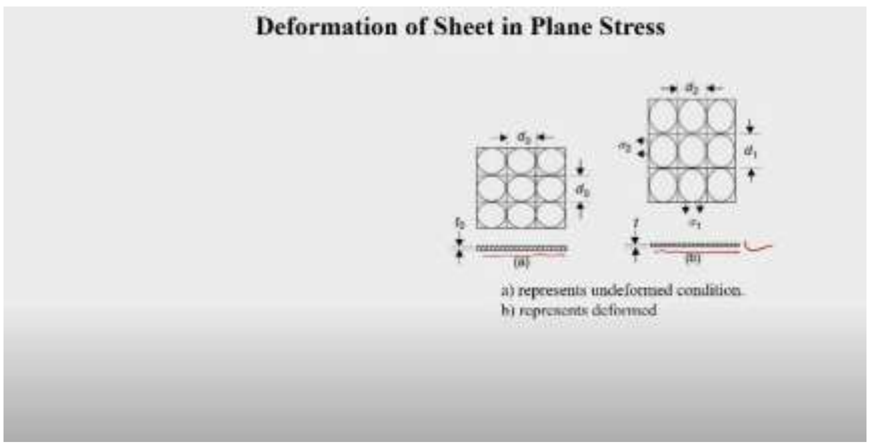
Plastic Working of Metallic Materials

Prof. Dr. P.S. Robi
Department Of Mechanical Engineering
Indian Institute of Technology – Guwahati

Lecture-32

Deformation of sheet in plane stress

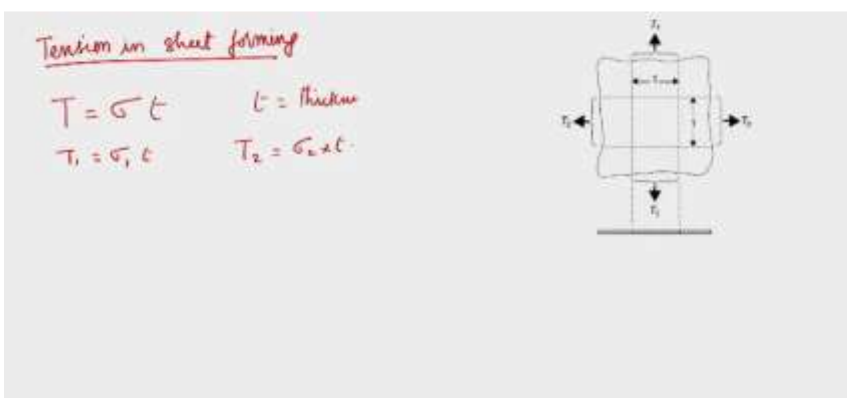
So earlier now we were discussing for that instant of a plane stress deformation of a work hardening material. Now what we will do is that we will just apply the theory to some region of a sheet metal which is undergoing say uniform proportional deformation. **(Refer Slide Time: 00:52)**



So suppose in that sheet you make some grids. A circular grid in a in a specific pattern or a square grid if you if you make it say like you can use it by different by electric catching we can do or if you make a say rubber stamp type thing you know your seal type thing with small this circular thing and then with that stamp pad now you can just put an impression on the surface that also a very simple thing. Maybe only thing is the diameter of the, of the circles or the square will be around 1 mm or so, something like that or 1. 5 mm we can make with the different sizes and that sheet is subjected to say your a proportional deformation maybe with a some value of strain stress ratio or strain ratio if you are maintaining it and then you try to deform. So this will be the take two cases okay the first one is A it represents a case where the material has not deformed so you will see circle is like a circle square is like a square. But whereas once you are applying a stress along the sigma 1 direction because the as I mentioned earlier the longest stress the highest stress direction is taken as the direction 1 and minimum one is taken as sigma 3. So, in this particular case after the deformation you will find that the circle has been distorted in the shape of an ellipse or if it is a square grid the square has become a rectangle. So, this is but the advantage is that in case of this you can always find out what is the strain along the sigma 1 direction and what is the strain along the sigma 2 direction that is easy that can be easily found out by just measuring this minor and major axis of the ellipse or by measuring the if it is a square grid then by measuring the length and breadth of this rectangle, so that is one.

Third is that after this you can also find out what is the thickness variation maybe initially what was the thickness because of this deformation was taking place what was the final thickness you will find that there is a small difference in the thickness. So strain is there in all the three directions those stresses we are considering is only along two direction okay. Normal to this thickness direction the stresses we are assuming it to be negligible. Now in the, when you consider this tension because there is always going to be a tensile force okay tension is there. So the deformation force per unit width is defined as the product of the thickness and the stress along that direction.

(Refer Slide Time: 03:44)



So because your sheet thickness is less, so if you take that along whatever direction so that is a per unit width if you are taking, what is that force? So that means that is what is called as the tension in sheet forming. So like if you say, So in sheet forming operation what happens there are mostly that there are tensile stresses okay and the force per unit width is defined as the so we can always tell what is the tension in these two cases as if you are taking the unit width the force per so that will be the tension can be along the stress into your thickness okay T is the thickness. So you can always write that T1 is equal to sigma 1 into T and T2 is equal to sigma 2 into T. So this is what we can always write. So in stretching when see always this direction 1 we are taking as the highest value so that will be naturally it will be a tensile stress. So and in in the stretching this operation T 2 will also be tensile okay.

