Plastic Working of Metallic Materials

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Lec 29: Analysis of Extrusion

In this lecture we will be discussing about the analysis of extrusion that means to determine what is the load necessary for the extrusion for an extrusion process. It may be called deformation or cold extrusion or it may be hot extrusion but the analysis is more or less similar.

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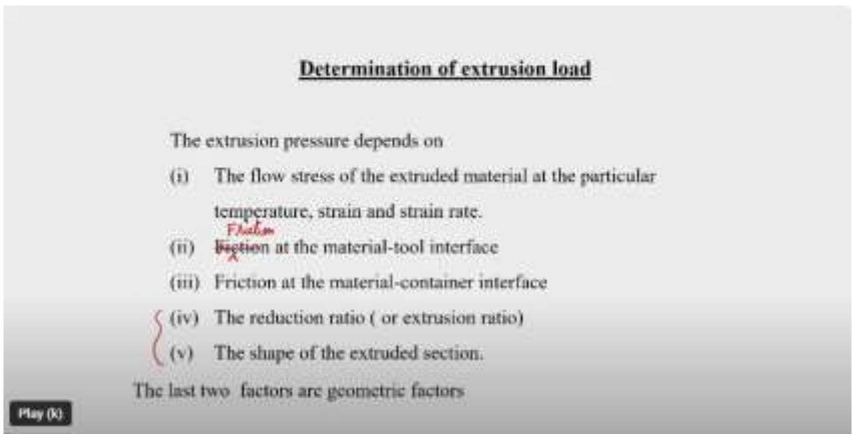

So when you wanted to find out the, determine the extrusion load, what is done is calculate the extrusion pressure and then you multiply by the cross sectional area at the inlet side, that is at the container side, okay. Cross sectional area of the inside, cross sectional area of the container, so that is what people do that. The extrusion pressure or the pressure required for extrusion process that depends upon the flow stress of the extruded material and the flow stress you know that that also in turn depends upon the temperature of deformation, the strain and the strain rate okay because if it is at a higher temperature naturally the flow stress will be lower but there is a strong dependency on the strain rate because strain rate sensitivity comes into picture. So though flow stress will be lower but that is there and then it also depends on the friction, on the friction at the material tool interface because when the metal is going to deform. Normally this is carried out in a conical die and you will find that most of the extrusion process the billet or inside the starting raw material we call it as a billet is normally cylindrical case, only very exceptionally only other sections are there because this is for the convenience for processing also manufacturing also and other things. So the invert or billet which is to be used is cylindrical in shape. And then okay this material will be deforming in the inside once it reaches the deformation zone that means where the die comes into picture. So the, it depends upon the material tool interface friction okay or friction at the material tool interface, or material die interface.

When another thing is there if it is a forward extrusion then you will find that there is going to be large amount of friction at the workpiece and the cylinder or the container. Because this part we have discussed in the last class and we also found out how the load versus the ram displacement will look like for both forward extrusion, backward extrusion and hydrostatic extrusion, all those things we have discussed. So the friction between the cylinder and the billet that also comes into picture. then what is the amount of reduction or normally in extrusion, if you do not use the word extrusion reduction ratio rather we use the word extrusion ratio okay. So but ultimately these two are related also we can directly so in a way we can say that what is the reduction ratio. So extrusion pressure depends upon higher the reduction ratio, higher the reduction, the higher will be the extrusion pressure which is required okay. Because one is internal shearing, everything will come into picture. Then the shape of the extruded section, that is very important. See, if it is a cylindrical to cylindrical piece, it is very simple, okay. Analysis also becomes easy. Cylindrical to square also, it is a problem.

But most of the case, especially non-ferrous materials and like aluminum, magnesium, they wanted some specific section, cross-sectional area, which is very complicated. So and the level of complexity when it increases the extrusion pressure also increases. Now another factor is that how many are there for, sometimes you may have multiple number of extrusion inside a die itself that also complicates the thing but the production is much faster so those type things comes into picture. So the last two these are the factors which are mainly due to the geometry. The other, the frictional stress, the frictional shear stress τ, Let it be at the work piece metal interface or let it be at the work piece die interface.

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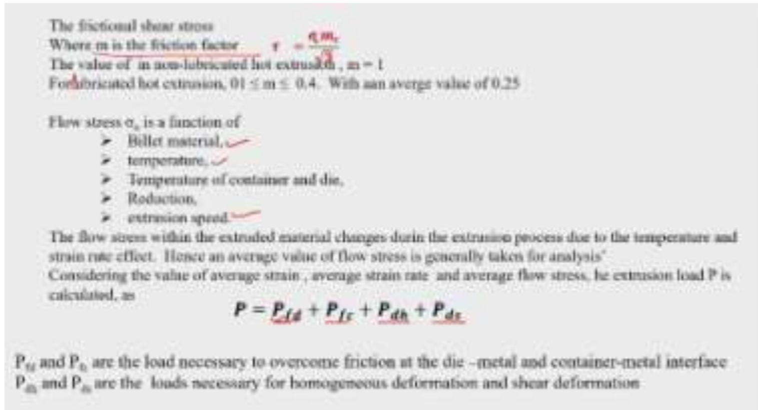

The frictional shear stress

Where m is the friction factor $\tau = \frac{\sigma_0 m_c}{\sqrt{3}}$

The value of m non-lubricated hot extrusion , m = 1

For lubricated hot extrusion, 01 ≤ m ≤ 0.4. With aan averge value of 0.25

Flow stress σ_0 is a function of

- Billet material,
- temperature,
- Temperature of container and die,
- Reduction,
- extrusion speed.

