Plastic Working of Metallic Materials

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Lec 33: Analysis of stamping

Yeah, today we will be discussing about the analysis of stamping operation and maybe in the next class we may go for the deep drawing operation. So, what is stamping? The stamping is extensively used mainly by automobile industries ok.

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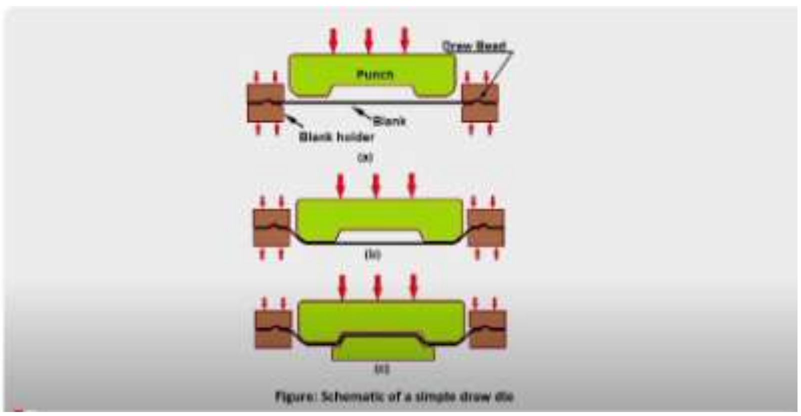

So, in this what is happening is that there is a it is a conventional draw die which is there you have a die and a punch this is a cross section which is being shown for a drawing die and punch okay for a stamping operation. So, in this you will find that the blank the blank which is here it is clamped by this, the help of a blank holder otherwise you know it will just move out. So you have to hold this blank blank blank in a proper way and when when it is held properly using a blank holder it can be just a piece by which there is a friction here but for very extensively what people do is that they have a draw bead also which will be incorporated so that you know it cannot just come out in so easily ok. So the blank holder it holds the sheet in such a way that it can be drawn inside.

So to some extent it can be drawn inside not that it is completely moving out. It can be drawn inside against the clamping action but still sufficient develop sufficient tension in this blank to stretch the sheet blank over the punch. So when you are looking like this see you you can just imagine that this this punch this punch is moving down and then okay so to some extent it should be able to move inside okay.

So it should be able to draw inside the sheet but at the same time that there should be sufficient tension should be developed in this side of the material or the blank. So that is what is the purpose of that. If that is not there, you will end up with the defects and other things. So to avoid that only, so that proper stretching is taking place on this, especially on the side walls and other things. For that purpose, the blank holder is a must.

And in this case, so when it is held between this blank holder and the punch moves down, you will see that there is a stretching which takes place. The material also will be just moving like this on the corner of the punch, the material will be moving like this. And at the same time, you will see the material will be moving here also on the die radius side also the material will be moving. So these things will happen. The resistance to drawing, so here when you look at it this side, the material will be drawing inward.

The resistance to drawing inward is due to friction between the sheet and this blank holder okay and this may be enhanced by the draw bits which is being provided okay. So, that is what it happens. So, here in this case what we will find that it is wrapped around this the corner and at the same time here also it is wrapped around to this the material.

Now, if you want a reentrant shape like say maybe an another shape is required like this, this can be formed by providing a punch from the bottom or you call it as not punch actually it is a lower die we can say you provide it either it can be stationary or it can be moving also okay. So, if it is moving you may provide for a counter punch that depends upon the machine which is there whether you are having a single acting machine or a double acting machine or a triple acting machine which you are having. Now most of the case industries you know they will have a triple acting machine. So that the this case also can be taken out okay taken care of but minimum double acting press will be required because one will be to provide the clamping force here at these two sides and the other is for the punch to move and get the shape for this okay for the drawing operation. One interesting thing observation is that the sheet during this stamping operation the sheet is not being compressed between the die and the punch though clearances are very less but the sheet is not being compressed like like in a forging press the in the forging you will find that the material is compressed here at nowhere the sheet is compressed and you will find that it is stretched most of the time it is especially at the corner the convex tool surface here and here the material is stretched and another thing is that here you will find that the material The workpiece material and the die or the punch, the contact is only on one side, not on both the sides. Even in forging also you will find that contact is on the two side, but here it is on one. For example, this punch and the metal you will find contact is here, contact is here or here. So, on the other side it is not there.

So, when you are having contact here this side there is no contact there is no contact here ok. So, that is what one should know. Similarly, when you are moving here there is a contact at one side only the other side it is not there. So, most of the case, but there are cases where it will be there there are exceptions that is there, but in general case we can say the contact between the tool and the work piece is generally at the only one side ok. And so there is no through thickness compression because there is only one side contact there is no through thickness compression.

So it is only by stretching how much the strain it is taking place. And you will find that though there are contact the contact pressure exerted by the die or the punch on the work piece material, it is very small compared to the floss stress of the sheet. So that is another interesting thing what you have to serve. So you considering this point we can say that you can neglect the throat thickness stresses. So in sheet material operation though it is a triaxial state of stress.

So the stresses normal to the sheet we can neglect it okay. So throat thickness stresses can be neglected and so that means most of the sheet metal forming operation we can consider it as a case of plane stress deformation. And you should also know that the stamping is one of the basic process for forming. And it is, you will find it extensive use in automobile industry, whether it is two-wheeler or three-wheeler or four-wheeler or six-wheeler. Most of the case in four-wheeler, you know, the body parts are being made by the stamping operation.

There is not too much of deep drawing operation is not there. It is generally by stamping operation. And in that case now the sheet is first formed to a shape in a draw die in a double acting press okay like what we are telling here. And that part after that initial drawing operation it is taken for the other secondary forming operation or blanking operation, forming operation. If you want the third also that may be done separately in a machine and then you may have to the second other portion includes blanking, okay shearing all those things also comes into picture because this part which is being held there now you may have to remove it. So, so all those things are taken in a different machine. And you will find that the process is a, is a three dimensional state of stress and it is very quite complex in this because at different places we have different types of stresses or strains which are going to take place. But maybe here this this I am referring from this book this complete this is taken from this chapter four of this book only okay I am referring to this book. Marciniak and Duncan and Hugh, mechanics of sheet material forming.

