Plastic Working of Metallic Materials
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Module 2
Lecture - 1
Mechanics of Metal Working

We will come to the next module, that is the mechanics of metal working. In mechanics of metal workings, we will be discussing mainly with the plasticity theory. So we will have to find out. So far, we were discussing about the simple tensile test, but in real case, when you are carrying out the metal working operations, may be, you may have to consider the two-dimensional state of stress or a three-dimensional state of stress. So let us go to that. Before that, the yielding criteria where the metal deforms plastically, all those criteria we have to look into that. So, we will go to this.

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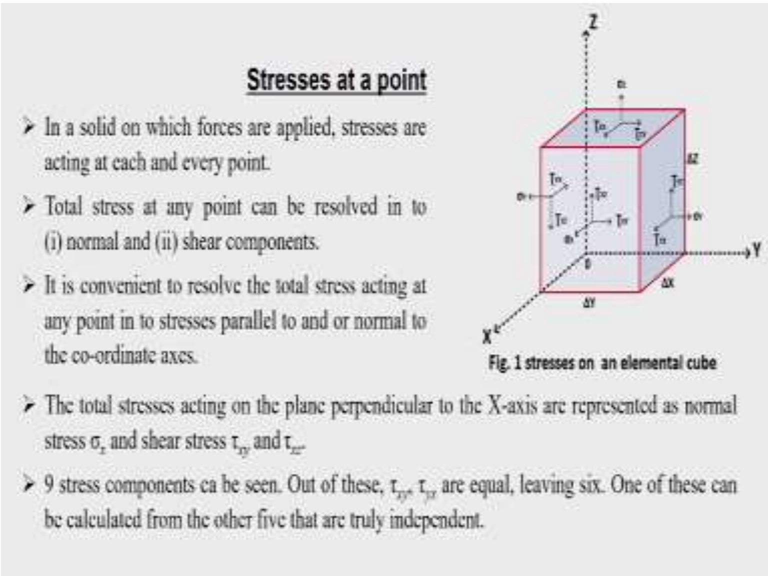

See any engineering component, when external forces are applied on the component, it induces stresses inside the body and then you will find that these stresses are acting at each and every point in the body depending upon the cross sectional area, the orientation and other things. So, we have to discuss with respect to what is the stress at a point, the state of stress at a point, how it can be resolved into various components okay.

So the total stress at any point can be resolved into, generally tend to resolve if you are considering a Cartesian coordinate system, the stress at any point can be resolved into say a normal component of the stress and also the shear component. So the shear components are coming into the picture. If you just take a box, an elemental parallelepiped region or a cube you should take it. We can see that on these faces, this plane, on this particular plane or any of the plane, we can find out what are the stresses and what are the component of that stress.

The component of that particular stress in a direction which is normal to that plane. For example, if you take a normal to this yz plane and what are the components of the shear stresses which are acting on the plane, say for example in this the component are we can say xy and xz are the shear components of this stress. So we can resolve into these, on each face no, we can resolve into 3 components and if you just look at it, say you will find that there are 9 stress components are coming into picture,.

Say one is the normal component σ_x, σ_y, and $\sigma_z$and the other is τxy, τxz on this front plane, and τyx and τyz on this side face, and on the horizontal plane τxz and τyz. So, you will find that so 3 normal stress components and 6 shear stress components are there. See out of this, you will find that τxy and τyx, see, for example τxy and τyx sorry this τxy and τ they are acting on these two sides and they are equal.

So when you are trying to explain it, only one is known, the other is automatically we will be knowing it because they are acting one is on this plane, one is on this plane okay. So, since the magnitude is same, you need only one. Similarly is the case that τyx and tau sorry τzy and τyz, see one is here, one is here, one is acting along this direction, one is acting along this direction. This causes the shear and so these two are same.

So that way when you look at it, finally now out of the 9, you will find that only 6 are left, that is sigma x which is a normal component normal stress sigma x, sigma y and sigma z and the shear component that is τxy, τxz, and τyz. So, these are the 6 components and out of that if one of these can see like out of the 6 to specify the stress at a point, we need only 5 because once you have the 5 components, the sixth can be derived out of that. Thus, that is the advantage with this.

So ultimately to know the state of stress, you need to have an idea about the 5 stresses which are acting on the component. Now let us just go through this, the state of stresses in 3 dimensions. I am not going to discuss in detail because this part is always you might have studied in your solid mechanics course, but only just to brush up this thing only this part I am discussing.