The flow stress within the extruded material changes durin the extrusion process due to the temperature and strain rate effect. Hence an average value of flow stress is generally taken for analysis'

Considering the value of average strain , average strain rate and average flow stress, he extrusion load P is calculated, as

$$P = P_{fd} + P_{fc} + P_{dh} + P_{ds}$$

P_{fd} and P_{fc} are the load necessary to overcome friction at the die –metal and container-metal interface

P_{dh} and P_{ds} are the loads necessary for homogeneous deformation and shear deformation

So we in most of the case particularly hot working operation when we look at it we define the τ as that is the frictional shear stress the interface near to the interfaces. So, sigma m c by root 3 basically it should be sigma 0 uniaxial process into m by root 3 c has no meaning actually c is whether it is at the container. or whether it is at the die billet when it is at the container billet you call it as MC when it is at the die and the billet interface you call just do it as but in general we can say the this is the thing uniaxial flow stress at that conditions of temperature and strain rate what is the uniaxial into M by root 3 because the this is mainly the sticking friction which is coming into picture and the value of M. value of M generally for a non-lubricated hot extrusion process, it is taken as perfect sticking friction that is M is considered as 1. And for a lubricated hot extrusion, for lubricated hot extrusion process, M will vary anywhere between the 0.

1 to 0.4, M will vary between 0.1 to 0.4. And for most of the analysis purpose we can take for a lubricated hot extrusion process we can just consider it as a value of 0.25 with an average value of 0.

$$\tau = \frac{\sigma_0 m_c}{\sqrt{3}}$$

25 we can take. The flow stress sigma 0 is a function of you know that billet material, flow stress depends upon the billet material. Say like whether it is non-ferrous materials like aluminum, magnesium, titanium or whatever it be or whether it is steel and steel itself whether it is a plain carbon steel or medium carbon steel or high alloy steel or

whatever it be. So that flow stress also depends upon the billet material and flow stress also depends upon the temperature, the temperature dependent properties comes because at a higher temperature, compared to room temperature, the flow stress will get reduced. The flow stress will be lower at a higher temperature.

And similarly, the temperature of the container and the die, that also matters because if your billet may be at a higher temperature, and the container temperature is lower then immediately there will be sudden cooling when it comes in contact with the when the billet comes in contact with the container extrusion container then it will get cool down. So, you will find that the flow stress changes may be and that due to that cooling outside surface temperature may be high may be low, but inside temperature may be high. So, all these things can come. So, it becomes very complicated. So, you should have an idea about what is the temperature of the container.

And similarly, what is the temperature of the die? Die, one has to be very careful. See, because with the continuous usage, the die should not get softened. So, die material is very important in these cases of extrusion, especially hot extrusion. Then, frost stress also depends upon the reduction. The higher the reduction, the frost stress will be high at the outlet.

And another is the extrusion speed because depending upon the extrusion speed, the metal moves through the deformation zone. And in that process, the strain depth keeps on changing and sometimes now you will find that there is an order of difference between the strain rate at the inlet to the die and outlet to the die. So, the speed also matters. So, There is some, in recent studies, you know, like deformation mechanism maps and other things, you will find that there is a process window over which strain rate and temperature over which you can deform the material. So, and with the higher speed, there is going to be defect.

But with the lower speed, that means lower strain rate, the defect may be less. So, the extrusion speed also has an important influence on the flow stress of the material. Now, the frost stress within the extruded material. The flow stress changes during the extrusion process due to the temperature and the strain rate effect. That is what I was telling.

Inside strain rate may be something, but outside strain rate, towards the end of the die, the strain rate will be very high. Hence, an average value, because these things are changing, so the flow stress based on that may keep varying from inlet to the outlet. So for analysis purpose, now we take the average value of the stress that is generally taken for the analysis. And with the reasonable to a reasonable extent our analysis with by taking this average value of strain or strain rate or effective strain and strain rate if you take it, you will find that it is very it is reasonably good okay and the extrusion load When you look at it, the extrusion load P depends upon the load necessary to overcome the friction at the die and the ingot, to overcome the friction at the die metal interface. It also depends upon the friction between the container and the billet, the interface at the friction between the container and the billet, and also the load required for homogeneous deformation of the billet inside the deformation zone and also because there is a change in the direction there is going to be a shear deformation which is taking place. So, the total load depends upon these 4 factors and let us now find out for each case how much it is coming.

$$P = P_{fd} + P_{fc} + P_{dh} + P_{ds}$$

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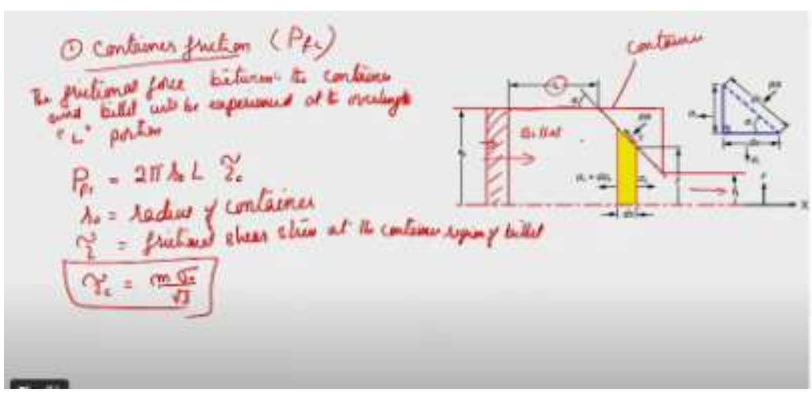

First, let us consider the container friction. So, if this is a section of that, you will find that this is your container, this part is your container which is there and this is your extrusion die which is coming here and this is your billet which is going out. When it is moving like this, it moves like this.