So exactly what I am going to teach is taken straight from that book, all the derivations and figures and other things, okay. So in this, we assume a two-dimensional process for simplicity. And we are assuming that the strain perpendicular to a plane of the diagram of this figure is zero and the deformation is both the plane stress as well as plane strain.

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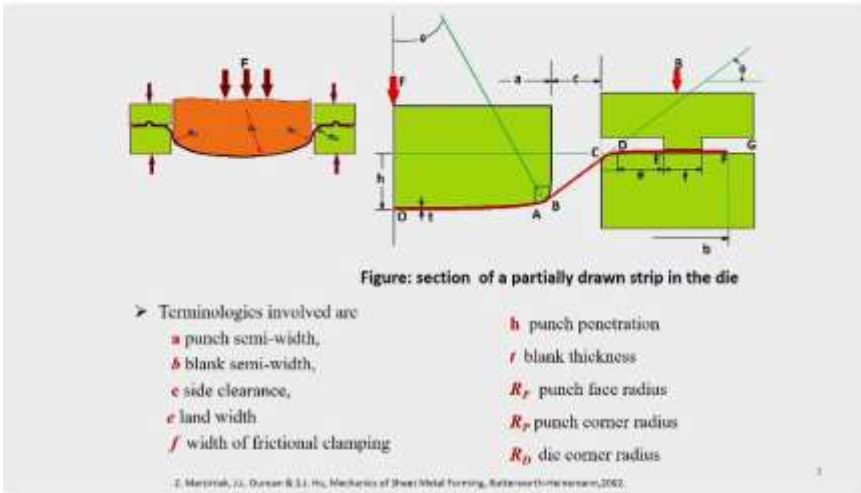

Now, let us come to the two-dimensional model of the stamping operation. For our analysis, we are going to consider a very simple die and punch system. So, we consider only very simple process material is being held between the draw bit or what you call blank holder and then punch is just moving down and then causing the stretching and then wrapping around the die and other things all those things are there where we can say that this This is the punch face radius we are assuming that from here there is a punch face radius through the entire distance and at the corner you will find that RP is the punch corner radius here, here also RP is the punch corner radius. Similarly, on this die side you have a radius here rd which is refers to die radius okay and you are applying a force here and here maybe a force we can say as B. for the for the blank holder force B and this is the force on the punch which you are going to apply okay. So, the terminologies which are used is as shown here see this A, A is this this distance A is A which is the punch semi width because because this is symmetric we can just take this and then have only a half of this part and then whatever comes you multiply by that. So, only this half part only we are taking.

So, considering that the our we have made some small assumptions have been made for the analysis to make it very simple and the terminologies are A is the punch semi width which is shown by this ok, punch semi width is there from here to the end of the punch. And the blank width initially you know blank width was kept from here to here from this distance was there okay. So, that is the blank width so that is this blank width you are just keeping it as here semi blank width and then so between the die and the punch the clearance is this side clearance and e is that this is the radius of on the die and from here to here this d to e. So, that distance e is the land width where movement of the material is not taking place. And from E to F that is a this F distance found by is the width of the frictional clamping.

Here we are not assuming a draw bead but just by pressing we are assuming that with the sufficient pressure here it is clamped here for simplicity of analysis only that has been done okay. So this is the F is the width of the frictional clamping. And at any instant, we assume that the punch has moved down from this distance by a distance h. And the thickness of the blank is t. RF is the punch phase radius and RP is the punch corner radius, RD is the die corner radius. These are the terminology which we will be using it.

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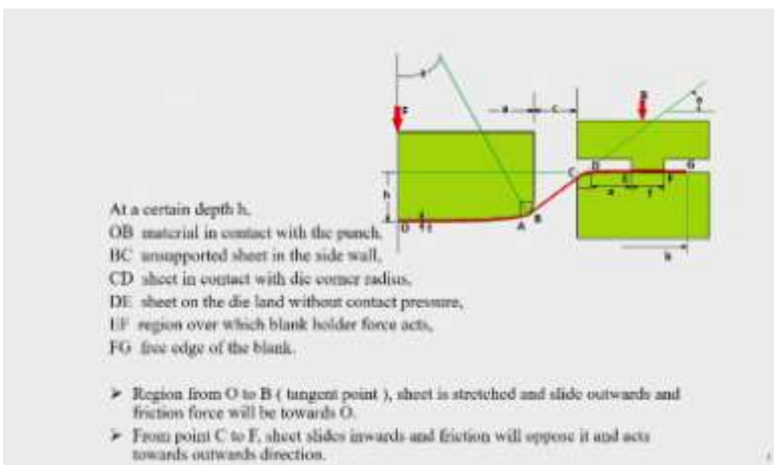

Now at some distance when the punch has advanced by a distance h from this part because at this point the punch makes the contact with the, with your blank. When it is at this point the punch makes contact with that and as the punch moves down. Suppose it has more traverse by a distance of h. So, for that configuration we are going to make the analysis.

So, we can say the material in contact with the punch is from say from O to A. to B. This is the part where the material is in contact with the punch, okay, this one. And from B to C, there is no contact. So, it is an unsupported sheet on the sidewall.

Though this is exaggerated view, but otherwise it should be very thin clearance only is there. So, from B to C is the unsupported sheet along the sidewalls which are there. And C to D is the is the part where the blank makes contact with the corner radius. So, from here to here it makes in contact similar to like A to B it makes a contact similarly from D to C also it makes a contact here ok. So, that is that with the corner radius and from D to E the sheet on the die land without contact pressure this is the part of the sheet on the die part without any contact pressure.

Whereas this distance E to F given by F that is this, this is the region over which the blank holder force act on the blank, okay. So that blank holder force F B is acting on the blank and F to G is the free edge of the blank maybe it may be allowed to draw inside and other things some distance may be kept there but normally some extra distance is always kept okay so that is that F to G that is. Now if you really look at that as the punch is moving down it is making contact with the blank is in contact with the punch from the distance O to O to B and the B is the tangent point to that punch okay.

