Plastic Working of Metallic Materials
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Module No # 01
Lecture No # 04
Friction and Lubrication

So today, this particular lecture on, it will be, we will be discussing about the friction and lubrication.

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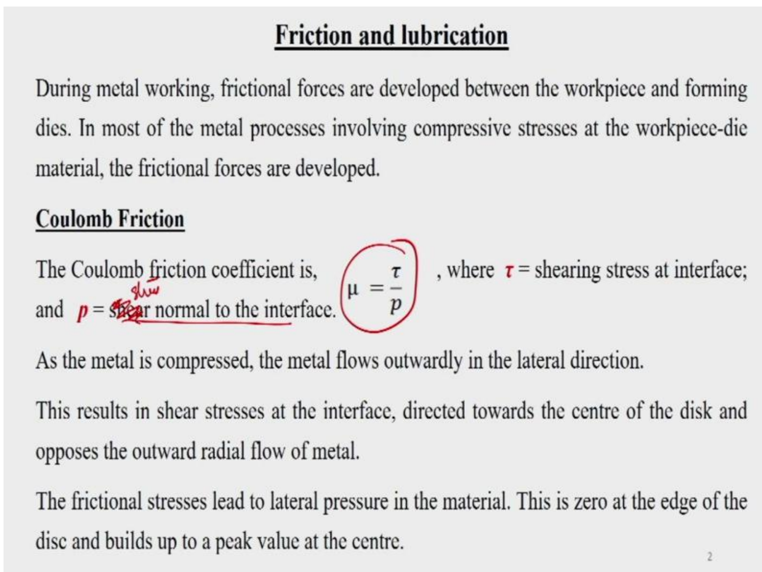

This friction and lubrication is very important in metal forming process, most of the cases specially, the deformation processes the friction and lubrication plays an important role. Because during this metal working, this because there is a relative motion between the die and the work piece, frictional forces are developed between the work piece and the forming die. And most of the metal working processes are specially deformation processing, it involves compressive stresses at the work piece and die interface or die material interface.

So there, this frictional forces are coming and like if, when you look at it suppose, you are just assuming a metal is being compressed between 2 patterns to maintain the constant volume relationship, metal will start to flowing on the radial direction. So, when it is flowing on the radial direction, this shear stresses are developed at the interface and this shear stresses are directed towards the center of the disk or work piece material and which opposes the radial flow of the material.

So, that is the frictional forces which are developed ok. So that, it is a type of shear stresses which are developed the interfaces and that will be opposing the radial flow of the material. Due to this, what happens is that the stresses increases. Otherwise if, there was no friction we may have a homogeneous deformation process, that what are the energy required for that or what are the forces required for that. And when you compare that there is a friction, then you will find that the frictional forces contributes to a great extent.

In some of the cases, you are in some cases your frictional forces may be as high as the energy required for homogeneous deformation itself and this. So we have to consider this friction. Let us look into them. There are 2 types of friction when it comes to the metal working process, one is a Coulomb friction and another is the sticking friction. The Coulomb friction or metal working terminology, we call it as a sliding friction, where the metal, there is a relative motion between the die and the work piece. And the Coulomb friction coefficient, Mu is represented by this Mu = tau / p, where p is the shearing stress at the interface and p is the shear the stress normal to the interface okay.

$$\mu = \frac{\tau}{p}$$

So as the metal is compressed, metal flows outwardly in the lateral direction as I have said. So let us just take a typical case of open die forging of a cylinder and to get a final shape you are going to get a very thin disk. So, if you have a cylindrical piece you are compressing it between.

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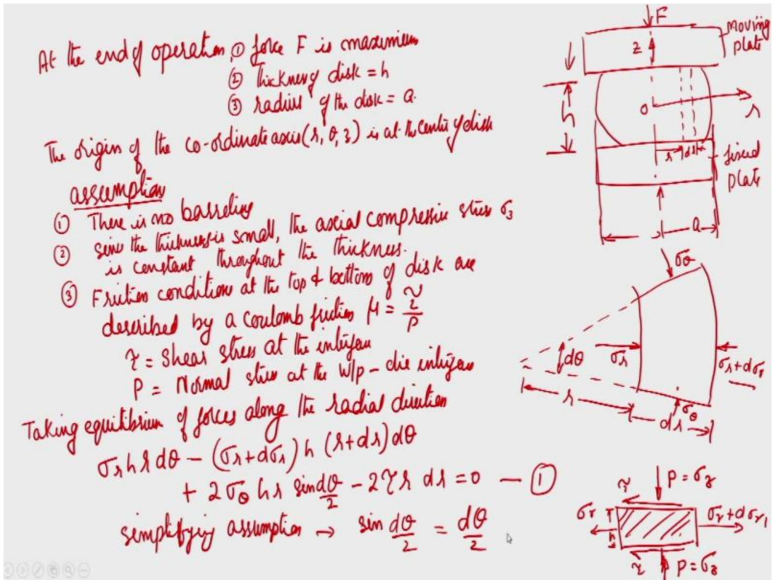

Say for example, if I just take that, this is a platen and this is a platen and in-between know you have your work piece material. When it is deforming you know there will be a barreling. So, the shape of a barrel is coming and you are having applying the force here and in the maybe in normal cases the bottom platen will be a fixed plate ok.

