Plastic Working of Metallic Materials

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Lec 36: Hydroforming

So, this this present lecture we will be discussing about the hydroforming and this hydroforming the main principle as we have discussed in the first lecture of this sheet metal forming. If you recollect that there is a sheet is formed against the die by fluid pressure and normally a flexible diaphragm is placed on the sheet and then it is formed into a female die cavity by the application of pressure.

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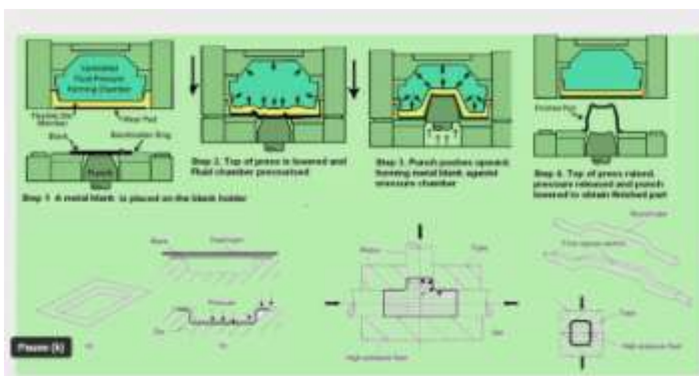

This is a typical example of that, where you can see that. See, the first step is that a sheet is made here, and then this, the whole control fluid pressure forming chamber is there. So above this sheet, you are going to keep a flexible die, flexible member. This flexible member can be, normally it is, synthetic rubber, rubber sheet will be there so that you know that is also flexible so that you can apply a hydrostatic pressure.

So, in hydrostatic pressure when you are applying this wrinkling and other things can be avoided that is the biggest advantage in this sheet metal forming using by hydro forming. So, the first step is a sheet is kept on the blank holder and then the top press is lowered down so that you know it holds this including the blank holder and it is very firmly held there. And then you are pressurizing this chamber by pumping in say high pressure fluid, okay. So, that will exert a pressure on this flexible die member which is shown in the yellow color this one.

So, depending upon what is the shape now this sheet material will just deform into that shape. Maybe in this particular case this is a separate case in this particular case a plunger is forcing it up so that whatever the way the shape of that the female cavity of that we will be getting it here, but here it is still pressurized a hydrostatic pressure is being applied, and after that you lower it and then eject the finished part. So this is the case. There are different versions and other things. This is just a schematic thing. So here also in a very simple way you can say there is a die cavity here, okay. A die cavity and on which you are keeping your blank. And above that, you are keeping the diaphragm. You can apply a pressure. It can be pneumatic. It can be hydraulic. Whatever it be, it can be there. But because of this diaphragm, which is flexible, normally it is upperized, sheet just deforms. The forming takes place and then get the shape of this die cavity.

The counterpart of the die cavity you are getting it in that shape. So this is one thing. This is also used for this forming of pipes and other things also with a high pressure fluid. So longitudinally you are applying a pressure on this side from here to here you are applying the pressure and then okay here also you can apply. So this can expand and get that particular shape. Now similarly you can get a final square section also you can get which is a pipe form you fill in the water or hydraulic fluid whatever be the type of fluid through this and then it expands into that and then once it comes and touches the die there further expansion stops it is constrained maybe the corner also will get it into that depending upon the pressure it can be filled up very accurately and other things so this is another method so you will find a large number of application for this hydro forming. So, basically it is forming under a type of hydrostatic pressure.

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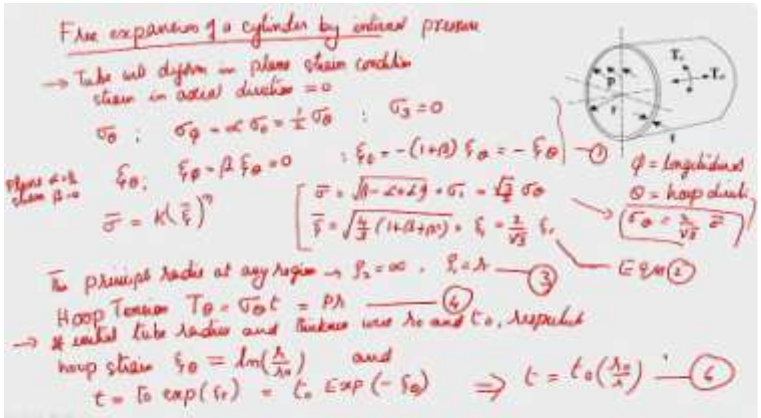

So, if you just the other parts are also brackets for bicycle parts, pipe fittings etcetera this is being used. Now let us consider some simple cases like free expansion of a cylinder by an internal pressure. If the cylinder is kept inside a die and then inside you are just applying the pressure, how it expands? So what are the conditions? So let us say the free expansion of a cylinder by internal pressure.

So here if you consider this expansion of a cylindrical tube okay a very long tube without change in length. So when it is very long we are assuming that there is not much change in length, along the longitudinal direction. So, but only the diameter only or radius only changes and maybe when the radius increases because it is being pressurized inside with this fluid under pressure P, then depending upon the yield criteria what is the pressure which is there, now this will start expanding. The diameter will keep on increasing or the radius will keep on increasing.

