Plastic Working of Metallic Materials

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Lec 35: Deep drawing

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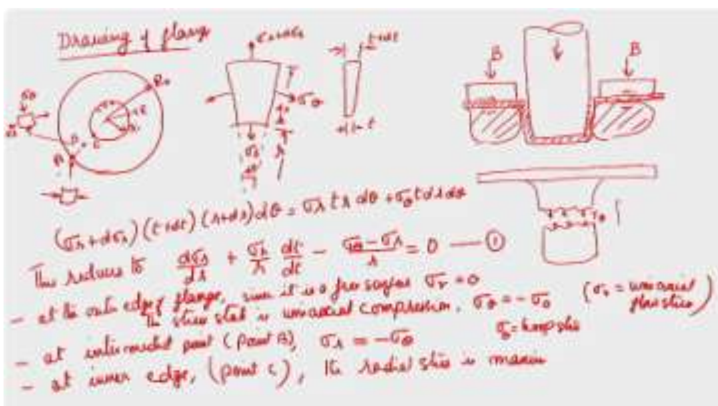

So today we will come to this last lecture in this module that is a cylindrical deep drawing. If you consider the cylindrical deep drawing, it is it is a very deep and that you have a punch which will be applying a force on a blank and maybe the blank will be deforming and the blank will be held between this blank holding devices and other things. This is the blank holder and we are applying a pressure B here. This is your work piece material, the blank. And in this if you just consider this deep drawing where the depth is very large there is a limit for this that is a drawing limit is there. So that when we come to that you will find that compared to the other small cup and other things. So here the depth is very large and this part we can consider consisting of 2 parts.

The one is see you have a stretching of a sheet and then another is a drawing and an annulus which is inwards okay. So if you just consider it is like this. These are the two things actually we have this tension here. Maybe this is T5 we can say, this is one part so here at this initial part you will find that the stretch this the drawing of an annulus part which is towards the inside and then okay it will be moving along it will be trying to move along this direction both the sides and then friction also will come into picture but whereas the second part is this stretching of the sheet over a circular path okay so circular circumference part around the punch so these are the two things and these two operations are connected to the cylindrical cup wall. So, because of these two things and the cup wall is not deforming, but the it transmit the forces between this both the regions one is the annular part where it is drawing in inwards and the other is the stretching of the sheet over the circular path. So, if you just consider the drawing of the flange part, so where it is drawing inwards. So, we can say drawing out the flange. So, you can consider that the flange part is a circular part.

We are only considering that part and where say your the flange radius is r 0 which is the initial radius and then ok maybe this is the r i the initial the internal radius of the flange part r i and then ok you you can just take at any arbitrary point r value ok. So if you just look at these conditions so there are stresses which are there ok inside stresses are also there and if you just look at this the flange of the shell any part of that can be considered as as an annulus part. The stress on the element of radius r if you just take an element on this flange part at an radius r. So, you will find the stresses acting at sigma r plus d sigma r the radial stresses and towards this direction it is a sigma r and then you have the hope stress that is the sigma theta which is there and this is your d r radius and if you come like this so this will be your r and this will be your d theta. So, this is the element and if you look at the thickness now you will find that the thickness is something like this t plus dt and here you are getting okay. So these are the stresses on the element of radius which is taking place okay. So the equilibrium equation when you write for this particular condition for this element we can just write here as sigma r plus d sigma r okay plus into t plus dt into r plus dr into d theta is equal to sigma r t r sigma r t into r d theta plus sigma theta t d r d theta. You are expanding and then eliminating that higher order terms and then manipulating it mathematically you are rearranging it.

$$(\sigma_r + d\sigma_r)(t + dt)(r + dr)d\theta = \sigma_r t_r d\theta + \sigma_\theta t dr d\theta$$

