Plastic Working of Metallic Materials

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Lecture 34

Instability in sheet metal forming

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So this lecture we will be discussing about the various instabilities that can occur during the sheet metal forming. So the first one as we have discussed earlier, so that is if you write that Load instability and tearing. So let us see that. The instability of the sheet to transmit the required force that is what or so that is what when you are transmitting the load at some stage and instability sets in and the metal finally end up in tearing okay or any other type of defect which can happen. See if you are considering the case of a deep drawing process where your material is like this it is being drawn. And maybe the punch was here Okay. And your die, this is your die. at this wall region you will find that the force required see two things are happening one is the metal will have to because you are you are holding here and the metal will have to be drawn inside from the this side and here another is that here the strain has to take place tensile straining will be taking place on the side walls. So if the force required to draw the flange invert if this force is it exceeds the strength of this the cup wall. then what will happen is that it can fail there okay. So this will happen when the tension around this circumference reaches a maximum and will be seen as a maximum pungent force. So that is what happens. So when the load reaches the maximum value what will happen is that at this region the failure will take place okay. So deformation becomes concentrated at some diffused neck region okay and it is no longer uniform. At some specific area the deformation takes place and this is called as the global instability. There are two things which are coming into picture. One is the diffused neck and the local necking. These two things are going to happen.

So if you take a cylindrical piece which is loaded in tension suppose this was a cylindrical piece a tensile sample and when you are loading it after reaching the maximum value you will find the necking which is taking place over this area. So the total diameter gets reduced okay. So similarly so if this is the cylindrical piece similarly if you take a rectangular a flat specimen generally which is used for sheet metal operations okay where this is your thickness. What you will find is that at some stage similar to this you will find that okay let me just draw like this. So at some stage you may find that okay similar to here you will find a decrease in the width. This is called as the diffused necking. Whereas in addition to that now you may find here which is called as this is like deformation taking place along very thin narrow band. Like in metallurgical terms now you call it as loader bands. So this is called as a local necking. So this diffused necking may end up with a local necking. So that in sheet metal operation. So that is these are the two differences. This is the diffused necking and this is the local necking. So this one should understand what is happening okay. So in a diffused neck once it happen now this is like a global instability criteria which is going to take place.

So we can just look at how this failure takes place is one is due to this diffused necking which is a global necking global instability criteria. So second is the localized necking. So when this happens in any localized area when this happen over an aeroband this will rapidly lead to tearing of the material and the failure will terminate there and that failure will terminate the forming process okay. So this is just a local instability criteria and another type of failure is your fracture, When you are deforming the material, it gets work hardened. When it work hardens, it gets brittle.

And at some stage, you will find that the material just fails in a brittle way. So that is called as the fracture mode, okay. So and another type of instability is wrinkling. So this is when one of the principal stresses in any element is compressive then what happen the sheet may buckle or it may wrinkle that means it thickness will get increase this is a compressive instability and resembles the buckling of a column so that is what. So the normal failure in sheet metal operation is instability which takes place is by any of these or it can be a combination of also but mostly it will be any one of this only okay.

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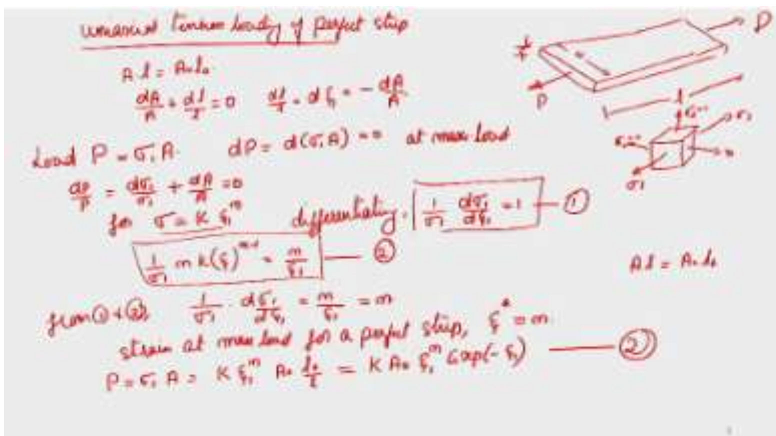

So now when we have to look at it let us take one by one. So what are the theories behind this localized necking and diffused necking? Let us consider a uniaxial tension of a perfect crystal, perfect strip. So if you have a strip like this, and you are applying your load over this area P, this is your length, this may be your width and this is your thickness. So, if you just look at this in such a case in a normal sheet metal operation, the type of stresses we can see that it is along this one you can call it a sigma 1. And along this direction sigma 2 is equal to 0, sigma 3 is also is equal to 0.

This is the normal assumptions which we take it. So we can always write that since the volume remains constant say A into L is equal to A naught into L naught. This part which I am going to explain was discussed in the earlier class but still I wanted to bring it here.

So that means you can just tell that from D A by A plus D L by L is equal to 0. Or DL by L we can say is equal to D epsilon 1 okay that is equal to minus DA by A okay.

$$A.l = A_0 l_0$$

$$\frac{dA}{A} + \frac{dl}{l} = 0 \quad \frac{dl}{l} = d\epsilon_1 = -\frac{dA}{A}$$

So, the load in the strip the load in the strip P is always equal to sigma 1 into A. say if you differentiate it, you can get it as dp is equal to d of sigma 1 into a, so is equal to 0, the maximum load. For the maximum load, this is the condition, okay. So if you just take that, that is dp by p is equal to d sigma 1 by sigma 1 plus d a by a, is equal to 0 for the maximum condition for at a maximum load. Equation number 1 and for a strain hardening material which follows that sigma is equal to k epsilon 1 raise to n which is the power power law we can substitute in this and we can get it as 1 by sigma 1 into if you differentiate it.

$$Load\ P = \sigma_1 A \qquad dP = d(\sigma_1 A) = 0$$

$$\frac{dP}{P} = \frac{d\sigma_1}{\sigma_1} + \frac{dA}{A} = 0$$

$$\sigma = k\epsilon_1^n$$

Differentiating we will get it as 1 by sigma 1 into d sigma 1 by d epsilon 1 is equal to 1. This is a general equation. So, this is one relation we are getting. This is the equation 1. And for this particular case, if you just take it as 1 by sigma 1 into n k into epsilon raise to n minus 1 is equal to n by epsilon 1.

$$\frac{1}{\sigma_1}\frac{d\sigma_1}{d\epsilon_1} = 1$$

$$\frac{1}{\sigma_1} nk(\epsilon)^{n-1} = \frac{n}{\epsilon_1}$$

So, we can get this equation number 2. These two cases we are getting. okay and if you substitute from 1 and 2 we can get 1 by sigma 1 into d sigma 1 by d epsilon 1 is equal to n by epsilon 1 that is equal to n. So that means the strain at maximum load which we have discussed earlier so that strain at maximum load for a perfect at maximum load for a perfect strip. So that means we can say that this epsilon star is equal to n because we have got that epsilon is equal to at maximum at this one is equal to n that was good.

$$\frac{1}{\sigma_1} . \frac{d\sigma_1}{d\epsilon_1} = \frac{n}{\epsilon_1} = n$$

$$\epsilon^* = n$$

So diffusion diffused necking cannot occur in one region that is another thing. This diffused necking cannot occur in one region. So in a perfect crystal or a strip the load to deform the strip we can get it as P is equal to sigma 1 a So you can write it as sigma 1 is equal to k epsilon 1 raise to n and a is equal to because a into l is equal to a naught into l naught. So we can say this a we can write it as a naught l naught by l. So that we can write it in this form as k a naught epsilon 1 raise to n l naught into l naught by l 1.

$$P = \sigma_1 A = k\epsilon_1^n A_0 \frac{l_0}{l} = kA_0 \epsilon_1^n \exp(-\epsilon_1)$$

So, we can write that it as exponential minus epsilon 1. equation number 2 that is because since epsilon 1 is equal to say log L by L naught. So for the tension in a perfect crystal you find that it is epsilon star is equal to n and you are getting this relationship p is equal to k a naught into epsilon 1 sorry epsilon 1 raise to n into exponential minus epsilon 1.

