Plastic Working of Metallic Materials

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Lec 31: Sheet deformation process

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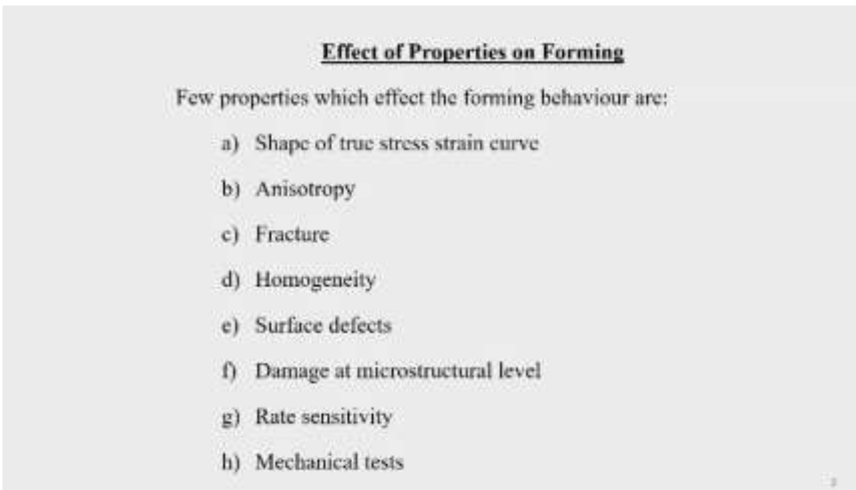

Today we will be continuing with with what we were discussing in the first lecture and after that we will be going to the sheet deformation processes, the basic fundamentals of sheet deformation processes. So the effect of properties on forming behavior of the metals especially in sheet metal is there. Let us look at what are these properties which affect the forming behavior of metals or metallic sheets. So some of them are the shape of the true stress strain curve. So from the true stress strain curve how we will have an idea how the material will behave. Anisotropic property of the material of the sheet because that also in a way depends upon the processing history of the material.

Then what is the fracture behavior of course that we cannot have any control on that. But at least we will know how it takes place and it is very difficult to have an idea about the fracture in sheet metal forming operation. Only thing is as a priori we will be having the knowledge about it. But still we will have an idea where the fracture is going to take place.

Then the homogeneity of the material, the surface defects. The damage at the microstructural level, rate sensitivity of the material or the workpiece material and the mechanical test we are doing it. What are the other test which is possible also we will have to see. So first let us discuss about it one by one. Firstly the shape of the true stress strain curve.

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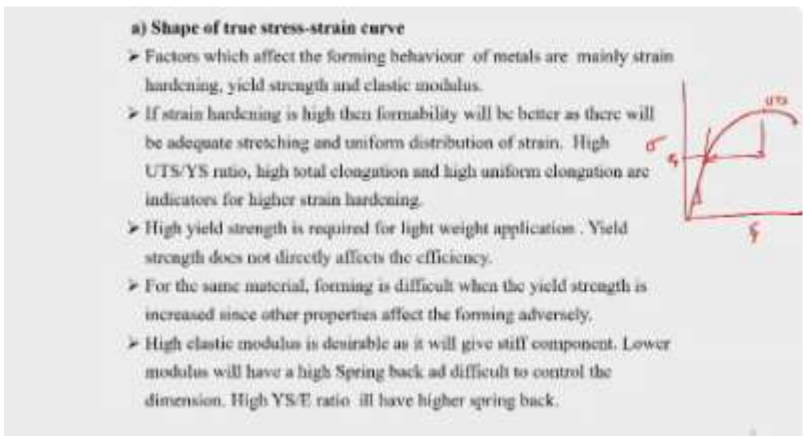

The factors which affect the forming behavior of metals are mainly the strain hardening, the yield strength and the elastic modulus. If you look at the stress-strain curve, you will have an idea like suppose it is like this, okay. So if this is your yield strength, and this is your ultimate tensile strength, this is your fracture point, this is your Young's modulus value of the material. So from here, from here to here, the material is the work hardening region, this is the uniform deformation region. And from this point to here, so from the yield point to the ultimate tensile strength, how much stress increases possible, okay and what is the strain during that period. So this gives an idea about the thing and say if the strain hardening is high, the formability generally is found to be better. Why? The strain hardening if it is high, there will be adequate stretching and uniform distribution of the strain. So you will find that the thinning effect will get reduced with the material having higher work hardening rate. Of course your stress increase will be very high for plastic deformation keeps on increasing but material with the higher work hardening rate will give you more elongation also mainly because the thinning effect is reduced.

Now the high ultimate tensile to yield strength ratio so this to this ratio that also will help in so that is an indicator of the work hardening rate or strain hardening okay. Then total elongation how much you are going to get before the material fails that is also if the elongation is very large then your forming behavior will be better okay. So that means because you are having a higher uniform elongation and these are all indicators of higher work hardening or strain hardening. Now high yield strength is required mainly for lightweight application because with the advent of the new material where people are just looking at lightweight application that means high strength to weight ratio materials especially that is where the light material light metals like aluminum and magnesium alloys are coming into picture or titanium also to some extent is coming into picture. Because with a lighter the material for a specific output if you want you need to have only very small weight of the material that means the efficiency can be increased okay.

So that is why the high yield strength material is required. And for the same material forming is difficult when the yield strength is increased. So, if you are by some means say for if you have a one material you can do some thermomechanical treatment and for thermomechanical treatment you can either increase the strength or decrease the strength. So, your yield strength if you increase it by some thermomechanical treatment naturally your yield strength will get reduced that is all. Forming it is better to have say reduce the yield strength and after after the deformation process you you do some heat treatment so that your yield strength is increased that is the biggest best advantage of doing it okay.

So it is difficult so when the yield strength is increased other properties like toughness and ductility will get reduced so that will be adversely affected so that one has to be careful about it. So, for from the point of if you have a higher capacity machine then it is better to have deformed the material in the higher yield strength value okay but if your this is capacity of the machine is less then you have no other alternative than to give heat treatment and reduce the yield strength and high elastic modulus is desirable as it will give the give stiff component. So, lower modulus have high spring back effect and difficult to control the dimension. So, with any material with the lower modulus value stiffness value it is very difficult to control the dimensions ok, but with the higher one though you may need to have higher stresses, but you can control the dimensions properly. And another is that high yield strength to Young's modulus value ratio will have higher spring back that is also people have been experimentally they have found that those material having high yield strength to Young's modulus value ratio they have a higher spring back effect.

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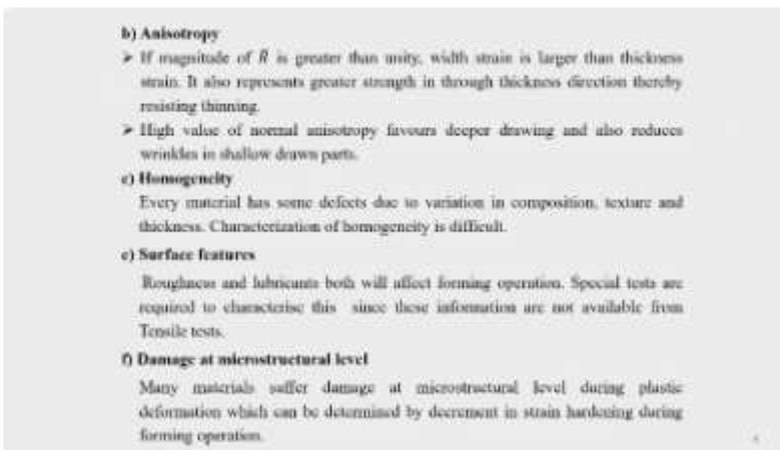

Now certain things is the anisotropy which we were discussing earlier if the magnitude of anisotropy is greater than unity then width strain is larger than the thickness strain. So, anisotropy is the ratio of width strain to the thickness ratio. So, if the higher the value, width strain is larger. So, it also represents greater strength in throat thickness directions. So, any material with the higher R value, you know, you will see that it is higher strength is there in the through thickness direction.

Thereby, due to that, your thinning is reduced. So, it means, so local necking and then fracture. So, that part will be reduced you can have a higher amount of strength for that. High value of normal anisotropy favors deeper drawing drawing operation. So, you can have a very deep cup and other things to be made and also higher normal anisotropy you know will reduce the wrinkles which which can form in shallow parts. So, these are some some of the advantages.