When the radius keeps on increasing, naturally there will be a strain along the thickness and other things. But this when you are considering this cases some of the assumptions are that the tube is the tube will deform in plane strain condition. That is one criteria. That means the strain in the axial direction is zero. Initially the tube will remain circular and the radius will increase okay and the stress and the stress states under those conditions are that we can say sigma theta that is a hoop stress okay. In this case sigma theta is coming and then your longitudinal stress is sigma phi so that is equal to say by alpha into sigma 0 that is equal to half into sigma theta. That is what we are getting and then along the axial direction it is 0 this is along the thickness direction along the that is sigma 3 is equal to 0 okay along the third direction it is 0 and for the when you are assuming that plane strain condition.

$$\sigma_\theta : \ \sigma_\phi = \alpha\sigma_0 = \frac{1}{2}\sigma_\theta : \sigma_3 = 0$$

For plane strain So you will say alpha is equal to half and beta is equal to 0. So we can say that the strain is epsilon theta. Then we can say epsilon phi is equal to beta into epsilon theta is equal to 0 because beta is equal to 0 and third is equal to along the thickness direction there will be a strain along the thickness direction that is equal to minus as per our earlier relationships under plane strain condition beta into epsilon theta 1 plus beta okay that is equal to minus epsilon theta. So epsilon theta is your principal stresses strains along direction 1 okay. So, that is along the hoop direction. So, now this this this theta as this figure is now if you just this is along the longitudinal direction and say theta is equal to hoop direction okay hoop direction. Now if you assume that material obeys the general power law relationship which is given by sigma bar is equal to k into epsilon bar raise to n okay.

$$\epsilon_\theta : \quad \epsilon_\phi = \beta\epsilon_\theta = 0: \ \epsilon_t = -(1+\beta)\epsilon_\theta = -\epsilon_\theta$$

$$\bar{\sigma} = k(\bar{\epsilon})^n$$

Then also if you just look at that the condition where sigma bar is equal to root of 1 minus alpha plus alpha square many times we have written this but still into sigma 1 that is equal to you can say root 3 by 2 into sigma theta. And similarly, we can say epsilon bar is equal to root of 4 by 3 into 1 plus beta plus beta square okay into epsilon 1 that is equal to 2 by root 3 epsilon 1. So here now we can just get this as sigma theta is equal to 2 by root 3 into this one we can get it okay. So this is maybe if you consider this as equation 1 and this as equation 2.

$$\bar{\sigma} = \sqrt{(1-\alpha+\alpha^2)} \times \sigma_1 = \frac{\sqrt{3}}{2}\sigma_\theta$$

$$\bar{\epsilon} = \sqrt{\frac{4}{3}(1 + \beta + \beta^2)} \times \epsilon_1 = \frac{2}{\sqrt{3}}\epsilon_1$$

So from that now we can say the principal radii at any region of the tube. So we can say the principal radii at any region if two radii are there one is it is infinity that is we can say sigma 2 rho 2 is equal to infinity and rho 1 is equal to r this is what you are getting along this direction longitudinal direction it is 0. So, from this one now we can get the hoops tension tension along the hoop direction that is hoop tension if you call it as t theta is equal to sigma theta into T a. So, that is equal to P r where P is the pressure internal pressure and r is the radius at any instant T is the thickness at any instant this is this whatever figure is there it is for a particular instant okay. So if you look at that if the tube is of initial thickness T0 and radius R0, it has been expanding from initial radius of R0 and thickness T0.

$$\rho_2 = \infty, \;\; \rho_1 = r$$

$$Hoop\ Tension\ T_\theta = \sigma_\theta t = pr$$

Now it has reached to a radius of R and T, okay. Then we can find out the hoop strain. So like if initial tube radius and thickness where r 0 and t 0 respectively where a hoop strain epsilon theta is equal log r by r naught. And the thickness t is equal to t naught into exponential epsilon t that is equal to t naught into exponential minus epsilon theta. So, that implies that T is equal to T naught into R naught by R okay.

$$\epsilon_\theta = \ln\left(\frac{r}{r_0}\right)$$

$$t = t_0 \exp(\epsilon_t) = t_0 \exp(-\epsilon_\theta) \qquad\qquad t = t_0\left(\frac{r_0}{r}\right)$$

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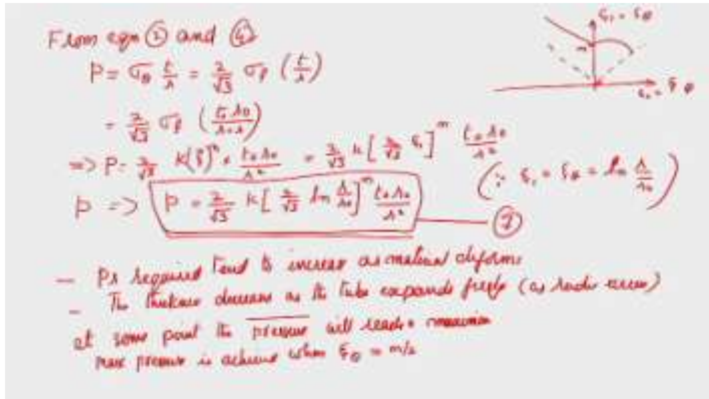

Now, see from this equation two and four. See what we can get it is the pressure which is necessary for this deformation at that at any instant is equal to sigma theta that is the hoop stress into T by R which is equal to 2 by root 3 into sigma f into t by r. See this is also there. So, that is equal to we can write it in this form that is 2 by root 3 into sigma f into T 0 R 0 by R into R because T by R we can write it in this form okay. So, that that is what we are getting.

$$P = \sigma_\theta \frac{t}{r} = \frac{2}{\sqrt{3}} \sigma_f \left(\frac{t}{r}\right)$$

$$= \frac{2}{\sqrt{3}} \sigma_f \left(\frac{t_0 r_0}{r \times r}\right)$$

So, that implies that P is equal to 2 by root 3 and we are having the relationship say k sigma f is equal to k epsilon bar to the power n into t 0 r 0 by r square. So, that will be finally, we are getting it as epsilon bar also we can write 2 by root 3 into k into epsilon bar we can write it as 2 by root 3 under plane strain condition that is epsilon 1 epsilon 1 is equal to epsilon theta. So, so that is a epsilon 1 to the power n into T 0 T naught R naught by R square or finally we can just on a simple mathematical manipulation we can get it as P the pressure which has to be applied for reaching this radius is P by root 3 into K into 2 by root 3 because epsilon 1 is equal to epsilon theta that is equal to log r by r naught. So log r by r naught to the power n into T0 r naught by r square because epsilon 1 is equal to epsilon theta is equal to log r by r naught.

$$P = \frac{2}{\sqrt{3}} k(\bar{\epsilon})^n \times \frac{t_0 r_0}{r^2} = \frac{2}{\sqrt{3}} k \left[\frac{2}{\sqrt{3}} \epsilon_1\right]^n \frac{t_0 r_0}{r^2}$$

$$P = \frac{2}{\sqrt{3}} k \left[\frac{2}{\sqrt{3}} \ln \frac{r}{r_0}\right]^n \frac{t_0 r_0}{r^2}$$

So, that is why we are getting this relationship. So, this is equation number 7. For the strain hardening material you will see that n is always higher, n is always greater than 0. So from that you will find that this is p. The pressure, it tends to increase as the material deforms.