So, this is because of the symmetry, isosymmetric case, so axisymmetric case, so we have shown only half of the arrangement. So, in this case you will find that this is the, this is the billet and this is the container. So if you are assuming that a cylindrical portion of the deforming body because most of the case this container is cylindrical in shape and the frictional force between the container and the billet, will be experienced at this length because this is the path which is moving forward from the ram, it is moving in this direction and it is pressing this billet here. So in this case what happened this is the length where the billet is in contact with the container this overhang length okay. So over length L which is given by this L that is the particular. So the frictional force, the frictional force, so first part we will just write it as container, in this the frictional force between the container and billet, will be experienced at the over length L portion.

And so, this the container friction P f c, we can say that this is the line, but since it is a cylindrical in piece you know you can say that P f c will be the total area which is coming is pi d into l what is l is this one sorry r 0 is coming here. So, you can say 2 pi say r 0 into l into your tau c where r 0 is the radius of the container and tau c is the frictional shear stress at the container region of billet. And normally you will find that this tau c is equal to say m into which we have discussed by root 3. So, this will be your uniaxial flow stress by root 3 where m is equal to and we discussed that m for lubricated cases. So for non-lubricated hot extrusion, if it is 1 and for lubricated hot extrusion, it will get reduced.

$$P_{fc} = 2\pi r_0 L \tau_c$$

$$\tau_c = \frac{m\sigma_0}{\sqrt{3}}$$

So M will be 0.1 to 0.4. So that is the thing. So this is how the container friction will take it. So that is the Pfc. Now as the forging advances this total frictional force keeps on reducing because L keeps on reducing okay. So this will be the initial case what we have said.

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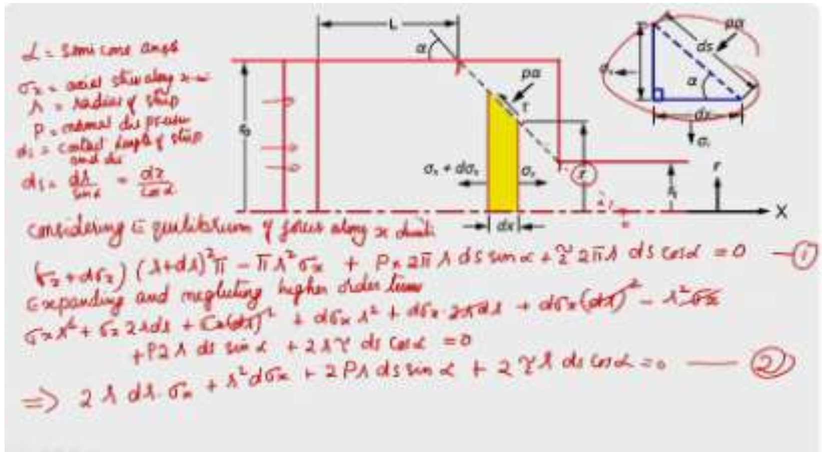

Now let us come to say force equilibrium in the deformation zone. We have to study at the deformation zone. So the deformation zone is basically a conical section which is shown by this. See, this is your die. So, your may be this from here is your deformation zone which is coming.

Other part it is not deforming. So, this region is the deformation zone. So, basically it is at the conical structure. We are assuming a conical die. And, so the deformation zone is a conical section with an included angle of 2 α. So, if you just expand this, this is alpha because it is symmetric that will be 2 alpha.

So, this is called as alpha is called as a semi con angle, semi con angle that is the thing. So, if you just consider a small strip. or a small disk strip or strip at the deformation zone because deformation zone is from here to here. So, from here to here is the deformation zone. So, if you just consider a small infinitesimally small disk of thickness d x and this is your x direction if you are telling and this will be your origin if you can just consider like that. So, this of d x which is shown by the shaded line that is the yellow colored shaded region is the disc and with the initial part which is r here this is the r and say you will find that and you will find that due to the pressure which is being applied at this region by the ram, there will be a variation in the stress from both the sides of the stress. So let us consider a small strip of thickness dx along the x direction, along this direction and then because of this effect of this extrusion pressure, extrusion load at this you will find that the stresses there is an imbalance of that, for if you are considering there is a variation in the stresses across this small strip. And these are the various forces which are acting or stresses which are acting.

You will find that there is this P alpha. P alpha is the or the normal pressure acting on the interface between the billet and the die. And the axial stresses are sigma x plus d sigma x on the left side and sigma x on the right side. and you can see that the various geometric consideration which are there. So axial stress is acting along this x direction and there is a radial stress acting on this strip also in this direction.