Now in homogeneity when you talk about it, see every material has some defect due to say variation in composition, it may be due to variation in the texture and it may be the variation in the thickness okay or maybe internal defects are there, maybe gas porosities or pinhole porosities or some inclusion all these things are there. So the homogeneity is something which is see which is only like you cannot quantify it homogeneity, okay. So it is only and so quantification of homogeneity is very difficult only by visual observation or observation or microscope you can say this is better homogeneous or maybe through microscopic technique by X-ray elemental mapping and other things you can say it is homogeneous, better homogeneity is there. But there is no quantification for so that way it is very difficult characterization of homogeneity but only thing is that you have to then that varies from person to person who is doing the analysis.

Now the surface features how good is your surface? The roughness and for that purpose many times now for metal deformation you may use lubricants also okay. So this roughness and the lubricants both will affect the forming operation and maybe you may have to conduct some special test to characterize this lubricants and roughness and what is its effect on the on the forming behavior okay and because the major thing is that this information is you cannot get it from a simple tensile test. Because if you are doing a bulge test it is like there the tensile test result will not give you an idea about the what is the friction between the workpiece and the die okay. So those difficulties are there. Now the damage at microstructural level also has also affected the deformation behavior or forming behavior of this thin sheet actually.

Because you will find that during the processing itself some materials might have resulted in some sort of damage inside itself, maybe at microstructural level or submicrostructural or submicron level during the plastic deformation. And these can be determined from the tensile test. So, how much is the damage which has come ok. Because if there is any damage which is taking place then ok this will show it will be reflected in a stress strain curve, it will be reflected in your work hardening behavior. If there any damage has come then you will see that there is a drop in the load.

So, that way you can have an idea about what is the how much is the damage which has already set in inside the material during the previous forming operation. So but if there is a damage then that also will affect your forming behavior sheet material forming because sometimes in some case this defect which has already been nucleated it may start growing and then you may not be able to get extensive deformation of your component. The next is the rain rate sensitivity that is strain rate sensitivity of the material.

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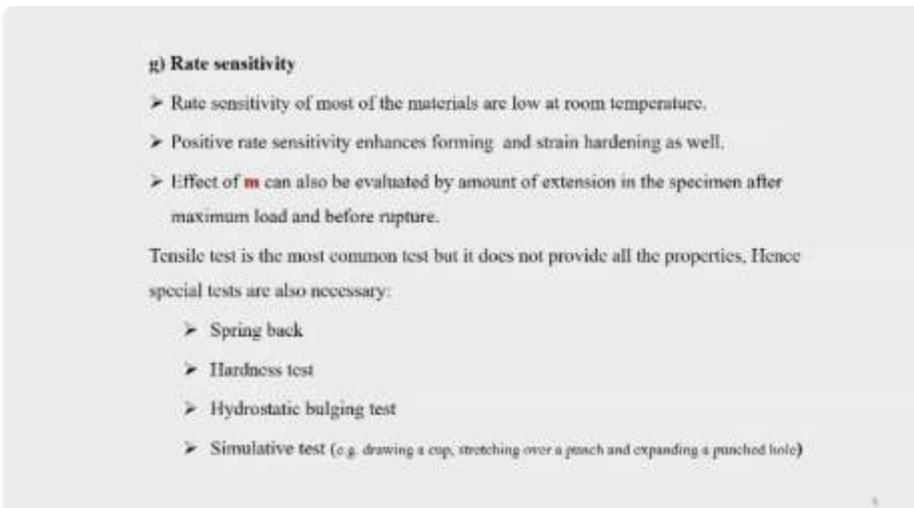

So these if you look at the strain rate sensitivity you will find that these are very low at room temperature. See strain rate sensitivity comes it it will be dominating at a higher temperature not at lower temperature. So so at lower temperature it is almost we can say very low. Now positive right sensitivity can be positive or negative. So positive right sensitivity enhances forming and strain hardening okay. And effect of M, this is that M can also be evaluated by amount of extension in the specimen after maximum load and before rupture okay.

So that we can find out this right sensitivity from the simple tensile test also. But many times that simple tensile test is most common test but it does not provide all the properties. Hence special tests are also required many times. What are the special tests?

There are some spring back test to find out the spring back effect. Then you may have to have a hardness test.

In steels, this hardness can be correlated to your yield strength also. Then hydrostatic bulging test, so that you will have an idea about how much is the amount of metal forming it can take.

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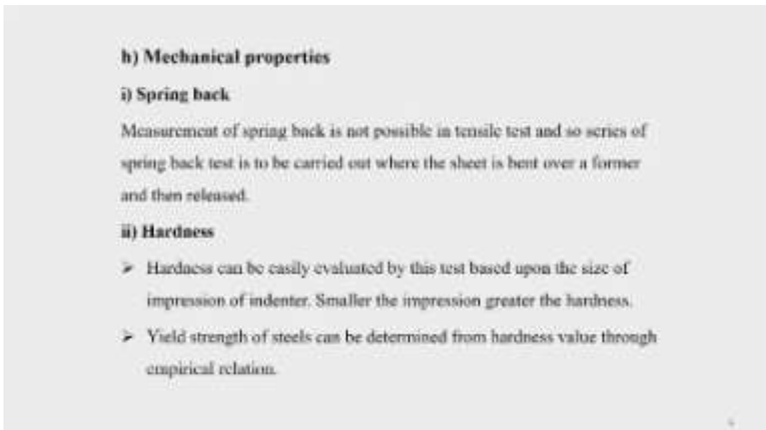

And another is the simulative test. So the spring bag test is the measurement of spring bag and it is not possible in a tensile test. So a series of spring bag test has to be done like you will be just it is bend over a former. So at certain angle you bend it and then release it and find out how much it is a spring bag. So plastically you deform it. Because of spring back it will just unwind by a small distance. So the sheet metal it will be wound around a former and you are allowing that while winding it undergoes some plastic deformation. And now the thing is that you release it you will find that it is unwinding by certain angle.

So this is the spring back faculty. And then depending upon the thickness of the sheet there is a limit up to which you can bend it. So, once you bend it without so that before that defect sets in and what is the maximum angle up to which you can bend without causing any deformation and then once you release it how much it unbends ok. So, that is what the spring back effect. So, for this you may have to work out with a large number of samples.

Now hardness is something which is the resistance of the material to indentation and either the indenter can be a hardened ball or it can be a diamond pyramid depending upon what type of test hardness test you wanted to do it. And this can be easily evaluated, okay. And the thing is that the smaller the indentation, when you are indenting with an intender on your workpiece material, if the material is harder, the impression which it has made on the surface will be smaller in size, okay. But if the material is very soft, the indenter can easily penetrate into that more and so that you will find that you are having a larger indentation okay.

So that is the indication of that. And most of the steels, the yield strength can be correlated to the hardness, but which is generally not practiced with in non-ferrous materials. These are basically indicated in steels. People have done carried out extensive work on trying to correlate between the strength and the hardness of the steel. Maybe when you do some different heat treatments and you get different hardness, then based on that your strength also will change. So, that information is already available.

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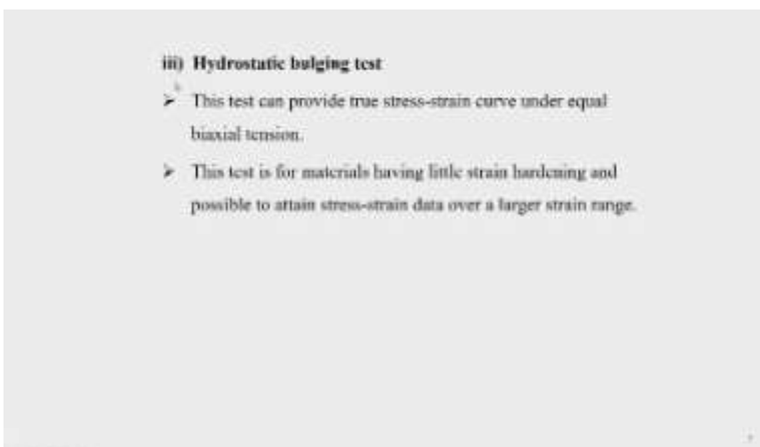

Third is the hydrostatic bulging test. So in the hydrostatic bulging test, so you are having a sheet metal and with a spherical intender now you are just pressing it on one direction okay and then so that from the center so that now there will be a biaxial tension which is there in all directions on the sheet metal okay. So the true stress strain curve under equal biaxial tension you will get it. So this test for materials having little strain hardening and possible to attain stress strain larger strain range.

