

Plastic Working of Metallic Materials
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Lecture – 27
Wire Drawing: Tutorial Problems

So, we were discussing about various relationships in wire drawing process and as mentioned earlier for the drawing stress itself know, you can look at whether it is the stress necessary for homogeneous deformation or stress necessary for deformation in considering the friction or at the same time, considering the redundant deformation and the different researchers have come out with the different relationships.

But we will find that many of the case when you do it, the difference is only marginal but within certain acceptable limits, so these equations can be used but so though there were several relationships which you have discussed in the last class, I thought we will have some tutorial problems, so that the students will be able to understand how to use this relationship, okay.

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Note: Following expressions may be used for solving the problems:

- Flow stress relationship for annealed copper is $\sigma = 320 \epsilon^{0.45} \text{ MPa}$.
- **Siebel's** relation for draw stress is $\sigma_d = \frac{F}{A_f} = \bar{\sigma} \left(\left(1 + \frac{\mu}{\alpha} \right) \ln \frac{A_0}{A_f} + \frac{2\alpha}{3} \right)$.
- Redundant Work factor ϕ_1 for copper is $\phi_1 = 1.0 + 0.27 \Delta$.
- The constants in the expression for redundant deformation ϕ_2 are:
 $C_1 = 2.25n^{0.28} K^{-0.10}$; $C_2 = 0.367n^{0.76} K^{-0.054}$

1. Copper rod of 10 mm diameter is drawn through a conical die to obtain a fractional reduction in area of 32%. The average coefficient of friction at the die-work piece interface during the drawing operation is 0.15.

(a) Determine the drawing load, in Newtons, if 45% of the total load is used in overcoming friction and redundant deformation (calculate using the mean flow stress).

(b) Determine the optimum die angle for the reduction.

(c) If the die angle (2α) is 40° , determine the drawing load, in Newtons, for the same reduction, using the relationship of Siebel.

So, in entire this tutorial session this data will be used, so for one particular material we will be using it for the annealed copper, where the stress strain relationship is given by 320 into ϵ raised to 0.45, it is 0.45, okay mega Pascal and the Siebel's relationship for draw stress which was not discussed which was just in a glancing way we have used for the draw stress considering redundant, considering the friction and die angle for a taper conical die angle.

So, this relationship is given where it also takes care of the redundant deformation, then redundant work factor of ϕ_1 for copper, the value is this which is obtained from the literature and the constants in the expression for redundant deformation ϕ_2 and is redundant work and this is redundant deformation ϕ_2 that $C1 + C2$ into Δ , so the $C1$ the constant is this, $C2$ is this one. So, this entire data will be used to be having some 3, 4 problems and the entire thing will be using this relation.

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Problem -1.
Fully annealed Copper rod of 10 mm diameter is drawn through a conical die to obtain a fractional reduction in area of 32%. The average coefficient of friction at the die-work piece interface during the drawing operation is 0.15.

(a) Determine the drawing load, in Newtons, if 45% of the total load is used in overcoming friction and redundant deformation (calculate using the mean flow stress).

(b) Determine the optimum die angle for the reduction.

(c) If the die angle (2α) is 40° , determine the drawing load, in Newtons, for the same reduction, using the relationship of Seibel.

$d_i = 10 \text{ mm}$ $\mu = 0.15$ $\lambda = 0.32$ $\xi_w = \lambda \ln\left(\frac{1}{1-\lambda}\right) = 0.15 \ln\left(\frac{1}{1-0.32}\right) = 0.3856$

$\bar{\sigma} = \frac{1}{0.3856} \int_0^{0.3856} 320 \cdot \xi^{0.45} d\xi$ $\sigma = 320 \xi^{0.45}$

$= \frac{320}{0.3856 \times 1.45} \left[\xi^{1.45} \right]_0^{0.3856} = 143.72 \text{ N/mm}^2$

for homogeneous deformation, $\sigma_d = \bar{\sigma} \ln\left(\frac{1}{1-\lambda}\right) = 142 \times 0.3856 = 55.43 \text{ N/mm}^2$

So, question number 1; first let us come to a fully annealed copper rod of 10 millimeter diameter is drawn through a conical die to obtain a fractional reduction in area of 32%, rates okay. The average coefficient of friction at the die work piece interface during the drawing operation is 0.15, so coefficient of friction; Coulumn's coefficient of friction, determine the drawing load in Newtons, if 45% of the total load is used in overcoming friction and redundant deformation.