The pressure, whatever is required for further deformation, you have to continuously increase it with, when the material deforms. And if the tube is allowed to expand freely, the tube thickness decreases. So, freely means, okay, there is no, it is not, there is no constraint, but it is not coming and touching on a die surface and other thing. That is, so the pressure, so like the conclusion from this is, the pressure, required tend to increase as material deform or expands. And second case is the thickness as the tube expands freely.

So, this is what we can get from this relationship. So, that means as as the radius as radius increases. Now, both effect will tend to decrease the pressure okay. So when the tube expands freely the tube thickness decreases and as the radius also increases. So when that happens is that at some point so only these are just contradictory things these 2 cases.

So what will happen is that both effect will tend to decrease the pressure okay. So in that case at some point the pressure will reach a maximum. At some point, the pressure will reach a maximum as a result of the opposing effects they try to cancel off okay. So like if you differentiate this P with respect to you can get it so if you are differentiating this P with respect to R you can say that the maximum pressure is achieved when the hoop strain is equal to n by 2. Beyond this pressure, the tube will swell locally like a ballooning it will take place and then there is a possibility that the tube wall will start necking and at some point it will just split within the bulged region.

So this is the main problem. So once it goes through the condition is this sigma theta is equal to n by 2 if the strain goes beyond that it is like instability sets in. At the localized area, it will start bulging because you are having a hydrostatic pressure from inside. So it will start bulging there and the necking will take and then the splitting because the thickness suddenly when that bulging is taking place at localized area, there the thickness will keep on decreasing and then it will start splitting. So since the cases of plane strain deformation, the loading path in the strain space is vertical. So that will come like this if you just look at it. So this is n. So the splitting is expected when the hoop strain is approximately having the value of n.

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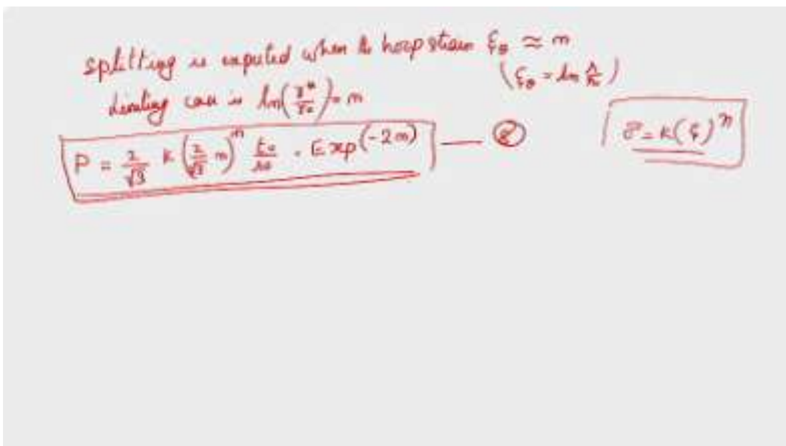

So that we can write here, expected when the hoop strain epsilon theta is approximately equal to your n. So, the limiting case is log r star divided by r naught is equal to n because epsilon theta is equal to log r by r naught. So, this star is the for the case for maximum condition or limiting case. So, and then substituting this in the equation for p we can get it as p is equal to 2 by root 3 into k into 2 by root 3 into n because log r by r naught no now you are getting it as n.

$$\epsilon_\theta \approx n$$

$$\epsilon_\theta = \ln \frac{r}{r_0}$$

$$\ln\left(\frac{r^*}{r_0}\right) = n$$

So, to the power n into t naught by r naught into exponential minus minus 2 n.

$$P = \frac{2}{\sqrt{3}} k \left(\frac{2}{\sqrt{3}} n\right)^n \frac{t_0}{r_0}.\exp(-2n)$$

So, this is the limiting case where you can get it because it purely depends upon your work hardening exponent or train hardening exponent n where you can say that is equal to k this one raise to n. So, this is the condition under that n is there okay. So, that is how you are getting that means this the limiting case r star that is the radius at which the splitting will take place and two materials are generally isotropic and if the longitudinal since because these tubes are just obtained by deformation either by extrusion or drawing or any any of these methods you can that is how it is obtained. And in such case now you will find that the tube material is anisotropic.

So if you are just finding out if the tensile testing for finding out the material property for this then if you are doing it based on the longitudinally if you are cutting the sample and doing then whatever you are getting the material may fail much below that because if you are you should know how along the hoop direction what are the properties and normally for small tube that will be very difficult. But whereas for large pipes and other things it is easy to machine it though it is lot little bit of inaccuracy you can do that. But if your this stress strain relationship if it is based on the sample stress strain under the longitudinal direction then you may get the pressure at which splitting which occurs maybe at a lower level than what you have been predicting by using equation 8.

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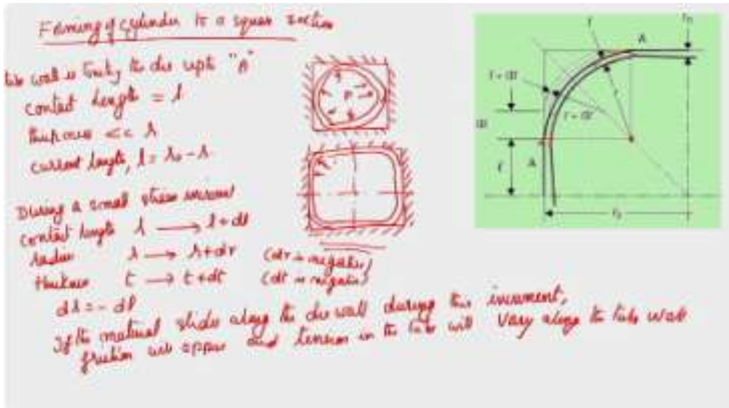

Now let us come to the forming of a cylinder to a square section. Many times this is generally used for this forming of the cylinder square section people go for that and the simple thing is that you have a die which is having a square section in which you just keep a cylindrical piece as a circular piece is a long pipe this is a section which is being shown okay my drawing is not that accurate so maybe we can just It is like this.