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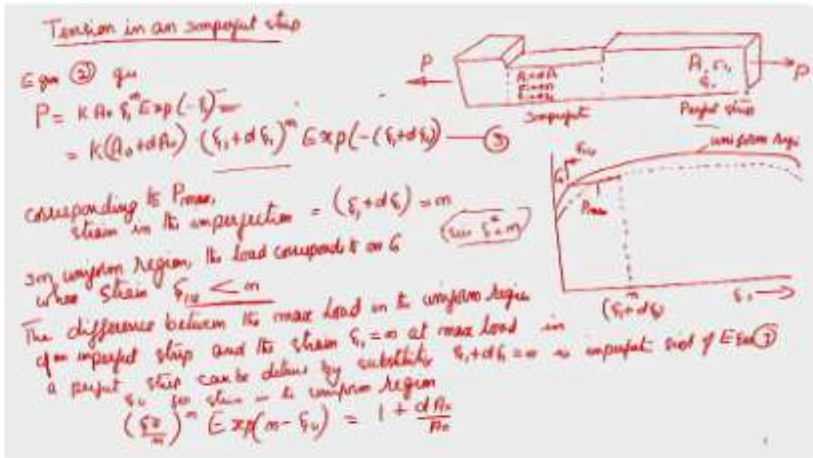

Now let us take the case of a tension in an imperfect crystal. See this tension in real material it is never perfect even if you just assume that okay it is well polished sample and other thing even surface also is never perfect when you look at the at a macroscopic level.

But most of the sheet metal when you look at look at this microstructure there are some imperfections inside. It can be voids, it can be very fine second phase precipitates which will behave in a different way. So you will find that no material is a perfect crystal. So let us for our analysis, let us assume a case where there is a small imperfection and we assume that imperfection is due to the minimum reduced area, cross sectional area where stress concentrations are taking place okay.

So if there is a void inside okay that that means effective cross sectional area that section is less okay. So that way we can just in a material let us introduce let us introduce a small imperfection. See basically if I just consider this as a material let me just consider like this. So, this is an imperfection area whereas here it is a perfect area. So, A 1 sigma 1 and epsilon 1 and we are loading along this direction P in a tensile sample.

We are using a tensile sample having a small imperfection. So, in this region let us say that A plus D A A1 plus dA1. So, sigma 1 plus d sigma 1 and epsilon 1 plus d epsilon 1. These are the conditions which are there. If you assume such a case, that means here a small reduction in area is there and you are applying a load P across this.

The same load is transmitted through equally through the two sections. So, one is the section is the imperfection here and this is a perfect it is not crystal we can say what is it called strip perfect strip perfect strip okay. So we are having this 2 region one is a perfect strip where the cross section area is large. So when this external load is applied along it is equally transmitted through this section A and B okay.

So, in such a case, this equation 2 will become, we can write for the two cases. Equation 2 gives, say one is p is a constant. So, we wrote that k a naught into epsilon 1 raised to n into exponential minus epsilon 1. So, that is for this perfect size. Now, that should be equal to whatever be the stress and strain at the imperfection area. So, we can say that K is a constant for the material. So, we can say A naught plus D A naught into epsilon 1 plus D epsilon 1 raised to N. into exponential minus of epsilon 1 plus d epsilon 1 okay. We are getting this okay. Here it is not clear I will write it. This is equal to k a naught plus d a naught where d A naught is negative into epsilon 1 plus d epsilon 1 raise to n into exponential minus of epsilon 1 plus d epsilon 1.

$$P = kA_0\epsilon_1^n \exp(-\epsilon_1)$$

$$= k(A_0 + dA_0).(\epsilon_1 + d\epsilon_1)^n \exp(-(\epsilon_1 + d\epsilon_1)$$

So, this is equation number 3. So this is for the case of imperfect strip and this is the case for that means at this section it this is this right hand side is true whereas for this section where the perfect strip is there this this one is true okay. So this is the thing. So if you look at the load versus strain curve okay if you just plot this load versus strain curve for the two sections. when your load is coming maybe one it will come like this okay this is for the uniform region see perfect region we are calling it as uniform area uniform region whereas for this part it will be in a imperfect region you will find that okay it is something like this. See what happens is that load increases and at some point here say let me just say it as G for the maximum load and the maximum load which can reach that is the maximum load which can be taken by the piece at any point.

So that will be this value corresponding to this whatever you are getting is the maximum load. So, but because here this is a cross sectional area when it reaches the maximum load no it will be the so that corresponding to that on your perfect crystal it will be you will have a strain of epsilon 1 u which will be less than the strain corresponding to this on your reduced to cross sectional area that strain will be corresponding to your n okay. So, this will be epsilon 1 plus d epsilon 1. So, when that value reaches this n it is.

So, your maximum load can be this P max. Because in your perfect strip or uniform region, the cross section is very large. It will not go beyond this. Rather, in this region, when the stress reaches, the stress corresponding to this maximum load reaches here, it will start deforming. And that will be the strain which you will be showing as epsilon plus d epsilon 1 here. Whereas that corresponding to that external load, your strain in the uniform region will only be epsilon 1 u we are just defining it as epsilon 1 u maybe u should be the suffix okay I am just putting like that that is maximum load u corresponds to the maximum load in that so that is the thing generally the notation which is used.

So the strain in the imperfection corresponding to this maximum load okay so corresponding to P max strain in the imperfection is equal to epsilon 1 plus d epsilon 1 okay and that will be equal to n because epsilon star since epsilon star is equal to n we previously got so that should be corresponding to your n. but that n value it will never reach for your perfect crystal okay. So that means in the in the uniform region the load corresponds to only g where the your strain the strain epsilon 1 u is less than your n okay. So, it will never reach that value. So, whatever you are trying to apply the load the deformation will take place at the imperfect region.

$$Corresponding\ to\ P_{max}\ ,strain\ in\ the\ imperfection = (\epsilon_1 + d\epsilon_1) = n$$

So this is the maximum uniform so you will find that this is always less than this your value of n okay because the load cannot go beyond that. So this is the maximum uniform strain since it is measured in the uniform region of the test. So uniform region cannot be loaded to a load more than g since any attempt to load beyond. It will be transmitted to the imperfection and the region of the imperfection only will get strained. Okay? It will deform there and finally in an uncontrolled way it will go as instability will set in and then the material will fail there. Since all the tension strips which contain, any of the tension strips you take, any strips in real material when you are taking, it will contain some sort of imperfection. There is no doubt in that. You cannot tell that material is there without any imperfection. Maybe that dA may be very small. That is all. So but in that case, it contains large number of imperfections in real materials. So the greatest imperfection will become the site for the diffused neck.