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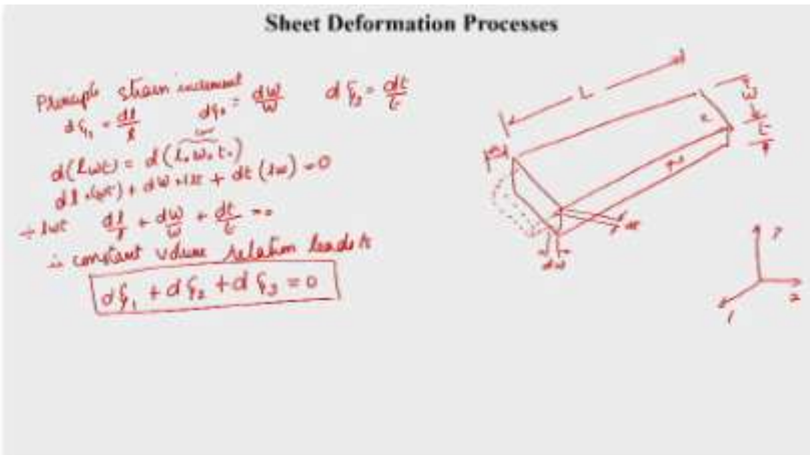

Now, let us come to this sheet deformation. So when you before we discuss about the mechanics let us just summarize the very basic fundamentals which we have discussed in our initial lectures and other things but here why we are discussing is that here again is that some terminology difference will be there and in sheet metal working they have a different terminology which is being followed that is why we will have to do it okay.

So for that let us assume a simple tension test, okay and in a simple tension test take the gauge length and do that. In sheet metal operation one important thing is there in sheet metal operation you will find that the stress which is perpendicular to the sheet surface that is very small. Since it is very small whereas the stresses are acting in other by axial direction but through thickness direction, the stresses are very small in sheet metal forming operation. And since it is very small, we can peacefully neglect it, okay.

So, that we will assume that the normal stress or stress normal to the surface of the metal sheet is 0. That means the sheet is forming the deformation process is a plane stress deformation case. Whereas those stresses can be assumed to 0, there will be strains in all the 3 directions strains will be there. Though thickness is less but strains are going to be there and you will find that thickness reduction is going to take place also because of this.

So in a tensile test if you take the gauge section suppose let us take the tensile test specimen for a sheet metal that will be a flat specimen and you take the gauge section how it will be the gauge section will look like this.

If it is like this and let us have the principal planes on principal directions, so let us say 1, 2, 3, this is the one direction. The direction 1 is the one having the highest stress that will be your tensile axis itself okay. So in that process let us say that the material has just deformed by a small amount okay. Let us assume that the material has just deformed and just drawing it in an exaggerated way. So, that this will be so, if I just say that this is your length direction and this is your width direction and this is your thickness direction. So, you will find that there is a small difference in this one what is called along the width length has increased.

So, this is your dw okay and maybe you may find that okay since it is coming here see this a small distance it is coming which is your dt and this is your delta L dL. So dimensions has changed. So let us assume that this is the direction 1, this is the direction 2 and this is your direction 3. So this is 1, 2, 3. So with respect to this coordinate system we can we can assume that in this.

So the principle strain increment in all the directions are say let us say principle strain increment. So d epsilon 1 along the direction 1 it will be dl by L okay and dw not delta it is d okay d epsilon 2 is equal to dw by w. d epsilon 3 is equal to dt by t.

$$\textit{Principle strain increment} \quad d\epsilon_1 = \frac{dl}{l} \quad d\epsilon_2 = \frac{dw}{w} \quad d\epsilon_3 = \frac{dt}{t}$$

This is the basic definition of this. So if you assume the constant volume relationship, okay, maybe like initially, so that means d of Lwt, if it is remaining constant, So this is equal to if you just take it as at any instant and which is equal to say the initial value L0 W0 T0 if you just take that. So you can just expand it and then you will find that DL into WT plus DW into LT plus dt into Lw so that will be equal to 0. So that is the basic assumption which is taking place. Now in all these things if you just divided by Lwt so we can say that d it will lead to dl by L plus dw by L plus dt. dW by W dT by T is equal to 0. The right hand side is 0 because this is a constant initial value and right left hand side also will be there but values the strain will be different. So, total strain will have to be equal to 0.

$$d(lwt) = d(l_0 w_0 t_0)$$

$$dl \times (wt) + dw \times \big(lt + dt(lw)\big) = 0$$

$$\div\, lwt \qquad \frac{dl}{l} + \frac{dw}{w} + \frac{dt}{t} = 0$$

So, constant volume relationship says so that means under constant volume relationship leads to because dL by L is d epsilon 1 so strain increment along direction 1 plus d epsilon 2 plus d epsilon 3 is equal to 0 this is your the strain increments along the 3 mutually perpendicular directions the sum of the strain increment will be equal to 0 so that is the constant volume relationship.

$$constant\ volume\ relation\ leads\ to$$

$$d\epsilon_1 + d\epsilon_2 + d\epsilon_3 = 0$$

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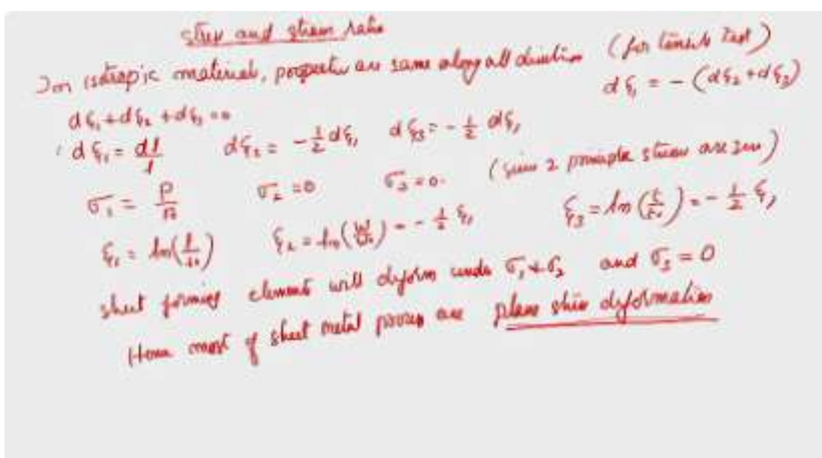

And when we come to this stress and strain ratios because it is always you will find that for isotropic material. the properties in isotropic material the properties are identical in all directions ok.So, in isotropic material properties are same along all directions. So considering there is no strain in the width and thickness direction will be of equal magnitude okay. So that means if you just take it as so d epsilon 1 plus d epsilon 2 plus d epsilon 3 is equal to 0. So the strain and thickness direction the strain will be strain increments will be same so that means d epsilon 1 is equal to dl by l and d epsilon 2 is equal to minus half d epsilon and d epsilon 3 is equal to minus half d epsilon 1 okay. So, this is because you are having a strain along the direction 1 then the sum of the strain in the other 2 will be equal to that is d epsilon 1 is equal to minus of d epsilon 2 plus d epsilon 3. So, and if this d epsilon 1 and d epsilon 2 are same then you will find that this is the case d epsilon 1 is equal to so that way we will get this relationship.

$$d\epsilon_1 + d\epsilon_2 + d\epsilon_3 = 0$$

$$d\epsilon_1 = \frac{dl}{l} \quad d\epsilon_2 = -\frac{1}{2}d\epsilon_1 \quad d\epsilon_3 = -\frac{1}{2}d\epsilon_1$$