Calculate using the mean flow stress, it is not man flow stress, it is mean flow stress, mean flow stress relationship because they found a different relationships but the easiest thing is the mean flow stress, though if you take the mean flow stress, the value will be somewhat less than what is actually required but has been told mentioned earlier as long as it is within 5%, it is okay, it is acceptable.

So, then determine the optimum die angle for the reduction we have the relationship for die angle and the α^* and if the die angle, 2α is 40 , determined the drawing load in Newton's for

the same reduction using the relationship of Siebel, so we will have a comparison between if you use these 2 relationship and find out how much it is varying, okay.

So, our case is d_i ; $d_i = 10 \text{ mm}$ and $\mu = 0.15$ and $R = 0.32$, 32% means R we are taking as 0.32 and so from this we can find out the strain, the strain for wire drawing is equal to $\ln \frac{1}{1-R}$ okay, so that is $= \ln \frac{1}{1-0.32}$ that turns out to be 0.3856, so now when we are using the; because calculate using the mean flow stress relationship, so we have to find out the mean flow stress.

$$d_i = 10 \text{ mm}, \mu = 0.15, r = 0.32$$

$$\epsilon_w = \ln \left(\frac{1}{1-r} \right) = \ln \left(\frac{1}{1-0.32} \right) = 0.3856$$

Mean flow stress here for a strain of 0.3856, so we can find out this, which is equal to $\frac{1}{\epsilon_b - \epsilon_a}$, $\epsilon_a = 0$, so that means divided by 0.3856 into integral from 0 to 0.3856 and the stress relation, so $\sigma = 320 \epsilon^{0.45}$, so we have to write 320 into $\epsilon^{0.45}$ d ϵ , so that comes out to be 3856 into 1.45 into $\epsilon^{1.45}$ from 0 to 0.3856, so that comes out to be 143.72 newton per millimeter square.

$$\bar{\sigma} = \frac{1}{0.3856} \int_0^{0.3856} 320 \epsilon^{0.45} d\epsilon$$

$$= \frac{320}{0.3856 \times 1.45} [\epsilon^{1.45}]_0^{0.3856} = 143.72 \text{ N/mm}^2$$

So, this is the average stress we are going to get it, okay for this particular range of strain that means, from 0 0.3856 or the reduction of 32% for a full annealed copper, the flow stress as per the data given the flow stress; average flow stress is 143.72 Newton per millimetre, so that if you are using this mean flow stress relationship, we can write it as $\sigma_d = \bar{\sigma} \ln \frac{1}{1-R}$, stress in the strain, this is for the homogeneous deformation okay.

So, for σ_d for homogeneous deformation, we can say for homogeneous deformation, if you ignore the friction; the effect of friction as well as the redundant deformation, this is what we are going to; the stress necessary for drawing $\sigma_d = 142 \ln \frac{1}{1-R}$ you are getting it as 0.3856, so that comes to me something around 55.43 Newton per millimetre, so your draw stress is this.

$$\sigma_d = \bar{\sigma} \ln \left(\frac{1}{1-r} \right) = 142 \times 0.3856 = 55.43 \text{ N/mm}^2$$

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Handwritten derivation showing the calculation of drawing force and angle α^* .

$$\begin{aligned} \text{Draw force } F &= \sigma_d \times A_f \\ &= \sigma_d \times \frac{\pi}{4} (8.246)^2 = 55.43 \times \frac{\pi}{4} (8.246)^2 = 2959 \text{ N} \\ &= 2.959 \text{ kN} \\ F_{\text{Total}} &= F_{\text{hom}} + F_{\text{(Friction + Redundant)}} \\ &= 0.55 F_{\text{hom}} = 2.54 \text{ kN} \\ \Rightarrow F_{\text{Total}} &= \frac{2959}{0.55} = 5.38 \text{ kN} \\ \alpha^* &= \sqrt{\frac{\mu \lambda}{1-\lambda}} = \sqrt{\frac{0.15 \times 0.32}{1-0.32}} = 0.265968 \text{ rad} \\ &= 15.21^\circ \end{aligned}$$