This reduces to the form d sigma r by dr plus sigma r by r into dt by dr minus sigma theta minus sigma r by r is equal to zero, equation number one. So, it reduces into this differential equation. So, now when you look at this, you will find that if you take this cases. So the the radius now along a line if I just take along this line. So you can have you can find out what are the forces at A at the extreme end of the flange then maybe at B which is an intermediate between the inner radius and this and the outer radius and then at C okay.

$$This\ reduces\ to\ \frac{d\sigma_r}{dr} + \frac{\sigma_r}{r}\frac{dt}{dt} - \frac{\sigma_\theta - \sigma_r}{r} = 0$$

At these points if you just look at the the stresses you will find that at the outer edge that means at the outer edge edge of the flange, since it is a free surface ok since it is a free surface your radial component of the stress sigma r is equal to 0. So you will find that it is a question at the outer surface the free surface at A it is a uniaxial compression stress in which sigma theta is equal to your uniaxial flow stress of the material. So that means current uniaxial flow stress of the material so this condition at this point it will be something like this okay. And it will be compressive in nature because if you look at that when it is drawing inwards, there will be thickening which is taking place. So you will find that this is the stresses which are acting whereas at the point B you will find that so here the stress state the stress state is uniaxial compression where sigma theta is equal to minus sigma naught that means sigma naught is the uniaxial flow stress at that point okay at that whatever be the condition wherever it is there.

$$\sigma_\theta = -\sigma_0$$

Now, at some intermediate point B, the radial stress will be equal and opposite to your hoop stress. So, that means at this point, if you look at it, you will find that the situation is this is sigma r is tensile and this will be sigma theta. It will be compressive in nature. Okay and so at B, at intermediate point, maybe point B, you will see that sigma r is equal and opposite to your hoop stress sigma theta, where sigma theta is equal to hoop stress and sigma r is the radial stress and at the inner edge that is point C you will find that the radial stress is the maximum okay the radial stress is maximum. So you will find that from outside towards the inside the radial stress keeps on increasing from the outer periphery to the inner radius RI of your this cup okay.

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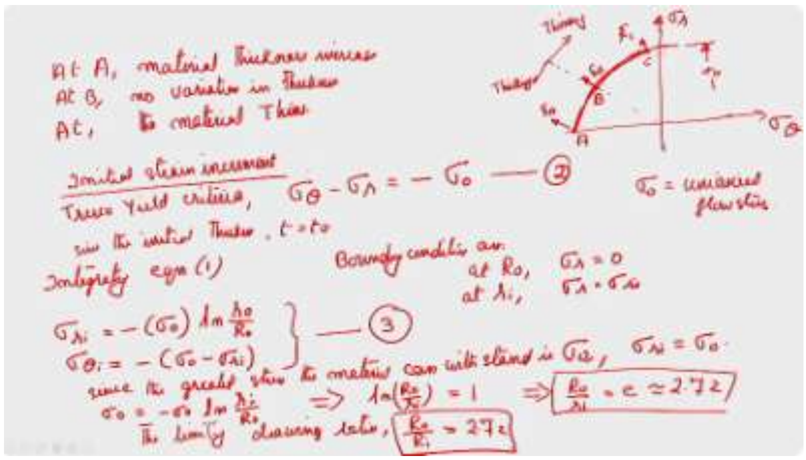

So if you just look at the strain diagram at these three points because strain diagram at these three points now assuming Tresca criteria it will be like this. So here you are having your what is that sigma theta and here it is your radial stress sigma r okay sigma 1 sigma 2 that is taking it you will assume that it is something like this okay where this is point A and your strain at point A is equal to epsilon A okay because when it is drawn inverse there is thickening which is taking place. So if you just look at this, this point where the alpha is equal to minus 1 and beta is equal to minus 1 also in that point if you like here it is thinning if you refer to earlier this one and this is thickening. Okay so so here you will get this point B here suppose at some intermediate point these two will be there so that is this condition and maybe so here you may find that this is your epsilon B and here it may be your epsilon C at some point and maybe if you just take this as your sigma 0. This is the instantaneous flow stress of the material, okay.