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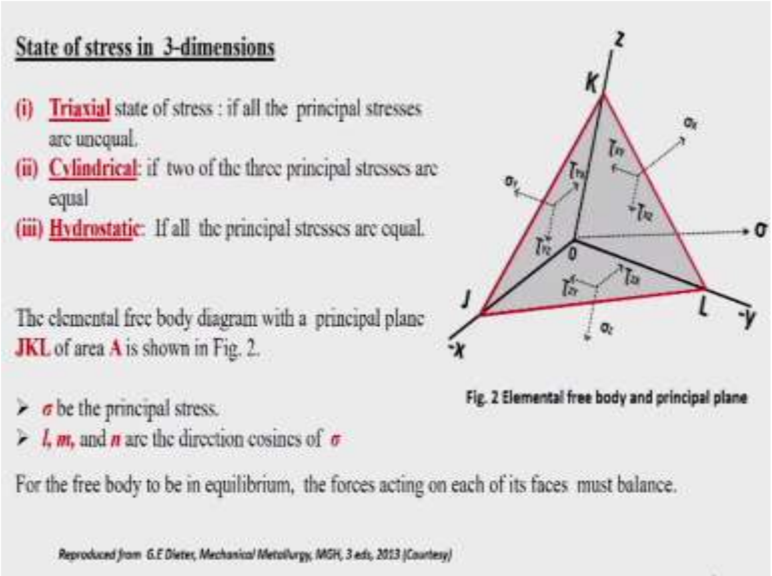

So the state of stress in 3-dimensions. See you can just assume that state of stress is only along one direction may be a uniaxial tensile stress of a wire, if you look at it. The stress which is acting on the wire is only along one direction, the other path we can ignore it. It is not that it is not there, but it is negligible, so we can ignore it. Suppose you take a sheet of paper and then pull it, then you will find that on two sides you pull it, then you find that the stresses are acting only along the plane of that sheet.

So we can have in any direction, may be x direction or y direction or combination of that we can have it, but that in that sheet of paper or thin sheet of metal when you are doing a foil, when you are applying a stress, the stress which is acting on that is only a biaxial state of stress provided you are applying the forces in two directions okay. So the triaxial state of stress is say like the principal stresses are unequal when you consider the principal stresses, they are all unequal, then you call it as a triaxial state of stress, a solid body when you are considering.

Now there is a cylindrical if the two of the three principal stresses are equal, then you call it as a cylindrical state of stress. Hydrostatic, if all the principal stresses are equal it is a hydrostatic stress, if they are compressive of tension, then you call it as a hydrostatic stress. So the elemental free body diagram when you wanted to consider the state of stress, you have to consider an elemental body, the same cube which was there in the previous case. If you take a section like this from here as a plane, and that back side path if you just take it, so and that is what this is shown here along the x, y, and z direction.

So this plane, when it is a cube bisection and then you are taking this along this plane, see we can just see what are the state of stress. So you are just considering a principal, we are assuming that JKL is a principal plane. Principal plane means that, principal stress is that stress where there is no, where the shear component is zero okay, so that is what, but here we are taking this JKL is a principal plane and its let the area of JKL be A, capital area, and σ be the principal stress acting on this plane okay.

So, that means, when it is a principal plane, the shear component on that is not there. So you can say that normal component is the σ which is coming in the picture and let l, m, and n are the direction cosines of this σ, that means it is the cosine of the angle between the σ and x that is l, the cosine of the angle between σ and y axis that is your m, and the cosine of σ and your z axis that is the value of n.

So, let l, m, and n are the direction cosines of σ and if you assume the body is in equilibrium, the forces acting on each of this faces must balance, the faces means one is KOJ, ZOL sorry KOL, LOJ, and JKL on this, forces acting on all these faces should be equal, that means that should balance.