That is where. So wherever that an imperfection is there and the cross sectional area is a minimum, that will be the site for the diffuse neck. And once the maximum load carrying capacity is reached in the neck region instability will set in. The material at the imperfection region it will just start expanding or growing and then finally it will result in the failure like tearing and other things. So the difference between the maximum load in the uniform region of an imperfect strip and the strain epsilon 1 is equal to n at the maximum load in a perfect strip can be determined. So that means so what I said is now it is the difference between the maximum load or load maximum in the uniform region of a perfect strip and the strain where epsilon 1 is equal to n at the maximum load. Sorry imperfect of an imperfect strip. In a perfect strip that can be determined. So that can be determined can be determined by substituting epsilon 1 plus d epsilon 1 is equal to n in the imperfection side of equation 3 and epsilon u for strain in the uniform region. So, that we can find it out epsilon u by n raise to n into exponential n minus epsilon u is equal to 1 plus d A naught by A naught.

$$\left(\frac{\epsilon_u}{n}\right)^n \exp(n - \epsilon_u) = 1 + \frac{dA_0}{A_0}$$

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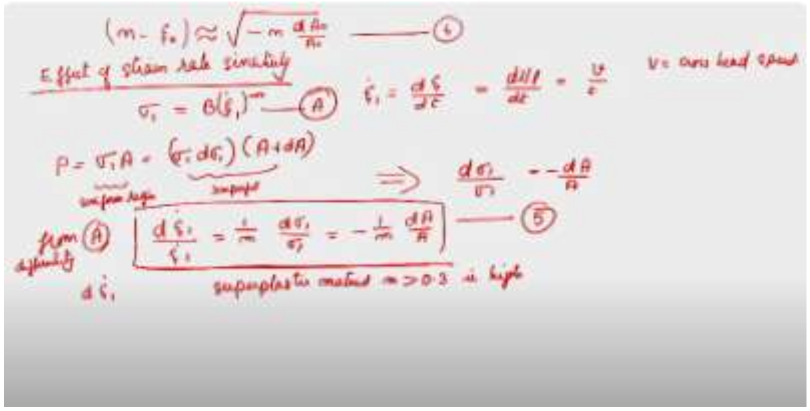

So in this real case, this n minus epsilon u and d A naught by A naught are very small quantities. So if you are just expanding this in the series expansion and taking the first term, you can get this relationship that is n minus epsilon u is almost equivalent to say minus n into d A naught by A naught. Now let us look at what happens to the strain rate sensitivity. So the effect of strain rate sensitivity. The initial lectures we have discussed that the strain rate sensitivity is very important bulk deformation of material at higher temperature because at higher temperature it becomes more of a strain rate sensitive.

$$(n - \epsilon_u) \approx \sqrt{-n\frac{dA_0}{A_0}}$$

So at higher temperature for deformation processing this is very important. Whereas in the case of sheet metal forming even at room temperature also though you may feel that strain rate sensitivity is not coming into picture but depending upon the strain rate sensitivity of the material that has an influence on your defect growth okay or maybe the strain to failure and how fast it reaches that has an important influence. So let us look at that. So in normal strain rate sensitivity that expression for the flow stress say maybe for uniaxial strain so that can be written as B epsilon dot 1 raise to n m okay, where epsilon dot 1 is equal to d epsilon by dt and that is equal to dl by l by dt. So, that we can say it is equal to v by t where v is the cross head speed cross head or ram speed cross head speed okay.

$$Effect\ of\ strain\ rate\ sensitivity \quad \sigma_1 = B(\dot{\epsilon_1})^n$$

$$\dot{\epsilon}_1 = \frac{d\epsilon}{dt} = \frac{dl/l}{dt} = v/t$$

So, it depends upon if you wanted to have a constant strength rate this is the expression which one has to get it. And the forces when you are transmitting it across these two sections you know we can still write it as P is equal to sigma 1 a. So, is equal to sigma 1 d sigma 1 into a plus d a. So this will be for an imperfect crystal and this will be for an imperfect strip and this is for a perfect strip. Or maybe this will be for the uniform region. So this is the case. So from this now we can write it as d sigma 1. So, load is the maximum load. So, d sigma 1 by sigma 1 is equal to minus d A by A. That is the difference between the stresses in the uniform region and the region where any perfection is there.

$$P = \sigma_1 A = (\sigma_1 d\sigma_1)(A + dA)$$

$$\frac{d\sigma_1}{\sigma_1} = -\frac{dA}{A}$$

So, that is what we are getting it. So, now from this from this relationship if I just put it as what let me put it as equation number A. So, from A you substitute it into this taking the derivative and other things we can get it as d epsilon dot 1 by epsilon dot 1 is equal to 1 by m into d sigma 1 by sigma 1 from this if you take the derivative differentiating. We will get this relationship, okay. So that is equal to, because d sigma 1 by dA1 is equal to minus dA by A, so minus 1 by M into dA by A. So this we can get it as five, equation number five.

$$\frac{d\dot{\epsilon}_1}{\dot{\epsilon}_1} = \frac{1}{m}\frac{d\sigma_1}{\sigma_1} = -\frac{1}{m}\frac{dA}{A}$$

Now, in this, if you really look at it, the difference for a given imperfection the difference in the strain rate that is a d epsilon dot 1 between the imperfection the region of imperfection and the uniform region. So you will find that that is that is that d epsilon dot 1 is inversely proportional to the strain rate sensitivity index okay so that is rate sensitivity index no that that that is one so it is inversely proportional that means if you have at some at room temperature if MA is very small and the difference in the strain rate is very large then the imperfection will grow very rapidly. So that is so if you are doing it at a very high strain rate and at a low strain rate and the M is a material property. So if M is small then it will have it will necking will take place at a much faster rate. The difference in the cross sectional area strain will be much higher.

So failure will localized necking will take place and it can fail. But in certain cases like super plastic deformation where you can elongate it to maybe 500 or 1000 times or more than that in certain cases you have obtained even up to 4000 also super plastic. If you just take a glass rod and heat it at a higher temperature and then you try to pull it you will find that the elongation is very large so that is a typical place of a super plastic deformation that is why in glasses you know you can use this glass molding to glass blowing to obtain a very large shape and other things. So, so in metals also in ceramics also this type of phenomena is observed. So, where in such case so in super plastic materials under that conditions of deformation super plastic material say M is high that is more than 0.3 that is it is high. So in such case now you can just deform to a very large extent. So so M strain rate sensitivity even at room temperature also if the M value is less then it cannot deform much okay because necking will take but if M value is high even at room temperature also you will find that you can have a more strain or higher strain.