So if you, so this is your sigma c, epsilon c, not sigma c. So at the outer edge the blank will thicken as it deforms because it is in this region because if you just consider if you refer to our strain diagram which we have discussed long back beyond this point on the left of this towards the second quadrant between the region A to B the material will be thickening whereas from B to C B to C the material will be thinning okay and so when it reaches to C there will be maximum thinning will be there. So, at point C the inner edge the sheet will just thin down and that will be the weakest point. The overall effect is that on one part you are finding at the edge see if you find here at this edge if it is thickening and here it is thinning the overall area of the flange between the you will find that it is remaining almost constant which is a very good approximation. So, at A, material thickens at B we can assume no variation in thickness at C the material thins.

okay as it deforms. This is during the deformation process which is taking place. So the overall effect to that is that in drawing the total area of the material initially in the flange region it remains constant okay. So there is not much change. So that is a very good approximation for arriving at some logical conclusion for us or maybe arriving at some good relationships.

Now if you consider the initial strain increment when when the material is trying to deform and apply the Truska criteria. So initial because you you may have to do all this by numerical methods and other thing but if you just consider the initial case and the final case that will be much much simpler for us purpose. So we let us consider the initial strain increment. So, Tresca, applying the Tresca criteria, yield criteria that is sigma theta minus sigma r is equal to the minus sigma 0 okay where sigma 0 is the uniaxial flow stress. Now as the initial thickness is uniform, since the initial thickness of the blank t is equal to t 0 since it is thickness because this expression is sigma 1 minus sigma 2 is equal to your yield stress. So, that is that is what the relationship which comes ok.

$$\sigma_\theta - \sigma_r = -\sigma_0$$

So, now when you come to this if you integrate this equation 1 applying these conditions the boundary conditions are integrating equation 1 and the boundary conditions are at outside radius r 0 your radial sigma r is equal to 0 and at the inner radius r i sigma r is equal to sigma r i okay. So, if you integrate and apply at this boundary conditions then you can get this relationship sigma r i I am not going to detail derivation of that but you will get it into sigma 0 into log r naught by r 0 that is 1 and sigma theta i is equal to minus of sigma naught minus sigma r i okay. So this you can get it as equation 3. Now the thing is that for a non strain hardening material the radial stress it is greatest at the beginning and then it will decrease as the outside radius diminishes. When when the radius keeps on decreasing the the radial stress keeps on decreasing okay.

$$\sigma_{ri} = -(\sigma_0) \ln \frac{r_0}{R_0}$$

$$\sigma_{\theta i} = -(\sigma_0 - \sigma_{ri})$$

The greatest stress that the wall of the cup which can withstand is sigma 0 okay it will decrease. So now the thing is that where sigma naught is the uniaxial flow stress. So, more than that now it cannot withstand then it will start deforming. So, if you substitute this since the maximum since the greatest stress the material can withstand is sigma 0 we can say that sigma 0 we can say sigma ri the maximum can be sigma 0. So, if you substitute it into this condition in equation number 3.

then we can get this sigma ri is equal to sigma 0 if you substitute that it will be sigma 0 is equal to minus sigma 0 log r0 by r0 where small r refers to your sorry log ri it refers inside radius and R capital R naught refers to the out radius of the blank okay.

$$\sigma_0 = -\sigma_0 \ln \frac{r_i}{R_0}$$

$$\ln \left(\frac{R_0}{r_i} \right) = 1$$

$$\frac{R_0}{r_i} = e \approx 2.72$$

So, this we can just write like that. So, sigma 0 and sigma 0 will get cancelled. So, log R naught by R i. So, this minus sign is there. So, is equal to E sorry is equal to 1. So, that means r naught by r i is equal to e that is approximately 2.72. So, this is the the limiting drawing ratio the limiting drawing ratio r naught by r i is equal to 2.72. This is a rough estimation it will give. But the thing is that this is an overestimate. You may not get that much of a drawing ratio in actual case. There are various reasons to that. One is the effect of strain hardening.