So in other cases T 2 can be negative many other method techniques are there T 2 can be there but if it is a stretching operation naturally T 2 also is tensile okay. So negative value of T 2 when it comes if T 2 is going to be negative that means it indicates compressive that is what okay and in such a case when whenever the T 2 comes to negative. So, it is compressive and there may be a wrinkling which is forming wrinkles can form and there may be a increase in the thickness also that also can happen.

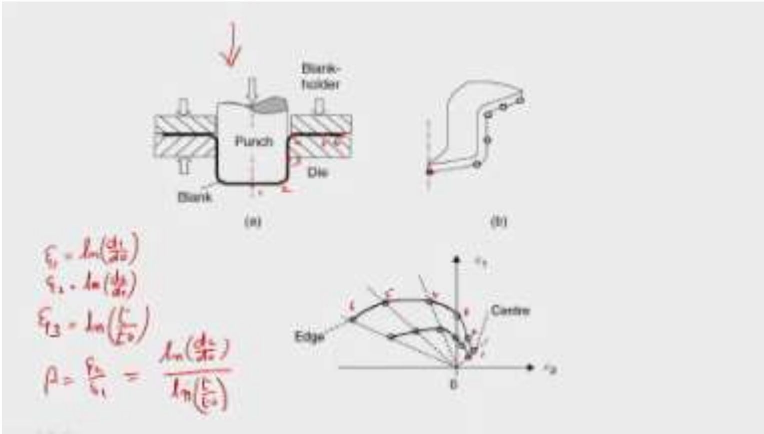
$$T = \sigma t$$

$$T_1 = \sigma_1 t \qquad T_2 = \sigma_2 \times t$$

So, when you look at the deformation analysis, the strain evaluation is done first. by measuring the the grids and after deformation the grids are the measurement of the grids are taken maybe along the 2 direction which is circular the major axis of the ellipse and the minor axis of the ellipse are measured and if it is a square rectangular piece the length and breadth is taken and plus we can also measure the thickness also after the experiment is done. And so the strains in after that it is plotted in the principal strain space okay.

So and then you will find that it provides the locus of all strains which are there. So for example, if you just take a simple example if you take, in this case.

(Refer Slide Time: 06:41)



You are just holding the there is a bottom punch there is a blank holder is there and a sheet material is kept and with a punch you know you are allowing it to move down and then causes this drawing operation this is the thing. So, at any instant when you look at it where what are the strains where where the strains have taken place.

If you are just looking all those things and try to plot the, the locus of the strains which are going to take place maybe at different places. So you can just get an idea about the deformation. So and this is being used for the deformation analysis okay. So this is also called as the strain signature. This one what we have shown now is for a proportional deformation okay so equally in all direction maybe it is work hardening and other things it is happening in all directions so that is what is happening so that is called as a strain diagram. So this is the strain diagram which is shown so you will find that say you find out the strain from the grid you measure what is this at the center so maybe at this point what is the strain. So from that grid pattern you can get it what is the strain at this point, so what is the strain at point this point, what is the strain at this point, what is the strain at this point. You are just measuring from the maybe your ellipse or from the rectangle after the deformation, maybe some amount of deformation has taken place and try to find out what is the type of strain there. So that if you just plot it maybe in this region, so you get this one. So, so at the center point you are getting here and at the edge you are getting here edge means this is the point 1 okay.

So, if I just say 1, 2, 3, 4, 5, 6 means this is 1 maybe 2, 3, 4, 5, 6 like that different place at different say You can just hold it and then increase the punch a moment down and then you will get it. So, only thing we are assuming that it is proportional. So, this may be the initial case and this is the later final case which you are getting okay and then trying to measure that epsilon 2 and epsilon 3 see epsilon sorry epsilon 2 and epsilon 1 epsilon 1 versus epsilon 2 this is what we are going to get it. So along the major dimension and the minor dimension along that axis you are just measuring and plotting at different cases.

So this is what you are getting the strain plot okay. So maybe if it is work hardening you will get this case. If it is just at the point of yielding or something you will get this. So that way maybe you can if it is a work hardening maybe one more you can find it out with a higher amount of strain. So that is the thing which you are going to get it.

So in the strain diagram the principal strains will be epsilon 1 you are going to measure it as in terms of d1 by d0 where d0 is your initial value okay and epsilon 2 is equal to log of D2 by D0 and epsilon 3 is equal to log of T by T0. You are measuring all these things because from the earlier case, we can measure this thickness also, we can measure all these dimensions. So this one, this you can measure, this you can measure okay and this also you can measure. so you can get all the 3 strains in that directions you are getting it.

$$\epsilon_1 = \ln\left(\frac{d_1}{d_0}\right)$$

$$\epsilon_2 = \ln\left(\frac{d_2}{d_0}\right)$$

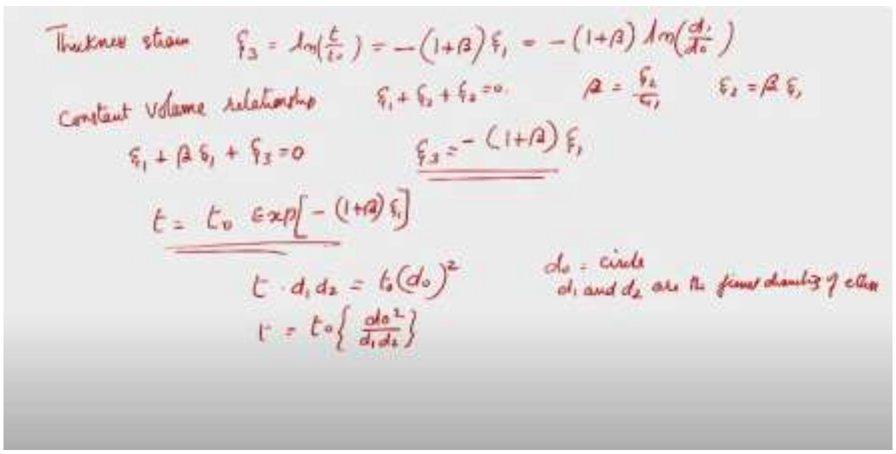
$$\epsilon_3 = \ln\left(\frac{t}{t_0}\right)$$