This is the fixed plate and this is the moving plate and this will be moving downwards ok. So let us take that the center of this as the coordinate axis and say you can just consider this case. This is the r radial direction and this is your origin of your coordinate axis. So, we can just to consider a case of a small element ok. Say at a distance r and an element of thickness dr. We can say that, this, the total length between this is equal to it a.

We can here also, we can just tell that is also a. That means, we are compressing it with the help of a force f and the height is h. When your process is complete, this is the condition for the last stage of the compression. When you are about to fix it because that is a stage, for the reduction when you are going to give it the stresses will be or the loaded requirement will be the maximum at that stage.

So, we wanted to find out, what is the maximum load required? So that, we may find it out by getting an overall finding out the variation of the stresses along this direction. So here, if you just take it as this is the z direction ok. This is the, you can consider it as the z direction ok. So, along that direction what are the, how is the variation of the stresses? And the integrate it and then find out the average pressure which is required multiplied by the cross sectional area will give you the total force required which is required multiplied by the cross sectional area will give you the total force required.

So, we have to assume that the restriction which is taking place. So, in this element the circular element, in an exaggerated view if, I just draw it will be something like that. Because, we are going to, so this is your, this one so this is the radius r and this is dr, ok. That element when you are looking so because of the flow in the direction, because of the shear stresses which are developed which is opposing the lateral flow of the material.

We found that, there will be an imbalance of forces. So, if you look at that here so, this is if you just consider that this is a σ_r in the radial direction it is $\sigma_r + d\,\sigma_r$ and you have this σ_θ in this direction. We are taking in this consideration, σ_θ this is also σ_θ, sigma suffix theta and this is going to be your $d\theta$, subtend the angle subtended by the center of the disk by this element ok.

So now, if you look at so this is the element looking from the top view, there will be a normal force, which is there. So if I just take the side view of this element, it will be something like this. So here, you have$\sigma_r + d\,\sigma_r$. I will draw this here, that element if I am drawing it here they top view. So $\sigma_r + d\,\sigma_r$ here it is σ_r. Then, you have the frictional force coming into here so that is equal to τ.

And you have your p, so that is a p which is equal to σ_z ok. So here, also it will come because on the 2 sides are there. So here, also sigma is τ is coming the shear stress and this will be the $t = \sigma_z$ ok. So, this is what and this height this is the height h so, that is what now if you look at them, so this is the typical case of a open die forging of a cylinder to obtain a circular disk. So, at the end of the operation the forces are maximum.

So, we can write this, at the end of the operation or process the forces, force f is maximum to the thickness achieved. Thickness of disk is h and radius of the disk = a ok. So, the origin of the coordinate axis, I am writing it the origin of the coordinate axis $(r\,,\theta, z)$ is at the center of the disk and this is that a small element, which of the disks subtending angle of $d\theta$. Sorry, this is not θ, this $d\theta$, a small element.

So $d\theta$ at the center, between the radius r and dr is shown here. So, that is this is that part we have taken it here and it is in an exaggerated view the thing. Now, when you are going for this analysis for this friction certain simplifying assumptions we have to take somewhere. Because somewhere have to take some assumptions, so assumptions are because that is very important. One, no barreling takes place, there is no barreling. Second, since the thickness is small, see since the thickness is very small, you can say that the axial compressive stresses ok.

σ_z, it is constant throughout the thickness of the material and also throughout the thickness since the thickness is small the axial compressive stress is constant throughout the thickness. Third, the friction conditions at the top and the bottom of disk are described by a Coulomb friction coefficient $\mu = \frac{\tau}{p}$ where, τ is the shear stress at the interface and p is the normal stress at the interface, τ is the shear stress at the interface and p is the normal stress at the work piece die interface.

So now, we can see that because of this lateral flow of the material the surface shear are directed towards the center. So, that is why because the surface shear stresses are directed towards the center and in that case and in the frictional shear stress leads to a lateral pressure in the material which is 0 at the edge. And that means at the free surface and then it builds you have a maximum value at the center. So, that is what is going to happen.