So, this is the die part. And then you are filling up with the pressure in all the direction, hydrostatic pressure. So that this will start expanding when it comes and touches the die wall okay there it stops and then it will move on the side. So later now towards the later part when the diameter is radius of this thing is changing and the deformation is taking place you may find that it has finally come to this stage okay at the corner at some intermediate point there is some radius which is coming okay. So you may find that okay it is like this and with this as your die surface So let us try to find out what is the pressure required for completely this cylinder to obtain the shape of the die itself as a square pipe okay. So now what we have shown is this maybe this figure is that it has been partially been expanded so that the wall is stretching up to some age.

So if I just take one fourth of that because of the symmetry I am taking one fourth of that and then that is shown here on the right hand side. So, you will find that it has reached an radius of r naught here, okay. That was the initial radius, r naught was the initial radius of that when you are taking, so that was the cylindrical piece. Now it has from here to here it has moved. So, you will see that there is a continuous reduction in the thickness also there.

So, this is an intermediate stage where this corner radius has become r and now for that configuration it is shown by this the solid lines here. from that point by dl so this was the initial contact length from at from up to point a see this is a point a and here also you can see the point there from r0 to here it has come so initial thickness was t0 now it has come to say t and now further with increasing the pressure inside and you are pressuring pressurizing it inside it still tries to deform and then plastically deform and get it into this shape. So from this position to this configuration shown by the dashed line, you will see that the change in length is dl. And the radius from r, it is going up to r plus dr, dr being negative. Because when it moves towards this side, the radius keeps on decreasing.

And similarly, the t, the thickness, that also changes to t plus dt, where dt is also negative. So from this figure, we can say that the wall is touching. The tube has been partially expanded. So the wall is touching up to a. wall is touching, tube wall is touching the die up to point A.

Contact length of the tube which is in contact with the die is equal to L and if you assume the thickness is far less than say your radius r, so current length l is equal to r0 minus r. So that is the initial case. And now from this radius r, you are giving a small strain increment. So when the small strain increment is coming from l plus dl, it expands.

The contact length increases from l to l plus dl. The radius changes from r to r plus dr dr being negative and correspondingly when it is deforming under plane strain condition the thickness also changes that is from t to t plus dt dt also being negative that means thickness is getting reduced radius is getting reduced okay.

$$current\ lenght, l = r_0 - r$$

$$contact\ length \quad l = l + dl$$

$$radius \qquad r = r + dr$$

$$thickness \qquad t = t + dt$$

So, the contact length increases during an increment during a small strain increment contact length from L it comes to L plus DL and radius from R to R plus DR as a DR is And you will say that thickness, say from t to t plus dt, dt also is negative. It happens. So from this from r to dr where dr is negative so we can write that because it is in the corner radius when it is decreases so we can say dr is equal to minus dl okay. So in the contact zone the tube will be pressed against the die by the internal due to the internal pressure p which is acting.

$$dr = -dl$$

So when that is happening from from this radius to it is moving up to this radius is getting shorter and there is an increase in the length that is possible where the material is sliding along the die wall. So when that is happening the friction will oppose the sliding and the tension will vary along the tube wall. So the material slides along the die wall during this increment, friction will oppose and tension in the tube wall will vary along the tube wall. As more and more it moves like this, whatever tension is on this part which is going to deform, you will keep on increasing. As the radius gets reduced and it is trying to fill up this, the corner part of the square piece, you will find that the pressure is continuously increasing.

So at some point, tension will be insufficient to stretch the wall because it keeps on increasing and then it may be insufficient to stretch the wall and at that point you will find that there will be a sticking friction zone which is taking place okay.

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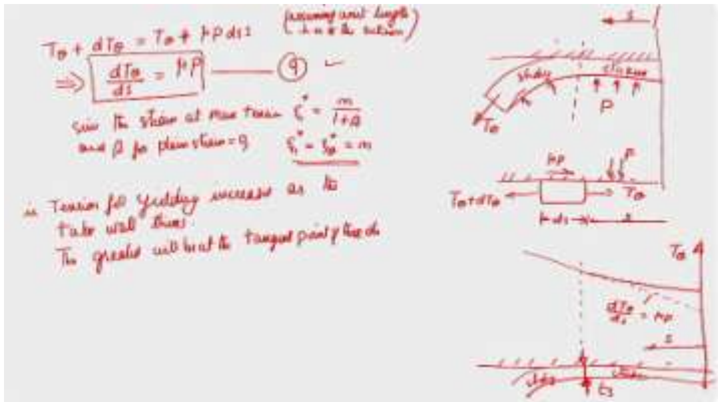

So, if you just look at this condition if I just draw this, this is the wall and maybe this is that part of your wall with tube wall okay. So, you will find that up to certain point So this is the sticking friction and this is the sliding friction okay. So we can say that if you just draw along this direction as S distance and your pressure is being acting along this direction P. So you will find that there is a tension the hoop tension is T phi So this tension keeps on increasing as more and more this one.

And maybe after some time, you will find that the tension which is inside the membrane or inside the tube wall is insufficient to stretch the wall. And so in that case, there will be sticking friction which is coming. So that if you just draw like this, you can just see that what is happening is, if there is an element you are just considering an element on this. So, for example, if this is the die wall and there is an element then you will find that on this there is an imbalance of forces and tension.