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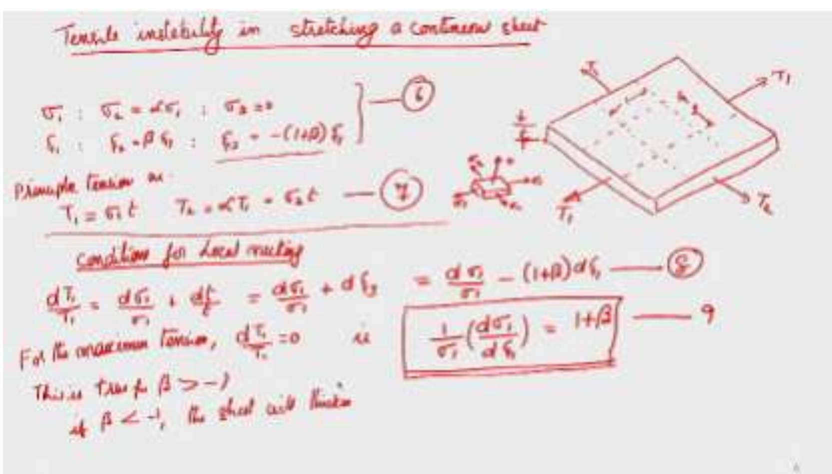

Now let us look at the next case that is tensile instability in stretching a continuous sheet stretching a continuous sheet. So in a tensile strip, if an imperfection is not there, is not present or it is not existing, then diffuse necking will start when the load reaches a maximum value.

So when the maximum load is reached, the diffuse necking will place. So in case of a sheet stretched over a punch this diffuse necking is not generally observed in practical case okay. So the tension may reach a maximum but the geometry of the punch why it is not happening is that the geometry of the punch because on one side you are coming in contact with that. So geometry of the punch that it imposes constraint on the strain distribution. So the constraint by the due to the geometry of the punch you will find that okay this diffused necking is not taking place. So by theoretically you may find that it can take place but when it is just when it is stretched over a punch you know that that is not taking place okay.

So, in a continuous state development of the local necks are necking are observed which are similar to that occur within a diffused neck of a tensile sample. So, when it is a continuous state this happens. So, width of this local neck necking which occurs is almost equal to the sheet thickness. So you can see that depending upon the sheet thickness more or less equal to that thickness will be coming okay that width of this because as I have mentioned localized necking means that like a band it is forming the width of that band will be equal to your thickness of the sheet though they do not contribute to the local necking they result in tearing of the material.

So global necking is your diffused necking. Diffused necking will be more or less, without the local necking will be almost about the sheet thickness, okay. And they do not contribute to global necking. They result in tearing of the sheet. That is what. So let us consider the condition for that when this can happen, okay. So how these things are formed. So we can consider a sheet deforming uniformly in a proportional process, okay. So like suppose you have a sheet like this. with a thickness t and this is your tension t 1 and this is your t 2 this is your thickness t okay so if you just consider a unit size where this is equal to 1 and this is equal to 1. So, in such case, you are assuming a proportional loading, deforming uniformly in a proportional process.

Proportional process means your alpha and beta are remaining constant. The stress ratio and the strain ratio, they are remaining constant. So, if you look at, I just take a small element from that in this case. okay and look at the stresses and stresses sigma 1 this is sigma 1 and this is sigma 2 this is sigma 2 and sigma this one will be 0 sigma 3 will be 0 okay so this is the condition so the the the deformation in such case can be specified as one is stress condition conditions if you take there are the stresses are sigma 1 Then sigma 2 is equal to alpha sigma 1 and sigma 3 is equal to 0 because it is a very thin sheet we are assuming the stress across the thickness is 0. Whereas the strains are sigma epsilon 1 then epsilon 2 is equal to beta into epsilon 1 and the third epsilon 3 along the axis 3. So that will be is equal to minus which we have earlier itself we have described it is equal to minus of 1 plus beta into epsilon 1.

$$\sigma_1 \quad : \quad \sigma_2 = \alpha\sigma_1 \quad : \quad \sigma_3 = 0$$

$$\epsilon_1 \quad : \quad \epsilon_2 = \beta\epsilon_1 \quad : \quad \epsilon_3 = -(1+\beta)\epsilon_1$$

So if you know that epsilon 1 and sigma 1 then you can always describe this condition. So the principal tension in the sheet are the principal tensions. Tensions are T1 is equal to say sigma 1 into T and T2 is equal to alpha into T1 and that is equal to sigma 2 into T because sigma 1 and sigma 2 are there. So, we can write this. So, maybe this we can write it as equation 6. This is equation 7 if you write like this.

$$T_1 = \sigma_1 t \qquad T_2 = \alpha T_1 = \sigma_2 t$$

So these are the state of stresses in this sheet. So the condition for local necking in this condition under these circumstances what are the conditions for local necking? The conditions which have been postulated is that local necking will start when the major tension reach a maximum. And see in this condition, since it is a proportional loading, alpha and beta remains constant. So, the local necking will start when the major principal tension reach a maximum value. So, that means by equation number 7, T1 should be the maximum or we can say if you take DT1 by T1 is equal to D sigma 1 by sigma 1 plus dt by t. So, that we can write it as dt by t is equal to epsilon d epsilon 3. So, d sigma 1 by sigma 1 plus d epsilon 3 we can write. So, this this can also be written as d sigma 1 by sigma 1 because epsilon 3 is this okay my into minus 1 plus b into epsilon 1 okay this is equation number 8.

$$\frac{dT_1}{T_1} = \frac{d\sigma_1}{\sigma_1} + \frac{dt}{t} = \frac{d\sigma_1}{\sigma_1} + d\epsilon_3 = \frac{d\sigma_1}{\sigma_1} - (1 + \beta)\epsilon_1$$

Now when the tension reaches a maximum that means derivative of that should be maximum condition it should be equal to 0. So that means so for for the maximum tension d t1 by t1 should be equal to 0. So, that means from equation 8 we can reach if this is equal to d sigma 1 by sigma 1 is equal to this is d epsilon 1 sorry ok. So, if you bring that d epsilon 1 to the left side, we can say that 1 by sigma 1 into d sigma 1 by d epsilon 1 is equal to 1 plus beta. And this is true for beta greater than minus 1 the strain ratio should be greater than minus 1 okay.

$$For\ the\ maximum\ tension, \frac{dT_1}{T_1} = 0\ is\ \frac{1}{\sigma_1}\left(\frac{d\sigma_1}{d\epsilon_1}\right) = 1 + \beta$$

So, if beta is less than minus 1 say what will happen is the sheet will start thickening. So, if beta is less than minus 1 the sheet will thicken. So because that is the ratio between sigma 2 and sigma 1 if it is less than minus 1 naturally it will be strain will be whatever it will start thickness will start increasing that is the thing. So and for a strain hardening material the tension will not reach a maximum for a general will not reach a maximum strain okay. So in such case where beta is less than 1. Since the sheet keeps on, thickness keeps on increasing due to a strain, due to a loading, it will never reach the value of sigma 1 because the stress inside that material will not increase.