So from this we can get the strain ratio beta is equal to epsilon 2 by epsilon 1 so that will be equal to sorry not here beta is equal to say epsilon 2 by epsilon 1 so that is equal to log d 2 by d naught by log t by t naught you can get that strain ratio.

$$\beta = \frac{\epsilon_2}{\epsilon_1} = \frac{\ln\left(\frac{d_2}{d_0}\right)}{\ln\left(\frac{t}{t_0}\right)}$$

So the strain ratio will remain same if the say maybe this to this is same okay. So that is what so it is proportional or linearly linearly if you can relate between these two then you call it as a constant strain rate okay.

(Refer Slide Time: 11:37)



So the thickness strain that is very important because that is what will decide your failure and other things. So you have to get it that epsilon 3 is equal to log T by T naught. So in terms of beta you can write it as it is 1 plus beta into epsilon 1 that is equal to minus of 1 plus beta into log d 1 by d naught is for a proportional straining.

$$Thickness\ strain\ \epsilon_3 = \ln\left(\frac{t}{t_0}\right) = -(1+\beta)\epsilon_1 = -(1+\beta)\ln\left(\frac{d_1}{d_0}\right)$$

So, for volume consistency relationship constant volume relationship that means a epsilon 1 plus epsilon 2 plus epsilon 3 this is a sum is always 0 and so strain ratio beta is equal to if you just put it epsilon 2 by epsilon 1 and epsilon 2 is equal to beta into epsilon 1. If you substitute that, we can write that epsilon 1 plus beta epsilon 1 plus epsilon 3 is equal to 0 or epsilon 3 is equal to 1 plus beta into epsilon 1 minus of. So, that we will get it, minus of 1 plus beta into epsilon 1.

$$Constant\ Volume\ relationship \quad \epsilon_1 + \epsilon_2 + \epsilon_3 = 0$$

$$\beta = \frac{\epsilon_2}{\epsilon_1} \qquad \epsilon_2 = \beta\epsilon_1$$

$$\epsilon_1 + \beta\epsilon_1 + \epsilon_3 = 0$$

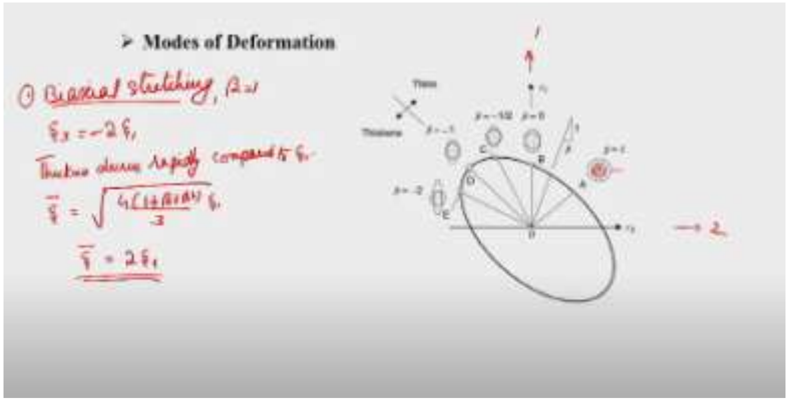
So, the thickness strain we can write it as this one and if t is equal to say we can write it as t naught because your log d by this one is coming. So, t naught into say exponential minus 1 plus beta into epsilon 1 so that we can get it. So if you look at the volume before deformation is equal to volume after deformation that means if you just say T into D1 D2 is equal to T into say D0 square because D0 since D0 is a circle and D1 and D2 are the final diameters of ellipse. So, from that we can write T is equal to, so this should be T0, T0 into d0 square by d1 d2. So that way we can get this relationship okay.

$$t = t_0 \exp[-(1+\beta)\epsilon_1]$$

$$t.d_1 d_2 = t_0 (d_0)^2$$

$$t = t_0 \{\frac{d_0^2}{d_1 d_2}\}$$

(Refer Slide Time: 14:55)



Now the mode of deformation when you look at it so it will come like this. So under different strain ratios so if you are plotting along the epsilon 1 and epsilon 2 axis along those directions you you will get this this type of a deformation mode. So the strain details we can get it in this case okay. So let us see that by the normal conventions we assign the major principal direction 1 to the direction of the greatest principal stress and consequently the greatest principal strain because those are parallel that is why so here it is your 1 direction and this is your 2 direction okay and if you plot it the various conditions at which you are going to get is. Let us look at this the first one this OA this is the strain diagram OA means beta is equal to 1.

So, first look is biaxial stretching in biaxial stretching beta is equal to 1. So in all the both the directions if it is stretching circle will remain circle that is why it is shown that see this was your initial case and this is the final case only thing is that the circle diameter has increased okay. So this indicates along the greatest principal direction all the strain points will lie on the point which is left of beta 1 beta is equal to 1 you will find that most of the for sheet metal application and the maximum value of a beta in this figure is for 1 and the minimum value is, is minus 2 in this direction okay. So that is OE see this is what you are getting if the principal stress sigma 1 tends to 0 then alpha tends to infinity so this is one thing which comes. So, you will find that in the biaxial stretching epsilon 3 is equal to minus 2 epsilon 1 since beta is is equal to 1.