So, t theta plus d t theta whereas here it is t theta. So, when this happens suppose this is the distance d as incremental distance which is taking place when it is going. So, and so your this is s. So, you will find that because this pressure is acting on the I wall now you will see there is a p here. So, when since it is trying to move in this direction your frictional force is mu p is acting in this direction. So, now under this case under this condition you can see that the if you take the equilibrium equation for the element of a small wall here then you can say that t theta plus d t theta is equal to t theta plus mu p is also in this direction plus mu p into d s.

$$T_\theta + dT_\theta = T_\theta + \mu P ds 1$$

This is the increment into 1. So, if you are if you are assuming say unit thickness unit length perpendicular to this figure, this sheet, the section, the section which is shown. Okay? So from this, we can just arrive at as d t theta. by ds, the tension how it varies? It is varying like this. So you are getting this equation number 9. So you will find that the deformation process is stable as long as tension increases with the strain.

$$\frac{dT_\theta}{ds} = \mu P$$

And the tension in the unsupported corner it will keep on continuing to increase as the radius becomes smaller and smaller. That is what is happening. As the radius keeps on decreasing, you will find that the tension keeps on increasing, okay. So, since the strain at maximum tension is earlier now we have come up since the strain at maximum tension Like if you look at our very initial equation itself, the maximum tension is epsilon 1 star is under this plane strain condition n by 1 plus beta.

$$\epsilon_1^* = \frac{n}{1+\beta}$$

And beta for plane strain is equal to 0. So you will get epsilon star is equal to epsilon star theta that is equal to n. So the maximum strain which you can get is equal to n.

$$\epsilon_1^* = \epsilon_\theta^* = n$$

So if you have a higher work hardening exponent exponent a material having a higher work hardening exponent then you can go for a large amount of strain. So that is what if it is less your amount of strain will get reduced. So in the present case the tension versus strain is positive up to say epsilon theta up to say n or maybe up to less than or equal to n that is what we get.

So, from this we can conclude that the tension for yielding of the tube the tension in the tube for yielding increases as the wall thickness decreases or tube wall tins. So, that is we can say that tension for yielding that is t theta increases as the tube wall thins. And the greatest tension will be at the tangent point of the die. The greatest tension that is here will be at the tangent point of the die. So, if you just start drawing like this maybe is a tangent point okay. So, we can say that at some point here you have this ticking and then you have this maybe if I just draw it here from this point. So, this in this direction if it is S, the S is in this direction. So, if you look at that t theta and you are plotting say t theta. So, it will just keep on this one. So, this will be d t theta by d s is equal to mu p, but in actual case it will not be like this, it will just keep on moving something like this.

Maybe that is due to some sort of material defects and other things which may be there, okay. So, if you look at that equation 9, this equation, due to friction tension decreases linearly so that is what you are getting this towards the center line so towards the center the tension keeps on decreasing but then when the tension is decreasing the thickness towards that direction is increasing okay and towards the right of the point where sliding ceases so this is your sticking and this is your sliding towards the right of this point. So, this thickness we can say this thickness is Ts if I just put this at that particular thickness it is Ts okay where sliding sees that the tension in the wall is less than that required for yielding okay. So, that is hence no further deformation in the sticking region is possible. So, once it has reached where up to wherever the sticking is there further now you will find that it is not thinning it will remain there because of the friction whereas if there was no friction it may just continue to slide that is the thing.

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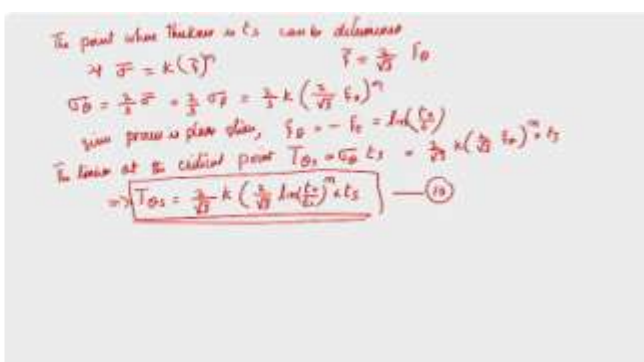

And the critical point where this thickness is Ts so that we can find out the critical point the point where thickness is T s that is the transition from sticking to sliding. So, that we can find out that is if sigma bar is equal to k epsilon bar to the power n is a material behavior and we can also that where this one epsilon bar effective strain is equal to 2 by root 3 into epsilon theta. So, we can write that sigma theta is equal to 2 by 3 sigma bar that is equal to 2 by 3 sigma f that is equal to 2 by 3 k which we have written earlier into 2 root 3 into epsilon theta to the power n. Hence, since the process is plane strain epsilon theta is equal to minus epsilon t where is the thickness direction is equal to log you can write it as t 0 by t. So, we can find out this tension at the critical point the tension because you know the the hoop stress.

$$\bar{\sigma} = k(\bar{\epsilon})^n \qquad \bar{\epsilon} = \frac{2}{\sqrt{3}}\epsilon_\theta$$

$$\sigma_\theta = \frac{2}{3}\,\bar{\sigma} = \frac{2}{3}\sigma_f = \frac{2}{3}k\left(\frac{2}{\sqrt{3}}\epsilon_\theta\right)^n$$

So, the tension at the critical point t theta s that is s refers to that at that point is equal to sigma theta into t s. So, that means t theta s is equal to 2 by root 3 into k into 2 by root 3 into log t naught by t s it should be t naught by t s to the power n. into Ts. So, this we can write it as it is 2 by root 3 into K into 2 by root 3 into epsilon theta to the power n into Ts.

$$\epsilon_\theta = -\epsilon_t = \ln\left(\frac{t_0}{t}\right)$$

$$T_{\theta s} = \sigma_\theta t_s = \frac{2}{\sqrt{3}}k\left(\frac{2}{\sqrt{3}}\epsilon_\theta\right)^n \times t_s)$$

$$T_{\theta s} = \frac{2}{\sqrt{3}}k(\frac{2}{\sqrt{3}}\ln\left(\frac{t_0}{t_s}\right)^n \times t_s)$$

So, this is what we will finally arrive at the tension at the critical point separating the sticking and the sliding region. So, that is what so in the sticking zone there is no sliding no sliding can take place and the slope of the tension curve will be less than mu p. So that is what if you look at that what I have run in the sticking region say if this is the slope of the tension curve d theta by d s which is equal to mu p but in this case the slope will be lower okay that is why that change in the or there is a change in the slope and it will be lower. And the distribution of thickness in the wall It can be determined by an incremental analysis that we can do but here we are not going to discuss much about that incremental analysis.