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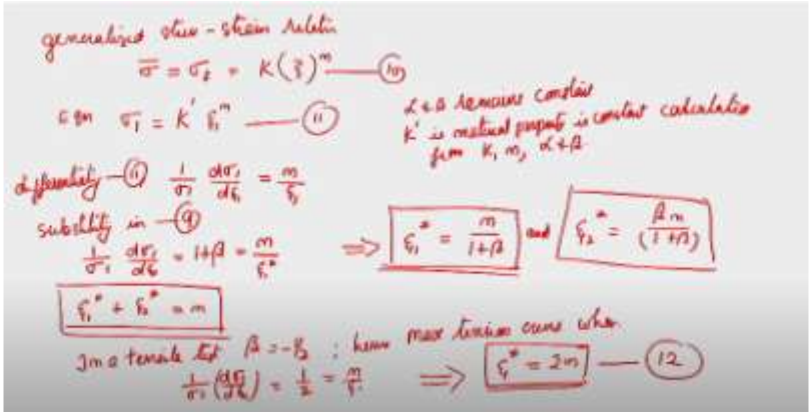

That is the main thing. So for a generalized flow stress, when you are having, we can have a generalized flow stress relationship. Generalized stress-strain relationship. is the effective stress we can write it is equal to your uniaxial flow stress is equal to k into effective strain raise to n. So if you just substitute the value of effective stress and effective strain from our previous derived equation and other things you know so this equation 10 will get the form of so it will that means that sigma bar is equal to say square root of sigma 1 square minus sigma 1 sigma 2 minus sigma 2 square or you can say 1 minus alpha plus alpha square into sigma 1 root of 1 minus alpha plus alpha square alpha square into sigma 1. or similarly epsilon bar no it is root of 2 by 3 into epsilon 1 square plus epsilon 2 square plus epsilon 3 square or you can say root under root 2 by 9 into epsilon 1 minus epsilon 2 square.

$$\bar{\sigma} = \sigma_1 = k(\bar{\epsilon})^n$$

So that relationship if you substitute and then get it we can get this in this form sigma 1 is equal to k dash into epsilon 1 raise to n okay. In this, This is the condition when you are having a proportional loading where alpha and beta alpha and beta remains constants. k dash is a material property and is a constant calculated from k, n, alpha and beta.

$$\sigma_1 = k^{'} \epsilon_1^n$$

You are just determining from the value of k, n, alpha and beta from that if you are getting. So, if you differentiate this equation one, we can write this 1 by sigma 1 into d sigma 1 by d epsilon 1 so that will be equal to n by epsilon 1 okay. So for the maximum tension if you are substituting this in the previous equation number 9 so substituting In equation number 9, 1 by sigma 1 into d sigma 1 by d epsilon 1 is equal to 1 plus beta. So, that is equal to n by, so now this is the maximum case, you know, when you are writing it epsilon 1 star. Star always refers to the case for maximum load condition. So that it implies that epsilon 1 star is equal to n by you are bringing 1 plus beta.

$$\frac{1}{\sigma_1}\frac{d\sigma_1}{d\epsilon_1} = 1 + \beta = \frac{n}{\epsilon_1^*} \; implies \; \epsilon_1^* = \frac{n}{1+\beta}$$

So if you know the value of beta and if you know the value of n the maximum strain along the principal direction 1 we can determine and also along epsilon 2 star is equal to beta into n by 1 plus beta. So, these two relationships you are getting. So, the strain at maximum at maximum tension that is epsilon 1 star plus epsilon 2 star is equal to n you are getting. So this value if you consider this epsilon 1 star and epsilon 2 the maximum strain in these two directions the sum of that should be equal to n that means epsilon 2 star is equal to 0 epsilon 1 star is equal to n that means for a uniaxial tension testing you will find the maximum strain is equal to n.

$$\epsilon_2^* = \frac{\beta_n}{(1+\beta)}$$

$$\epsilon_1^* + \epsilon_2^* = n$$

But for biaxial and other conditions under different strain paths the total value should be equal to n. So let us see these two conditions okay. So in a tensile test when you look at it for a tensile test beta is equal to minus 1. We earlier discussed about tensile test beta is equal to minus 1. Hence, the maximum tension occurs when 1 by sigma 1 d sigma 1 by d epsilon 1 is equal to 1 by 2. So, that is equal to n by epsilon 1 or that indicates that in such a case epsilon star is equal to 2 n.

$$\frac{1}{\sigma_1}\frac{d\sigma_1}{d\epsilon_1} = \frac{1}{2} = \frac{n}{\epsilon} \; implies \; \epsilon^* = 2n$$

This is a very important point one has to look at it So in the maximum tension condition it signifies that the onset of local necking that is the local necking strain in a uniaxial tension is epsilon 1 star is equal to 2n which is twice the strain for a maximum load and start of diffuse necking. So for a uniaxial tension it is twice the strain for a maximum load and where the start of diffuse necking takes place. So that is the condition which is coming okay. So you have to find out in this particular case you will find that in sheet metal operation you may get a strain which is much higher.

So you are going to get it at 2n and other. So by drawing say grid circles on a sheet, this is how the sheet metal experiments are done, this module itself lecture number 2 if I am right that grid circles we have demonstrated. So if you are drawing this grid circle we can experimentally determine the strain at the onset of local marking because under different conditions it is straining. So strain history at different places may be different okay.

So there we can study the minor and major axis of your ellipse and then find out what is the strain along epsilon 1 and epsilon 2. So we have discussed that maybe during if it is a circular circular grid it may just extend like this like an ellipse. So you measure this and this so after this so wherever the failure has taken place adjacent to that you measure that you may not be able to find it exactly at that split but very adjacent to that you measure the strains okay. So that will be the, because that is a region very adjacent to a local necking. So we can approximate that that is the region where the maximum straining has taken place or very near to that.

So and in a similar way you measure this strain for different strain paths. Because you can always by measuring the circular this one you can measure the strain path at different places and in both the quadrants of a strain diagram okay we can establish a foaming limit diagram. So after measuring this for different strain paths in quadrant 1 and quadrant 2 of the strain diagram. We can just look at a what is called as a forming limit diagram which delineates the boundary of uniform deformation and onset of nexus. So the point which you are going to get it very adjacent to the failure or even some means if you can measure that the failed region also the minor and major axis if you can measure though it is very difficult at least that point you you know that okay this is the boundary between the for the strain path particular strain path with the epsilon 1 and epsilon 2 combination of that this is the region where the failure will take place. So at different places if you measure it and then you plot all these things you can just get a two region one where the failure can take place and the other where failure cannot take place. The boundary between that is the safe region so that we can get it. So this is called as the forming limit diagram.