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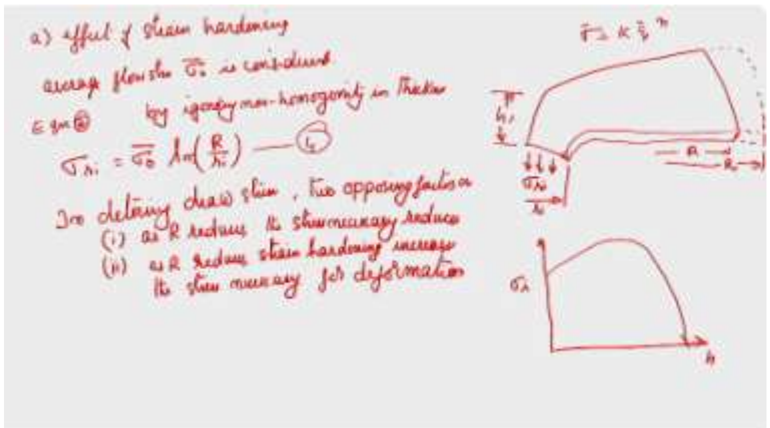

So say A is the effect of strain hardening. As the material is drawn inwards plastic deformation takes place and then the stress necessary to draw the flange will keep on increasing because with the more and more of strain which is happening say suppose if you take sigma is equal to k epsilon raise to n maybe in this case as with the more and more of strain happening your stress will keep on increasing. So what will happen is with the deformation the stress necessary for drawing keeps on increasing okay. The during the drawing of the flange the outside radius will decrease at any instant okay. So, maybe like if you just look at this case if this is the thickness okay.

So, let us say that this is h 1 and initially this was so this was R naught now it is R any intermediate point R okay and you will find that here the stresses are acting that is sigma R i. and this is your R i. So, if you consider this at any instant from R0 it just come to R so the material has been drawn inside when the material has been drawn inside it work hardens and due to that your stress will be the stress necessary for deformation will keep on increasing. So that is continuously varying but for most of the deformation studies we assume for simplicity that we can take an average stress flow stress.

So average flow stress like average flow stress sigma naught is considered for, for simplicity okay. So over the whole flange and though this deformation is taking place we have found that at the outer extreme edge as the metal is drawn inwards thickness will keep on increasing whereas at this inside radius it keeps on thinning. But we also assume that more or less the area volume of the material is going to remain the same okay. So we can assume that for our analysis if you assume that the non homogeneity is avoided in the thickness is ignored then equation three that means by ignoring non homogeneity in thickness then we can write this equation three as sigma r i is equal to sigma naught bar into log say r is at any instant by r i.

$$\sigma_{ri} = \overline{\sigma_0} \ln\left(\frac{R}{r_i}\right)$$

We can write this. So, it keeps on changing with the r. and then if you just plot that value of ri versus h the the thickness at which it has been drawn. So, sigma ri you will find that it gradually increases reaches a maximum value and then suddenly it decreases to 0 ok. See in determining the draw stress you will find that there are two opposing factors which are coming one is as the r becomes smaller and smaller the stress reduces okay and see that as per this equation the stress necessary it reduces but the thing is that at the other side the opposite is happening that is the increasing the stress due to strain hardening so these are two opposing trends so in determining draw stress the two opposing factors are one as r becomes smaller as r reduces the stress reduces, stress necessary reduces then as R reduces strain hardening increases the stress necessary for deformation okay.

And you will find that okay these two conditions are there so at some this opposite or competing factors are happening so it will initially increase to a maximum value and then it further decreases. See now the thing is that though we mentioned that drawing limit is there is a limit for that and it is whatever we have assumed is 2.72 is mainly due to these factors which are coming okay. One is we have ignored the effect of friction.