Here the extreme cases, we can discuss two cases at the extreme cases. One is either with no friction at the die wall, there is no friction or second case is the other extreme cases with a 100% shaking friction along the entire contact length. So we are going to discuss about these two cases.

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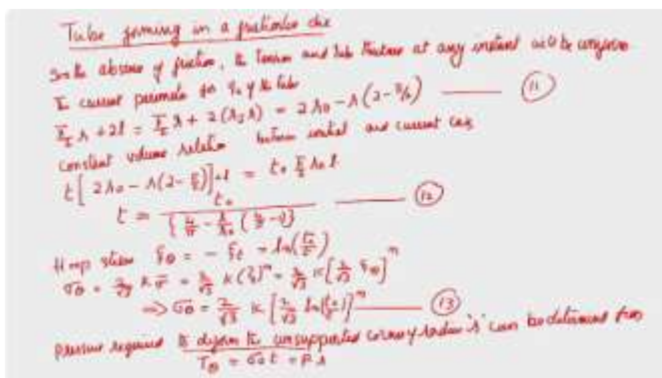

So, first let us consider this tube forming in a frictionless die. The friction at the die, tube, contact, interface, it is not present. If that is not present, if you are assuming that there is no friction at the contact between the die and the tube, so at any instant, the tension and also the thickness at any point in the tube will be uniform.

If friction is there, it will have a constraint on the movement of the material, but if there is no friction and only completely 100% sliding is taking place, and there is no friction at all. Then what will happen uniformly it will thin okay and you can get it. So the thickness at any point at any instant if you look it will be uniform okay. So the current perimeter so we can say that in the absence of friction. Friction means friction at the tube die interface. The tension and tube thickness at any instant will be uniform okay so when you look at that current perimeter from our previous figure the, the at any instant the perimeter the current perimeter for say maybe one by fourth of the tube So, because it is axisymmetric that is what we are taking.

 So, that will be pi by 2 into r. So, 1 by 4 into your 2 pi r no plus 2 l where l is the contact length. So, that comes to pi by 2 r plus l is equal to r minus r naught minus r. So, that we have written earlier. What is this? so that is equal to we can write it as 2 r naught minus r into 2 minus pi by 2 we can write in this form okay so that if it is equation number 11

$$\frac{\pi}{2}r + 2l = \frac{\pi}{2}r + 2(r_0 - r) = 2r_0 - r\left(2 - \frac{\pi}{2}\right)$$

But one thing is there we are assuming there is the material the volume remains constant for the tube the since the volume of the tube remains constant so under constant volume relationship from the initial stage to the instantaneous value, constant volume relationship from the between initial and current case. So, if you write that, so you can write t into 2 t is the instantaneous value r naught into your r into 2 minus pi by 2 you can say that into what is this into 1 because you are assuming a unit thickness perpendicular to this may be your screen.

 So, that is equal to say t naught into pi by 2 into r naught into l. So, from that we can write t is equal to just and do a simplification you will get it as t naught divided by 4 by pi. So, it is very simple simplification you can write by r by r naught into 4 by pi minus 1. you can get the thickness for any, any instant provided you know the r and r naught, okay. So in such case, since it is plane strain condition, the hoop strain is epsilon theta is equal to minus epsilon t is equal to log t naught by t.

$$t\left[2r_0 - r\left(2 - \frac{\pi}{2}\right)\right] \times l = t_0 \frac{\pi}{2} r_0 l$$

$$t = \frac{t_0}{\left\{\frac{4}{\pi} - \frac{r}{r_0}\left(\frac{4}{\pi} - 1\right)\right\}}$$

$$Hoop\ strain \quad \epsilon_\theta = -\epsilon_t = \ln\left(\frac{t_0}{t}\right)$$

So, for the material obeying say your power law equation the hoop stress. So, sigma theta under this condition is equal to 2 by root 3 k sigma bar is equal to 2 by root 3 into k into epsilon bar the same thing to the power n that is equal to 2 by root 3 into k into 2 by root 3 epsilon theta to the power n.

$$\sigma_\theta = \frac{2}{\sqrt{3}}k\,\bar{\sigma} = \frac{2}{\sqrt{3}}k(\bar{\epsilon})^n = \frac{2}{\sqrt{3}}k\left[\frac{2}{\sqrt{3}}\epsilon_\theta\right]^n$$

So, that means that the value you are getting is sigma theta is equal to 2 by root 3 k into 2 by root 3 log T naught by T to the power n. So, the pressure required to deform the unsupported corner of the radius we can get it at the pressure required required to deform the unsupported corner of radius r so that we can take it from can be determined from. So, t theta is equal to sigma naught t is equal to p into r per pressure into instantaneous values of the radius.

$$\sigma_\theta = \frac{2}{\sqrt{3}}k\left[\frac{2}{\sqrt{3}}\ln\left(\frac{t_0}{t}\right)\right]^n$$

$$T_\theta = \sigma_0 t = p.r$$

So, this is what. from this relationship you can get that value you can calculate the the pressure which is required okay sigma this is sigma theta okay sigma theta into t sigma theta is given by the relationship into t then from that now divided by r you can find out the pressure okay.

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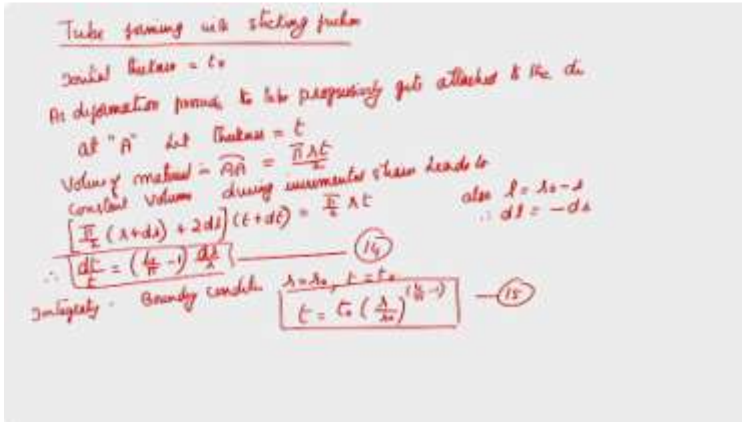

Now say let us take the other case where there is a with a sticking friction very high sticking friction. So tube forming with the sticking friction at the die tube interface. We will say assume that friction is very high. So, in this case if the tube sticks to the die wall because under the high pressure because the large amount of friction is there and due to the hydrostatic pressure it will be sticking to the die wall and when the contact is made. So, let us assume that the initial thickness is g 0. So, as more and more of the wall gets progressively attached to the die wall at each stage the thickness keeps on reducing. So, at the tangent point will decrease. So the thickness at the tangent point will decrease. So at A, let the thickness be t.