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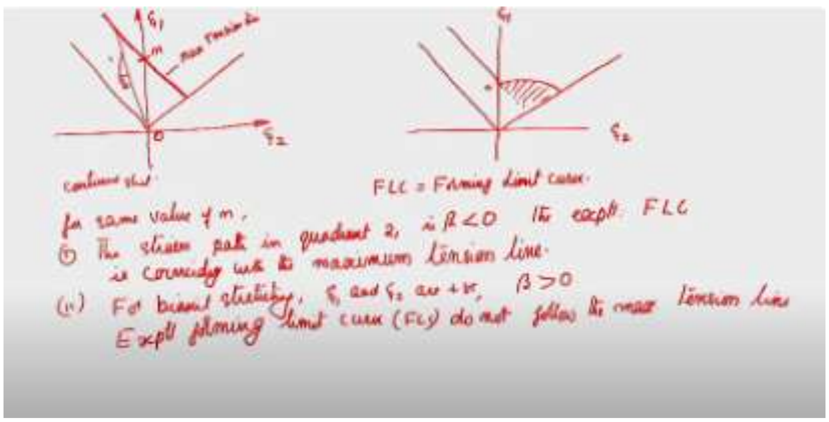

So, that if you look at it, it will be something like this. So, this is for your epsilon 2, this is for epsilon 1 and you will find that this is for 0. is a continuous sheet and you will find that the you can just draw this maximum tension line like this for a combination of epsilon 1 and epsilon 2 maximum tension line. The maximum tension line will intersect it at the value of n and if you just join this maybe for any condition no you can just draw this for any condition so that means this you will find that the slope is 1 and a half here okay. So whereas for the actual case so in the second quadrant no it is okay you are finding it safe but if you look at the actual case real material you will find that in the first quadrant it is not so okay here you are getting up to here it is true up to this is up to n value this is epsilon 1 and this is epsilon 2 but here instead of directly coming like this so you will find that it is it is coming something like this.

So there is a region here where it is not matching. So this is the value of n. So in this case when you look at it for materials with the same value of n the forming the following observations can be made one is the four same value of n the strain path in the second quadrant. That is this is the second quadrant here this is the second quadrant the strain path In the second quadrant is that is for beta is equal to less than 1, the experimental FLC is coinciding with the maximum tension line. The strain path in quadrant 2 that is for beta less than 0, the experimental FLC that is where FLC is equal to forming limit curve is coinciding with the maximum tension line whereas, for both epsilon 1 and epsilon 2 positive that is in the biaxial stretching. That is in quadrant 1, epsilon 1, sorry, not sigma, epsilon 1 and epsilon 2 are positive and beta greater than 0.

This experimental forming limit curve or FLC, it do not follow the maximum tension line. That is what we can see here. See here, it is not following the tension line. The maximum tension line is this dotted line. But you will find that it is at a higher value.

So that means this indicates that there is some process which retards the necking in the biaxial tension condition. The biaxial stretching condition, there is some reason which retards or which slows down the necking process after it has reached the maximum tension line. So, that is one thing. So, how that is taking place we will have to look at it.

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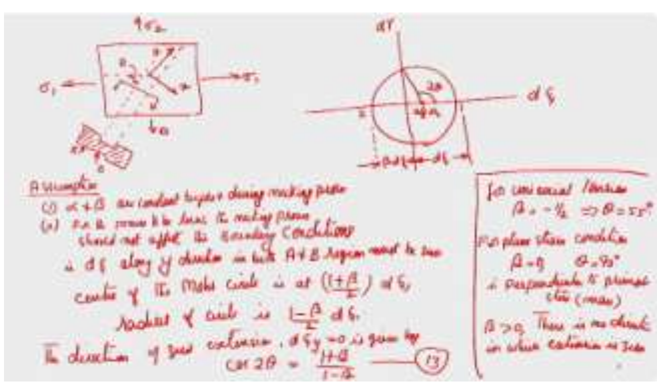

So, let us for that particular case let us just assume a case in a material say a sheet like this the let us assume that the necking is taking place in along this region. So, whereas this is your sigma 1 and this is your sigma 2 and if I just take a section along this cross section area you may find that okay just do like this this is the section if you just take It is a reduced section that necking has taken place and if you just look at this as y and this as x the local coordinates axis if you take like that and see if this is inclined at theta okay. So, we identify the uniform region as A and an imperfection as B so that means this region as A and maybe this region as B if you identify that. For analysis of the necking process the assumptions are if you just consider assumptions. One the stress and strain ratios remain constant alpha and beta are constant before and after and during necking process. Second assumption is for necking process to to be local the necking process should not affect the boundary conditions okay.

The necking process the process to be for the process to be local the necking process should not affect the boundary conditions. That is the strain increment in both the region A and B along the y direction that must be 0 that is the strain increment along y direction that is along the groove if you assume it is a groove along that groove in that direction y direction in both A and B region must be 0. It is going to be the same, So from a Mohr circle representation now we can just draw the Mohr circle representation for the state of stress in this case and strain in this case sorry the strain and d gamma if you just find out this this is the circle circle will come like this okay and this is the center so so center we can just write this as 2 theta okay and this is 1 this is 2 So, beta d epsilon 1 and this is equal to d epsilon 1.

So, this is the, so from this mahar circle, we can find that center of the circle. So, this is equal to 1 plus beta by 2 d epsilon 1 that is center of the Mohr circle is equal to is at 1 plus beta by 2 into d epsilon 1. and the radius of the circle from this itself radius of circle is 1 minus beta by 2 into d epsilon 1. So the direction of 0 extension d epsilon y is equal to 0. So that that you can get it from the circle okay. So that the direction of 0 extension of 0 extension that is d epsilon y is equal to 0 is given by cos 2 theta is equal to 1 by 1 minus beta we are getting equation number 13 ok. From this what we are getting for a uniaxial tension beta is equal to minus half for beta is equal to minus half hence theta is equal to 55 degree. So, for a plane strain condition beta is equal to 0. For plane strain condition beta is equal to 0.

$$Centre\ of\ the\ Mohr\ circle\ is\ at\ \left(\frac{1+\beta}{2}\right) d\epsilon_1$$

$$Radius\ of\ circle\ is \frac{1-\beta}{2} d\epsilon_1$$

$$\cos 2\theta = \frac{1+\beta}{1-\beta}$$

So, theta is equal to 90 degree. So, that is perpendicular to the principal stress. That is maximum principal stress, okay. And if beta is greater than 0, there is no direction in which the extension is 0. This is the thing. There is no direction in which extension is 0. So that is from this we can get that. So if there is a direction which there is no extension then local necking along this direction is possible when the tension reaches a maximum.

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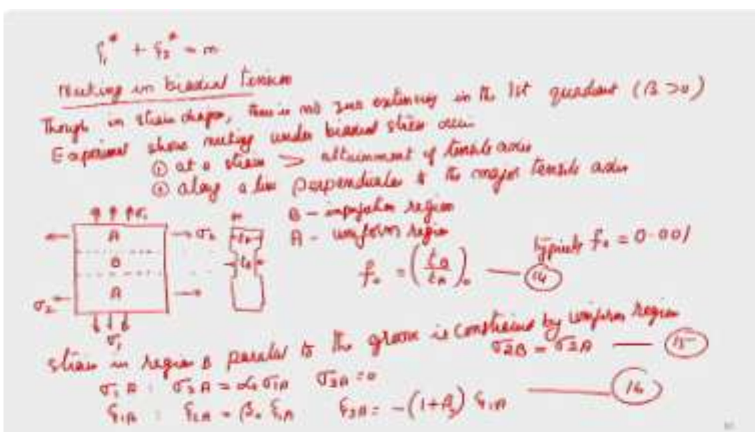

If there is no direction of 0 extension that is by a stretching process where beta is greater than 0, The strain at which tension is maximum is given by earlier we found that it is given by epsilon 1 star plus epsilon 2 star is equal to n. So that means there is no direction of 0 extension. That happens when beta is greater than 0 for the stretching, biaxial stretching process. Strain at which tension is maximum is given by this one.