So and various geometrical relationships we can get it from this. Now if you consider that strip is under equilibrium under a steady state condition if it is under equilibrium and the equilibrium forces along the x direction. So considering, considering the equilibrium of forces along x direction. We can write sigma x plus d sigma x that is on the left side into the radius is r this is the radius which is the r plus d r square into pi the total area it is going to come. So, minus pi r square on the other side pi r square sigma x plus you will see that p into 2 pi r , because there is a normal pressure is acting its component you have to take it along the x axis.

Similarly, there is a the shearing stress shear stress at the interface between the die and the punch which is represented by toy is also there. So, here if you look at the normal stress 2 pi r into d s sin alpha So, you can just look at d s sin alpha is coming here plus tau into the shear stress component if you are taking taking the component along the x axis 2 pi r into d s cos alpha, sorry this is sin alpha not theta cos alpha is equal to 0 we can write equation number 1. where sigma x is the axial stress. I will write sigma x is the axial stress along x axis and r is the radius of the strip. P is the normal die pressure. d s is equal to the contact length of strip and die. And from this geometry we can find it as d s is equal to d r by sin alpha is equal to d x by cos alpha. So, that way we can write it or we can also tan alpha is equal to that is dr by dx. So, that way also we can find it out. So, now expanding this equation and neglecting the higher order terms.

$$(\sigma_x + d\sigma_x)(r + dr)^2\pi - \pi r^2\sigma_x + P \times 2\pi r ds \sin\alpha \ + \tau 2\pi r ds \cos\alpha = 0$$

$$ds = \frac{dr}{\sin\alpha} = \frac{dx}{\cos\alpha}$$

So, expanding and neglecting higher order terms, we can write this say like if you expand it now you will find it as a sigma x into r square plus sigma x into 2 r dr plus sigma x into dr square plus d sigma x into r square plus d sigma x into two r d r plus d sigma x into d r square minus r square sigma x plus two p into two r d s sin alpha plus 2 r tau into d s cos alpha is equal to 0. So, this will go this will go this will go sigma x d r square also will go because these are the higher order terms and this also this and this will go. So, this this many terms will get removed. So, we can finally, write it as 2 2 r dr into sigma x plus r square d sigma x r square d sigma x plus the last two terms that is 2 p r ds sin alpha plus 2 tau r ds cos alpha is equal to 0.

$$\sigma_x r^2 + \sigma_x 2rdr + \sigma_x (dr)^2 + d\sigma_x r^2 + d\sigma_x . 2rdr + d\sigma_x (dr)^2 - r^2\sigma_x + P2rds \sin\alpha + 2r\tau ds \cos\alpha = 0$$

$$2rdr.\sigma_x + r^2 d\sigma_x + 2Prds \sin\alpha + 2\tau rds \cos\alpha = 0$$

So this is equation number 2. Now let us consider this is equation number 2. Now let us consider the equilibrium of forces in the radial direction because here also there is a radial direction which is coming.

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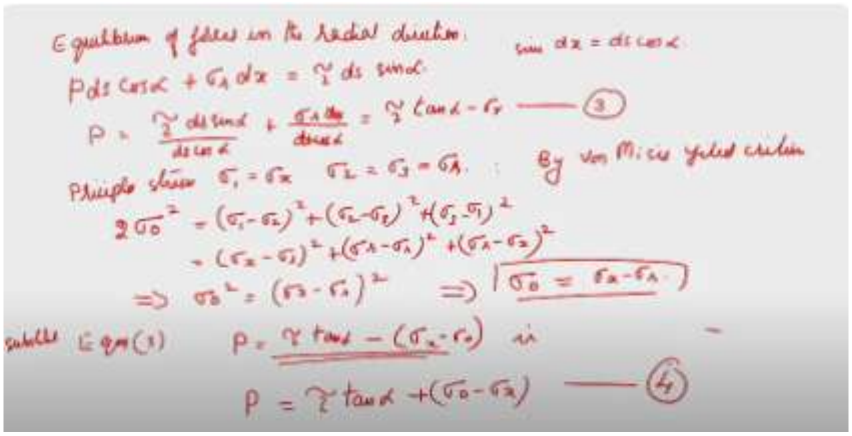

So the equilibrium of forces of forces in the radial direction. So, that if you look at it there are what are the terms which are coming. So, one is the p ds cos alpha. So, this this part if you resolve it in this direction, that is coming then sigma r this sigma r component also coming and then you will have this component of this frictional force at the interface.

So, that these three if you write it you can write like that p ds cos alpha plus sigma r dx that is equal to tau ds sin alpha. So, that is because say like if you write dx is equal to ds cos alpha. That way we can write okay. Since dx is equal to ds cos alpha we can write that p is equal to in a simplifying simple mathematical manipulation we can write it as tau ds sin alpha by ds cos alpha plus sigma r into d x by d s cos alpha. So, this and this will get cancelled is equal to tau tan alpha.

$$Pds\cos\alpha + \sigma_r dx = \tau ds \sin\alpha$$

$$P = \frac{\tau ds \sin\alpha}{ds\cos\alpha} + \frac{dr\ dx}{ds\cos\alpha} = \tau\tan\alpha$$

So, this is equation number 3. Now if you look at, we were just considering the case for a cylindrical billet and extruded pieces also cylindrical in shape. This is the simplest case we were considering. So in that case, because this is axisymmetric. So you will find that the principal stresses can be assumed as, so this is at principal stresses, we can assume as sigma 1 is equal to sigma x sigma 2 is equal to sigma 3 that is equal to sigma r okay. Sigma 2 is equal to sigma 3 that is the radial stress which is coming and then considering assuming by Von Mises criteria, in our earlier class lectures, yield criteria for yielding, we will write that 2 into your uniaxial yield strength 2 sigma square is equal to sigma 1 minus sigma 2 the whole square plus sigma 2 minus sigma 3 the whole square plus sigma 3 minus sigma 1 the whole square. So, that if you substitute this values that is sigma x minus sigma 1 the whole square plus sigma r minus sigma r the whole square plus sigma r minus sigma x the whole square.