So as deformation proceeds, the tube progressively gets attached to the dial. So, let us say at a point A let thickness be be equal to T. So, for unit length the perpendicular figure the volume of the material in say in this distance a which was shown in our earlier figure is equal to pi RT by 2. So this volume will remain constant during the incremental strain whatever be the volume of the material that will remain constant during the incremental strain. So when you are remaining so constant volume during incremental strain leads to we can write that pi by 2 into r plus dr plus 2 dl into t plus dt is equal to pi by 2 r t.

$$Volume\ of\ material\ in\ \widehat{AA} = \frac{\pi rt}{2}$$

$$Constant\ volume\ during\ incremental\ strain\ leads\ to$$

$$\left[\frac{\pi}{2}(r+dr) + 2dr\right](t+dt) = \frac{\pi}{2}rt$$

So, this is the relationship we are getting. also we can get the earlier we wrote L is equal to R0 minus R and so hence DL is equal to minus DR. So, if you substitute into this equation and get it so therefore, we can get it as DT by is equal to 4 by pi minus 1 into dr by r. A very simple simplification you will get this relationship which is equation number 14 okay. And you integrate this equation with applying the boundary conditions boundary condition or at r is equal to r 0, t is equal to t 0.

$$\frac{dt}{t} = \left(\frac{4}{\pi} - 1\right)\frac{dr}{r}$$

So you just so log t is equal to and then the log r 4 by pi minus 1 to log r. So under this boundary conditions when you put it it you can say that from t 0 to t, you will get from r 0 to r. So, that is what we will get it and then with a simplification we can reach this as t 0 into r by r naught to the power 4 by. So, we can get it as t is equal to t naught into r by r naught to the power 4 by pi minus 1 this is the relationship you are getting.

$$t = t_0\left(\frac{r}{r_0}\right)^{\left(\frac{4}{\pi}-1\right)}$$

So, the hoop strain and the pressure to continue the deformation can be determined as shown in the previous case itself that the previous case also we have to so you can find out the the the epsilon theta and your the pressure by this relationship in the same manner if you go you can find out the final relationship and you can get it so now the thing is that we have to See that failure which takes place in forming a square section. So in plane strain, during the plane strain forming of a tube of uniform section, you can do that by having a uniform section.

This process is possible. It is not that it is not possible. You can have a plane strain forming of a tube with a uniform cross section. But the problem is that the tension in the deforming wall continues to increase that is what is possible. So if the tension reaches a maximum value then necking and failure take place because that you can find out. So if the strain is greater than the value of n then naturally your this failure will taking place maybe by localized deformation like a like a ballooning it will take place at a local area and then there it will puncture. So, for plane strain the limit for power law hardening material is when hoop strain is equal to the value of hoop strain reaches the value of n then is the condition where the failure can take place.

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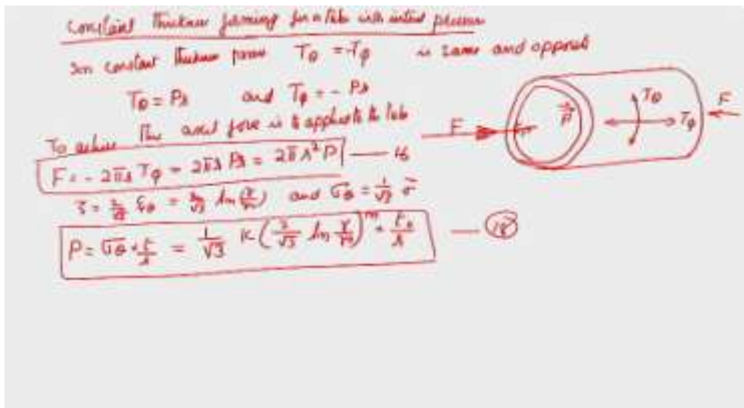

Now if you look at the constant thickness forming, the pressure required will increase as the radius near the corner decreases. So if you just take that constant thickness forming. So, during the plane strain expansion, as soon as the hoop strain reaches n, the material will start splitting or it will start failing. So, that is the condition if you look at it, see here is your hoop strain, that is your principal strain one and this is your longitudinal strain, okay. So, when you really look at it, if this is the zero value, so it may come up to here and then it will so this is your n okay.

So this is that as long as your strain keeps on increasing till it reaches n but beyond that when once the strain crosses that value splitting will take place okay. So that is the case. So the strain path in that case is shown like this. To obtain the required strain in the path, you better choose a high strain hardening material. If you are choosing a high strain hardening material, then the thing is that more and more strain you can get it okay because n value will be high instead of 0.1 or 0.2 you may get a n value of 0.3, 0.4 if you can get a material of that high strain hardening then large amount of deformation is possible in that case okay. But the problem is there with the n value increasing if you look at your previous equations pressure which will be required will be very high so that we cannot help it it will be so high. So now let us consider for the case for a tube expanded with the internal pressure okay thickness forming and deformation for a tube with internal pressure, you are having a tube. So, in a constant thickness process, the hoop and axial tension, the hoops tension and the axial tension or the stresses are equal and opposite.

So, we can say that in in constant thickness process, T theta is equal to T phi that is the tension in the hoop direction and tension in the this is phi they are same but they are all opposite okay so we can write that they are that is same and opposite if one is tensile other is compressive so so we can write that t theta is equal to p r and T phi is equal to minus P r since it is compression. So, to achieve this the axial force has to be applied. So, if you look at the first figure pipe you are going to have an axial force in this direction.

$$T_\theta = P_r \quad and \quad t_\phi = -P_r$$

So, that is what you are going to get it. To achieve this axial force is to be applied to the tube. So, if you just consider this is the tube circular tube with some wall thickness and we can say this is your T phi and this is your T theta. So, internal pressures are there, P is there. You are going to apply an axial force like this F and maybe from here F.