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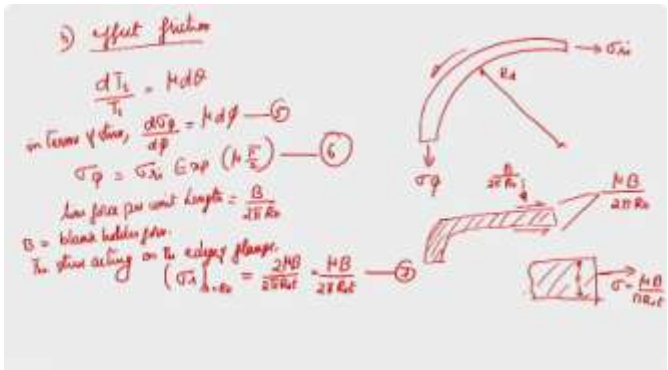

So in the stamping analysis say B the effect of friction. First one we consider that strain hardening, second is the effect of friction which is taking place. So in your stamping analysis now we due to sliding around the radius and the when it is drawing when it is sliding around the radius we arrived at that dT1 by T1 is equal to mu d theta. If you refer back to that you will find that. So this can be written in terms of the stress that means in terms of stress we can say d sigma phi by d phi is equal to mu d phi.

$$\frac{dT_1}{T_1} = \mu d\theta$$

$$In\ terms\ of\ stress, \frac{d\sigma\phi}{d\phi} = \mu d\phi$$

So, we can write this and if you just look at what is happening when it is moving around to this one the die because that it will be coming like this with the thickness. So, here you will find that sigma ri is there, here you will find that sigma phi is there okay. The material when you are drawing it will be moving around this your die is there. So, this is what is that rd radius of the die or maybe I can put rd radius of the die okay. So considering this if you just at the, this is the condition at the die radius and if you integrate it we get this equation sigma phi is equal to sigma ri exponential mu into pi by 2.

$$\sigma_{\phi} = \sigma_{ri} \exp\left(\mu \frac{\pi}{2}\right)$$

So, this is what we will be getting. So, you will find that the friction between the blank holder and the flange will increase the drawing stress. And it is a reasonable approximation that the blank holder force B will be distributed around the edge of the flange. Completely when you are applying that it will be distributed around the flange as the line force of and we can consider that it is a line force of magnitude b by 2 pi r 0. So, because if you look at that the below the flange though this will be may be it is like this.

So, we can say here your frictional forces are there so the normal force which is going to act so that will be we can consider as B by it is like a like a line force of magnitude B by 2 pi r naught because the total force B and because it is thickening at the edge only that thickening is coming okay because there it is thickening. So, it will be it will be in contact at the edge that is one. So, whatever force is there it is like a line force on the periphery that is why that we are coming. So, it is b by 2 pi r 0 is a, is a line force which is coming and a per unit length okay. So, that means the line force per unit length is equal to b by 2 pi r naught where b is equal to your the blank holder force.

$$Line\ force\ per\ unit\ length = \frac{B}{2\pi R_0}$$

Now when you look at this so there is a friction which is coming into picture so at the both the sides if you look at it this will be equal to mu B by 2 pi r0 because 2 mu B and 2 this one is coming into picture so we can assume that okay this is sorry if the edge of this if you consider this will be a line of sigma is equal to mu B by pi r0 t where this t is your thickness okay. So that way we can just consider so this is your flange part. So, this can be expressed as a stress acting on the edge of the flange that means so the stress in this direction the stress because it is a line force you know stress acting on the edge of the flange that is sigma r at r is equal to r naught is equal to 2 mu B by 2 pi r0 t. So, that is equal to mu B by 2 pi r0 t. So, this is where b is your blank holder force and t is the thickness r 0 the initial radius of the your material okay.

$$(\sigma_r)_{r=R_0} = \frac{2\mu B}{2\pi R_0 t} = \frac{\mu B}{2\pi R_0 t}$$

So, both this a and b what we have effect of friction as well as the earlier case work hardening this will increase the stress required to draw the flange okay.