So in this region the sheet is stretched it is stretched in this direction and it tries to slide out outside from O to B. So it is just trying to slide outside okay outward. So when it is trying to move slide outward at the contact region the frictional forces will be there so you will find that the frictional forces are acting in this direction okay. So that means frictional forces will be directed towards the center of the punch that is towards O in this figure. Whereas when this drawing operation is taking place if you look at the die side from C to F we can say not F actually yeah metal will be sliding inwards depending upon your force metal will try to slide inward, into the die so that means in the direction of O but then frictional forces will be developed wherever the contact is there and this will be opposing this movement and so the frictional force in the die will be towards the outside surface so remember that this is symmetrical or symmetrical and then that is what we are considering here. So in this case here the frictional force is directed inwards at the punch side and at the die side the frictional forces are directed outward because the metal will be sliding inside in the die towards the center so frictional force will be outside.

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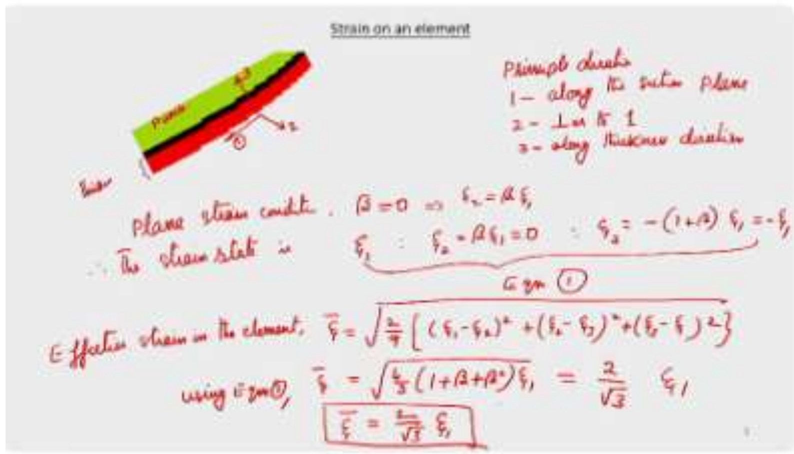

Now based on this let us try to do an analysis of this material okay. So let us try to find out the strain on the element a part of this which is given this is your sheet okay thickness sheet thickness or we can I can say the thickness and this is the punch.

So, if you look at the strain on an element, if you take a small element like this, you will find that at any instant the thickness and also the stress and the tension in the sheet will keep on varying across the part throughout the entire part. So from here to here the stresses will change, the thickness will change, the tension in this sheet will change and the stresses also will change. So if you assuming if you assume that principal direction along the direction of your material okay that is along the section and plane. So, if you assume that this is the principal direction 1 and so normal to that will be direction 2 and say in this direction it will be 3. So, direction 2 we have to write it in this and this will be the direction the principal direction 3.

So, that is what we have. So, 1 is the sectioning plane the principal direction 1 along the section plane 2 is perpendicular to that and three along the thickness direction. So, these are the three direction fielding and we are assuming that this, the process is in plane strain condition. So in the plane strain condition in the last lecture we discussed that beta in that case will be 0 where beta is your strain ratio. Beta is the strain ratio that is epsilon 2 by epsilon 1 that is epsilon 2 is equal to beta epsilon 1 that is what we discussed.

So strain ratio is 0. So the strain rate a strain state hence the strain state in the material is one is epsilon 1 ok. Next is epsilon 2 is equal to beta into epsilon 1 that is equal to 0 and third is epsilon 3. So, epsilon 3 is equal to the same relationship last last lecture we have come across 1 plus beta into epsilon 1. So that is equal to minus epsilon 1.

$$Plane\ strain\ condition, \beta = 0 \quad \epsilon_2 = \beta\epsilon_1$$

$$The\ strain\ state\ is\ \ \epsilon_1 \quad : \quad \epsilon_2 = \beta\epsilon_1 = 0 \quad : \quad \epsilon_3 = -(1+\beta)\epsilon_1 = -\epsilon_1$$

This is what we are going to get. So this relationship let us say equation 1. Now the effective strain in the element. In our previous lecture, We just had this is equal to 2 by 9 into epsilon minus 1 minus epsilon 2 square plus epsilon 2 minus epsilon 3 square plus epsilon 3 minus epsilon 1 square. This we have so that will be equal to say root of substituting this okay using equation 1 epsilon bar is equal to 4 by 3 into 1 plus beta plus beta square into epsilon 1. So, that is we can if you substitute this value into this expression we will get that that comes to 2 by root 3 into epsilon 1. So, in this is equal to 2 by root 3 epsilon 1 that is equation number 2 okay so the strain in the element we got and alpha is the stress ratio.

$$Effective\ strain\ in\ the\ element, \bar{\epsilon} = \sqrt{\frac{2}{9}\{(\epsilon_1-\epsilon_2)^2+(\epsilon_2-\epsilon_3)^2+(\epsilon_3-\epsilon_1)^2\}}$$

$$\bar{\epsilon} = \sqrt{\frac{4}{3}(1+\beta+\beta^2)\epsilon_1} = \frac{2}{\sqrt{3}}\epsilon_1$$

$$\bar{\epsilon} = \frac{2}{\sqrt{3}}\epsilon_1$$

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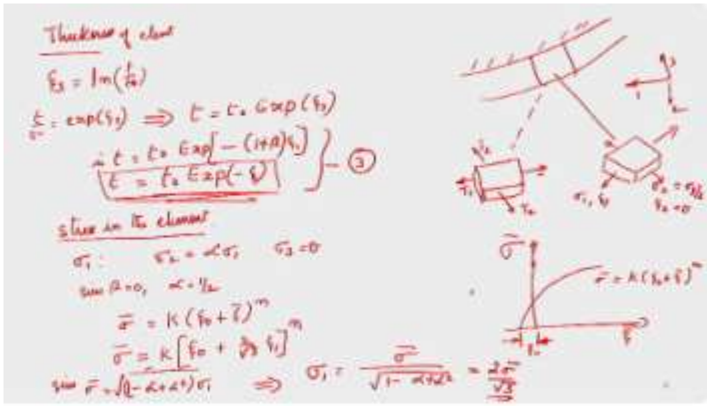

Now the thickness of the element because if you look at this here we can just consider this case and if you take a small element here that is what we are discussing. So, here you will find that 1 is along this direction. So, this is sigma 1 epsilon 1. This is sigma 2 is equal to sigma 1 by 2 and epsilon 2 is equal to 0. Now if you look at the tension on this case you will also see that the tension is also like this T 1 and T2 the tension also we can get. So these are the principle directions. So one is here, one is this, this is 2 and this is 1. So sorry this is 3. So this is the directions you are getting okay.