So, what are the stresses and strains so that means if you look at it sigma 1 is equal to your p by a where a is your cross sectional area sigma 2 is equal to 0, and sigma 3 is equal to 0. I am telling this is for the tensile test okay okay for a tensile test for a tensile uniaxial tensile test that is the condition. So that means you will find that these are the conditions. So if you integrate this equation d epsilon 1 d epsilon 2 and d epsilon 2 you will always get epsilon 1 is equal to log L by L naught 2 is equal to log w by w naught and that is equal to minus half epsilon 1. Similarly, epsilon 3 is equal to log t by t naught that is also equal to minus half epsilon 1. Because for a uniaxial tensile test we are we are seeing the stresses are only along one direction the stresses at the other two directions are 0 that is why you have some geometric constraint to get have a uniaxial tensile stress okay. So and if at all stresses are there those will be equal also.

$$\sigma_1 = \frac{P}{A} \quad \sigma_2 = 0 \quad \sigma_3 = 0$$

$$\epsilon_1 = \ln\left(\frac{l}{l_0}\right) \quad \epsilon_2 = \ln\left(\frac{w}{w_0}\right) = -\frac{1}{2}\epsilon_1 \quad \epsilon_3 = \ln\left(\frac{t}{t_0}\right) = -\frac{1}{2}\epsilon_1$$

So maybe I can write that since this is because since the two principal stresses principle stresses are 0. So in actual in typical sheet metal processing most elements will deform under the membrane stresses sigma 1 and sigma 2 because that is not will not be equal to your uniaxial tensile test okay. So in typical sheet process in sheet forming elements will deform under sigma 1 and sigma 2, sigma 3 is we can assume it is a 0 ok along the normal direction we have earlier mentioned it is almost 0 because it is totally negligible we can just you know. So, under this condition for a sheet metal forming you have sigma 1 and sigma 2. So, the conditions of sheet metal processing will be like of a plane stress deformation. Hence most of the sheet metal process are plane stress deformation whereas bulk deformation we always go for plane strain conditions okay here it is plane stress condition. It is always convenient, it is convenient for deformation of the element under sheet metal processing to describe in terms of strain ratio and strain stress ratio.

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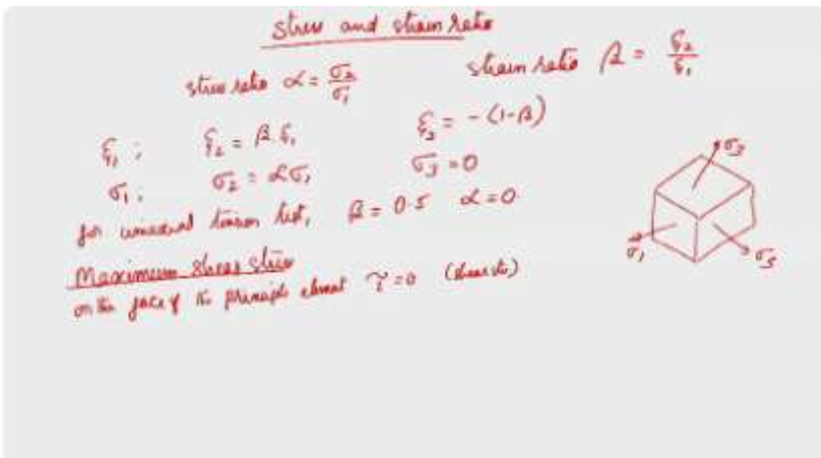

So let us see what is that. See because it has its own advantages, it becomes very convenient and become very simple for all calculations, stress and strain ratios. So the stress ratio alpha is because you have only two stresses so alpha is equal to is defined as alpha is equal to sigma 2 by sigma 1 remember sigma 1 in all our cases in henceforth whatever we are discussing sigma 1 will be the direction along with the highest tensile stresses are there.

$$stress\ ratio\ \alpha = \frac{\sigma_2}{\sigma_1}$$

Sigma 2 can be either tensile or it can be compressive, okay. But we are always taking that highest stress in the positive direction that is a tensile stress as sigma 1, okay. And then this is the strain ratio is defined as as epsilon 2 by epsilon 1. When a sheet is subjected to deformation, what are the stresses along these two directions? That ratio gives you a stress ratio and the strain which is taking place in these two directions are the strain ratios okay. So that means we can always say that in strain ratio you know when when it is along the direction one sigma 1 if it is there what are the other strains epsilon 2 is equal to beta into epsilon 1 and if you substitute beta into epsilon 1. And if you substitute that epsilon 3 will be equal to 1 minus of 1 minus beta into epsilon 1 because you substitute this you will get it the strain condition and you have this sigma 1 is there and your sigma 2 will be equal to alpha into sigma 1 and sigma 3 in the third direction the normal to a surface that is 0.You will find that for uniaxial tension test from this you can always find it out beta is equal to 0.5, and alpha is equal to 0 because sigma 2 by sigma 1 we are taking okay. So for uniaxial tension sigma 2 is equal to 0, sigma 3 is also equal to 0. So that is why if you substitute that you can directly get this and next we will come to the maximum shear stress.

$$strain\ ratio\ \beta = \frac{\epsilon_2}{\epsilon_1}$$

$$\epsilon_1; \quad \epsilon_2 = \beta\epsilon_1 \quad \epsilon_3 = -(1-\beta)$$

$$\sigma_1; \quad \sigma_2 = \alpha\sigma_1 \quad \sigma_3 = 0$$

$$for\ uniaxial\ tension\ test, \ \ \beta = 0.5 \quad \alpha = 0$$

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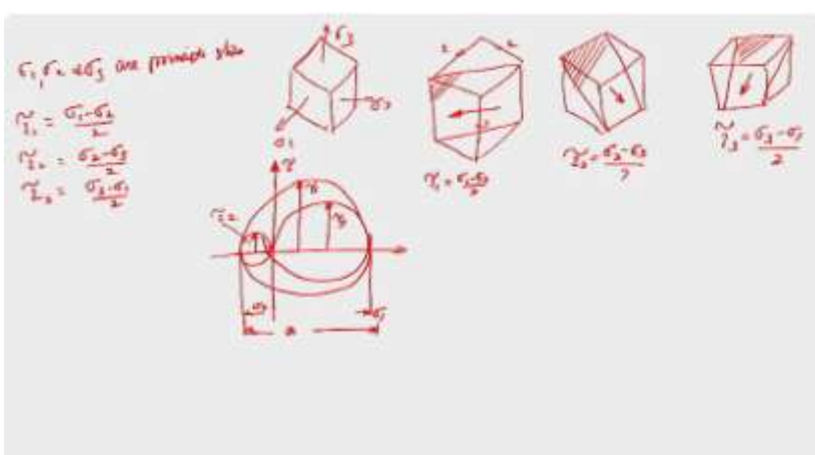

Let us find out what is this maximum shear stress. See if you just take a cuboid like this or any and you look at the general state of stress if you just assume that on these planes on the planes of this cube there is no shear stresses only normal stresses are acting okay. So along this direction sigma 1 and this is along sigma 2 and this is sigma 3. So, along the on the on the face of the cube of the element this is a principle element tau is equal to 0. So, that means shear stress. But if you consider planes even slightly inclined to that any of the faces you will find that on those planes there is both your normal stress as well as shear stresses are acting in the picture okay. And with more and more inclination you will find that the shear stress keeps on increasing and the stage will come when the shear stress is maximum, okay so that shear stress planes are inclined at 45 degree to the principal direction.

So for example if I just take this case for illustration this is sigma 1 sigma 2 and sigma. The shear stress by the maximum shear stresses if these are the principal stresses if sigma 1, sigma 2 and sigma 3 are principal stresses. If they are the principal stresses, then the maximum shear stress are given by so, you can get say different corresponding to the one plane you just tilt that plane and see and keeping the other two parallel to other two know then you can find out what is this planes ok. So, we can get this tau 1 is equal to sigma 1 minus sigma 2 by 2, tau 2 is equal to sigma 2 minus sigma 3 by 2 and tau the shear stress. So, 3 this is equal to sigma 3 minus sigma 1 by 2.

$$\tau_1 = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_2 = \frac{\sigma_2 - \sigma_3}{2}$$

$$\tau_3 = \frac{\sigma_3 - \sigma_1}{2}$$

So, this that way we can get it. So, for example, if you look at what are those planes, say if this is one plane maybe okay I have just it is not equal to a size. So, this part along with this directions if you look at it this plane, okay in this plane you have this tau here. So, in this case now you will find that tau 1 is equal to sigma 1 minus sigma 2 by 2 the maximum okay the sigma 2 and sigma. So, this will be you will find that it is equally inclined to your sigma 1 and sigma 2 see sigma 1 direction see this direction it is equally inclined to this is the one direction okay direction 1 and this is the direction 2. So with it will be equally inclined to the 1 and 2 and parallel to that. So this is the direction of this maximum shear stress.

