $\phi_2 = k_1 + k_2 \sin \alpha / \epsilon_w$ you can get it this is suffix w okay.

$$\epsilon^* = k_1 \epsilon_w + k_2 \sin \alpha$$

So if you just a plot that ϵ^* versus ϵ_w plot we will have k_1 as the slope we will have slope as k_1 and intercept on ϵ^* axis as $k_2 \sin \alpha$. This advantage of this method is that only 2 data points are required for one particular value of α because it is straight to an equation. So one particular α no you just take care so for 2 reduction and then get the value and then you can get the value of k_1 and so it becomes very simple. Hence ϕ_k can be determined for any die angle so that way it is very convenient.

$$\phi_2 = k_1 + k_2 \frac{\sin \alpha}{\epsilon}$$

(Refer Slide Time: 47:01)

ϵ_w eqn (15) and (19) identical and using equation (9)
 $c_1 = k_1$ and $4c_2 = k_2$ for practical wire drawing reduction

Method (iv)
 Measure the hardness of wire across the transverse section.
 The equivalent strain for the whole cross section ($\bar{\epsilon}^*$) can be determined by graphical integration.
 The ratio $\bar{\epsilon}^*$ to the nominal wire drawing strain will give the value ϕ_2 .

$\bar{\epsilon} = A(H)^m$

So and if you look at that from equation 15 and in 19 are identical and the using equation 9. So we can get $c_1 = k_1$ and $4c_2 = k_2$ a very good approximation we can get it for all practical wire drawing operations. Now next is their method 3 that means that also is a very convenient form of

determining the work function. In this you have measure the hardness of the drawn wire across the cross section okay.

$$c_1 = k_1 \text{ and } 4c_2 = k_2$$

Measure the hardness of wire across the transfer section in a continuous form you are measuring at equal spacing and then get a plot of that okay but before that the material no you just homogeneously you do from do it and sell test maybe for some reduction of 5% 10% 20% 30% 40% like therefore different reductions you just get a series of experiments done. So deform it you say different reductions so that because it is a tensile pulling then you can easily get the equivalent strain there and section 8 after each reduction find out the hardness of that material because it is deforming in a homogeneous form. So find out the hardness of that material.

So you will get a plot of hardness versus strain for homogeneous strain for the hardness verses homogeneous strain you can get a plot and now in your sample after drawn wire you section it and across the section like if you a circular piece along the diameter you just to find out what is the hardness here at this different points and if this is a centre. So you can say this is say 1,2,3,4 like that you can say 1,2,3,4.

So along the across the if it is rectangular piece it across the thickness you measure that so then at this point you just take the value of this hardness and correlate it to its equivalent strain from maybe you are so for this homogeneous strain with the hardness may be like something it will come. So from this value you can just find out what is the hardness at 1,2,3,4 okay. So in that case you can always find the equivalent strength for the cross section.

The equivalent for the whole cross-section that is ϵ^* can be determined by graphical integration. So from this master plot you measure this and then you find out what is this epsilon star 1 epsilon. I will draw it in a bigger way so you can say $\epsilon^* 1 \epsilon^* 2 \epsilon^* 3 \epsilon^* 4$ like that you can get it the value it was our drawn wire and then this value the equivalent strain for the whole cross sectional area.

You can find out by graphical integration okay and the ratio of epsilon dot star to the nominal wire drawing strain will give the value of ϕ_2 see once you get this relationship know this hardness versus this one. So we can say that the strain versus so that can be you fit a curve for this it can be plotted as AH ratio n m we can n we are using this so here we can say H and this is so this similar to see what is addressed similar to that.

$$\epsilon = A(H)^n$$

So this H instead of the strain we can convert it to H. So this become very easy only you have a master curve you develop it to first rate conducting some experiment and after that note for any value you can find out what is this equivalent strain. The ratio of epsilon dot bar star we can get it and from that from this ratio to your nominal strain will give you the value of ϕ_2 once ϕ_2 is known you just look at it whether B into e is large or small and then from that we can find out.

Okay this technique is very satisfactory for a single pass wire drawing operation for single pass wire drawing operation this is satisfactory but for a multi pass this is not found that much suitable okay or satisfactory. The reason which is still not being established so with this for today we will talk the next we will you have some practical classes. So some problem using this part which you had discussed.