So, you are; now you have to calculate the load; load is draw stress into the cross sectional area at the output okay, so that means the total frictional stress, so $F =$ draw force, F for homogeneous deformation = σ_d into A_f , so we have to find out the A_f , so because we know that this is equal to strain is equal to $\ln \frac{A_0}{A_f}$, okay that is equal to $2 \ln \frac{d_0}{d_f}$, okay but now we have to find out d_f , so that means d_f is equal to and this is equal to 0.38566.

$$\epsilon = \ln \left(\frac{A_0}{A_f} \right) = 2 \ln \left(\frac{d_0}{d_f} \right) = 0.38566$$

$$d_f = \frac{d_i}{e^{\left(\frac{0.38566}{2} \right)}} = 8.246 \text{ mm}$$

So, from this we can get a $d_f = d_i / e$ raised to 0.38566 divided by 2, so that comes to this is e raised to; so that comes to 8.246 mm, so d_f is there, so this is equal to σ_d into $\pi/4$ 8.246 square, so that is equal to $\sigma_d = 55.43$ into $\pi/4$ 8.246 square, so that comes to say your draw stress for homogeneous deformation is 2959 Newton; kilo Newton, so this is equal to 2959 Newton, so this will be equal to kilo Newton value.

$$\begin{aligned} \text{Draw force } F \text{ (homogeneous deformation)} &= \sigma_d \times A_f \\ &= \sigma_d \times \frac{\pi}{4} (8.246)^2 = 55.43 \times \frac{\pi}{4} (8.246)^2 = 2959 \text{ N} \\ &= 2.959 \text{ kN} \end{aligned}$$

Now, the question is; so this is for the homogenous deformation, the force necessary for homogeneous deformation, in the question if you look at it; if 45% of the total load is used in overcoming friction and redundant deformation, determine the drawing loads, so that means

this is for homogeneous deformation, so if you write in the general equation that means, if suppose $F_{total} = F_{\text{for homogeneous deformation}} + \text{the force necessary for overcoming friction} + \text{redundant deformation}$, okay.

$$F_{total} = F_{homo} + F_{(Fric.+Red.defor.)}$$

So, whereas this redundant deformation F force for friction plus redundant deformation is equal to 0.45 of F_{total} , so that way we can write that if you bring it to this side, so that is 0.55 $F_{total} = 2.959$ kilo Newton or from that we can calculate the $F_{total} = 2.59$ divided by 0.55, so that comes out to be to 2.59 divided by 0.55, this for point; 2.9, sorry 959; 2.959 divided by 0.55 = 5.38 kilo Newton.

$$F_{(Fric.+Red.)} = 0.45 \times F_{total}$$

$$0.55 F_{total} = 2.59 \text{ kN}$$

$$F_{total} = \frac{2.959}{0.55} = 5.38 \text{ kN}$$

So, this is thing for question number 1a, okay the total force is equal to this one because out of that you will find that redundant deformation, the energy for; the force for redundant deformation is 0.45 of this and for homogeneous deformation is 2.959, so this is how you calculate because if you are using that efficiency term, then this is how you can calculate okay.

Now, the question number b, is what is the determine the optimum die angle, so that we have got it, α^* it depends only on your coefficient of friction and the redundant deformation sorry, coefficient of friction and the reduction, it is never dependent upon the flow behaviour of the material, so that is what we have discussed last slide, so it will be under $\frac{\mu r}{1 - r}$, so that comes to 0.15 into 0.32 divided by; r was how much; 0.32, divided by $1 - 0.32$, so that we are getting it as 0.15 into 0.32 divided by 0.68 and under root point that is equal to 0.265968 radians.

$$\alpha^* = \sqrt{\frac{\mu r}{1 - r}} = \sqrt{\frac{0.15 \times 0.32}{1 - 0.32}} = 0.265968 \text{ rad}$$

$$\alpha^* = 15.21^\circ$$

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$\alpha = 20^\circ$, $\sigma_d = \frac{F}{A} = \bar{\sigma} \left\{ \left(1 + \frac{\mu}{\alpha} \right) \ln \left(\frac{A_0}{A_f} \right) + \frac{2\alpha}{3} \right\}$ Siebel's relationship
 $\bar{\sigma} = 143.72 \text{ N/mm}^2$, $r = 0.32$, $\mu = 0.15$, $\alpha = 20^\circ = 0.349 \text{ rad}$
 $F = \left[\frac{\pi}{4} (8.246)^2 \right] \times 142.726 \left\{ \left(1 + \frac{0.15}{0.349} \right) 0.38566 + \frac{2 \times 0.349}{3} \right\}$
 $= 53.4 \times 142.726 [0.55146 + 0.2326] = 5975 \text{ N} = 5.975 \text{ kN}$
 $\% \text{ draw} = \left(\frac{5.975 - 5.38}{5.38} \right) \times 100 = 11\%$
 $d_0 = 10 \text{ mm}$
 $d_f = 8.246 \text{ mm}$