Now if you look at the tau 2, okay, so it will be this plane, okay. So and the direction will be along this it can be this way or the reverse okay. So this is your taught to the maximum shear stress in this particular plane. So that is that is given by Sigma 2 minus Sigma 3 by 3 and you will find that it is inclined at 45 degree to Sigma 2 and Sigma 3 okay in along the 2 and 3 direction. So because this is 3 if this is 3 2 and 3.

So it will be equally inclined to 2 and 3. And similarly, you will find the next one as the maximum shear stress tau 3 along another plane. So this will be equal to say maybe we can say this direction. So this is tau 3 is equal to sigma 3 minus sigma 1 by sorry this is 2 not 3 by 2 and you will find that this plane in this plane know it is equally inclined to sigma 3 or direction 3 and direction 1 and this will be the direction at which the maximum shear stress will be acting in these two things. And you can always find out that from the if you and it is very easy to calculate this from the your what is called a Mohr circle.

Mohr circle representation if you look at it, it becomes very convenient. So, suppose you are having this here and you are having here and this is your tau direction So you can draw this bigger circle it looks like an ellipse maybe okay assume that okay it is a circle it is something like this. So this will be your tau 2 tau 3 because here this one is your sigma 1 okay this is sigma this is your sigma 2 and this the diameter of this A is equal to sigma 1 minus sigma 2. So, your maximum tau will be this one. So, what is that tau 1 and this is tau 3 and this is tau So, you will find that the maximum shear stress tau 1 is equal to sigma 1 minus sigma 1 plus sigma 2 by 2. So, that is what you are getting. So, the all the three you can get it by your Mohr circle this one.

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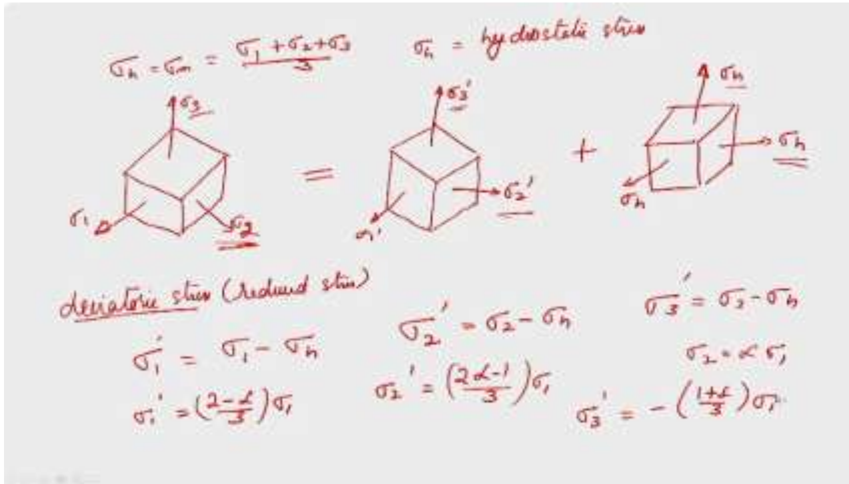

Now when you go to that the stresses at any point when you look at it earlier also we have discussed there is a hydrostatic stress which is coming into picture which is defined by your sigma h in some cases it may be sigma m okay that will be based on the principal stresses it will be sigma 1 plus sigma 2 plus sigma 3 by 3 okay. So, where this is the sigma h is the or sigma m is the mean or hydrostatic stress. Since it is hydrostatic, we will use, sometimes it is used as a sigma m also, hydrostatic stress.

$$\sigma_h = \sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \qquad \sigma_h = hydrostatic\ stress$$

See the hydrostatic stress means it is of equal dimension that is acting in all directions. So, like as if you have kept something instead of water and a fluid pressure is there. So, around all the directions this will be under equal this one that is something similar to a pressure, but here it is only what is induced inside that. So, that is what stress at any point you can just. Consider that and then at along the three principle say the principle stress state can be written as a combination of the hydrostatic and deviatoric stress okay.

So, for example, if I just say that okay this is sigma 1 which I think this slide has been shown earlier also. So, if this is sigma 1, this is sigma 2 and this is sigma 3. So this we can write as a sum of in all the three directions, hydrostatic stress will remain the same. So that will be a deviatoric stress. Say for example, sigma 2 dash, sigma 1 dash, and sigma 3 dash plus a hydrostatic component that means equal in all direction sigma h sigma h.

So that way along sigma 1 if you just look at it sigma 1 the principal stress sigma 1 is the sum of a deviatoric stress sigma 1 dash plus the hydrostatic component. Similarly sigma 2 if you take sigma 2 is equal to a sigma 2 dash plus the hydrostatic components. Similarly sigma 3 is the deviatoric stress along the direction 3 and the hydrostatic stress. So, along the 3 mutually perpendicular directions you are getting it. So we can say the deviatoric stress is also called as the reduced stress or reduced stress

Why a reduced stress? Because from your from all the three principal stresses you are just removing the hydrostatic stress which is an equal value about that. So, that we can say we can write the sigma 1 dash is equal to sigma 1 minus sigma h. Similarly, sigma 2 dash is equal to sigma 2 minus sigma h. and sigma 3 dash is equal to sigma 3 minus sigma h.

$$\sigma_1' = \sigma_1 - \sigma_n \qquad \sigma_2' = \sigma_2 - \sigma_n \qquad \sigma_3' = \sigma_3 - \sigma_n$$

So, that way we can just write like this along the 3 directions. But if you are writing this substituting the value of sigma 1 plus sigma 2 plus sigma 3 and say if you say sigma 2 is equal to alpha sigma 1. So, if you are using this sigma 2 is equal to alpha into sigma 1 and you substitute under these conditions in a similar way you can write that sigma 1 dash in terms of alpha if you can write it as 2 minus alpha by 3 into sigma 1 we just substitute in this sigma h is equal to plus sigma 1 plus sigma 2 plus sigma 3 by 3 and then you do it you will get it. Similarly sigma 2 dash you will get it as it is equal to 2 alpha minus 1 by 3 into sigma 1 because you can easily measure the sigma 1 and then sigma 3 dash is equal to minus of 1 plus alpha by 3 into sigma1. So basically the deviatoric stresses are the difference between the actual the principal stresses and its hydrostatic stress.

$$\sigma_2 = \alpha\sigma_1$$

$$\sigma_1' = \left(\frac{2-\alpha}{3}\right)\sigma_1 \qquad \sigma_2' = \left(\frac{2\alpha-1}{3}\right)\sigma_1 \qquad \sigma_3' = -\left(\frac{1+\alpha}{3}\right)\sigma_1$$

So that is what. So now let us look at these conditions and apply this on your yield criteria. Because these are the relationship between the stress and strains. Now we have to using this relationship we have to look into the yield criteria.