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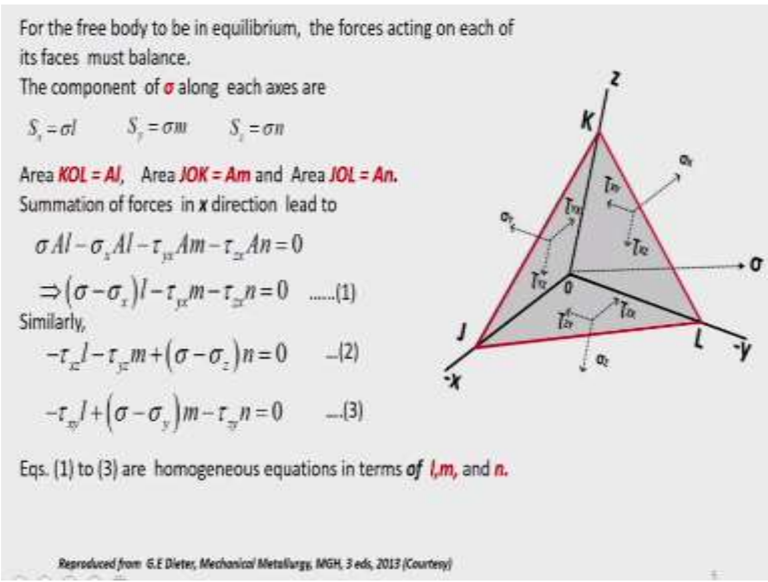

So for the free body to the in equilibrium, forces acting on each of its faces must balance. The component of σ along each axes, say this component of σ along each axes to be just considered that we can specify it as $Sx, Sy\ and\ Sz$, that means along the x axis it is Sx, Sy it is σ m, and Sz it is σ n okay because l, m and n are the direction cosines. So areas of KOL, you can say that Al your direction cosine, similarly JOK = A into m, and JOL = A into n where A is the area of JKL.

$$S_x = \sigma l$$

$$S_y = \sigma m$$

$$S_z = \sigma n$$

Now, if you just summation of the forces in the x direction, along this direction, because that has to be under equilibrium, that should be equal to 0. So that way, we can write it as σAl, $\sigma$is the principal stress on plane JKL – σ_x into AL okay, so that is along the direction which is coming, the direction cosine you are taking, then $-\tau_{xy}$ into Am okay, so that is along the y direction the component you are taking it and $-\tau_x$, the shear component so you are taking it τ_{xz} into An = 0. So this, if you just you can write it in this form itself is equal to that is equal to σ –sigma x into $\mathbf{1} - \tau_{yx}$ into m– tau zx into n = 0.

$$= (\sigma - \sigma_x)l - \tau_{yx}m - \tau_{zx}n = 0$$

So that is one equation. Similarly, you can just sum the take the summation of the forces in the y direction, along this direction, and then you will get this equation – tau xz into l– tau yz into m + z –z z into n that is = 0 and the next equation is that if you are just summing the equations along the z direction, so the same thing you are getting this equation.

$$-\tau_{xz}l - \tau_{yz}m + (\sigma - \sigma_z)n = 0$$

$$-\tau_{xy}l + (\sigma - \sigma_y)m - \tau_{zy}n = 0$$

So similarity is there, just shifting, one is y –yz, okay sorry, this is along the z direction, this is along the y direction, I just I am sorry, it got interchanged okay. So, these are the thing. So these 3 equations if you look, it is a homogenous equation in terms of l, m and n. So, you may have to solve it.

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Solution to Eqs. (1), (2) and (3) can be obtained by setting the determinant of coefficient of ***l***, ***m***, and ***n*** equal to zero. i.e.

$$\begin{vmatrix} \sigma-\sigma_x & \tau_{yx} & \tau_{zx} \\ -\tau_{xy} & \sigma-\sigma_y & \tau_{zy} \\ -\tau_{xz} & -\tau_{yz} & (\sigma-\sigma_z) \end{vmatrix} = 0 \qquad \text{........ (4)}$$

The solution Eq. (4) results in the cubic equation

$$\Rightarrow \sigma^3-(\sigma_x+\sigma_y+\sigma_z)\sigma^2+(\sigma_x\sigma_y+\sigma_y\sigma_z+\sigma_x\sigma_z-\tau_{xy}^2-\tau_{yz}^2-\tau_{xz}^2)\sigma-(\sigma_x\sigma_y\sigma_z+2\tau_{xy}\tau_{yz}\tau_{xz}+\sigma_x\sigma_z-\sigma_z\tau_{xy}^2-\sigma_x\tau_{yz}^2-\sigma_y\tau_{xz}^2)=0 \qquad \text{..........(5)}$$

The three principal stresses σ_1, σ_2 and σ_3 can be the roots of Eq. (5)

Substituting the principal stress in Eq. (5) and we can solve for l, m, and n.