So now the thickness of the element at any instant So, the current thickness now see we have just how much it has strained. So, from the straining now you can just get it as your epsilon 1 is equal to log T by T naught. So, from that now you can get it as T by T naught is equal to exponential epsilon 1. Sorry epsilon, this thickness is along this one, epsilon 3 or which implies that t is equal to depending upon what is the strain epsilon 3 is equal to t naught into exponential epsilon 3. So, epsilon 3 the relationship you got it as so that is a t is equal to t naught into exponential minus 1 plus beta into epsilon 1. So or we can write T naught is equal to T naught into exponential minus epsilon 1. So these things we can get okay.

$$\epsilon_3 = \ln\left(\frac{t}{t_0}\right)$$

$$\frac{t}{t_0} = \exp(\epsilon_3)\ implies\ t = t_0 \exp(\epsilon_3)$$

$$t = t_0 \exp[-(1+\beta)\epsilon_1]$$

$$t = t_0 \exp(-\epsilon_1)$$

So maybe this is equation number 3. So we got the effective strain in the element. okay and we also found out the thickness variation depending upon that effective strain we can find out the thickness depending upon the strain 3 we can find out this because that way we were able to find it out. Now let us look at the stress in the element. So in this element if you look at the state of stress we can write that is the state of stress Sigma 1 is there along this direction 1 that is along this direction it is sigma 1 okay and sigma 2 is equal to in terms of your stress ratio alpha into sigma 1 and we are assuming that along the third direction because through thickness stresses are 0 that is assumption we are doing it so that sigma 3 is equal to 0 okay. That is because beta is equal to 0 since so once this is there since beta is equal to 0 from this our earlier equation was there in the last class now we arrived at something ok.

$$Stress\ in\ the\ element$$

$$\sigma_1 \quad : \quad \sigma_2 = \alpha\sigma_1 \quad : \quad \sigma_3 = 0$$

So, we can find that under this condition alpha is equal to 1 by 2 because this is what sigma 1 by 2. sigma 2 is equal to sigma 1 by 2 so alpha is equal to 1 by 2. So this effective stress strain law we have to develop which we are going to use what we have discussed one of the equation which was there was that effective stress is equal to K into epsilon naught plus epsilon effective stress into raise to n. This what we used was where if you look at this that because it depends upon your sheet whether it was earlier undergone some prior deformation.

$$since\ \beta = 0,\ \ \alpha = \frac{1}{2}$$

$$\bar{\sigma} = K(\epsilon_0 + \bar{\epsilon})^n$$

So, it will be like this. So, that is what works. So, sigma bar is equal to k into epsilon dot not plus epsilon bar raise to n and where this corresponds to your epsilon naught. So, this was the, this, this relationship we are going to use it. So, in that now if you substitute the value of effective strain from our equation number 2 I think equation number 2, 2 by root 3 into epsilon 1 we can write that this relationship is epsilon bar is equal to k into epsilon naught plus 2 by root 3 into epsilon 1 because epsilon 1 you can always calculate it to the power n okay. So, from this you can get the major stress also by using the relationship like the sigma bar is equal to root of 1 minus alpha plus alpha square which we have discussed earlier into sigma 1. So, from this we can find out the sigma 1 where it gives actually from this maybe I will write this. Since in the last lecture we have discussed sigma 1 is equal to 1 minus alpha plus alpha square into sigma 1. So, that if you substitute here, so from that this implies that sigma 1 is equal to sigma bar by root of 1 minus alpha plus alpha square. So, that is equal to 2 sigma bar by root 3. We are getting this. So that where this the stress on the element sigma 1 also we can find out okay.

$$\bar{\sigma} = K\left[\epsilon_0 + \frac{2}{\sqrt{3}}\epsilon_1\right]^n$$

$$\sigma_1 = \frac{\bar{\sigma}}{\sqrt{1-\alpha+\alpha^2}} = \frac{2\,\bar{\sigma}}{\sqrt{3}}$$

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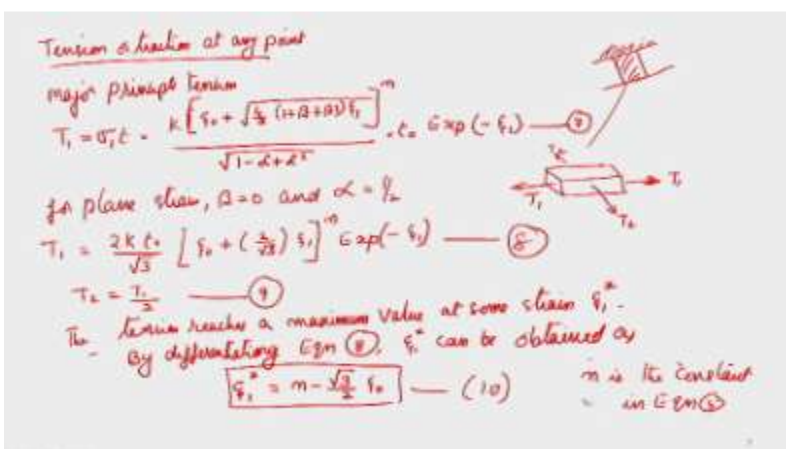

Now the thing is that we have to find the traction force at a point okay. So referring to our previous figure itself the traction force if you look at it T1 and T2 okay. So this is T1 and this is T2 when you wanted to find out this T1 and T2. For the given material and sheet thickness, the tension or force per unit width at a point can be expressed as a function of the strain at that point. So, major principal tension, so major principal tension, sorry if you are just doing like this. This is T1 and this is T2 okay. So, so this T1 the sectioning plane now we can just find it out as T1 the last class we discussed it is equal to sigma 1 into T and sigma 1 we have found out from here sigma 1 relationship and that sigma 1 if you substitute into your this one we can get this relationship as k into your epsilon naught plus root of 4 by 3 into 1 plus beta plus beta square into epsilon 1 all raise to n. divided by root of 1 minus alpha plus alpha square into thickness. So, thickness is equal to t is equal to t naught exponential we found it as minus epsilon 1.