So that converting into angle, it will be around 15.21 degree, so alpha star = 15.21 degree now, the third part is if the die angle is 40 degree, if that means alpha = 20 degree, determine the drawing load in newton for the same reduction using the relationship of Siebel's, so Siebel's relationship in the first image itself is given to us, I will write it as sigma = sigma d = F/A which is equal to sigma bar that is a mean flow stress which we have already found it into 1 + Mu/alpha.

This is very widely accepted relationship that is why we had taken, into log A0/af + 2 alpha/3 but this is the Siebel's relationship and from that we already know that the mean flow stress is equal to how much; 143.72 newton per millimetre square okay, so r = 32; 0.32 and Mu = 0.15, right, 0.15, alpha = 20 degree, if you convert it into radians it will be 0.349 radians, alpha should be in radians, okay.

$$\alpha = 20^\circ, \quad \sigma_d = \frac{F}{A} = \bar{\sigma} \left\{ \left(1 + \frac{\mu}{\alpha} \right) \ln \left(\frac{A_0}{A_f} \right) + \frac{2\alpha}{3} \right\}$$

$$\bar{\sigma} = 143.72 \text{ Nmm}^{-2}, \quad r = 0.32, \mu = 0.15, \alpha = 20^\circ = 0.349 \text{ rad}$$

And we know that what is a A0 and Af, A0; sorry, d0 = 10, df = 8.242; 8.246, okay, so it is a very straightforward question, so that is F = pi/4 into 8.246 that is the area; cross section area into the mean flow stress, 142.726, this is F, into 1 + 0.15 divided by alpha; alpha in radians is 0.349 into log A0/af, so that is your strain, log A0/af = 0.38566 + 2 into 0.349/3, okay.

$$d_0 = 10 \text{ mm}, d_f = 8.246 \text{ mm}$$

$$F = \left[\frac{\pi}{4} (8.246)^2 \right] \times 142.726 \left\{ \left(1 + \frac{0.15}{0.349} \right) 0.38566 + \frac{2 \times 0.349}{3} \right\}$$

So that comes out to be 0.15 divided by 0.349 + 1 = into 0.38566 = 0.55146; 1416 + 2 into 0.3492 into 0.349 divided by 3, 0.2326, so it comes out to be 5975 newton or that is equal to 5.975 kilo Newton, the force, you see that by this 5.38 kilo Newton and now, 5.975 kilo Newton okay, the difference is only very less okay, the difference if you calculate it say for example, the percentage deviation in the value is equal to say 5.975 minus the other case we got it as 5.38; 5.38 divided by 5.38 into 100, so deviation is very less.

$$F = 53.4 \times 142.726\{0.551416 + 0.2326\} = 5975 \text{ N} = 5.975 \text{ kN}$$

So that comes to about 5.975 – 5.38 divided by 5.38, so it is only 11%, it is approximately 11% that is acceptable actually, in this case because that is because we are taking the average flow stress, okay and in one case, you are taking the value of; exit value of redundant die angle and the this one now, why this is high is because this is not the optimum die angle, optimum die angle we have got it as 15.21.

$$\% \text{ deviation} = \left(\frac{5.975 - 5.38}{5.38} \right) \times 100 \approx 11\%$$

So, maybe if you take that the optimum die angle as alpha star and then substitute here you may find the slightly lower values, it will not even come to 11%, it may be within plus or minus, it will be difference will be only 10%, okay. Now, with what you are comparing; you are comparing with that of the initial case and with the Siebel's case, if the base is change then it will show that a difference in values, still lower value.

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Problem-2
 For the material in question-1, during drawing using a conical die of semi die angle 20° , the average coefficient of friction at the die-work piece interface is 0.15

(a) determine the drawing load using the expression $\sigma_d = \bar{\sigma} (1+B) \phi_1 \ln \frac{A_0}{A_f}$.