So, then we can get this condition here because when you are applying on this wall know. So, if you are giving on the wall. So, here the T phi will be a compression and here because of the pressure it will be tension. So, that it can go further. So, so that means the F is equal to minus 2 pi r T phi is equal to 2 pi r into P r is equal to 2 pi r square p this is axial force you have to apply. And your same condition like that if you write epsilon bar is equal to 2 by root 3 many times you have written into theta epsilon theta is equal to 2 by root 3 into log r by r naught okay and your hoop stress sigma theta is equal to 1 by root 3 into sigma bar.

$$F = -2\pi r\, T_{\phi} = 2\pi r P_r = 2\pi r^2 P$$

So, for a material following the power law equation since the thickness remains constant we can write that P is equal to sigma theta into t by r because tension is equal to p into r is equal to sigma theta into t that is what we are getting. So, if you substitute the value of sigma theta into that. So, you can get 1 by root 3 into k into equivalent strain 2 by root 3 into log r by r naught okay raise to n into T naught by R this is the relationship we are getting okay. So, from that now from this the pressure which is required we can find out okay. Now, we can consider the case where effect of friction on an axial compression this was the case where there was no friction now let us see that because here we are applying that axial pressure only.

$$\bar{\epsilon} = \frac{2}{\sqrt{3}}\epsilon_{\theta} = \frac{2}{\sqrt{3}}\ln\left(\frac{r}{r_0}\right) \quad and \quad \sigma_{\theta} = \frac{1}{\sqrt{3}}\bar{\sigma}$$

$$P = \sigma_{\theta} \times \frac{t}{r} = \frac{1}{\sqrt{3}} k \left(\frac{2}{\sqrt{3}} \ln\frac{r}{r_0}\right)^{n} \times \frac{t_0}{r}$$

And the friction when it is coming in contact with the die we are not discussing. So it is a free tube where there is an axial pressure and we are filling it up with the hydrostatic pressure inside.

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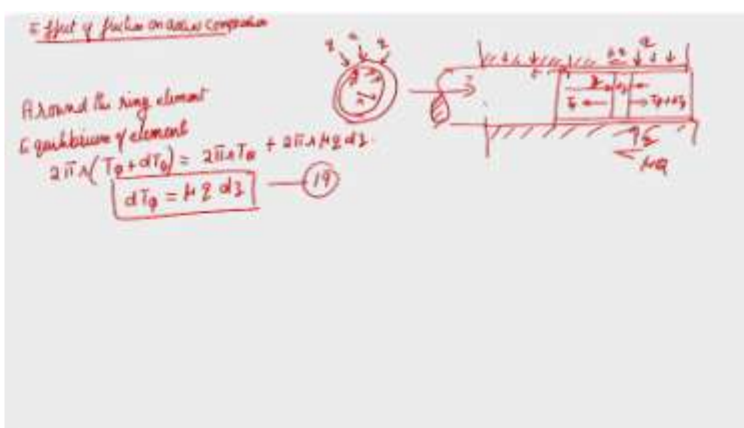

But now let us case where it is in contact with the die wall and there is going to be a pressure what happens. So if you just look at that so here effect of friction on axial compression. Maybe this is your die wall and your plunger is here.

This is the wall thickness and you take an element here, a cylindrical element. So, here you will find that it is longitudinal value t phi in this case t phi plus d t phi you are you are having the pressure here and friction is there. So, if I just take a case say maybe here we can say mu one is q is here if this is your q and mu into q is your frictional force sorry mu into q is your frictional force and this wall thickness is t okay. So, if you just consider this this is the die wall. This is the plunger by which you are going to apply a force F, okay, in this case. So, when you look at that piece, your tube piece, it will be like this.

So, this is your Q on the circumferential direction, the pressure on the die wall. Whereas inside you are finding out P and this is your radius R. This is the condition, equilibrium condition. So the force F is applied at the end of the tube to achieve the axial compression okay. So because you need that it should be a compressive in stress actually T5 should be a compressive in nature then only we can equate it as equal to T into R okay.

So effect of the force is the local due to this you will find that there is a local effect because continuously when it is coming the friction force at each and every point it continues to vary. So if you look at this point. The plunger on the left applies a compression to the tube and at some distance z, at some distance z, the tension on one side of the element is t phi and on the other element is t phi plus dt phi. So, around the elemental ring, if you put it out, around the element, d z the tension on one side of the element is T phi and on the other side is there. So, equilibrium of the element if you take it if you write it as 2 pi r into T phi plus d T phi is equal to 2 pi into t phi plus 2 pi r mu q into dz okay your frictional force is coming into picture.

$$Equilibrium\ of\ element$$

$$2\pi r\left(T_{\phi} + dT_{\phi}\right) = 2\pi r T_{\phi} + 2\pi r \mu q dz$$

So that means from this you will get this equation d t phi is equal to mu q into dz. So that means as z increases the tension or the traction increases and becomes more tensile. So initially towards the plunger side you will find that with the z is approaching to 0 the the stresses are compressive in nature but when it moves when the z with the distance z in the opposite direction when you look at it because here also you can find and then here also you can find mu q and this is q okay. So with the more and more towards the left the nature of the compressive stress it decreases. So it becomes more and more tensile and after certain stage it becomes completely tensile.

$$dT_{\phi} = \mu q dz$$

So when the tube length is very large the axial force will not have any effect especially at the center of the tube. But for short tubes it will have an effect because there will be a compressive stresses for short tube. So it all depends upon your L by D ratio of the tube. So long tube you may not have much effect but short tube it will definitely you can deform it with friction and other things and then as shown in our first figure in the photograph that can be obtained. So now so that means what I am telling you that from this plunger end this is the plunger end from here it is compressive in nature and towards that it becomes this one. For this today it is fine.