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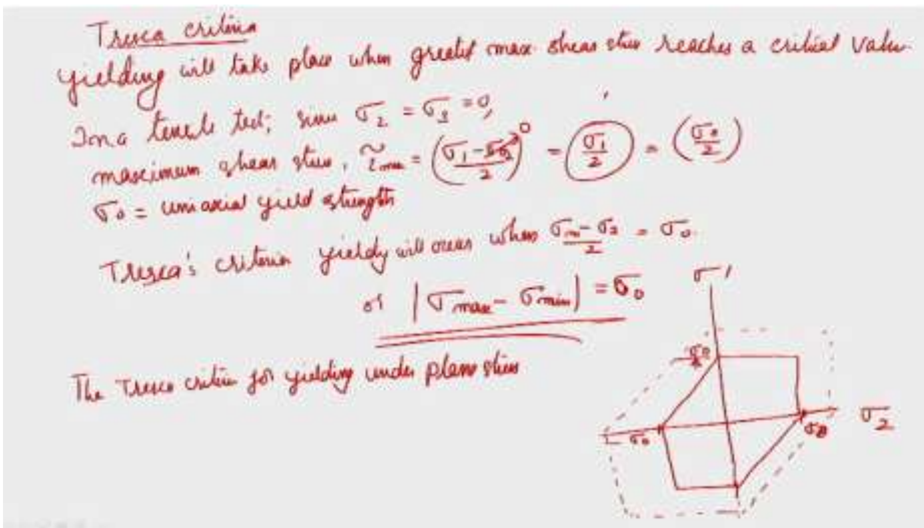

So let us say first is the Tresca criteria. So in the Tresca criteria, if you say a uniaxial state of stress in sigma 2 is equal to sigma 3 is equal to 0, maximum shear stress at yielding is equal to your flow stress. So that means for a you have to find out which is the greatest maximum shear stress. So Tresca criteria says that yielding of the material will take place when because you have three maximum shear stresses the greatest of the maximum shear stress when it reaches a critical value yielding will take place okay. So yielding or plastic deformation will take place when the greatest maximum shear stress reaches a critical value and what is this critical value you have to find out. The simplest thing for finding out the correlating is to get a tensile test that is because you can always correlate it and correlate with that one. So in a in a tensile test Since sigma 2 is equal to sigma 3 is equal to 0, then the maximum shear stress is tau max is equal to sigma 1 minus sigma 2 by 2 where sigma 2.

$$In\ a\ tensile\ test, since\ \sigma_2 = \sigma_3 = 0,$$

$$maximum\ shear\ stress, \tau_{max} = \left(\frac{\sigma_1 - \sigma_2}{2}\right) = \frac{\sigma_1}{2} = \frac{\sigma_0}{2}$$

So this is 0 so that means it is equal to sigma 1 by 2. So that means in a uniaxial tensile strength you are conducting what is the yield strength you are getting so that is that. So this we can write that this is equal to say sigma 0 by 2 is a uniaxial where sigma 0 is a uniaxial yield strength of the material. So, the Tresca's yield criteria will occur as per Tresca's criteria yielding will occur sigma max minus sigma 2 by 2 is equal to sigma 0 or we can say the absolute value of sigma max minus sigma minimum is equal to sigma 0. So under plane strain condition, the plane strain yield criteria, the Tresca's criteria for yielding under plane stress, it will be a hexagon, okay.

So that if you just plot it like this, this is sigma 2, sigma 1. So this is your sigma f and this equal to your sigma f. So we can just say that it comes like this and from maybe here it comes okay and so this will come down and it will reach here and this is your hexagon. So you will find that this is sigma f or sigma o and this is equal to sigma o. So this is minus sigma o and similarly so it looks like a hexagon. So if you look at under all these conditions sigma 1 and sigma 2 if you just take all these conditions you will find the locus of that point is a hexagon okay that indicates the state of stress as the stress ratio changes.

$$Tresca's\ criteria\ yielding\ will\ occur\ when \frac{\sigma_m - \sigma_2}{2} = \sigma_0$$

$$or\ |\sigma_{max} - \sigma_{min}| = \sigma_0$$

So when stress ratio changes how it goes. When the material is work hardening, this hexagon it will expand outwardly. Because you are flow stress sigma 0 that will keep on changing accordingly the so it may come like this here and it may go like this. So, when the deformation is taking place, you may find that but still it will be a hexagon the same shape okay. So this first the dash the straight line which has been drawn is for a fully annealed material when you are just deforming it okay.

But when the deformation is changing and if you are maintaining the stress ratio constant then you will find that okay this the hexagon size keeps on increasing because your new sigma not flow stress will come into picture.

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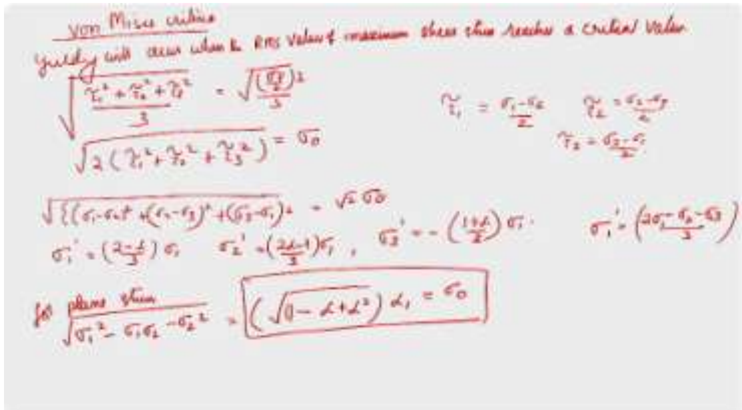

So now let us look at the Von Mises criteria. According to the Von Mises criteria, earlier we have discussed in depth for bulk deformation process. Because these I am again telling this because the terminology used in sheet material forming is different that is why you have to always represent in terms of stress ratio and strain ratios that is why it is coming. So yielding will occur when the root mean square value of maximum shear stresses reaches occur when the root mean square values of maximum shear stress reaches a critical value.

So what is that critical value you have to find out. So for that now at when you are doing a tensile testing during the, at yielding 2 of the maximum shear stresses will have a value sigma f and the third one is 0. So, in a sheet material work so sorry in a tensile testing. So, the criteria here what we have mentioned we can explain in this way that is tau 1 square plus tau 2 square plus tau 3 square by 3 is we can say is equal to sigma f by 2 the whole square by 3. Why because sigma 1 and sigma 2 is equal to minus of this value. So that way so two values which will be there it will be the your uniaxial flux plus by 2 that is what is going to happen.

$$\sqrt{\frac{\tau_1^2 + \tau_2^2 + \tau_3^2}{3}} = \sqrt{\frac{\left(\frac{\sigma_p}{2}\right)^2}{3}}$$

So this is what you are going to get it or we can also write so in this condition so root of 2 into tau 1 square plus tau 2 square plus tau 3 square is equal to sigma f sigma naught uniaxial flow stress of the material ok. So, this is one advantage if you have conducted uniaxial flow stress we can just correlate with this value what is the von Mises criteria and Ruska criteria that is one advantage with this. And if you use this for tau 1, tau 2, tau 3 you know if you are using your tau 1 is equal to sigma 1 minus sigma 2 by 2 ok and tau 2 is equal to sigma 2 minus sigma 3 by 2 and 3 is equal to sigma 3 minus sigma 1 by 2. If you substitute in this now you will get this is equal to root of sigma 1 minus sigma 2 square plus sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square. So that is equal to because this 2 will come here so it will be root of 2 sigma 0. So this is what you are going to get because here all this 2 is coming below so that is why it will you will come to this.

$$\sqrt{2(\tau_1^2 + \tau_2^2 + \tau_3^2)} = \sigma_0$$

$$\sqrt{\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \sqrt{2}\,\sigma_0$$

So if you just substituting the in terms of deviatoric stresses you will say that sigma 1 dash is equal to 2 minus alpha by 3 into sigma and sigma 2 dash is equal to 2 alpha minus 1 by 3 into sigma 1 and sigma 3 dash is equal to minus of 1 plus alpha by 3 which we have got a Taylor into sigma 1 and sigma 1 dash is equal to 2 sigma 1 minus sigma 2 minus sigma 3 by 3. we are getting you for the plane stress the criteria is for plane stress the Von Mises criteria comes to sigma 1 square minus sigma 1 sigma 2 minus sigma 2 square.

$$\sigma_1^{'} = \left(\frac{2-\alpha}{3}\right)\sigma_1 \qquad \sigma_2^{'} = \left(\frac{2\alpha - 1}{3}\right)\sigma_1 \qquad \sigma_3^{'} = -\left(\frac{1+\alpha}{3}\right)\sigma_1$$

So, that is equal to you will get it as 1 minus alpha plus alpha square into sigma 1 that is equal to sigma 0. So this is your Von misses.

$$for\ plane\ stress,$$

$$\sqrt{\sigma_1^2 - \sigma_1\sigma_2 - \sigma_2^2} = \left(\sqrt{(1-\alpha+\alpha^2)}\,\alpha_1\right) = \sigma_0$$

So both these expressions are valued for isotropic materials.