However, geometric constraint permits, prevents the instantaneous growth of the local neck. So in this previous figure, if the line shows maximum tension in both quadrants but only indicate local strain in the second quadrant where minus strain is epsilon 2 is negative. So also for plane strain beta is equal to 0 and epsilon 2 is equal to 0 for the plane strain condition. The major strain at necking is the minimum. This is the condition where you are getting okay.

Now if you look at for a biaxial tension necking and biaxial tension. So, in the strain diagram, we found that for epsilon 1 and epsilon 2, both are positive. There is no zero extension in the first quadrant. That is for beta is greater than zero. However, when you do the experiment, still necking occurs under biaxial tension. So, though in the strain diagram, there is no zero extension in the first quadrant. First quadrant means this is greater than zero. Experiment shows necking under biaxial stress. So, at a strain greater than the attainment of maximum tension along a line perpendicular to a major tensile axis. So, this experience shows necking under a biaxial tension occurring one at a strain greater than that of the attainment of tensile axis and along a line perpendicular to the major tensile axis.

So, that if you look at if you just consider this as a figure below here if you just consider a figure like this. So, this is your sigma 1 and say maybe this is sigma 2 and if you take a region where there is a imperfection say area this is the uniform region and B is the imperfection region this is the uniform region. So, we can say that B is the imperfection region and A is the uniform region. And if you assume that the imperfection as a group shown by this dashed line with the thickness TB.

So, if I just take the cross section of that, this is TA and this is TB. cross section and this imperfection see it is very difficult to characterize an imperfection but let us assume that imperfection is characterized by a homogeneity factor given by F0 which is equal to Tb by Ta at the initial case okay at 0 and this normally you know typically we can assume that F0 is of the order of 0.001 that also we can assume.

$$f_0 = \left(\frac{t_B}{t_A}\right)_0$$

So when you are loading it strain in the region B parallel to the group would be constrained by the uniform region A. So that the compatibility condition sigma 2B is equal to sigma 2A. So strain in region B parallel to the group will be constrained by uniform region. So, that now we can say sigma 2 b is equal to sigma 2 a which is parallel to the group. That is one thing. So, in that case now we can just assume that the proportionality condition if it is there we can say sigma 1 a is the and sigma 2 a which is equal to alpha 0 into sigma 1 a and the third is sigma 3 a is equal to 0. Now, the strain epsilon 1 a Another is epsilon 2a is equal to beta naught into epsilon 1a and epsilon 3a is equal to minus of 1 plus beta is not represent the initial condition epsilon 1a.

$$\sigma_1 A \;:\; \sigma_2 A = \alpha_0 \sigma_1 A \quad \sigma_3 A = 0$$

$$\epsilon_1 A \;:\; \epsilon_2 A = \beta_0 \epsilon_1 A \quad \epsilon_3 A = -(1+\beta)\epsilon_1 A$$

So, you can write this as 16. okay but the case is not true when you are loading it in this direction 1 this is your direction 1 and this is your direction 2 so this is 1 and this is 2 so when you are loading it in the direction 1 the strain will be different in both the places stresses also will be different okay.

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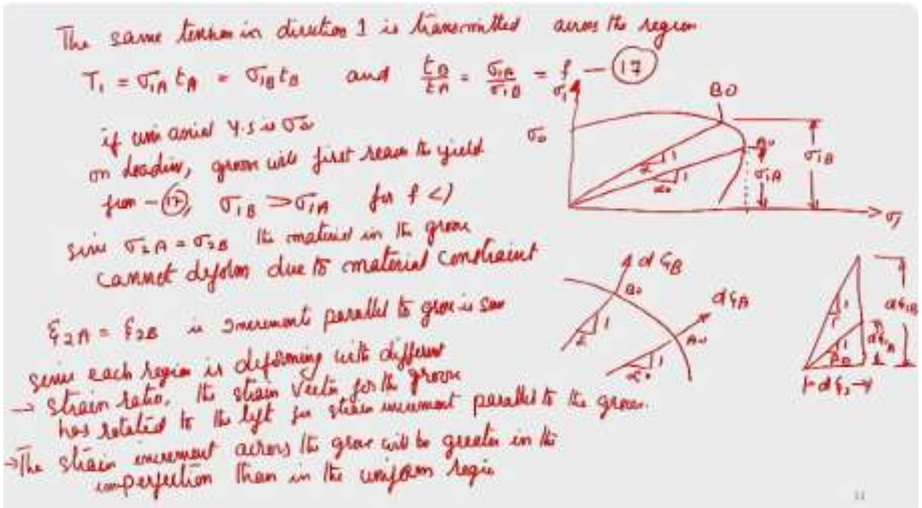

So the same tension in direction 1 is transmitted across the region both the region so that the same tension is directed in direction one is transmitted across the region such that T1 is equal to sigma 1 A into T A and that should be equal to sigma 1 B into T B. So, stress will be different in both case and T B by T A is equal to you will get it as from this T B by T A is a sigma 1 A by sigma 1 B. So, that it comes T A by T B is equal to F know. So, you get it as equation number 17. So, if you consider the initial yielding know, if you just consider the stress diagram, if this is sigma 2 and this is sigma 1, and if this is your yield locus you may get it here this is this will correspond to your sigma 1. So, here it is equal to 1 by alpha 1 by alpha naught and this point if you give it as a, this is a uniaxial stress is equal to sigma 0 if you put it here. Then on loading the groove will first reach the yield. So groove is going to reach first the yield okay. So that means in this condition so from 17 you can get it as so in the uniaxial yields if uniaxial yield strength is sigma naught on loading will first reach the ill surface because their cross sectional area is less.

$$T_1 = \sigma_{1A} t_A = \sigma_{1B} t_B \quad and \quad \frac{t_B}{t_A} = \frac{\sigma_{1A}}{\sigma_{1B}} = f$$

So, so that means from from 17 equation number 17 what we will find the sigma 1 b is greater than sigma 1 a for f less than 1. So, that that is what so since sigma 2 a, is equal to sigma 2 b in the both the case material in the groove cannot deform that was a constraint condition due to a geometric constraint. So, that is what happens since so you will find that so this is your b naught and here it is 1 by alpha and this will be your sigma 1 b your stress in sigma 1 b will be much higher than in sigma 1 a so that is what it is coming so you you will find that this one this slope also is changes okay so since sigma 2 a is equal to sigma 2 b the material in the groove cannot deform due to material constraint. Therefore, as the stress in A increase to reach the yield locus, when the stress in the A increase to reach the yield locus, the point representing B will move around the yield stress yield locus to B naught. So that means when this in area in the region A where there which is a uniform region when the stress is trying to reach the yield locus which is represented by A since a cross sectional area in region B which contains the imperfection so it will reach the value of B. So that means it has to just rotate okay it will move around the yield locus to B naught So if you just consider the incremental deformation which is taking place this is your d epsilon a which is normal to your locus this is a0 and maybe we are just finding it okay it is something like this okay and maybe here this is d epsilon b which is B not here maybe this has to be normal to this okay.