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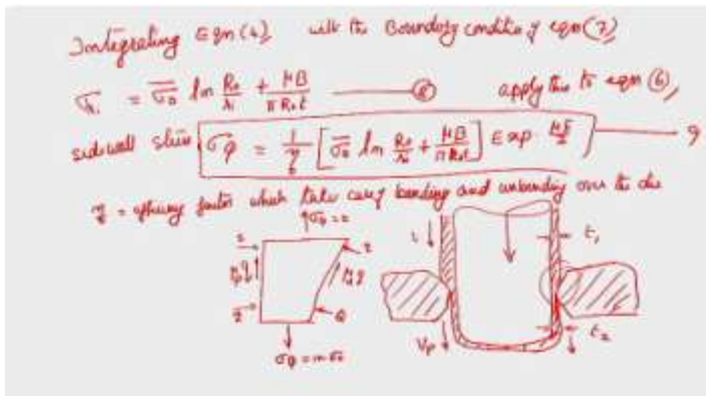

So, integrating equation 4 we can get with the boundary condition of equation 7, we can write the expression for the drawing stress is equal to and you are using the average flow stress log r naught by r i plus mu b by pi r naught t. And if you apply this to equation 7, equation we will get the side wall stress sigma phi is equal to we can finally express it in this I am not going into the detailed derivation but we can get it sigma naught bar log r naught by r i plus mu b by pi r naught t into exponential mu pi by 2 where this is an efficiency factor which takes care of bending and unbending.

$$\sigma_{ri} = \overline{\sigma_0} \ln \frac{R_0}{r_i} + \frac{\mu B}{\pi R_0 t}$$

$$sidewall\ stress, \sigma_\phi = \frac{1}{\eta}\left[\overline{\sigma_0} \ln \frac{R_0}{r_i} + \frac{\mu B}{\pi R_0 t}\right] \exp \frac{\mu\pi}{2}$$

Due to that, no work hardening will take place. At the same time, friction also will come over the die. So, that is what we are going to get it. Now there are some cases where we can discuss about the wall ironing because after deep drawing you may give an ironing process you may apply on that so that it becomes its thickness will get reduced and then you get a very uniform thickness with a good surface quality also. So, that process is called as ironing.

So, wall ironing of deep drawn cups. So, if you look into that, this is a process but I am not, okay, I will explain that. See, if See, suppose this was the material and by ironing you found that the thickness has reduced. We have to assume that this is uniform thickness. I can roll it up to this way.

So, here you have a die like this. So this is the ironing die so if you just really look at this part see it will be like this the forces will be like this you are having this normal force Q here. So these two will be different because the frictional force at the punch workpiece material and the die workpiece material will be different okay. So and this is your thickness we can say that this is T2 okay and initial thickness let us say it is T1. So, these are the forces which are going to come into picture. So, you will find that when the punch moves down it velocity it moves down with the velocity of the punch that is v p whereas it may not be the same here the material may not be moving with the same velocity here. So, that means v 1 at the initial stage will not be the same as the output velocity output velocity will remain depending will be a constant as the punch velocity. But here it will be different, it will be less than that.

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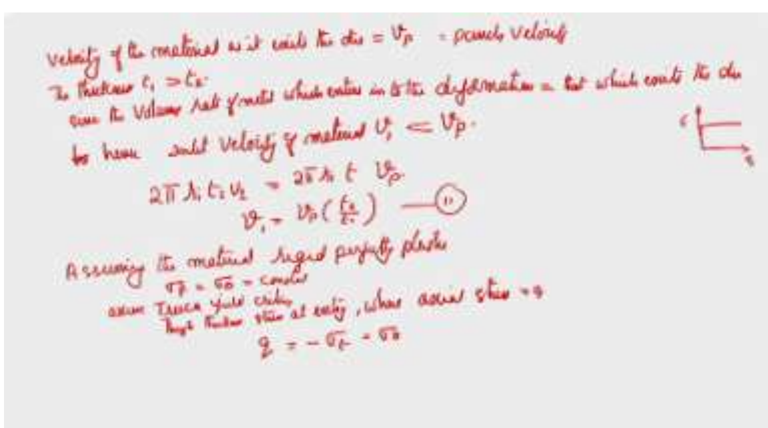