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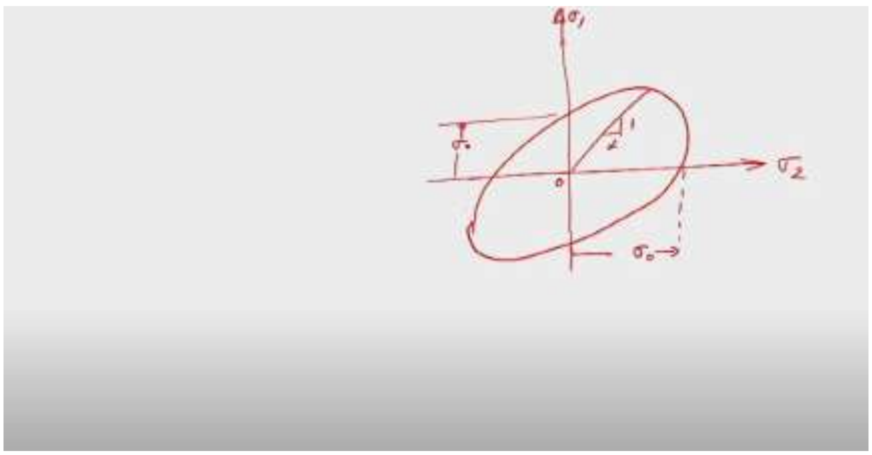

So if you look at the Von Mises criteria, how the yield locus look like. So that also we have discussed earlier but let us just revise it here. So this is your sigma 2 and this is your sigma 1. So it will be an ellipse okay somewhere like this it will go. So this is your all the 2 directions if you take it now you will find that this is your sigma 1 sorry sigma f or uniaxial flow stress sigma naught and this will be your uniaxial flow Okay. So it will look like this and this is a case for alpha is equal to 1 somewhere here at 45 degree if you look at it this is equal to alpha is equal to 1 okay. So this is the condition you can always take at any point on the strain history if you look at it or you can find out what is the criteria for yielding at this which is taking place. Now when the material is work hardening yourself plastically deforming it this will again expand but the shape will still remain the same. So, so yield criteria helps to predict the stresses at which a plastic deformation occurs in plane stress condition where the ratio of stress in plane of the sheet to flow stress of the material if it is known, so it will be able to predict the stresses.

In elastic deformation region, see whatever is inside this ellipse is the elastic deformation region. Stress and strains are directly related and can be calculated very easily because it follows the Hook's law and other things. So it is very easy to find out in direct relationships you get it but the thing is that when you come to plastic deformation region there is no relation which exists and it may happen that at some stage material may be under the under state of stress but there is no change in the shape that also it can happen. So magnitude of deformation depend upon the boundary movement and not on the stress stresses. So that is the thing which happens.

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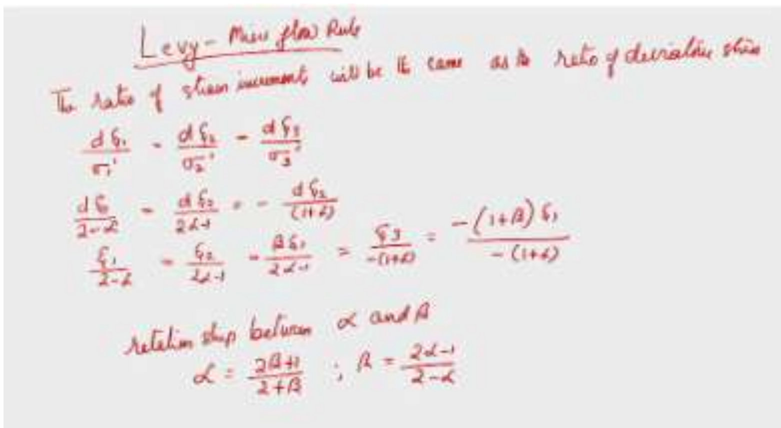

Now let us come to say Levy-Mises in plasticity this we have to this is very important. Levy-Mises flow rule which also we have discussed but still now it is time that we have to look at it. Because we have already discussed the actual state of stress, actual stress state in a material is a combination of the hydrostatic and the deviatoric state of stress. In that the hydrostatic stress will not influence the shape change of the component. It will just tell maybe its volume increases or decreases but shape change is not being caused due to hydrostatic stress.

Only the size of the component only will change due to effect of the, because equally in all direction it is coming. Whereas what causes the change in the, alters the stress state sorry shape of the component is your deviatoric stress. So this, Levy-Mises rule states that the ratio of strain increments will be the same as the ratio of the deviatoric stress. So, the ratio in the of the strain increments will be the same as the ratio of deviatoric stresses. So, that means strain in say for example, strain increment d epsilon 1 by sigma 1 dash is equal to d epsilon 2 divided by on the deviatoric stress in that direction that is equal to d epsilon 3 into sigma 3 dash. So, in terms of your if you write that sigma 1 sigma 2 sigma 3 in terms of your alpha value the stress ratio we can write this as d sigma 1 divided by 2 minus epsilon is equal to d sorry 2 minus alpha, d sigma 2 divided by 2 alpha minus 1 that is equal to minus of d epsilon 2 divided by 1 plus alpha we can write in this term in terms of alpha okay.

$$\frac{d\epsilon_1}{\sigma_1'} = \frac{d\epsilon_2}{\sigma_2'} = \frac{d\epsilon_3}{\sigma_3'}$$

$$\frac{d\epsilon_1}{2-\alpha} = \frac{d\epsilon_2}{2\alpha-1} = -\frac{d\epsilon_2}{(1+\alpha)}$$

And see the under plane stress condition what happened the proportional conditions if it is deforming under proportional condition in all direction the stress ratio that means The stress ratio is maintained a constant. Then we can integrate this above equation and to get the strain itself. Instead of strain increment, so this 1 by 2 minus alpha is equal to epsilon 2 by 2 alpha minus 1 is equal to this is equal to beta epsilon 1 by 2 alpha minus 1 because epsilon 2 is equal to beta into epsilon 1 is equal to epsilon 3 by minus of 1 plus alpha. So, that is also equal to you can write in terms of epsilon 3 as see this one 1 plus in strain increment if you write it as epsilon 1 by minus of 1 plus alpha.

$$\frac{\epsilon_1}{2-\alpha} = \frac{\epsilon_2}{2\alpha-1} = \frac{\beta\epsilon_1}{2\alpha-1} = \frac{\epsilon_3}{-(1+\alpha)} = \frac{-(1+\beta)\epsilon_1}{-(1+\alpha)}$$

So, we can write. The Levy-Mises rule follows this one but this we were discussing about the stress ratio and the strain ratio. These are correlated the relationship between alpha and beta that we can correlate that is alpha is equal to 2 beta plus 1 by 2 plus beta. Or we can write it as beta is equal to 2 alpha minus 1 by 2 minus alpha this way also we can write. But this relationship between the stress and the strain it will not give the exact value of the strain that is that is true okay.

$$relationship\ between\ \alpha\ and\ \beta$$

$$\alpha = \frac{2\beta + 1}{2 + \beta} \qquad \beta = \frac{2\alpha - 1}{2 - \alpha}$$

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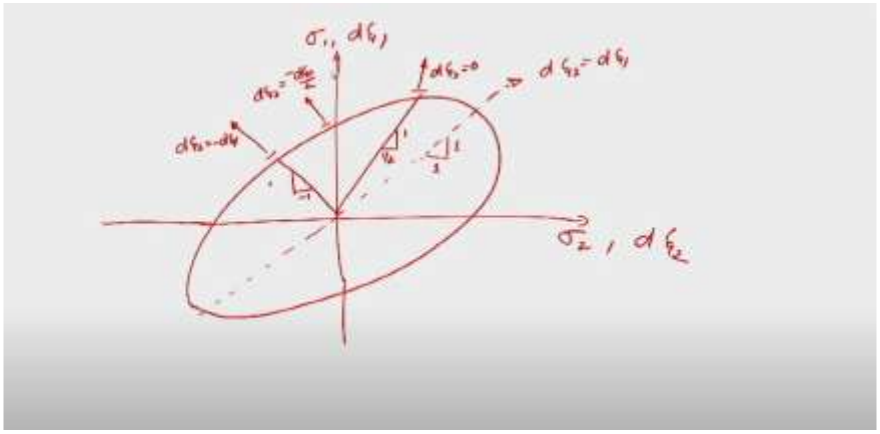

It is only we can correlate with it will not give you the exact value of this thing. So if you look at the flow rule, Levy-Mrs. flow rule, it gives the relationship between the stress and the strain ratios. It will not indicate the magnitude of the strain. So that is one thing it will not tell you the exact value of the strain. If the element deforms under a given stress strain or if alpha is known the ratio of the strain in the previous case and this case that we can find out from along the two directions but it will not give an absolute value.