So, that will lead you to sigma 0 square is equal to sigma x minus sigma r the whole square or sigma 0 that uniaxial yield strength of the material is equal to sigma x minus sigma r. So, this relationship we are getting as per the Von Mises criteria.

$$\sigma_1 = \sigma_x \quad \sigma_2 = \sigma_3 = \sigma_r$$

$$By\ Von\ Mises\ yield\ criteria,$$

$$2\sigma_0^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

$$= (\sigma_x - \sigma_r)^2 + (\sigma_r - \sigma_r)^2 + (\sigma_r - \sigma_x)^2$$

$$\sigma_0^2 = (\sigma_x - \sigma_r)^2 \qquad \sigma_0 = \sigma_x - \sigma_r$$

So, from that if you substitute into equation 3, so equation 3 if you substitute that we will get it as p is equal to tau tan alpha see minus sigma r tau tan alpha minus sigma r is equal to sigma x minus sigma r is equal to sigma x minus sigma 0. So, this is the relationship we are getting or that is p is equal to tau tan alpha plus sigma 0 minus sigma x this we are getting. I will write it as equation number 4 that is p is equal to tau tan alpha plus sigma 0 minus sigma x.

$$P = \tau \tan\alpha - (\sigma_x - \sigma_0)$$

$$P = \tau \tan\alpha + (\sigma_0 - \sigma_x)$$

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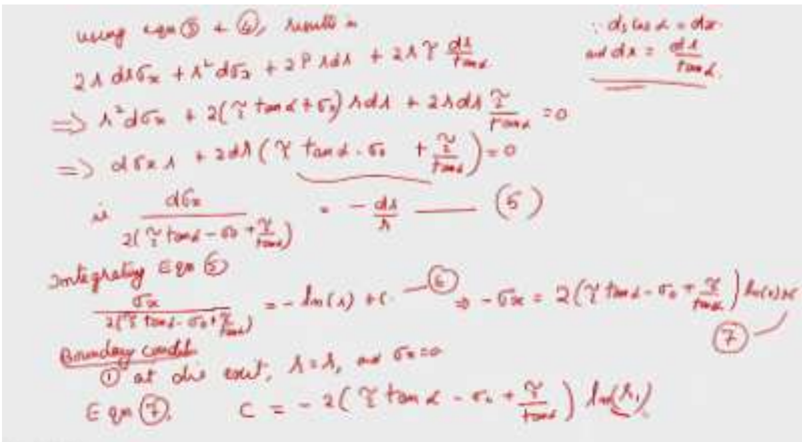

So, this is equation number 4. Now, substituting equation 4 into equation 3, using equation 3 and 4, we will get results in 2 r d r sigma x plus r square d sigma x plus 2 p r d r 2 p into r d r plus 2 r into tau into d r by tan alpha. This is basically d s cos alpha is equal to d x and d x is equal to d r by tan alpha. So, when you substitute this we can we will get this term from that geometric relationship itself which I have written earlier. So, just to remember here I am just writing it. So, that means this relationship from that we will get it as r square d sigma x plus 2 into tau tan alpha plus sigma 0 r d r plus 2 r d r into tau by tan alpha is equal to 0, just rearranging. So, that means we are getting d sigma x r the r is common in everything if you cancel it is equal to tau d r into tau tan alpha minus sigma 0 plus tau by tan alpha is equal to 0. So, that means in the differential form we can write it as d sigma x by 2 into tau tan alpha minus this term we are bringing at the denominator sigma 0 plus tau by tan alpha is equal to minus d r by r. This is the differential equation you solve for this and we can get the value. That means, if you integrate it integrating equation one in equation five. So, you will get it as sigma x by 2 tau tan alpha minus sigma 0 plus tau by tan alpha is equal to minus log r plus c or sigma x is equal to 2 into tau tan alpha minus sigma 0 plus tau by tan alpha into log r if you put this as minus sigma x.

$$2rdr\sigma_x + r^2 d\sigma_x + 2Prdr + 2r\tau\frac{dr}{\tan\alpha}$$

$$r^2 d\sigma_x + 2(\tau\tan\alpha + \sigma_0)rdr + 2rdr\frac{\tau}{\tan\alpha} = 0$$

$$d\sigma_x r + 2dr\left(\tau\tan\alpha - \sigma_0 + \frac{\tau}{\tan\alpha}\right) = 0$$

$$\frac{d\sigma_x}{2(\tau\tan\alpha - \sigma_0 + \frac{\tau}{\tan\alpha})} = -\frac{dr}{r}$$

$$Integrating, \frac{\sigma_x}{2\left(\tau\tan\alpha - \sigma_0 + \frac{\tau}{\tan\alpha}\right)} = -\ln(r) + c$$

$$\sigma_x = 2\left(\tau\tan\alpha - \sigma_0 + \frac{\tau}{\tan\alpha}\right)\ln(r) + c$$

Now you apply the boundary condition applying the boundary condition one is that the, what are the boundary conditions. See if you look at the boundary condition say at r is equal to r 1 ok when the radius is equal to say r 1 which is at the exit. So, this is the exit side this is equal to exit side. So, at the exit when you are applying the axial stresses are 0. Only inside the die, only the stresses will come, but once it comes out of the die, the axial stresses are 0.