(b) determine drawing load using the expression $\sigma_d = \bar{\sigma} \left(\frac{1+B}{B} \right) \phi_1 \left(1 - \left(\frac{A_f}{A_0} \right)^B \right)$ and find out the percentage deviation in the value obtained for the sub-section (a).

$\bar{\sigma} = 143.72 \text{ N/mm}^2$, $\alpha = 20^\circ$, $\mu = 0.15$, $\ln \left(\frac{A_0}{A_f} \right) = f = 0.38566$
 $B = \mu \cot \alpha = 0.15 \cot 20^\circ = 0.15 \tan(90-20)^\circ = 0.412$ $\beta = 0.412$
 $\phi_1 = 1 + 0.27 \Delta$ $\Delta = \left(\frac{d_0 + d_f}{d_0 - d_f} \right) \sin \alpha = \left(\frac{10 + 8.246}{10 - 8.246} \right) \sin 20^\circ$
 $\phi_1 = 1 + (0.27 \times 3.558)$ $\Rightarrow \Delta = 3.558$
 $\phi_1 = 1.961$
 (a), $\sigma_d = \bar{\sigma} \left(\frac{1+B}{B} \right) \times \phi_1 \left[1 - \left(\frac{A_f}{A_0} \right)^B \right] = 143.72 \left[\frac{1.412}{0.412} \right] \times 1.961 \times 0.38566 = 80.79 \text{ kN}$

So, now let us come to a second problem; for the material in question 1 during the drawing using a conical semi die angle of 20 degrees, the average coefficient of friction at the die work piece interface is 0.15, same problem. Determine the drawing load using the expression σ_d is equal to; because we also found using that redundant work and redundant deformation we have just taken it, okay.

So, one is that Φ_1 is the redundant work, so let us just because though we discussed about that we should know how to use it that is the main purpose of that and then make a comparison with it, so let us just do that. So, in the same problem when you look at it, we do not have to do the calculation because $\bar{\sigma}$, the mean flow stress we already got it as 143.72 Newton per millimetre square mega Pascal we can say.

And die angle α is equal to; semi die angle is 20 that is $\alpha = 20$ degree, die angle is 40, semi die angle is 20, coefficient of friction $\mu = 0.15$ and $\log A_0/A_f = 0.38566$, so $\log A_0$ that is a strain by A_f is the strain that is equal to 0.38566, this is what; so we have to just find out for; first we have to calculate what is this Φ_1 ; Φ_1 okay, so and in this B is there B , you know that what is $\mu \cot \alpha$, so that is equal to 0.15 into $\cot \alpha$ is 20 degree.

$$\bar{\sigma} = 143.72 \text{ Nmm}^{-2}, \alpha = 20^\circ, \mu = 0.15, \ln \left(\frac{A_0}{A_f} \right) = \epsilon = 0.38566$$

So that is equal to 0.15 into $\tan 90 - 20$ that is $\tan 70$ degree, so that will come to $\tan 70$ into 0.15, so 0.412, so $B = 0.412$, B you have got now, we have to find out Φ_1 is; Φ_1 in the initially, itself it is given as $1 + 0.27 \Delta$, where $\Delta =$; so d_i that is a deformations on geometry, $d_i + d_f$ for this particular wire drawing operation, we have written that in the last lecture it was mentioned $d_i - d_f$ into $\sin \alpha$, okay.

$$B = \mu \cot \alpha = 0.15 \cot 20^\circ = 0.15 \tan (90 - 20)^\circ = 0.412$$

$$\phi_1 = 1 + 0.27\Delta$$

So that is equal to d_i is $10 - 8.246$ divided by a plus, this is plus, $10 - 8.246$ into $\sin 20$, so $\Delta = 3.558$ value, so $\phi_1 = 1 + 0.27$ into 3.558, so that comes to; into $0.27 + 1.961$, so a is equal to; for a, $\sigma_d = \bar{\sigma}$ into $1 + B / B$ into ϕ_1 into $1 - A_f/A_0$ raised to B , so it is equal to 143.72 into B is 0.412, so 1.412 divided by 0.412 into ϕ_1 is 1.961 into $1 -$; so I can write it as; into $\log A_0/A_f$ is equal to your strain only.