So now if you take the case of equilibrium of the forces in the along the radial direction. Say because this is the free body diagram. So, if you are considering the equilibrium forces along the radial direction, we can just write this, taking equilibrium of forces along the radial direction. So, we can say that here $\sigma_r h$ into r into $d\theta, rd\theta$ is that arc length - $\sigma_r + d\sigma_r$ that is, this part into r, sorry into h into $r + dr$ into $d\theta + 2$.

So, we have to take this $\sigma_r hrd\theta$ on the 2 sides are $\sin\frac{d\theta}{2} -,$ because, the frictional forces when you look at it is there at the 2 surfaces here and here. So, that is why, we have to write it 2 τrdr = 0 ok. See, this is your equation number 1 now, if you assume that for small value of theta, a reasonable assumptions simplifying assumptions is a $\sin\frac{d\theta}{2} = \frac{d\theta}{2}$.

$$\sigma_r hrd\theta - (\sigma_r + d\sigma_r)h(r + dr)d\theta + 2\sigma_\theta hr\sin\frac{d\theta}{2} - 2\tau rdr = 0$$

$$Simplifying\ assumption\ \sin\frac{d\theta}{2} = \frac{d\theta}{2}$$

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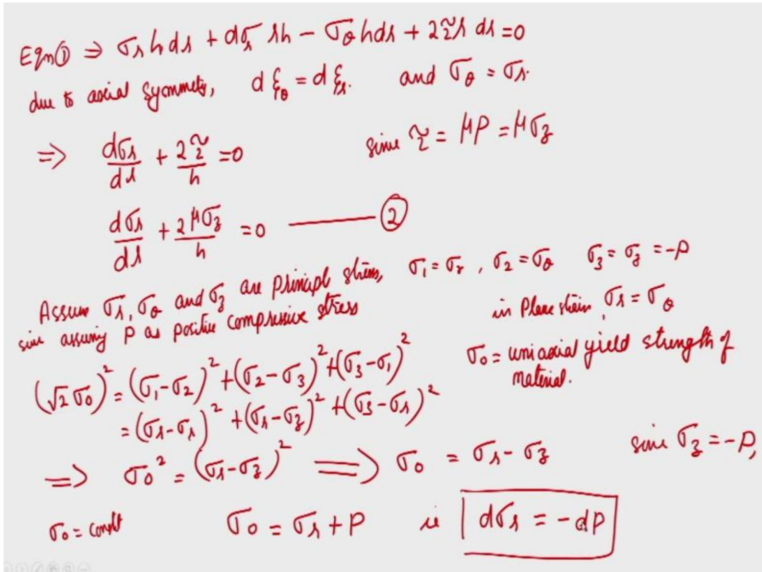

So, in that condition equation number 1, we can simplify it like $\sigma_r hdr + d\sigma_r rh$ - $\sigma_r hdr$ + $2\tau rdr$ = 0 ok. And, when you look at the axial symmetry, due to axial symmetry the strain, $d\varepsilon_\theta = d\varepsilon_r\ and\ \sigma_\theta = \sigma_r$. So, if you consider this case, this above equation, so the equation above equation, so ok we can say if you substitute into this we will get that we can simplify it in this form $d\sigma_r$ divided by dr + $\frac{2\tau}{h}$ = 0.

So this equation, we are getting and this tau, we can just say our earlier assumption. Because, since $\tau = \mu p$. So, where p is the vertical pressure the normal to the surface. So that, we can write it as $\mu\sigma_z$. So, this equation will come to $d\sigma_r$ by dr + 2 $\mu\sigma_z$/ h = 0 equation number 2, we are getting. Now the thing is that if you assume that the σ_z, sigma 0 and σ_r the principle stresses, though I have not discussed it in the subsequent lectures in the module 2.

$$\sigma_r hdr + d\sigma_r rh - \sigma_\theta hdr + 2\tau rdr = 0$$

$$due\ to\ axial\ symmetry, d\varepsilon_\theta = d\varepsilon_r\ and\ \sigma_\theta = \sigma_r$$

$$\frac{d\sigma_r}{dr} + \frac{2\tau}{h} = 0, since\ \tau = \mu p = \mu\sigma_z$$

$$\frac{d\sigma_r}{dr} + \frac{2\mu\sigma_z}{h} = 0$$

This part will be coming explaining about the principle stresses and other things. And these Von Mises criteria, all those things are coming but, here since I have to discuss about this friction here. It will be very difficult to explain that here. So, but you have to take it to one has to assume that, so assuming that and in your B.Tech, course on strength of materials. You know this principle stress and strains are already discussed.