So, as the punch must remain in contact with the base of the cup, velocity of the material as it exits the die is v p, velocity of the material as it exit the die is equal to V p and so that is equal to the punch velocity you can say punch velocity. But during the ironing there is no change in the volume, volume remains constant but the thickness at the end ray is higher the thickness, thickness t 1 is greater than thickness t 2. So, the rate at which because of this the rate at which the material enters into the deformation zone that should be the same as the rate at which it leaves the die Okay so for that case what happens is that because the velocity of the material at the exit of the die is Vp the velocity at the inlet to the deformation zone will be lower than Vp. Okay so hence, okay we can say that since the volume rate of metal which enters into the deformation zone is equal to that which exits the die. Hence, the V i inlet velocity of material V i will be less than V p. So, since a constant volume relationship is maintained you can say that if you write this equation 2 pi ri ti into vi is equal to 2 pi ri into t1 this is t1 v1 we can say t2 vp. So from this we will find that V1 is equal to Vp into say T2 by T1.

$$2\pi r_i t_1 v_1 = 2\pi r_i t v_p$$

$$v_1 = v_p \left(\frac{t_2}{t_1}\right)$$

So that we are getting. So now the punch is moving down faster than the incoming material and the friction force between the punch and the material is downward so that is one case. So if you look at that the punch is moving down and the friction force is also there so that will be moving down at the punch material interface but it will be moving but if you look at that frictional force at the die and the punch it will be upward it is in the opposite direction because more material is coming and filling up that area so it will always have a what you call it as backward flow of material and okay due to that your frictional force will be in the opposite direction okay. So so initially the friction at the die at the punch and the material basically at this region this region so that is equal to mu p since it is mu p is downward it will be assisting the deformation process whereas in this case you will see that this friction it will be downward mu d that will be moving upward not downward it will be moving upward so that will be opposing the deformation process. So for this case because mu P that is the frictional force at the punch metal interface is assisting the deformation process.

If you have a higher frictional force then it will be very useful. So many times now people make increase the surface area of the punch surface but of course you cannot increase too much because you may face problem in removing it also. So if the surface whereas at the die material interface your friction should be less for that now people may add large amount of lubricants at their surface. So that now the the resistance will be reduced in this okay.

So it is advantageous to have high punch side friction at mu p. Now if you assume that the material behaves in a assuming the material the material your blank is rigid perfectly plastic rigid perfectly plastic means it is this way okay so that means sigma f is equal to your yield strength sigma y naught you know actually yield strength is a it remains a constant and no work hardening if it is there and if you apply the Tresca criteria Tresca yield criteria and the truth thickness stress through thickness stress at entry where axial stress is equal to 0 we can write that Q is equal to minus sigma t is equal to sigma 0. So, that means at the exit the axial stress is less than sigma to ensure deformation occurs only inside the die, okay. So as shown in figure, you can also come out with certain cases like the axial stress.

$$q = -\sigma_t = \sigma_0$$

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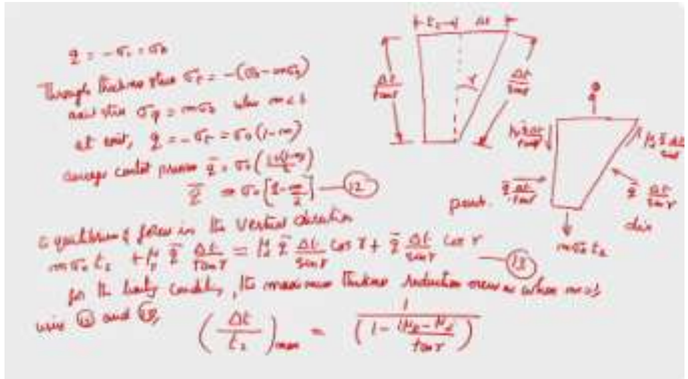