$$\Delta = \left(\frac{d_i + d_f}{d_i - d_f} \right) \sin \alpha = \left(\frac{10 + 8.246}{10 - 8.246} \right) \sin 20^\circ$$

$$\Delta = 3.558$$

$$\phi_1 = 1 + (0.27 \times 3.558) = 1.961$$

$$\begin{aligned} \sigma_d &= \bar{\sigma}(1 + B) \times \phi_1 \left[1 - \left(\frac{A_f}{A_0} \right)^B \right] = 143.72(1.412) \times 1.961 \times 0.38566 = 8079 \text{ N} \\ &= 8.079 \text{ kN} \end{aligned}$$

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Handwritten calculations:

$$\begin{aligned} \sigma_d &= \bar{\sigma} \left(\frac{1+B}{B} \right) \phi_1 \left(1 - \left(\frac{A_f}{A_0} \right)^B \right) \\ &= 143.72 \left(\frac{1.412}{0.412} \right) \cdot 1.961 \cdot \left[1 - \left(\frac{8.246}{10} \right)^{2 \times 0.412} \right] = 140.329 \text{ N/mm}^2 \\ F &= 140.329 \times \frac{\pi}{4} (8.246)^2 = 7494.2 \text{ N} = 7.49 \text{ kN} \\ \% \text{ diff} &= \left(\frac{8.079 - 7.49}{7.49} \right) \times 100 = \underline{\underline{7.86\%}} \end{aligned}$$

So that is 0.38566, so that comes to value of 8.079, so this is equal to 8.079 newton, so this is kilo Newton by the first problem, so if you use the second expression that is sigma d = sigma bar into 1 + B/B, now here your first relationship is this is, okay so here there is a mistake, this 1.412, sigma bar 1.41251 into this one, that is what we are getting okay. Now, the second expression is 1 + B / B into Phi 1 into 1 – Af/A0 raised to B.

So that is equal to 143.72 into 1.41, d is 412, 1.412/0.412 into phi 1; phi 1 is 1.96; 1.961 into 1 – Af; Af is final know, so it is equal to 8.246 divided by 10 raised to B, so 2B we have to put here, so 2B will be pi and pi; pi/4 and pi/4 will cancel, so that is a 2 into 0.412, so this value comes out to be 140, so approximately it is equal to 140.329 Newton per millimetre square.

$$\begin{aligned} \sigma_d &= \bar{\sigma} \left(\frac{1+B}{B} \right) \times \phi_1 \left[1 - \left(\frac{A_f}{A_0} \right)^B \right] \\ &= 143.72 \left(\frac{1.412}{0.412} \right) \cdot 1.961 \left[1 - \left(\frac{8.246}{10} \right)^{2 \times 0.412} \right] = 140.329 \text{ N/mm}^2 \end{aligned}$$

So, that means force is equal to 140.329 into pi/4 into df; df is a 0.; sorry, 8.246 square okay, so that you will get it as, so we are getting 7494.2 Newton, so that is approximately 7.49 kilo Newton, see the difference between these two; one is 8.079, so difference is percentage deviation is equal to 8.079 - 7.49 divided by 7.49 = see, around 7.86%, this is what we are; see, it is a less than 8% day.

$$F = 140.329 \times \frac{\pi}{4} (8.246)^2 = 7494.2 \text{ N} = 7.49 \text{ kN}$$

$$\% \text{ deviation} = \left(\frac{8.079 - 7.49}{7.49} \right) \times 100 = 7.86\%$$

So, these things you know when you look at it by different approaches, you are getting a different value but since it is within that limit, it is acceptable okay.

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Problem-3
For the data given in Question 1 and the value of drawing load obtained by the expression given in question 1(c), determine the value of the axial stress at the centre line during drawing.

$$\sigma_{max} = \frac{F}{A_f} = \frac{2 \times \sigma}{3 \left(1 + \frac{r}{r_0}\right)} \times \frac{1}{\ln\left(\frac{A_0}{A_f}\right)}$$

By Siebel's Relation, $\frac{F}{A_f} = \frac{5975}{\frac{\pi}{4} (8.246)^2} = 111.88 \text{ N/cm}^2$

$$\sigma_{max} = 111.88 + \frac{2 \times 0.349 \times 14372}{3 \times \left[1 + \frac{0.15}{0.349}\right]} \times \frac{1}{0.38566} = \underline{\underline{172.64 \text{ N/cm}^2}}$$

Now, let us come to the third problem, for the data given in question 1 and the value of drawing load obtained by the expression given in 1(c) that is the Siebel's, determine the value of the axial stress at the centre line during that one, so in the last class we discussed that during drawing through a conical die, there is a variation in the longitudinal stress across the cross sectional area.