$$T_1 = \sigma_1 t = \frac{K\left[\epsilon_0 + \sqrt{\frac{4}{3}(1+\beta+\beta^2)\epsilon_1}\right]^n}{\sqrt{1-\alpha+\alpha^2}} \times t_0 \exp(-\epsilon_1)$$

So, that way we can find out the tension T1 along this and for the plane strain case beta is equal to 0 and alpha is equal to 1 by 2. So, that is what we have found strain ratio is 0 and stress ratio is 1 by 2. So, from that T1 we can just find it out as 2 k t0 by root 3 into epsilon naught plus 2 by root 3 into epsilon 1 raise to n into exponential minus epsilon 1. This is equation number 8. And so since T1 you got it so T2 is equal to T1 by 2 this also we have discussed under the plane strain condition.

$$for\ plane\ strain, \beta = 0 \quad and \quad \alpha = \frac{1}{2}$$

$$T_1 = \frac{2Kt_0}{\sqrt{3}}\left[\epsilon_0 + \left(\frac{2}{\sqrt{3}}\right)\epsilon_1\right]^n \exp(-\epsilon_1)$$

$$T_2 = \frac{T_1}{2}$$

Now the thing is that differentiating this equation 8 that indicates the tension reaches a maximum at some strain value. So the strain reaches sorry the tension reach the tension reaches a maximum value at some strain epsilon 1 star okay. So this can be found out by differentiating this by differentiating equation so by equation eight epsilon one star can be obtained as n minus root three by two epsilon naught where n is the work hardening exponent, n is the term in equation 6, 5.

$$\epsilon_1^* = n - \frac{\sqrt{3}}{2}\epsilon_0$$

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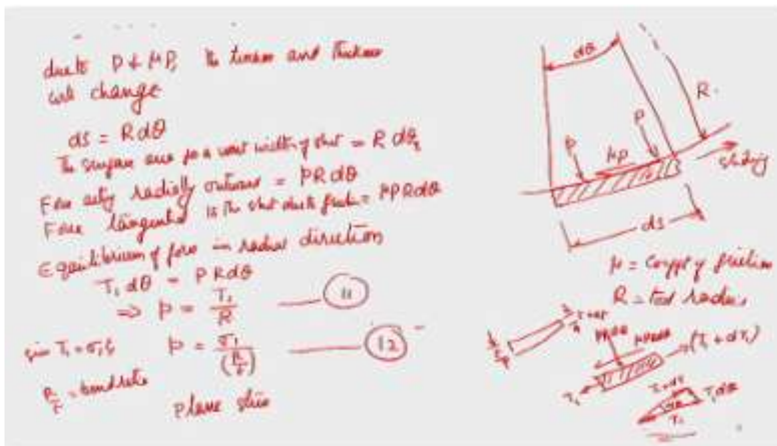

Now we will have to look at the the element the echelon of the element when it is sliding on a curved surface we found that from O to say what is that called B, O to B the material was sliding and similarly on the die surface also. that is from D to E the material was sliding even up to F the material will be sliding and except that free side free edge you know it is not sliding all other part it is sliding which is in contact with either the die or the punch. So we have to find out that equilibrium of the element during the sliding of a curved surface.

So, that we can find. So, if you just consider this an element like this where this is your d theta and this is your radius of the punch. And since the material is may be this distance this is d s So if you just consider now the tool surface is curved that is what from O to A it is curved. So there will be a contact pressure P and if the sheet is sliding along the surface. So you will have a contact pressure P here, normal pressure here, contact pressure P will be there here on the surface okay and if the sheet is sliding along the surface there will be a frictional stress so we found that at the on the punch side the frictional stress will be moving in this direction okay.

So this is mu p this is the normal pressure p and this is the mu p so here also you know you are having p so these things are there where mu is the coefficient of friction sliding friction and material is sliding along this direction okay. Because it is sliding and there is friction also is coming into picture the normal pressure the pressure will be there the contact pressure at the metal and the punch interface is p and there is a frictional force mu p you will find that both the tension and the thickness will change because of this frictional force. So, due to P and mu P the tension and thickness will change okay. So if you look at that the length of the element can be expressed in terms of the tool radius where r is the tool radius we can say r is the tool radius. So this ds is equal to r into d theta okay this is r into your d theta and then if the force acting on the element radially outward okay is the and the or or we can say the surface area for a unit width is equal to r into d theta 1. So, the force acting on the element the radially outward force acting is equal to P r d theta. So, that is this total force which is acting. So, because this is the pressure, pressure into this distance d s that is equal to r d theta.

$$ds = Rd\theta$$

$$The\ surface\ area\ of\ a\ unit\ width\ of\ sheet = Rd\theta_1$$

$$Force\ acting\ radially\ outward = PRd\theta$$

So, that is what you are getting and the force which is acting tangentially due to friction is equal to your mu p into your R d theta 1. So, that is equal to we can say R d theta. If you really look at this variation of thickness, it will be looking like this.

$$Force\ tangentially\ to\ the\ sheet\ due\ to\ friction = \mu PRd\theta$$

Maybe from here you may be having some variation is there. This is not uniform. Let us just draw as if here it is more and here it is less, okay, variation of thickness. So we can say that this is t and this is t plus dt. okay so if you look at the variation because of this the variation the tension is coming in the picture the same way we can look at that variation in the tension t1 and here you will get t1 plus dt dt1. you are getting that and so on this you are finding out that the frictional forces mu P r d theta this is mu P r d theta and this is what is that P r d theta. This is the equilibrium forces acting on this. So so one is that so you can just see that if you just this is the free body diagram of that element whatever it is there. So if you look at the forces which are acting on this the T1 is acting like and T1 plus delta T1 is there. So it will be like this, d t1 and this will be t1 d theta okay. So, since this is d theta, so when it is just wrapping around this, you will find there is a change in the direction okay. So, you because of that the direction of the tension forces differs by an angle of d theta that is what is shown here in this case. I do not know this drawing here at this corner These are the directions of these forces. It changes by, the direction changes by d theta value. So that is the thing.