So, epsilon 3 and is equal to minus 2 epsilon 1 you can get it so that means epsilon 3 is equal to minus epsilon 1. Minus sorry 2 into minus 2 epsilon 1 as the ratio is 1 since beta is equal to 1 and it will clearly show that the thickness will decrease more rapidly compared to epsilon. So, here now thickness decrease rapidly compared to epsilon 1. Your effective strain is equal to root of 4 into 1 plus beta plus beta square by 3 into epsilon 1. So, that is what we are getting in this case.

$$Biaxial\ stretching, \beta = 1$$

$$\epsilon_3 = -2\epsilon_1$$

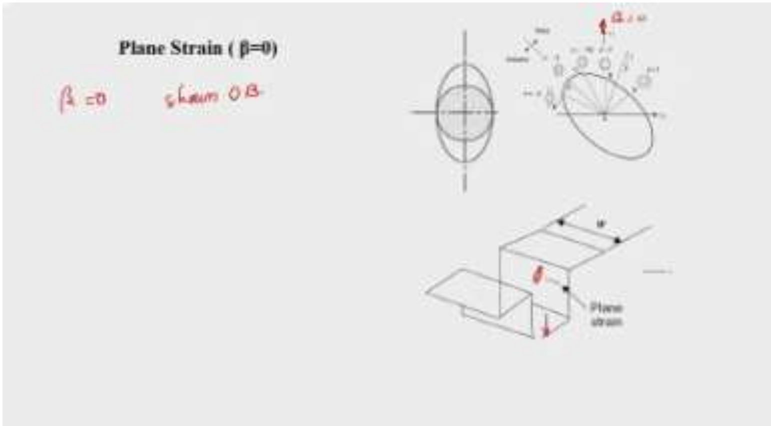
$$\bar{\epsilon} = \sqrt{\frac{4(1+\beta+\beta^2)}{3}}\epsilon_1$$

So, you will find that epsilon bar will be is equal to 2 epsilon epsilon 1 effective stress you will get it this way.

$$\bar{\epsilon} = 2\epsilon_1$$

So work hardening of sheet occurs very rapidly in this process where there is equal biaxial stretching.

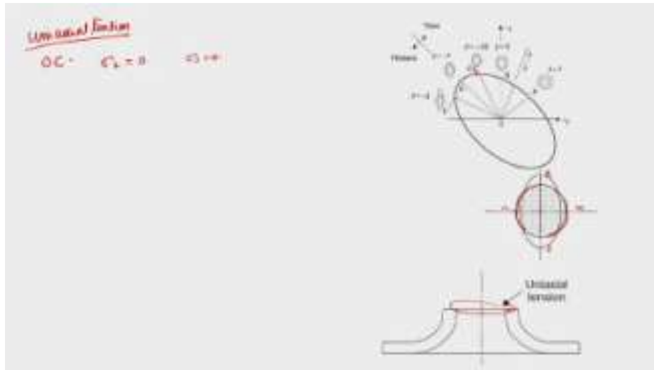
(Refer Slide Time: 18:48)



Now the next case is plane strain. In the plane strain condition beta is equal to 0. So that is this condition beta is equal to 0 okay. So when it is so and this is shown by line OB So, in this case what happens is that the sheet is stretched only in one direction. The other one it remains the diameter remains the same but along the axis 1 it gets stretched okay. So that is why it becomes an ellipse with the minor axis unchanged but the major axis it increases. So this is basically for this type part when it is stretching in this direction now if it was a circle here it will come like this here okay. So when with the punch you know when it is moving down when it is moving down you will find that a circle was which was here now it has come like this.

So it is just got elongated one. So this is a deep drawn or a trough like part which is happening only this elongation is taking place this is the plane strain condition.

(Refer Slide Time: 20:10)

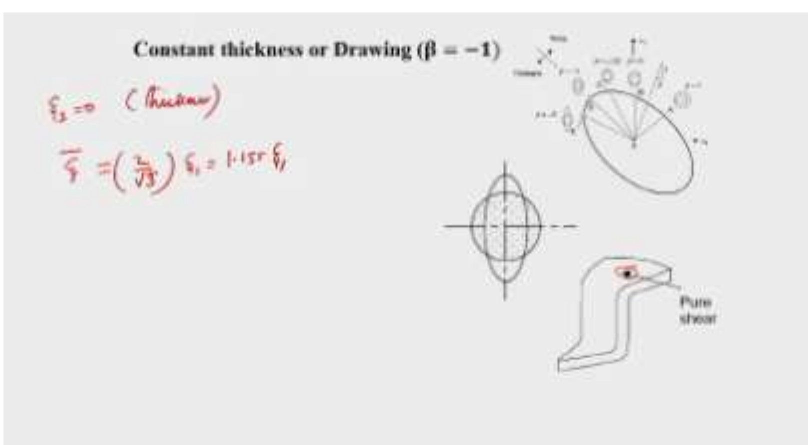


Now when you look at say the uniaxial tension, this is nothing but point C that is OC. In this, it is almost equal to a tensile test in a sheet when minor axis is 0 and when sigma 2 is equal to 0. So, in this case sigma 2 is equal to 0 and minor axis minor stress is 0. So, that is sigma 3 is also equal to 0.

$$Uniaxial\ tension \quad \sigma_2 = 0 \quad \sigma_3 = 0$$

This is a condition for what we are getting it. So the sheet stretches in one direction and contracts in the other direction. So you will see that so this was your initial diameter but in this case now there is a contraction in this direction and there is an elongation in this direction. This is what is happening. This is a case for a whole extrusion when the sheet is stretched at this part okay you are just stretching it that for a whole extrusion part of what happens this is that part case on the side window surface you are you are going to get it actually okay.