When you section it and then look at the across the diameter, you will find there is a variation in the cross sectional area, see if strain is not sufficient, the deformation at the centre will not proceed that is all, so at the centre in that case, you will find that it has not strained whereas, in the surface it has strain but most of the case in the strain is very large at the wire surface, the stress is less than the value of the average drawing stress.

This refers that is what you will find okay, while at the centre of the wire it is greater than the applied stress, the reason is mainly when the reduction is large, metal is constrained so at the centre, it will have a higher strain, so you will have a larger amount of the longitudinal stress at the centre and at the centre of the wire, it is always greater than the applied, so you are not going to give only very less strain, you are going to deform it in a large extent in one pass itself okay.

So that is why this is happening and the excess stress at the centre of the wire, at the centre line, the excess stress above that is required for drawing is given by the difference between the centre line stress σ_{max} and the average drawing stress, so that means if you write that the centre line stress is σ_{max} because that is the maximum stress you will be having, okay minus your average stress A_f .

So that relationship with the last day we have discussed is $2\alpha \bar{\sigma} / 3(1 + \mu/\alpha)$ into $1/\ln(A_0/A_f)$, this they are getting because of the using the Siebel's relationship only okay, so into $1/\ln(A_0/A_f)$, this is the relationship, so there by Siebel's relationship because we have mentioned the centre expression for this, this one know, this is by Siebel's, so Siebel's relationship; F/A_f , we straight got it as 5975 divided by A_f , 8.246.

$$\sigma_{max} - \frac{F}{A_f} = \frac{2\alpha \bar{\sigma}}{3(1 + \frac{\mu}{\alpha})} \times \frac{1}{\ln\left(\frac{A_0}{A_f}\right)}$$

$$\text{By Siebel's relationship, } \frac{F}{A_f} = \frac{5975}{\frac{\pi}{4}(8.246)^2} = 111.88 \text{ N/mm}^2$$

So, let us just look at that the how much it is; is equal to 111.88 Newton per millimetre square, so from this we can just calculate a $\sigma_{Max} = 111.88 + 2 \text{ into } \alpha = \text{in radians}$, α in radians is; α in radians is 0.349 into 0.349 into $\bar{\sigma}$ that you have one, what is the value of $\bar{\sigma}$; 143.72 divided by $3(1 + 0.15 \text{ divided by } 0.349)$, so that comes out to be 143.72 into 0.349 into 2.

No, A_0/A_f is 0.38566, so 0.38566, so that is 172.44 Newton per milli metre square mega Pascal, okay, so this is the maximum stress that is the longitudinal tensile stress at the centre of the (()) (36:10) is there this is the value we are getting.

$$\sigma_{max} = 111.88 + \frac{2 \times 0.349 \times 143.72}{3 \times [1 + \frac{0.15}{0.349}]} \times \frac{1}{0.38566} = 172.44 \text{ N/mm}^2$$

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Problem 4

(a) For annealed copper wire, determine the limiting reduction possible for the homogeneous deformation (i.e. by assuming no friction or redundant deformation) by drawing operation.

(b) If there is a 50% increase in drawing load due to friction and redundant deformation, compared to the load necessary for homogeneous deformation, determine the limiting reduction possible.

Handwritten notes:

a) $r_{max} = 1 - e^{-\eta(n)}$
 For homogeneous deformation, $\eta = 1$
 $r_{max} = 1 - e^{-(0.47)} = 0.7659$
 $n = \text{work hardening exponent}$
 $\sigma = K \epsilon^n$
 $\eta = \text{efficiency factor}$
 $n = 0.47$

b) 50% increase in load due to friction + redundant deformation
 is Total load is 1.5 times than for homogeneous deformation
 $r_{max} = 1 - e^{-\eta(n)}$
 $r_{max} = 1 - e^{-(0.667 \times 1.47)} = 0.619$
 $\beta = 1.5$
 $\eta = \frac{1}{\beta} = 0.667$

Now, let us come to the question number 4; problem number 4; see this is for annealed copper wire, determined the limiting reduction possible for homogeneous deformation that is by assuming no friction or redundant; whenever it is a homogeneous deformation means it is just for pulling in a tensile testing machine, so that is what the homogenous deformation for this particular reduction, okay.