So the equilibrium of forces, we can write it in this one. T 1 d theta this is T 1 T 1 d theta is equal to in the in the radial direction P r d theta or that implies that P the normal pressure or radially outward pressure P is equal to T 1 by R okay. And if you recollect since T 1 is equal to sigma 1 T the last class we have discussed so this may be 11. So, we can write it as P is equal to sigma 1 by R by T, where R by T is the bend radius, bend ratio.

$$Equilibrium\ of\ force\ in\ radial\ direction$$

$$T_1 d\theta = PRd\theta$$

$$P = \frac{T_1}{R}$$

$$since\ T_1 = \sigma_1 t, \quad P = \frac{\sigma_1}{\left(\frac{R}{t}\right)}$$

Now the contact pressure you will see that it is inversely proportional to the bend ratio R by T and the radius of the curvature of the punch is is many times several orders of magnitude greater than the thickness of the sheet compared to because thickness of the sheets are generally very small. So compared to the radius of the punch you will find that the punch radius is much much higher than that of the sheet thickness and even the thickness and even at the most corners areas if you look at it this will be 5 to 10 times the thickness okay so that is the even corners the radius of curvature is less. So even in that case also you will find that the radius of the die or the punch it is maybe 5 to 10 times the thickness sheet thickness which is.

So then the principal stresses sigma 1 is normally around 15 percentage higher than the flow stress. So due to that the contact pressure will be a small fraction of the flow stress only. It will not be the full flow stress and we are assuming that otherwise because many times if the lubricants are there it will still be lower okay. So we can assume that this condition is of plane stress okay.

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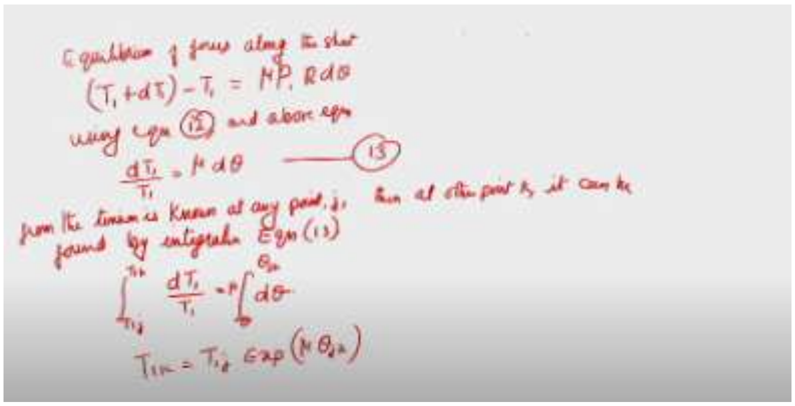

And if you look at the equilibrium condition for forces along the sheet that is along principle direction 1 along the sheet if you write it as t1 plus dt1 minus t1 is equal to mu p1 into rd theta. So, using equation number 12 and above equation, we can get dt1 by t1 is equal to mu d theta. This is the relationship with differential equation we are getting it. And you will see that the contact pressure depends upon the radius ratio.

$$Equilibrium\ of\ forces\ along\ the\ sheet$$

$$(T_1 + dt_1) - T_1 = \mu P_1 R d\theta$$

$$\frac{dT_1}{T_1} = \mu d\theta$$

But the change in the tension, it is independent of the curvature. There is no relationship between the curvature. And it is a function of the coefficient of friction and the angled, angle turned by the sheet on the punch or the radius. It is basically on the punch. How much? That is what you call it as warp angle. How much it just returns that is what is called as the warp angle. So, if the tension at one point on the sheet on the section is known then the tension at other point k can be found out from the tension tension is known at any point j. Then at other point k, it can be determined, found by integrating equation 13. That is integral from t i j to t i k t 1 j to t 1 k d t 1 by t 1 is equal to integral from 0 to theta j k between j and k what was the the angle turned through ok.

$$\int_{T_{1j}}^{T_{1k}} \frac{dT_1}{T_1} = \mu \int_{\theta}^{\theta_{jk}} d\theta$$

So, d theta. So, from this we can always calculate it. So, so from that is say T 1 k is equal to T 1 j into exponential mu theta j k by how much angle it has turned. So, that way we can find out the tension. If you know that the angle turned between the two points how much it has turned between the two points so that way we can just get this value if you know at one point. So, from at point j if you know what is T 1 then at K you can find out what is the T 1.

$$T_{1k} = T_{1j}\exp(\mu\theta_{jk})$$

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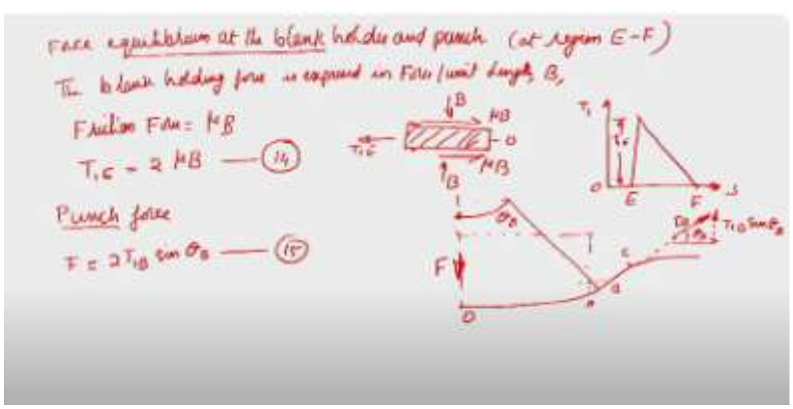

And now if you look at the force equilibrium at the hole blank holder and the punch that is basically at region EF okay. So, at that point it is clamped by these two flat surfaces that is what we have shown in our initial figure itself. See here it is it is clamped by these two flat surfaces okay and that is held by the blank force holder force B okay. So, if the force is expressed the blank hold holding force is expressed in force per unit length as b then the friction force is mu b that is if you just take that portion at that area in one hand you are having say t 1 e and outside it is 0. So, you have this mu b like this friction force mu b on this surface as well as on this surface mu b mu into b. This is the friction force because metal is trying to move inward here. So, frictional force on the die side it will be outside and then your force acting here is B which is also will be acting here as B. The normal force is b per unit length. So, in this case now we can write this equation as T 1 e because at point e is equal to 2 into mu b because at both the surfaces there is friction at the top as well as the bottom.

$$Friction\,force = \mu B$$

$$T_{1E} = 2\mu B$$

See here as well as here the frictional forces are there. So, T 1 e is equal to mu b that we can write it like this equation number 14. So this is a very good relationship and we can very see on one one hand we are having T1 e on the T1 e and the other side the tension is 0. So that means we can assume that it is going as a like a reasonable approximation will be like the force is going say along maybe at some point from here it is just moving linearly so that way we can assume that this is E and if this is F the variation in the force is there this is T1 E and here it is 0. So we can so that is a straight line relationship we can always assume okay. So across this distance F it falls off to 0 at the when it reaches the surface it is 0 and beyond that the free surface there also there is no tension or anything which is coming into picture okay. Now the next part is the punch force we have to find out. So next we look at the punch force. For the punch force say if you just consider this the same if this is a wrap angle theta B So this is OAB and here it is C. So if you just draw this. So from this, this is the force F which is acting on this. The force acting on the punch that is in equilibrium with the tension in the sidewall.