(Refer Slide Time: 21:56)



So, next is the constant thickness drawing where beta is equal to minus 1. So, in the other case now you will find that beta is equal to minus 1 by 2 you are getting So here beta is 0 uniaxial testing. Now let us have the next case constant thickness or drawing operation where beta is equal to minus 1 which you are going to get it. So this is the case where a case of a drawing okay.

The membrane stresses and strains are equal and opposite and the sheet deforms without change in the thickness. There is no thickness dimension but you will find that the along this direction there is a contraction along the minor axis sorry axis 2 and along the axis 1 there is an elongation which is taking place and the thickness strain is 0. So you will say that epsilon 3 is equal to 0 that is thickness that is that is 0 and the effective stress effective strain epsilon bar you can get it as 2 by root 3 into epsilon 1 that is equal to 1.

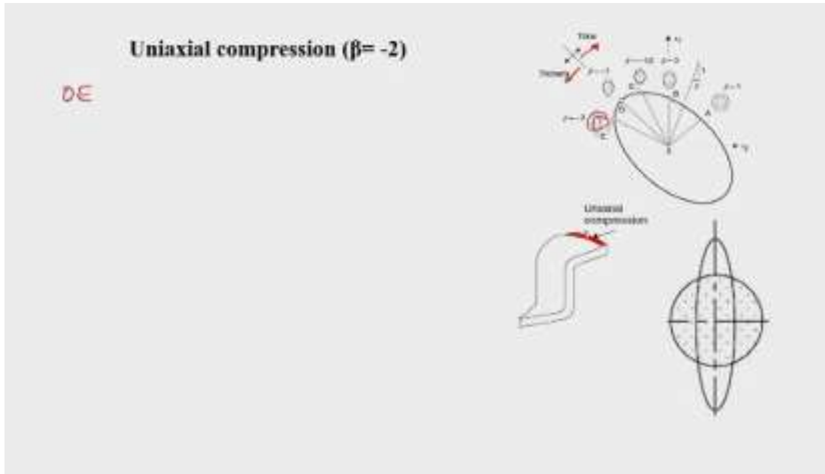
155 epsilon 1. The work hardening in this case is gradual.

$$\epsilon_3 = 0$$

$$\bar{\epsilon} = \left(\frac{2}{\sqrt{3}}\right)\epsilon_1 = 1.155\epsilon_1$$

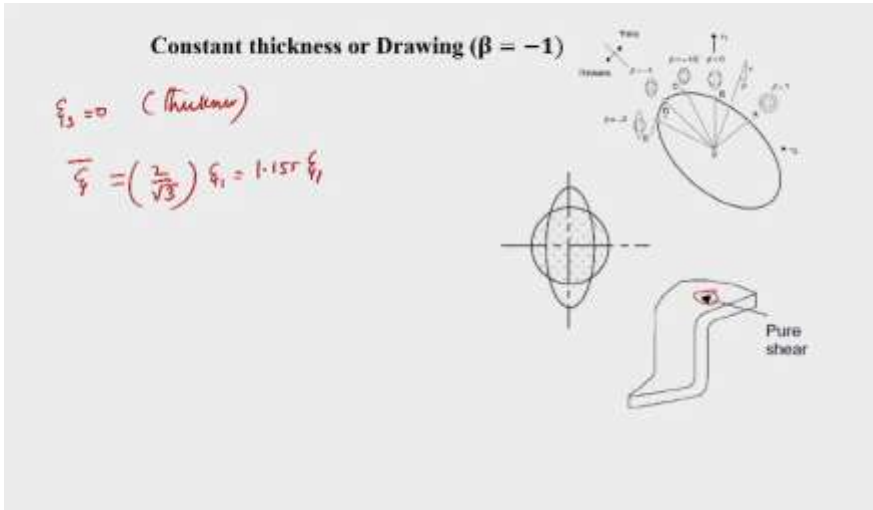
It's not very abrupt. But in this case you may find a large amount of strains which can take place that is one advantage because for carding is only variation splitting may not take place under this condition. If it takes place it will be at a later stage only so that is one advantage so constant thickness or drawing operation that is one thing which is which happens at this region okay of your this one.

(Refer Slide Time: 24:02)



Then the last case is the uniaxial compression which you are having here beta is equal to 2 so given by OE you will find that there is a extensive this compression which is taking place. So in this, beta is equal to minus 2 or even beta is equal to minus 1 also can take place. So, from beta to minus 1 and upward you will see that the sheet can thinning will take place, but from when beta is minus 1 to this downward keeps on the minus change increases you will see that the material keeps on thickening. And when this thickness changing is going to take place okay. Some cases you may end up with wrinkling and other things okay. So this problem may come because there it is going to be the uniaxial compressions or compression stresses in your this one is not there so but basically you can get this condition at the at this tip of this part so there it may not be a much of a problem.