So, you should be familiar with that assuming that $\sigma_r, \sigma_\theta\ and\ \sigma_3$are the principle stresses. Then, we can say $\sigma_1 = \sigma_r, \sigma_2 = \sigma_\theta, \sigma_3 = \sigma_3 = -p$. Because, we are assuming p as a positive compressive stress, assuming p as positive compressive stress ok. So, if you apply the Von mises yield criteria that though it will be discussed later.

And here also we can say σ_r in plain strain condition equal to σ_θ in plain strain condition. Plain strain condition $\sigma_r = \sigma_\theta$ and so the Von Mises criteria says that root 2 into sigma 0 square = $\sigma_1 - \sigma_2$ the whole square +$\sigma_2 - \sigma_3$ the whole square + $\sigma_3 - \sigma_1$ the whole square ok. So, where sigma 0 is the uniaxial yield strength of the material, means work piece material.

So, if you substitute these conditions you know we will get it as a sigma 1 = sigma r = sigma theta. So, we can say sigma 1 - $\sigma_r - \sigma_r$ we can write sigma r or sigma theta – theta. Also we can write both are same the whole square + $\sigma_r - \sigma_r$- σ_z= -p ok. So, we can say - $\sigma_z$the whole square + σ_z - σ_r the whole square. So, this implies that ${\sigma_0}^2 = \sigma_r - \sigma_z$ the whole square or that we can write it as sigma 0.

$$\sigma_0 = \sigma_r - \sigma_z$$

and since we have written that$\sigma_z = -p$. Since, $\sigma_z = -p$ positive compressive stress we can write

$$\sigma_0 = \sigma_r + p$$

So, that is if you take the derivative we can write it as d sigma r = -p. If, you take the derivative because this is a constant ok. So, sigma 0 is a constant so sorry $d\sigma_r$= -dp you are getting this.

$$Assume\ \sigma_r, \sigma_\theta\ and\ \sigma_3\ are\ principal\ strain, \sigma_1 = \sigma_r, \sigma_2 = \sigma_\theta, \sigma_3 = \sigma_3 = -p$$

$$Since\ assuming\ p\ as\ positive\ compressive\ stresses$$

$$in\ Plane\ strain\ \ \sigma_r = \sigma_\theta$$

$$\left(\sqrt{2}\sigma_0\right)^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

$$= (\sigma_r - \sigma_r)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_3 - \sigma_r)^2$$

$$\sigma_0{}^2 = (\sigma_r - \sigma_z)^2$$

$$\sigma_0 = \sigma_r - \sigma_z$$

$$Since\ \sigma_z = -p$$

$$\sigma_0 = \sigma_r + p$$

$$d\sigma_r = -dp$$

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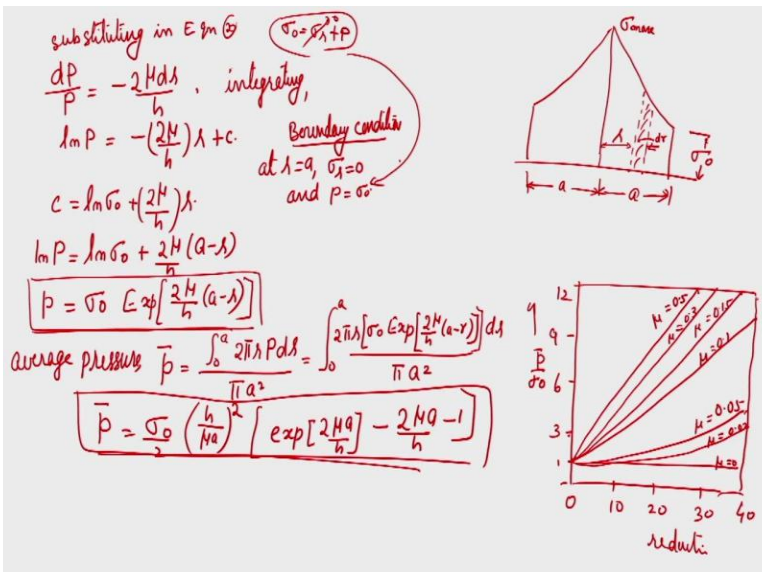

Now, if you substitute this in equation number 2. In equation 2, if you substitute it so, you can get it as dp / p = -2 Mu dr / h. That is what, you will be getting now you are directly integrate it. So integrating we can write it as log p = -2 Mu / h into r + c this is what, we are getting. So now, if you just look at the boundary conditions what are the boundary conditions, the boundary conditions are that is at the edge of the disk. At the edge of the disk means the radius = a ok.