So, that is one boundary condition. So, that means, at the entrance to the die, at 1, at r is equal to r 1, that is, sorry, at die exit, r is equal to r 1 and sigma x is equal to 0. So, this is this this you write it as equation 7 and this is equation 6. So, equation 7 if you substitute from that you will find that c is equal to so this is c is equal to minus 2 into tau tan alpha plus minus sigma 0 plus tau by tan alpha log r 1.

$$Boundary\ condition, at\ die\ exit, r = r_1, and\ \sigma_x = 0$$

$$c = -2\left(\tau \tan\alpha - \sigma_c + \frac{\tau}{\tan\alpha}\right)\ln(r_1)$$

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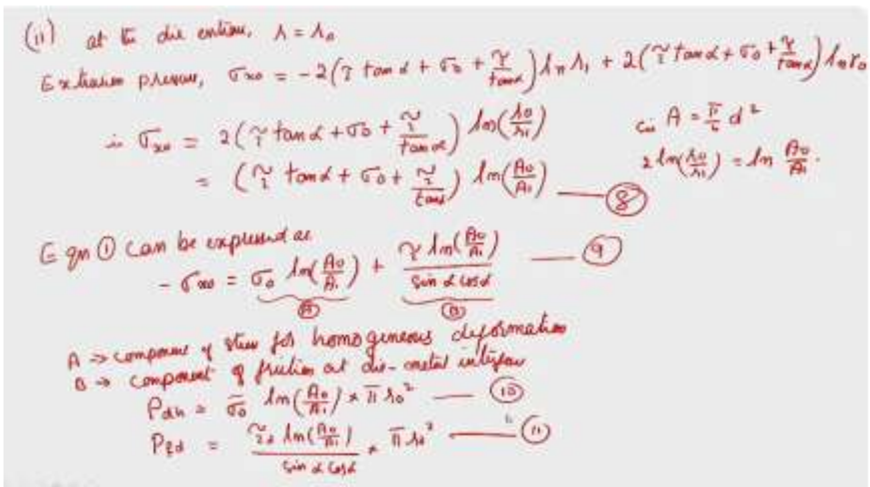

So, this value and you apply the second condition that is the second boundary condition is that, at the entrance to the die at the die entrance r is equal to r 0, that is the billet diameter billet radius, the radius is equal to billet radius and the extrusion load. because that is the load which you have to apply or extrusion pressure when you wanted to extrusion pressure we have to apply, sigma x0 is equal to if you substitute the value of this one in equation number 6, you will get that is equal to minus 2 into tau tan alpha plus sigma 0 plus tau by tan alpha log r 1 plus 2 into tau tan alpha plus sigma 0 plus tau by tan alpha into log r 0 here this should be 0 plus that was a mistake, or or that is sigma x 0 which is the extrusion pressure which you required we can write it as 2 into tau tan alpha because just doing a simple manipulation mathematical manipulation rearrangement you will get it as by tau tan alpha tau by tan alpha into log r 0 by r 1. 4. So, that is also equal to 2 in terms of area wise tan alpha plus sigma 0 plus tau by tan alpha. So, 2 log r 0 by r 1 is equal to log a 0 by a 1 because a is equal to pi by 4 d square from that you can since we can write.

$$at\ the\ die\ entrance, r = r_0$$

$$Extrusion\ pressure, \sigma_{x0}$$

$$= -2\left(\tau \tan\alpha + \sigma_0 + \frac{\tau}{\tan\alpha}\right)\ln r_1 + 2\left(\tau \tan\alpha + \sigma_0 + \frac{\tau}{\tan\alpha}\right)\ln r_0$$

$$\sigma_{x0} = 2\left(\tau \tan\alpha + \sigma_0 + \frac{\tau}{\tan\alpha}\right)\ln\left(\frac{r_0}{r_1}\right)$$

$$= \left(\tau \tan\alpha + \sigma_0 + \frac{\tau}{\tan\alpha}\right)\ln\left(\frac{A_0}{A_1}\right)$$

So, that means log 2 log r 0 by r 1 is equal to log a 0 by a 1. So, this is your equation number 8 the relationship which you are getting. Now, this equation number 8 can also be written in this form, minus sigma x 0 is equal to sigma 0 log a 0 by a 1 if you just split it and write a 0 by a 1 plus tau log a 0 by a 1 by sin alpha cos alpha. See in this way we can write it so which is so from this the first part of this equation this is a stress for homogeneous deformation if I just write it as A and this is B so A is the component of stress for homogeneous deformation and b is the component for die of friction at die metal interface.

$$-\sigma_{x0} = \sigma_0 \ln\left(\frac{A_0}{A_1}\right) + \frac{\tau \ln\left(\frac{A_0}{A_1}\right)}{\sin\alpha\cos\alpha}$$

So, we can say that that is p d h is equal to sigma naught we can just say use it as an average value also because it will vary from inside to outside. So, we can write sigma naught log in the generalized A 0 by A 1 this is the strain which is there. So, effective stress into effective strain you can say and into the cross sectional area that is pi r square r 0 square. So, this is your equation number 10 and P F D. So, this is the frictional force is tau at the die area log a 0 by a 1 by sin alpha cos alpha into pi r square.

$$P_{dh} = \overline{\sigma_0}\ln\left(\frac{A_0}{A_1}\right) \times \pi r_0^2$$

$$P_{fd} = \frac{\tau d \ \ln\left(\frac{A_0}{A_1}\right)}{\sin\alpha \cos\alpha} \times \pi r_0^2$$

So, r 0 square that is 11. So, these two components we got third component also we got now only component which is left is the shearing stress it is a shear stress which we wanted to draw.