So, tension in the sidewall is T1b. So, we can write on the both the sides it is we have taken only half of that part. So, if you take on the left hand side also we can just write that F is equal to 2 into T1b sin theta b the component of that. So, because this part this is will be T 1 B T 1 B sin theta B. If you if you just resolve this into vertical and horizontal component, so this is it.

$$Punch\ force\ F = 2T_{1B}\sin\theta_B$$

So, you just take that F is equal to the twice one the both the sides are there. So, that way we can get this relationship for that okay. The tension distribution across the section if you look at it okay. So the tension at each point along the strip can be obtained okay as if so that we can just see that how much how the tension along the strip varies okay. If the strain so like you are just considering this is the point okay and maybe it.

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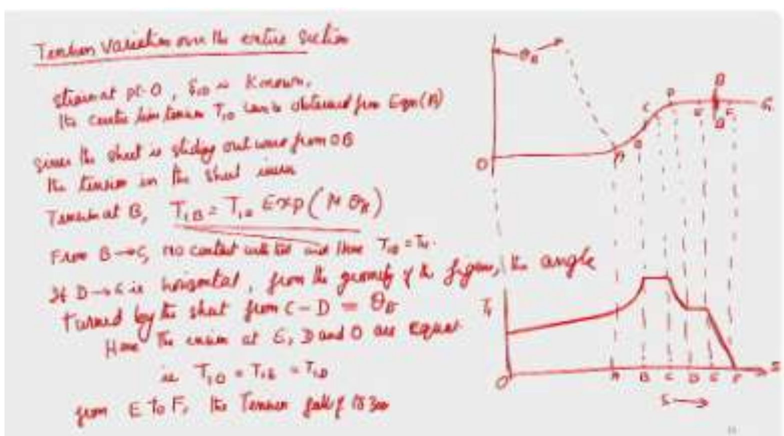

This is your O and it is going like this and then it is going like this and maybe you are just getting this as your theta B Okay. This is O, A, B, maybe C and your D. C is this C and D and maybe here you are getting all these things so your B is acting here so maybe E is there and F is there okay and at the end G let us see that how the variation the tension is taking place. So the distribution of the tension over the section so the tension variation over the entire section. If you look at that variation, if the strain at this point at the middle point all can be obtained. So, that means, the strain at midpoint, point O epsilon 1 naught is known the tension the center line tension T 1 naught can be obtained from our earlier relationship what is that equation okay this let me say maybe the 13 14 I have written so maybe a maybe let me write it as a from equation a from equation a. So, as the sheet between O and B is sliding outwards against the opposing force of friction from B to O, the tension in the sheet will increase. Since the sheet is sliding outward from over to B, the tension in the sheet will increase. And angle of the the wrap angle theta B can be obtained by the geometry and the distance at which it has moved down that distance x. So, tool geometry you can find out. So, that tension at B that is t 1 b we can write it as t 1 o if you know the tension at o t 1 o into your exponential mu theta b this is from equation a ok. So, from that we can find it out. Now, if you look at it from the side wall between b and c the sheet is not in contact with the tooling and the tension is constant.

$$T_{1B} = T_{1O} \exp(\mu\theta_B)$$

So, from B to C, no contact with the tool and hence T1B is equal to T1C. If the surface of the sheet under the blank holder is horizontal as shown in this figure, the angle turned between C and D because there also there is an angle which is turning place C and D will be the same as theta B. So, because this all this and this will be if this is horizontal from maybe we can said D to G if D to G is horizontal from the geometry of the figure, the angle turned by the sheet from C to D is equal to your theta B or angle of wrap. So, and hence the tension D and also at E will be that equal to the center line.

Hence, the tension at E D and O are same are equal that is T 1 O is equal to T 1 E is equal to T 1 D. Now, from E to F, we have seen that it will fall off in a straight line from E to F. The tension falls off to zero. So, if you really look at the variation in the tension at each and every point, if you look at it, how it will come? If I just plot like, what happens? So, if you draw the variation, say O in terms of t1, So this is O. So from here a slight increase will be there because when it moves from the center to a surface because the thickness is varying there will be an increase in this case and when it is turning across A to B you will find that there is a sharp increase across the radius it reaches like this and after that from B to C because tension in the strip is remaining constant. So, you will find that it is remaining constant here and again from here C to D it is falling off down maybe it may come down something like this depending upon that radius and other things and then from D to E it remains constant okay D to E it remains constant and E to F it will fall off to 0. I will just draw this as 0 instead of here. Okay, not necessary. It falls off to 0. So, this is this is the variation of the tension from if you know at one place, you can always calculate from the relationship at all other places.

So this is for A, this is for B, this is for C, this is for D, this is for E and F and beyond that F to G it is 0 okay. So this if you just draw this distance S. So this is the total variation in this. So all you need is that you want at one place what is the what is the tension so other places we can get it okay. So that means initially there is a slight variation and increase from A to 0 to A from there because the radius is changing it is wrapping around that so there is a sharp increase and from C to D because there is no contact with the tool so the tension at C and tension at D B and C is going to remain the same and then because again there is a fault because there the metal is moving inward so frictional forces will be outside so here you will find that these are changing like this okay.