If you just draw this figure, so this is T0. This is t2 and this is delta t the change in the thickness okay. So you will see that okay this is gamma and you can say that the forces which are acting also you can write this what is this dimensions delta t by tan gamma. So, if you just take this delta T by sin gamma. So, in this case, if you look at the forces which are acting on that. So it will be say you have the frictional force here, the normal force here okay. So this is similarly you have this normal force here and you have the frictional force in this okay. So this is the die side, here die side and here it is punch side okay.

So if you just look at that here at the top it is 0. Whereas at the down you will see that M into sigma naught into T 2 okay. So similarly here you will find that shear stress is mu D where the stress frictional stress at the die metal interface into Q bar into delta T by sin gamma and here it is equal to mu p into q bar into delta t by tan mu tan gamma okay tan gamma and here you will find that this is fungicide. So, here this will be Q bar average flow stress Q bar is basically is equal to into delta T by tan gamma this is die side and here it is equal to Q bar into delta T by sin gamma okay you are getting. So you will find that Q is equal to minus sigma t is equal to sigma 0 okay. So if you assume that through thickness thickness thickness delta t t sigma t the stress through thickness stress sigma t is equal to sigma 0 minus m sigma 0. Because the axial stress sigma phi is equal to m sigma 0 where m is less than 1.

$$q = -\sigma_t = \sigma_0$$

$$\sigma_t = -(\sigma_0 - m\sigma_0)$$

$$\sigma_\phi = m\sigma_0$$

So, at the exit you will find that q is equal to minus sigma t that is equal to sigma 0 into 1 minus m. So, the average contact pressure if you take it the average contact pressure q q bar is equal to sigma 0 into 1 plus 1 minus m by 2. So that is equal to sigma 0 into 1 minus m by 2.

$$q = -\sigma_t = \sigma_0(1-m)$$

$$Average\ contact\ pressure\ \bar{q} = \sigma_0\left(\frac{1+(1-m)}{2}\right)$$

$$= \sigma_0[1-\frac{m}{2}]$$

Now if you take the equilibrium forces in the vertical direction okay so taking equilibrium forces, of forces in the vertical direction on this element what we have shown is equal to m into sigma 0 t 2 plus mu p into q bar mu p into q bar into delta t by tan gamma is equal to mu d q bar delta t by sin gamma into cos gamma plus q bar into delta t by sin gamma into sorry cos gamma.

$$m\sigma_0 t_2 + \mu_p\ \bar{q}\frac{\Delta t}{\tan\gamma} = \mu_d\ \bar{q}\frac{\Delta t}{\sin\gamma}\cos\gamma + \bar{q}\frac{\Delta t}{\sin\gamma}\cos\gamma$$

Now the substituting this 12 in 13 and then do a mathematical simplification and considering the fact that the limiting condition for the limiting condition limiting condition the maximum Thickness reduction occurs when m is equal to 1 using 12 and 13 we can get delta t by t2 the maximum is equal to 1 minus 1 minus mu p minus mu d by tan gamma. So this is what you will be getting.

$$\left(\frac{\Delta t}{t_2}\right)_{max} = \frac{1}{\left(1-\frac{\mu_p - \mu_d}{\tan\gamma}\right)}$$

So you can just do this with by having the surface quality okay surface phenomena or the punch and the die so that now your friction can be changed accordingly and then you can have the maximum reduction based on this mu p and mu d and what is this angle at which it is going to take the die angle what is that you can find it out okay. So that is the maximum limiting condition for maximum thickness reduction so that way you can get it okay. Thank you.