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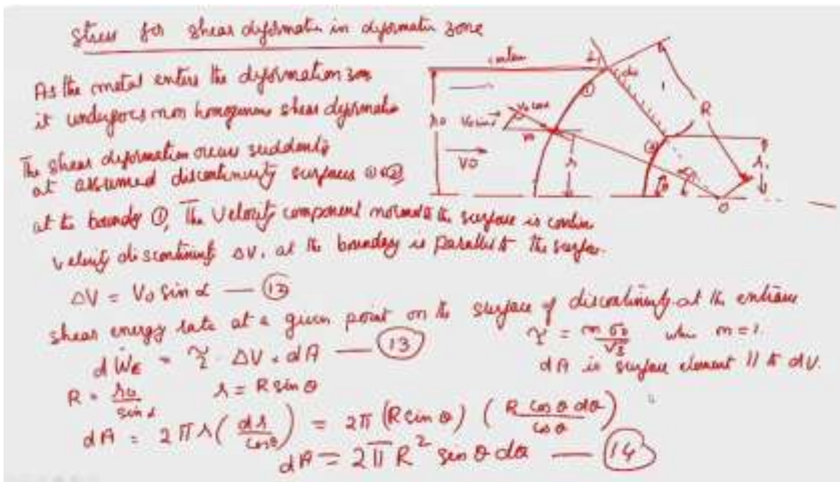

So, the stress necessary for shear deformation in the stress for shear deformation in your deformation zone. So, let us look at that because the metal is flowing there is going to be a change in the direction of flow. So, and metal will flow parallel to the die work piece interface.

So, if you just consider this, as the piece which is there and here. So, if you just take this as the alpha value. Let me extend it somewhere here so that it comes this we can assume as the origin of this. So, here it is r 0 that is the billet radius container diameter also, we can say and this is your r at the exit r 1 at the exit. So, this distance, so this is the die, let us say. This length if it is r this is your alpha and when the metal is flowing in this direction when it comes here somewhere it will change its direction of flow and it will you we assume that it is flowing parallel to the die surface die interface die work piece interface. So, we can say there and the material undergoes non-homogeneous shear deformation.

Shearing is taking place with the center as O and we can just assume for our simplicity this is the area. The moment it crosses this region there is going to be a change in the direction. So, if it comes like this and then you will see that change in the direction. If it comes here, there is a change in the direction.

If it comes here, there is a change in the direction. So, this is how the, so there is going to be a velocity discontinuity when you cross this region and the discontinuity surface. This is the discontinuity surface which is there, which is shown by this. So, across this surface, there is going to be a non-homogeneous shear deformation and the energy and the load necessary for this we have to estimate. And similarly when it comes to the exit here also we are assuming there is another velocity discontinuity is coming here because after this it is assumed that there is no metal flow is in the initial direction. So, our horizontal velocity of metal flow when it is moving here it is your V0.

So, let us take an arbitrary point which is there. So, where the radius is r, to get the relationship where the radius is r and normal to this at the you take this component one is normal to the surface and another is parallel to the surface. So, this is your v 0. So, this is v sin alpha v 0 sin alpha and this is v 0 cos alpha. So, this will be the diagram which is coming. One velocity discontinuity is taking place at this discontinuity surface and say I will just at this surface and this surface two discontinuity.

So, there is a as the metal enters the deformation zone it undergoes non-homogeneous shear deformation and the shear deformation occurs suddenly at the assumed discontinuity surfaces say 1 and 2. So, at the entrance boundary that is at a at the boundary one the velocity component normal to a surface is continuous, the velocity component, component normal to the surface is continuous, that means it is this this is straight, it is continuous there is no problem but whereas if you look at the velocity discontinuity at the boundary which is parallel to that, so that you will find that it is not continuous. So, this there is a small mistake in the way I have written drawn it will be here this is that it is like this 90 degrees here. So, the component at the the velocity component delta v, velocity discontinuity delta v at the boundary is parallel to the surface.

So we can say that delta V is equal to V0 sin alpha maybe the equation 12. So whenever it is crossing this boundary there is going to be a shear energy rate. So the shear depending upon the velocity shear energy rate. at a given point on the surface at the entrance at the entrance region one is at the exit and another is at the entrance we can say d w e dot is equal to your tau into delta v the velocity discontinuity into area into d a this is 13 and we can see that tau is equal to m into sigma 0 by root 3 where for perfect sticking m is equal to 1 and dA is a surface element parallel to dV, where the shear occurs.

$$\Delta V = V_0 \sin\alpha$$

$$d\dot{W}_E = \tau.\Delta V \times dA$$

$$\tau = \frac{m\sigma_0}{\sqrt{3}} \quad where\ m = 1$$

So, we can from this geometry this r is equal to say we can write r r naught. So, we can write this geometric relationship your r is equal to r naught by sin alpha and r is equal to r sin theta. This is alpha and if you take this as theta So, this relationship we can write ok. So, and d i is equal to 2 pi r into d r by cos theta we can write this relationship. So, that is equal to 2 pi into r is equal to r sin theta.