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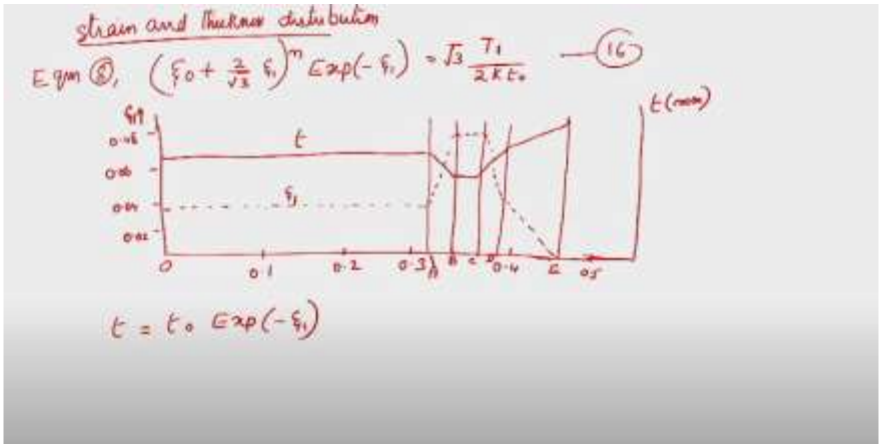

Now if you look at it another point is that you have to find the strain and the thickness distribution. So from the distribution of strain, we can see that from the previous figure, we can and from the equation 8 also we can find out the relationship. So that means writing in the form equation 8. See, this equation if you write it, we can write in this form. epsilon 0 plus 2 by root 3 into epsilon 1 raised to n into exponential minus epsilon 1 is equal to root of 3 into t1 by 2k t0, 2k into t0. okay so this from this relationship epsilon must we had to find out the epsilon by numerical methods

$$\left(\epsilon_0 + \frac{2}{\sqrt{3}}\epsilon_1\right)^n \exp(-\epsilon_1) = \sqrt{3}\frac{T_1}{2kt_0}$$

so that means it will be slightly complicated but still if you just plot that say one value if you get it all other things you can get it that is the biggest advantage okay and you will find that the variation the strain is like this okay so maybe from 0 to maybe 0.1, 0.2, 0.3, 0.4 and maybe 0.5. So, strain 1 if you plot like this, you may get the test 0.02, 0.04. 0.06, 0.08 if this is the strain axis. So, maybe we can just say that here this is your A is B, C, D and maybe E. So, in this side if it is thickness in millimeter, we can say So the thickness variation it is more or less remaining constant up to here and then here it comes down. So it remains almost constant then again it comes down and maybe thickness will increase here because there now because of the d and t there is no contact.

So that will almost only small variation thickness only will be there. So this is your t. And if you look at epsilon 1, it will be strain will remain same because the radius if it is very large, we can say it remains same here and then maybe strain increases some value, it remains constant here, then again it will come down okay and then strain will fall off to 0 here okay different slope. So this is epsilon 1 this will be the variation okay and we can also have the variation pressure also in the same way okay we can calculate it and find out. Now the thickness at any point see T is equal to is given by T0 into exponential minus epsilon 1. So if you are finding out the epsilon 1 at any point you can find out the thickness how much it is from this from the initial thickness.

$$t = t_0 \exp(-\epsilon_1)$$

And you can also find out from the volume at each section what was the initial volume and finally what was the later volume. So if you are maintaining that then from that also you can find out the thickness. The total sum at each section if you take it and then you can find equate it with the initial volume and what is the final volume that way we can find out.

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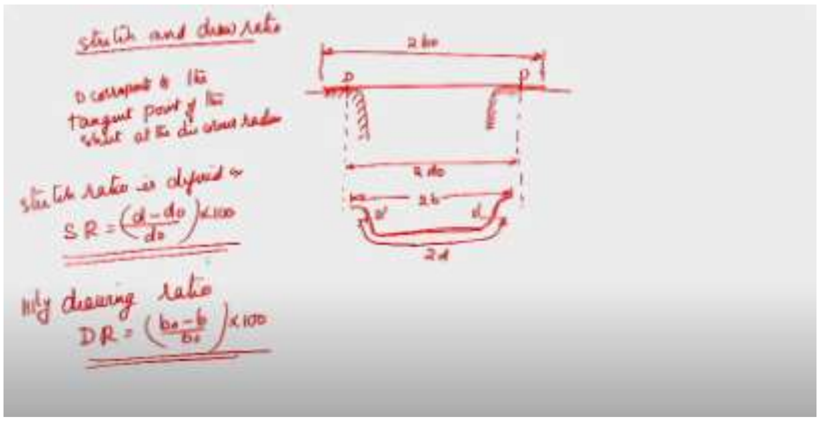

Now to the last part we have to know the terms which are there stretch ratio and draw ratio. This term also we will just tell because this is also very important because people should know what is the stretch ratio and what is the strain ratio. Later when we go to this limit diagrams and other things now we will have to come out with that. So in that case let us say that there is a blank is here and it is in contact with your die here okay. So this is the die part. May be initially your blank was having some distance to B naught where this d corresponds to the tangent point of the sheet at the die corner radius. You will see that as the blanking is taking place when you are pushing it down from this it will make a mark on your see this point D where it is where it is in direct to contact after that it is in tangential to that.

So, from here only that initial point is there. So, when it is going to wrap around it, it will make a mark on this a marking will take on the sheet slightly. So, initially when the position was at D, it will move somewhere inside. So, you will find that okay, it has, now if this is a material which has been after drawing operation by some distance, if you look at it and if you look at the sheet and then you will find that maybe initially it was at this point, but now it has come to say D dash here. Because of the marking you can always find it out. So, this distance we can say as 2D and that this has moved may be up to some distance here, because drawing has taken place. So, we will say that okay this this is 2 b. So, from 2 b naught it has moved to say 2 b because of the drawing operation and then this 2 d naught it has come to 2 d this total length okay. So, the stretch ratio S R is defined as S R is equal to D minus D naught by D naught into 100. Similarly, drawing ratio drawing ratio dr is equal to b naught minus b by b naught into 100 okay.

$$SR = \left(\frac{d - d_0}{d_0}\right) \times 100$$

$$DR = \left(\frac{b_0 - b}{b_0}\right) \times 100$$

So these two definitions are there because most of the analysis when you are carrying out the experiments it will be discussing regarding the drawing ratio and the stretch ratio okay. So it is often found that the problems will occur in stamping if these ratios change too rapidly with successive sections in the tool okay. So with this today's class is over, lecture is over. Thank you.