So initially if you know the value and then okay under a constant alpha it is moving it is deforming then we can have the strain that is all. So the relationship can be illustrated for different load paths as shown here in the, if you just take this, this is your sigma 2 and d epsilon 2, this is your sigma 1 and the same So that way if you do that then so you can just get that. If I just draw this, this is a case where d epsilon 2 is equal to d epsilon 1 okay. So this slope is 1 and 1 and this is a case where you have half slope is half and then maybe you can just from here you can just draw it as d epsilon 2 is equal to 0 and this is d epsilon 2 d epsilon 2 is equal to minus d epsilon 1 by 2. So, if I just draw like this sorry. So this is the condition when d epsilon 2 is equal to minus d epsilon 1 and here it is minus 1 and 1. So these are the conditions which we are getting. It will look like this.

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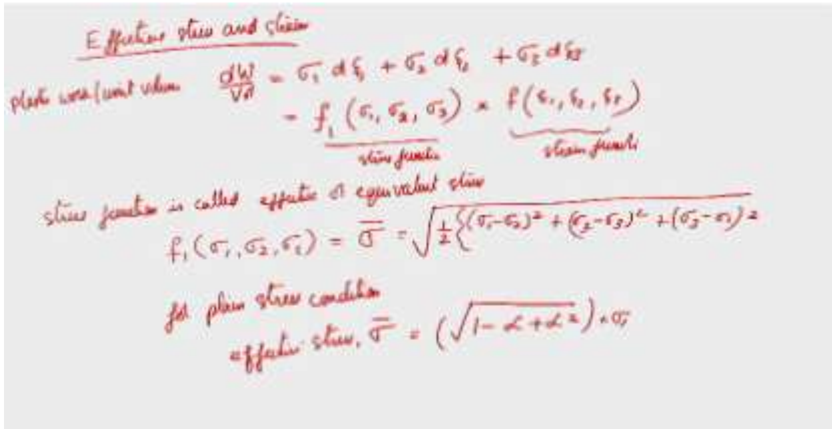

Now when we discuss about this deformation, it is very convenient to express the stresses as effective stress and strain as effective strain because earlier also we discussed the say for example effective stress and strains, okay. The plastic work per unit volume, if you just calculate it, it is dW, this plastic work, dW by whatever volume is there, it it was expressed on sigma plus d epsilon 1. plus sigma 2 into d epsilon 2 plus sigma 3 into d epsilon 3. So, so this we can always write it as a as a two function a function of maybe sigma 1 the principal plane sigma principal stresses sigma 2 and sigma 3 and maybe another if I just write this as f1 another function into another function which is nothing but your epsilon epsilon 2 and epsilon 3 we can write like this. So this this is a stress function and this we can call it as a strain function. So this total plastic work per unit volume we can write in terms of a stress function multiplied by a strain function so that is what we can write.

$$plastic\ work\ by\ unit\ volume\ \ \frac{dW}{V} = \sigma_1 d\epsilon_1 + \sigma_2 d\epsilon_2 + \sigma_3 d\epsilon_3$$

$$= f_1(\sigma_1, \sigma_2, \sigma_3) \times f(\epsilon_1, \epsilon_2, \epsilon_3)$$

So the stress function is called , is called the effective or equivalent stress. So that is we can say that F of principle stress sigma 1, sigma 2, sigma 3, we can write it as sigma bar which is effective stress. So, we can write as in terms of your Von Mises criteria that is sigma 1 minus sigma 2 square plus this sigma 2 minus sigma 3 square plus sigma 3 minus sigma 1 square. we can get it. So for plane strain condition sorry plane stress condition because in sheet metal we are using the plane stress condition. So for plane stress condition we can substitute that condition for sigma 1, sigma 2, sigma 3 and other thing in terms of sigma 1 and sigma 2 so that the effective stress we can say sigma bar is we can write it as root of 1 minus alpha plus alpha square into sigma 1.

$$f_1(\sigma_1, \sigma_2, \sigma_3) = \bar{\sigma} = \sqrt{\frac{1}{2}\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\}}$$

$$effective\ stress, \bar{\sigma} = (\sqrt{1 - \alpha + \alpha^2}) \times \sigma_1$$

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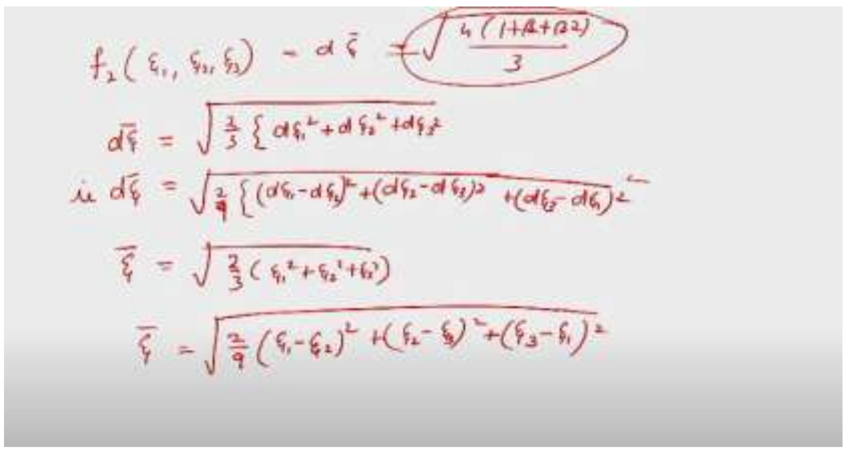

So, that is what we can get it. Similarly, the strain function, representative strain function sigma 2 in terms of epsilon 1, epsilon 2, epsilon 3, we can write it as, for plane strain condition, we can write, so this is equal to d epsilon bar, so that we can write in the form as 4 into 1 plus beta plus beta square by 3. We can write it in this form. Why? The general state of stress, if you look at it, it is d epsilon bar is equal to for the general state if you just 2 by 3 into it is equal to d epsilon 1 square plus d epsilon 2 square plus d epsilon 3 square ok. So, that is which is nothing but you can always write it as 2 by 9 into d epsilon 1 minus d epsilon 2 the whole square plus d epsilon 2 minus d epsilon 3 the whole square plus d epsilon 3 minus d epsilon 1 the whole square.

$$f_2(\epsilon_1, \epsilon_2, \epsilon_3) = d\ \bar{\epsilon} = \sqrt{\frac{4(1 + \beta + \beta^2)}{3}}$$

$$d\,\bar{\epsilon} = \sqrt{\frac{2}{3}\{d\epsilon_1^2 + d\epsilon_2^2 + d\epsilon_3^2\}}$$

$$d\,\bar{\epsilon} = \sqrt{\frac{2}{9}\{(d\epsilon_1 - d\epsilon_2)^2 + (d\epsilon_2 - d\epsilon_3)^2 + (d\epsilon_3 - d\epsilon_1)^2\}}$$

So, in this way we can write it. So, if this is going on a proportional process the equivalent strain increment. So if it is on a proportional process where the alpha and beta remains constant in that case you know from the so this above equation can be integrated from the natural or true strains and substituted to incremental strain. So we can get it as you integrate it you will get it as like this. So 2 by 3 into epsilon 1 square plus epsilon 2 square plus epsilon 3 square So that is equal to you can write say 2 by 9 into yeah so that is what we are getting this okay. So the same thing when you are integrating you will get it as 2 by 9 into epsilon 1 minus epsilon 2 square plus epsilon 2 minus epsilon 1 square plus epsilon sorry epsilon 2 minus epsilon 3, epsilon 3 minus epsilon 1 square.

$$\bar{\epsilon} = \sqrt{\frac{2}{3}(\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2)}$$

$$\bar{\epsilon} = \sqrt{\frac{2}{9}(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2}$$

We will get it in that form. It is always convenient to represent terms of equivalent strain and equivalent stress and this is and that relationship for equivalent strain we can get it as here and for the equivalent stress we can or effective or equivalent stress we can get it as this, this is sigma 1. Thank you.