So, at r = a the radial stresses equal to 0 at the free surface, ok since p = sigma 0, sorry and p = sigma 0 ok. So, that means if you substitute into this, so we will get it as a log p sorry c = log sigma because sigma r is 0 means sigma is from the previous equation you know. We are getting that what is r? sigma 0 = sigma r + p when it comes you know this becomes 0 that is what here it is coming ok.

$$\frac{dp}{p} = \frac{-2\mu dr}{h},$$

$$integrating,$$

$$\ln P = -\left(\frac{2\mu}{h}\right) r + c$$

So, we will get p = sigma so we can put that log sigma 0 c = +2 Mu / h into r. So, substituting into this equation, we can write it as log p = log sigma 0 + 2 Mu / h into a – r. So, if you substitute the value of c here and write it you will get like this. So, that is p = because you can bring it that log p / log sigma 0 and then take it as p / sigma 0 equal to exponential. This 1 take antilogarithm so, we can write like that this = sigma 0 into exponential 2 Mu / h into a – r.

$$at\ r = a, \sigma_r = 0\ and\ P = \sigma_0$$

$$\ln P = \ln \sigma_0 + \frac{2\mu}{h}(a - r)$$

$$P = \sigma_0 Exp\left[\frac{2\mu}{h}(a - r)\right]$$

So, this is the expression for the variation in the die pressure from the center to the surface where a is the surface of the disk and r is at any distance. We substitute the value of r, you will get that what is the variation of p? So, if you plot it you will get something like this. Say so, this is sigma 0 and this is sigma max or say sigma max maximum value and this is a. So this is the variation of here and this is the small elemental ship which we have taken at any instant.

If, we just take it so this is that small so this is r and this is dr ok. Now, basically we are interested in the average pressure during the forging operation or during the material working operation. So, average pressure, by the average pressure p bar equal to integral from 0 to a 2 pi r into p into dr divided by pi a square. So that, we will get it as integral from 0 to a 2 pi r into if you substitute this equation 0 to a sigma 0 into exponential 2 Mu / h into a - r dr divided by pi a square.

So, that value you will get it as something like this p bar equal to sigma 0 / 2 into h / Mu a the whole square into exponential 2 Mu a / h - 2 Mu a / h – 1. This is here it is square this is the average pressure distribution average pressure. We can now average pressure into your total area will give you the total force which is required. So now, if you look at this the variation of say p / sigma naught. You will have as per this equation you will have a feel for that how the friction influences the forces of the material for deformation.

$$Average\ pressure\ \bar{P} = \frac{\int_0^a 2\pi r P dr}{\pi a^2} = \frac{\int_0^a 2\pi r \left[\sigma_0 Exp\left[\frac{2\mu}{h}(a-r)\right]\right] dr}{\pi a^2}$$

$$\bar{P} = \frac{\sigma_0}{2}\left(\frac{h}{\mu_0}\right)^2 \left[exp\left[\frac{2\mu a}{h}\right] - \frac{2\mu a}{h} - 1\right]$$

So, if you look at itself may be from 0 to 40 ok. This is the reduction, if I say 10, 20, 30 reduction and if I just plot it here as p bar / sigma naught, uniaxial sigma naught is your uniaxial yield strength and p is the die pressure if it is plotted like this. So, you will find that it is a very big amount is going to come so maybe if I am just putting it as a 12 means here it is 6. So this is 3, so 9 if you just plot it something around here you will get that from.

So, it should be straight almost straight ok. Similarly, you may get something like this the symbolic symbolically I am just drawing it. So here, μ = 0. Here for different values of μ, μ = 0.02 so here, μ = 0.05 for different reduction. Now, if you do like that and here it is μ =0.1, here μ =0.15 and μ = say 0.3 and here this μ =0.5. You see that the friction forces due to the friction forces when μ changes from 0 to 0.5.

So even, for a reduction of 25%. now, you will find that the die pressure is very high 12 times than what is required ok. So, that is what for the case where μ = 0 with the 0 or when μ is very less with the μ =0.02 which you can get it by very good solid lubricants like molybdenum di sulfide or graphite. Now, where you get μ value of very small value you will find that the frictional total force.

So, because this value divided by your uniaxial yield strength. Uniaxial yield strength will give the average pressure distribution. So, that is going to be 12 times see that is a very big value. So, that is what always one should look at reducing the friction but it is not that in some cases you need friction also for a proper metal flow, otherwise you know there if it is completely flowing also then there will be a problem.

So, those are at some places you may require sticking friction some places you may require sliding friction. So, in the next lecture, we will discuss about the sticking friction. So, though here we were discussing about the sliding friction ok. Thank you.