So, it is equivalent to that but inside a die when it is deforming that what will be the draw stress that is what without friction and redundant deformation assuming that homogeneous rate is deforming, a square after deformation converts into a rectangle that is a very basic assumptions which we have come. So, for this we have derived that relation for r_{max} , so r_{max} the maximum reduction which is possible is $1 - e^{-\eta(n+1)}$, where n is the work hardening exponent of the relationship $\sigma = K \epsilon^n$.

$$r_{max} = 1 - e^{-\eta(n+1)}$$

$$\sigma = K \epsilon^n$$

So, this is that of; and η is the efficiency factor so, in this case we can say that 1 minus; here when it is only a homogenous deformation, so for homogeneous deformation $\eta = 1$, so r_{max} in this case, it is very simple; $1 - e^{-n}$ is equal to; what is its value, 0.45, so $n = 0.45$ in the equation itself, $0.45 + 1$, so that comes out to be 1.45, so $1 - e^{-1.45}$, this is the

maximum reduction which is possible when you are considering the homogeneous deformation.

$$r_{max} = 1 - e^{-(0.45+1)} = 0.7659$$

Now, for considering this question number b; if there is a 50% increase in drawing load due to the friction and redundant because of the friction and redundant deformation, your drawing load will increase, so your efficiency goes down, so compared to the load necessary for homogenous deformation, determine the limiting reduction possible, so that is what. So, 50% increase in load due to friction and redundant deformation.

That means, the total load is a say 1.5 times that for the homogenous deformation is 1.5 times, then for homogeneous deformation, so that means beta is equal to say, 1.5 or eta = 1/1.5 which is = 0.667, okay. So, in such case we can find that your r max; maximum reduction possible is equal to 1 - e raised to - eta into n + 1, so 1 - exponential of - eta into n + 1.

So that is equal to 1 - exponential of eta is 0.667 - 0.667 into 1.45, so that comes out to be 0.619, so in the due to the redundant deformation and the friction at the die metal interface, the maximum reduction possible now comes out to be only 61.7% reduction in cross sectional area whereas, in the other case it was 76.59%, so there is a big difference due to this 50% increase in the drawing load is there due to as a result of this the maximum reduction.

$$\beta = 1.5, \eta = \frac{1}{1.5} = 0.667$$

$$r_{max} = 1 - e^{-\eta(n+1)} = 1 - e^{-(0.667 \times 1.45)} = 0.619$$

Say, this comes out to be somewhat let us see how much is the difference that is 0.7659 – 0.619, so divided by say 0.7659, if you are taking the base as the homogeneous, so around 19% reduction is reduced, so that is the effect of this friction and redundant deformation, okay. Today's class is; I think if you look at these results, which are possible by using any different relationship you are going to get the for practical purpose or any of these relationships if you use also, you will find that for practical purpose it is satisfactory.

See, you cannot; one cannot expect the exact value because many other factors are also coming in the picture like say, effect of temperature, the effect of strain rate, so that we have many other factors have come, we have usually, ignore all those things and then only we have arrived at this, so that is the main reason for this variation, so if but for all practical purpose, these relationships are reasonably good.

And not differing by more than say plus or minus 10%, so that way these things are good and for practical purpose, for industrial applications these are all; because your machine, you are not going to make a machine as per your requirement of the work piece, one thing you have to look at it is whether that machine can do this purpose that is what is important, if it cannot meet that requirement then do not try it.

But otherwise it will not matter whether your total force is going to be say, 7.49 kilo Newton or 8 kilo Newton because when you buy the machine, machine capacity is almost fixed, so it is on certain standard, so you know, so maybe from 5 tonne you may get it to 10 tonne, then naturally, to 50 tonne, then 100 tonne like that only the machine capacity goes, so that way this is fine but designing the die is very important.

Because when you are designing it, then your power requirement will be less, so if you are using the optimum die design, so that is the most important thing which is necessary and for that your lubricating conditions has to be there, the surface of the die has to be made properly with all heat treatment and with very good surface finish, so that frictional effect can be reduced plus avoid redundant deformation, so that die angle has to be an optimum and then you can reduce it, okay.