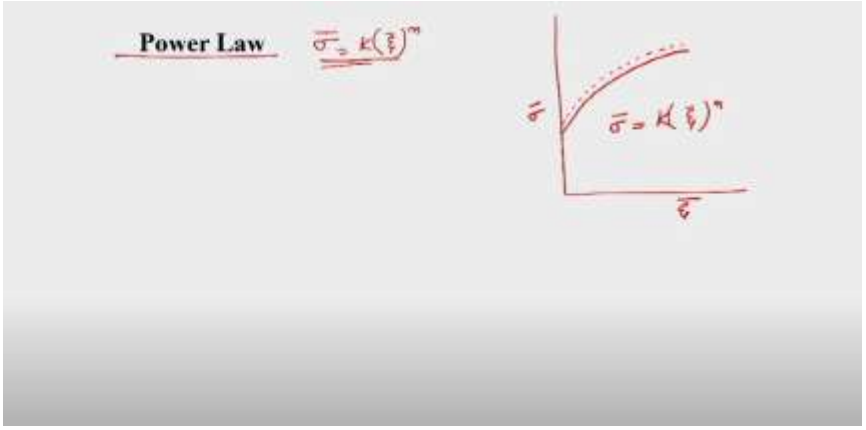
(Refer Slide Time: 25:19)



Now when you come that this whatever we were showing in this, these things were based on the true strains, but if you just plot on the engineering strain curve, then you will find that only A is straight, others are also curved things. So, due to this, people generally do not prefer this plotting on the, this is E2 and this is E1. On the using the axis as engineering strain axis they do not take it because this is the case of biaxial stretching okay. This is a plane strain condition, this is a uniaxial tension, pure strain. So here now you may find it difficult to get the proportional deformation.

The other case it becomes very easy because these curves are very different. So that is why whereas if you plot on the logarithmic scale a logarithmic sorry natural sorry logarithmic strain then it becomes straight say like what we are getting here. So that way it becomes very this one.

(Refer Slide Time: 26:24)

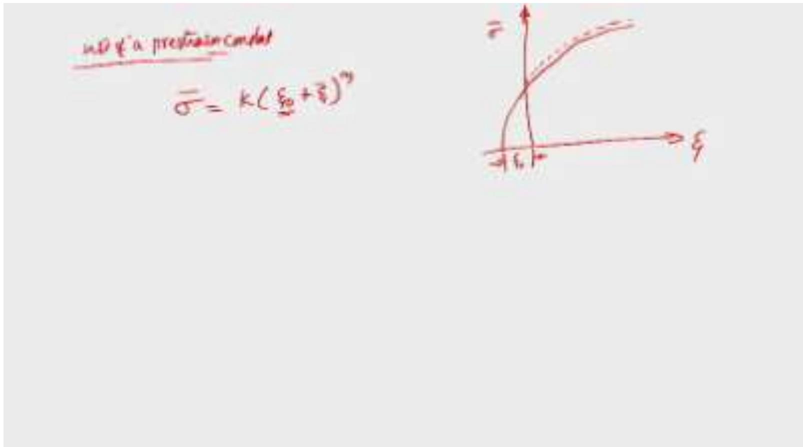


Now the thing is that we also need to know the effective stress and strains in material working operations. So let us look at that part how it is coming. The stress strain relationships are very much necessary okay. Then only we can get the strain distribution which is taking place. The effective strain we discussed in the last class and what is the in the last lecture and what is the effective stress also we discussed in the and if you are able to correlate between this effective stress and effective strain and then get a mathematical relationship between this it becomes much easier. So, the best and the most simplest is the power law equation which is given by sigma bar is equal to k into epsilon bar raise to n okay. This is the most convenient method and the curve will look like this.

$$\bar{\sigma} = k(\bar{\epsilon})^n$$

So this dashed line is your experimental work and the other one is your fitting curve okay and then with that fitting curve only you are getting this equation. This is a much very convenient relationship which is obtained. So the only disadvantage of this law is that at zero strain it predicts zero stress and an infinite slope to the curve. So that is one main problem with this okay. It does not indicate the actual initial yield strength of this material.

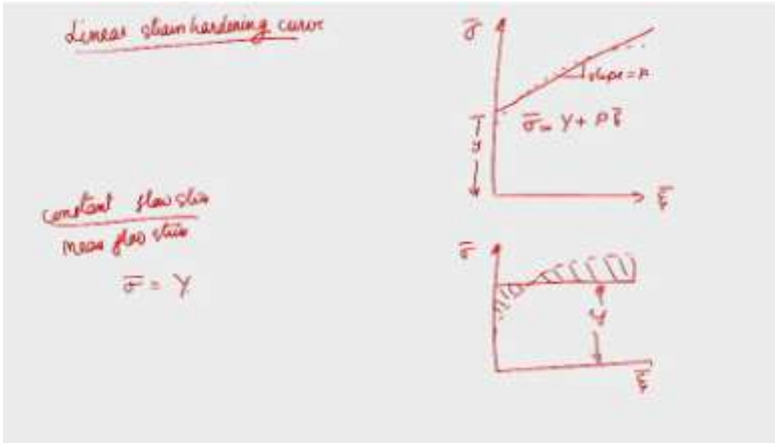
(Refer Slide Time: 28:35)



Now the secondary relationship is the pre-strain constant okay so your offset strain constant it is only similar type thing only small difference the relationship it will be it is expressed as sigma f the initial value is equal to k epsilon dot raise to n sorry yeah this when you are getting it it is as if the material you got it was after some some deformation which has been taken a pre-strain constant you can say use of a or offset stress. So, that figure is it is similar to this only thing is that in the, your this is your experimental data points and when you are fitting it because this has already been deformed by some amount no. So, that you have to find out what is this pre strain which is given that is this epsilon So, this relationship will be instead of this, you can just write it as sigma bar is equal to K into epsilon naught plus epsilon bar raise to n. That is the only difference because you wanted this epsilon bar because it is already given a pre-strain before you procured the material.

$$\bar{\sigma} = k(\epsilon_0 + \bar{\epsilon})^n$$

(Refer Slide Time: 30:26)



So, that is what in this case. So this is the now next is your the linear strain hardening. So that means if your curve was something like this data points experimental data points are like this you are just going to get fitted with a straight line equation like this okay. So there in that case the relationship will be epsilon bar is equal to y plus p into epsilon dot where p is the slope of this straight line and y is the intercept which you are going to get it so this is y okay and this will be your slope is equal to your p value. So, in some case this is also used and maybe in another case you know we can also use the constant flow stress.