So let me just draw like this. So that is this is 1 by alpha slope is 1 by alpha and this is 1 by alpha not you are getting this. So this is the strain field which you are getting d epsilon b and d epsilon b. So if you consider the increment in the deformation now d see epsilon 2a is equal to epsilon 2b. The increments parallel to the groove is the same that is strain increment parallel to the groove is same that is true. So, we can see that strain vectors are perpendicular to yield surface in this case. Since each region is deforming with a different strain rate, so you will find that both the regions are deforming with a different strain rate, the strain ratio, not strain rate.

The strain vector for the groove has rotated to the left. So since each region is deforming with a different strain ratio, strain vector for the groove has rotated to the left and for strain increment parallel to the group. Whereas the strain increment across the group will be greater in the imperfection than in the uniform region. So, if you just draw that part, you will find that in this, if this is d epsilon 2, d epsilon and this is equal to d epsilon 1 b.

So, this is 1 by b naught b naught beta naught and this is equal to 1 by b 1 by beta. So, this is what you are going to get the strength.

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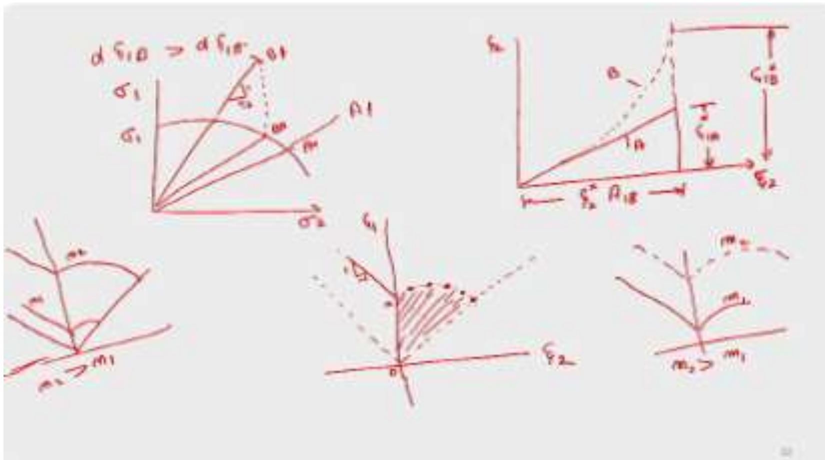

So, you will see that in such a condition homogeneity will be greater that is the F will finish that means you will find that D epsilon 1 B is greater than D epsilon 1 A. The strain in the groove will increase because basically if you really look at your cross sectional area is just that. So strain in the groove will increase compared to that in the uniform region.

And only slightly when the tension is increasing. See, when you increase the tension, you will find that it is increasing very slightly only. So, if you just really look at that, in this condition, when you do that, see, if it is like this, you have A naught the stress field, it will move like this. And this is your, this one. So, when the, as the tension is increasing slightly, you will find that it is increasing and the strain is increasing. So, if you just draw like this, if this is the case of epsilon 2 A 1 B and this is the epsilon 1 A star I will just draw like this in region A and B how the strains are taking how the stresses are happening.

So, you will find that at some area it slowly increases slightly and then after some time now it just keeps on increasing and reaches this value okay. So this is the point for B and this is for the guys for A the strain field is like this okay. So strain in the groove it will increase compared to that in the uniform region only slightly when the tension is increasing okay when the tension is increasing it will increase slightly gradually it increases after the tension maximum has reached and then continuous till the groove reaches a state of plane strain that is at B f. that is once it reaches here BF it reaches a state of plane strain. At this stage the strain parallel to the groove stops.

So that is what is happening and the groove will then continue until failure and the strain is uniform in the uniform region it ceases. So that is what happens at this. The strain state just outside the neck is the maximum strain that can be achieved and the strain epsilon 1 star A and epsilon 1 star B are known as the limit strains. The analysis repeated for different values of alpha naught and beta naught and a diagram for biaxial strain region can be established for this case. So if you do some experiments like this by doing for different strain history and other and plot this epsilon 2 and epsilon 1, you will get this forming limit curve.

So, when this is 0, this is epsilon 1, this is epsilon 2. So, depending upon this, this is n. So, here it is minus 1, it is plus So, in actual case what happened it instead of going that maximum tensile strain it follows this path. So, this may be your experimental things for different strain paths you can just get it and you will get this product. So this is the forming limit curve. So in this what happens is that this gives, it delineates the region between area where deformation is taking place.

So here it is a failed region whereas this is a sound region. If you have deformation, the combination of stress and say epsilon 1 and epsilon 2 is within this boundary which is drawn by this line, shaded line. So that is safe. The other region once you crosses the any of the strain it is higher than it will fail. So once you have for a particular material this forming limit curve is there if you know the value of n and then you get this then you can always restrict your deformation to such a value that the strain in the sheet is not approaching this limit curve bounded by this region.

So that is what is coming. So the second quadrant if it is there you can always tell the limit is always on the maximum tension line. Whereas in the biaxial stretching you will find that it is not so you have a better more value region you are getting it. It is not otherwise no, it should have come like this. But here you are getting higher.

You will get a higher strain in this case. But only thing is what is the maximum strain you can get as a combination of epsilon 1 and epsilon 2. So that limit should not be crossed. That is the advantage of using this. Now if you look at what are the factors affecting the forming limit curve, the two important factors, there are many factors but the most important factors are one is the strain hardening. So if you look at that the strain hardening n strain hardening say like this is your 0 and it is this when you are having a low strain hardening value it is like this okay maybe it will be drawn properly.

So for low strain hardening value, you will find it is like this. So this is n1. Whereas for high strain turning material, you will have still higher value. So this is n2, where n2 is greater than n1. So for biaxial stretching high n value is required so that now you have a large amount of strain which you can get it. So the material should have high n value or hardening exponent it should be higher. So if you are having a material having low or hardening exponent then for biaxial stretching it will create problem and then tearing will take place okay. When n is equal to 0, the plane strain forming limit along vertical axis, that will be 0. But it can be stretched in fully biaxial stretching. That is the biggest advantage. So in plane strain condition, it may be difficult.

But in biaxial stretching, you can always deform to a very large extent. Another is the rate sensitivity. If you have, say, rate sensitivity also, so this is for one case. And the other case is like this. So you have a larger area. So this is high say m1 m2 m1 okay I will put follow the same m1 m2 so m1 m2 is greater than m1 in that case also you will have so in biaxial stretching this m will delay the growth of the neck higher m value will delay the growth of the neck that is one advantage with high strain rate sensitivity material.

There are other factors also but see like the other factors are you cannot characterize what is called as a defect. It is very difficult to characterize or imperfection. You cannot characterize it properly. Then failure of the material because it gets forgotten that also is very difficult to use in this cases. With this today it is over. Thank you.