$$R = \frac{r_0}{\sin\alpha} \qquad r = R\sin\theta$$

So, you can say capital R sin theta into d r by cos theta is cos theta. So, we can write r cos theta d theta by cos theta or that is equal to 2 pi r square sin theta d theta. So, this is d a.

$$dA = 2\pi r\left(\frac{dr}{\cos\theta}\right) = 2\pi(R\sin\theta)\left(\frac{R\cos\theta d\theta}{\cos\theta}\right)$$

$$dA = 2\pi R^2 \sin\theta d\theta$$

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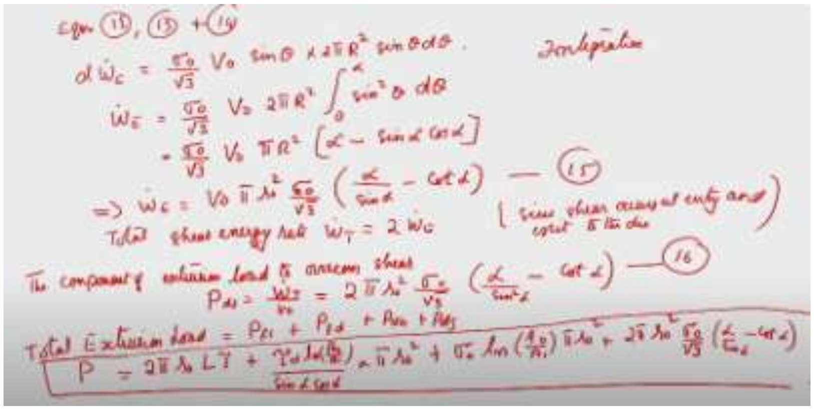

So, from this 12, 13 and If you do if you substitute it in this equation from this we can just find it as d w dot e is equal to sigma 0 into root 3 into v 0 sin theta into 2 pi r square sin theta d theta. So, you integrate it you will get the energy required. So, that is w dot e is equal to strain energy right is equal to sigma 0 by root 3 into v 0 into 2 pi r square integral from 0 to alpha sin square d theta.

$$d\dot{W}_E = \frac{\sigma_0}{\sqrt{3}} V_0 \sin\theta \times 2\pi R^2 \sin\theta d\theta$$

$$\dot{W}_E = \frac{\sigma_0}{\sqrt{3}} V_0 2\pi R^2 \int_0^{\alpha} \sin^2\theta d\theta$$

$$= \frac{\sigma_0}{\sqrt{3}} V_0 \pi R^2 [\alpha - \sin\alpha\cos\alpha]$$

So, that you will get it as sigma 0 by root 3 into v 0 into pi r square and simplifying it you can get alpha minus sin alpha cos alpha or w dot e is equal to v 0 pi r 0 square sigma 0 by root 3 into alpha by sun squared alpha minus cot alpha. So, that is 15. Since the shear occurs at the entrance and exit at 2 region the velocity discontinuity is taking place. So shearing is taking place once it at the enter entry to the deformation zone and at the other is at the exit to the deformation.

$$\dot{W}_E = V_0 \pi r_0^2 \frac{\sigma_0}{\sqrt{3}} \left(\frac{\alpha}{\sin\alpha} - \cot\alpha\right)$$

So this is 1 and this is 2. So these 2 regions are taking place. So since it is taking place twice the total shear energy, total shear energy rate w dot t is equal to 2 w dot e since at entry and exit to the die. So we can say that the portion of extrusion load, the component of extrusion load to overcome work on the shear p d s is equal to w dot t by v 0 per unit volume. So, this is that so we will get it as 2 pi r square into sigma 0 by root 3 into alpha by sin squared alpha minus cot alpha, this is what we are getting. So, total extrusion load is the sum of all these four total extrusion load is equal to p f c as we have written the very first one p f d plus p d h plus p d s. So, that is equal to if you write all these things it will be 2 pi r 0 l into tau plus tau d log a 0 by a 1 by sin alpha cos alpha into pi r 0 square plus sigma 0 into log a 0 by a naught by a 1 pi r naught square plus 2 pi r naught square. It is a long equation sigma 0 by root 3 into alpha by sin squared alpha minus cot alpha. All these four components are taken. So, that is how we get it.

$$\textit{The component of extrusion load to overcome shear,}$$

$$P_{ds} = \frac{\dot{W}_T}{V_0} = 2\pi r_0^2 \frac{\sigma_0}{\sqrt{3}}\left(\frac{\alpha}{\sin^2\alpha} - \cot\alpha\right)$$

$$Total\ Extrusion\ Load = P_{fc} + P_{fd} + P_{dh} + P_{ds}$$

$$P = 2\pi r_0 L\tau + \frac{\tau d \ln\left(\frac{A_0}{A_1}\right)}{\sin\alpha\cos\alpha} \times \pi r_0^2 + \sigma_0 \ln\left(\frac{A_0}{A_1}\right)\pi r_0^2 + 2\pi r_0^2 \frac{\sigma_0}{\sqrt{3}}\left(\frac{\alpha}{\sin\alpha} - \cot\alpha\right)$$