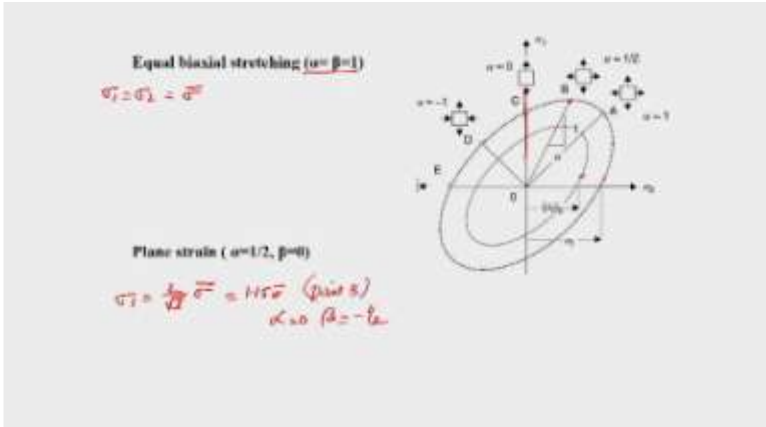
$$\bar{\sigma} = Y + P\ \bar{\epsilon}$$

For computational purpose or calculation purpose this is the most convenient thing constant yield stress or constant flow stress. So that is if your curve was like this, this is epsilon bar and this is sigma bar. So this is also sigma bar and if your data point was something like this, you are just taking that like this. So maybe this region this is that main flow stress value which we take it, okay. So, that is the main flow only thing is that for total energy what is required then this may be much convenient to determine.

So that means here it will be sigma bar is a constant y where this is equal to y. You take the area under the curve and then integrate it within the limits and get the so numerical integration technical integration of that only is coming. So that is the, and take the average value so that is. This is for many material working especially bulk forming process this is much more convenient to do it okay.

$$\bar{\sigma} = Y$$

(Refer Slide Time: 33:14)



Now when you go for the next is your the stress diagrams when you do it. The diagram in which the strains are plotted was shown earlier okay. Now we have to see that how the stresses are plotted okay. Variable study this is very important in this a diagram in which the stress state associated with each strain point is very useful in understanding the forces involved in the deformation process okay. So this figure is not of any particular this is not of any particular process okay it is a general process which is general diagram which has been given under different conditions. The controls of equal effective stresses are shown this is the case where it was the initial case and the one which is last one is the case where maybe in a proportional way it was just work hardening it was plastically deformed.

So, when the plastically deformation takes place work hardening is taking place or your yield strength will increase and that is why from initial point here to here it has increased. So, only thing is that that ellipse, remain an ellipse only its major and minor diameters only keep on changing and otherwise you are getting it. So, initially the initial flow stress was this sigma f naught. But when work hardening has taken place, then your yield strength has become this final yield strength sigma f.

So that is what it represents. So let us look at under various conditions for this. So first is that equal biaxial stretching where alpha is equal to beta is equal to 1, okay. So we can say that in this case, This is it is equal to a biaxial stretching where sigma 1 is equal to sigma 2 and that is equal to your sigma bar.

$$\alpha = \beta = 1$$

$$\sigma_1 = \sigma_2 = \bar{\sigma}$$

Next you have the plane strain condition where alpha is equal to half and beta is equal to 0. So this is that alpha is equal to half and beta is sorry this is alpha is equal to what this is alpha is equal to half and beta is equal to 0 that is your plane strain condition. So you will find that in this there is a 0 strain in the in the two direction and the stress rate is indicated by the point B. So this point. So, here in this condition sigma 1 is equal to 2 by root 3 into sigma bar or that is equal to 1.15 sigma bar.

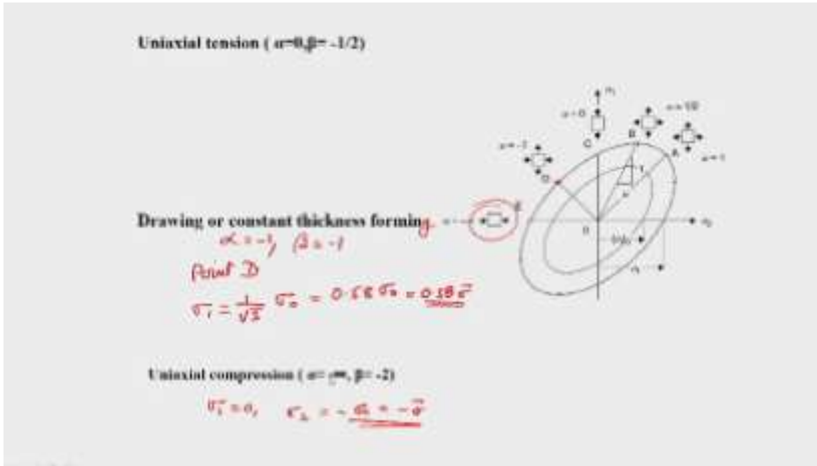
$$\alpha = \frac{1}{2}, \quad \beta = 0$$

$$\sigma_1 = \frac{2}{\sqrt{3}} \ \bar{\sigma} = 1.15 \ \bar{\sigma}$$

The equivalent stress under plane strain condition when you look at it this is what you are going to get it. So, this is shown by the line point B. Now, if you look at this condition which is your uniaxial uniaxial tension there you will find that say alpha is equal to 0 and beta is equal to minus half alpha is equal to 0 and beta is equal to minus half and the process occurs in the tensile test and as mentioned it is at a free edge.

$$\alpha = 0 \quad \beta = -\frac{1}{2}$$

(Refer Slide Time: 36:41)



So that is what we can do. Now drawing or constant thickness forming where alpha is equal to minus 1 and beta is equal to minus 1 okay. In such a case you know you will see that this is that condition okay that is D point D that is the condition okay. So, there you will find it sigma 1 is equal to 1 by root 3 into sigma f or sigma f uniaxial flow stress that is equal to 0.58 sigma naught or you can say that 0.58 sigma bar.

$$\alpha = -1, \quad \beta = -1$$

$$\sigma_1 = \frac{1}{\sqrt{3}}\sigma_0 = 0.58\sigma_0 = 0.58\,\bar{\sigma}$$

Now the last case is the uniaxial compression which is taking place where alpha is equal to minus infinity beta is equal to minus 2. So you will find that sigma 1 is equal to 0 and sigma 2 is equal to minus sigma naught is equal to minus epsilon bar.

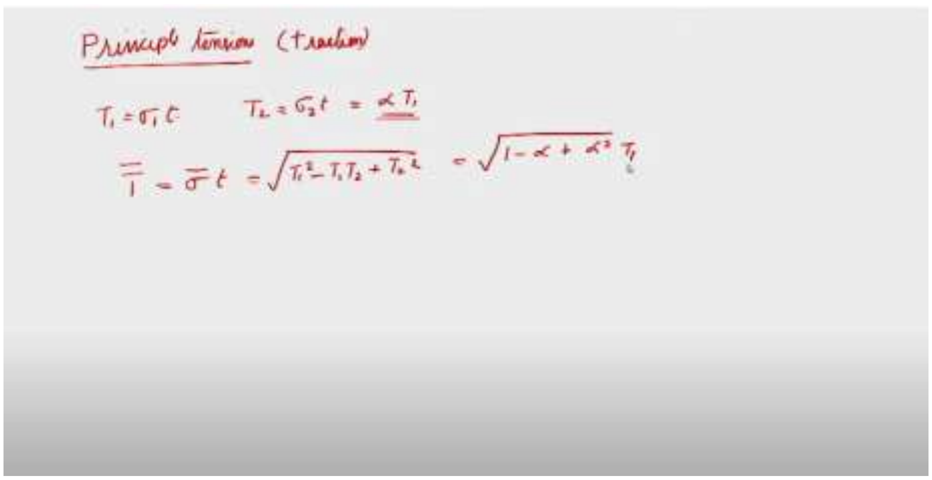
$$\alpha = -\infty, \quad \beta = -2$$

$$\sigma_1 = 0, \qquad \sigma_2 = -\sigma_0 = -\bar{\sigma}$$

So this is what you are going to get it okay. So these are the different conditions of the stresses, and it is this the stress plot and the strain plot this this will give you an idea or stress diagram and the strain diagram these are very important sheet metal working because under various condition you wanted different shapes and you wanted different sizes you should have an idea where the maximum stresses are coming where the maximum strains are going to take place which region it is going to deform. So you it is always convenient to find out if you have an idea at this point the strains will be the maximum. You can just do some experimental work and then find out how much is the strain.

If the strain is too large whether thinning will take place. If thinning is taking place there is a chance of rupture to take place. So we finally will end up with limit diagrams okay. or forming limit diagrams we may end up with that. So that way this type of an especially the strain diagram will be much more important in in these cases.

(Refer Slide Time: 39:19)



See when you look about the principle tensions and tractions or maybe tractions, some people use the word traction other people some people use the word tension okay. So the tensions govern the deformation of the sheet and the forces acting on the tool that is very that that is what is happening. So we also found that tension per unit width you know it is nothing but sigma into your thickness at that at any particular point whether it has deformed or not so that is how you calculate okay. So, it is found more convenient to model process in terms of tension rather than the stress and this for this reason tension on different process are calculated actually. So, we can say that T1 is equal to sigma 1 into T and T2. is equal to sigma 2 into t. So, sigma 2 into t. So, sigma t is equal to alpha into t1.

$$T_1 = \sigma_1 t \quad T_2 = \sigma_2 t = \alpha T_1$$

See that is the relationship is equal. So, if the material obeys. A von Mises yield criteria the principal tension in the street at yield will be generalized and so effective yielding tension we can find it out the effective tension which you can get it is by sigma bar into T. So, that is equal to so sigma bar it will be T 1 square minus T 1 T 2 plus T 2 square. we are getting. So, that in terms of stress ratio we can get alpha 1 plus alpha plus alpha square into T 1.

$$\bar{T} = \bar{\sigma}t = \sqrt{T_1^2 - T_1T_2 + T_2^2} = \sqrt{1 - \alpha + \alpha^2}T_1$$

So, that way you can get all those things. So with this, this today's class is, today's lecture will stop and next we will go to the some practical cases of sheet material working, we will come to that. Maybe next day, next lecture we may discuss about stamping or I think before stamping we should discuss about the forming limit diagram. So next lecture we will go to the next chapter actually, okay. Bye.