There are three combinations of stress components in Eq. (5).

since these coefficients determine the principal stresses, they do not vary (invariant coefficients with the co-ordinate axes

So you can write in the matrix format the solution of equation 1, 2, and 3 can be obtained by setting the determinant of the coefficient of l, m, and n equal to zero 0. So that means, we can write it in this form. So, sigma – sigma x, tau yx,– tau xy, tau zx,– tau xz, then tau yz, okay by mistake this part has come. We can see these two are same and these two are same and you can write in the determinant form, the matrix format. So solution of this equation results in the cubic equation.

If you just do that, the value of the determinant when you take it, you get a cubic equation okay. There is sigma cubed – sigma x + sigma y + sigma z in the sigma square + sigma x sigma y + sigma y sigma z + sigma x sigma z – tau xy square – tau yz square – tau xz square x sigma, so you are getting this cubic equation and three principal stresses if you assume that instead of sigma x, sigma y, sigma z is there, then you can write as if you write it in terms of principal stresses, this shear components vanishes.

$$\begin{vmatrix} \sigma-\sigma_x & \tau_{yx} & \tau_{zx} \\ -\tau_{xy} & \sigma-\sigma_y & \tau_{zy} \\ -\tau_{xz} & -\tau_{yz} & (\sigma-\sigma_z) \end{vmatrix} = 0$$

So then from that, now we can find out the roots of this equations okay. If you are assuming the principal stresses as considering σ when so substituting the principal stresses in equation 5 five and we can solve for l, m, and n. Then, you will find that there are 3 combination of stress components which is coming and this specialty is that since its coefficients determine the principal stresses they don’t change, they don’t vary, it is a invariant. So, with respect to if you are just giving your cube a rotation and if you are writing the principal stresses, it remains the same.

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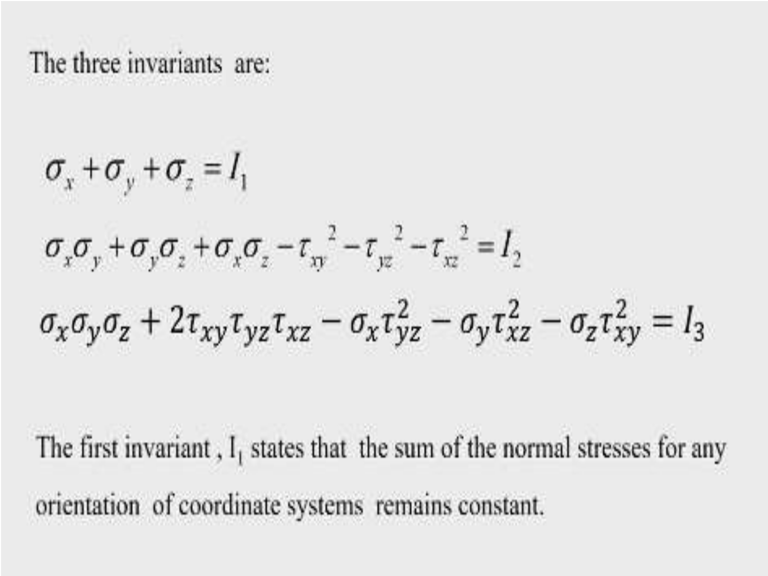

For example, these are the 3 invariants are sigma x + sigma y + sigma z = 0, is equal to you call it as invariant 1, this is the invariant 1, I1 and the second part if you look at that this term when you take it that is invariant 2 because there the shear component is coming, so this is the invariant 2 and the third one is invariant 3, we call it as this and these three are there. The specialty is that in the first invariant, that I1 states that the sum of the normal stresses for any orientation of the coordinate system remains same.

So, sigma x + sigma y + sigma z = I1. If you write it as sigma –x + sigma –y + sigma –z, these are rotating the coordinate system and that is equal to if you write sigma 1 + sigma 2 + sigma 3 that is not going to change, it is always going to remain the same, so that is why it is called as invariant. Similarly is the case for this also, irrespective of the coordinates, you can always write it. So you can write this in terms of principal stresses and get these values.

$$\sigma_z + \sigma_y + \sigma_z = l_1$$
$$\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_x\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{xz}^2 = l_2$$
$$\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{xz} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{xz}^2 - \sigma_z\tau_{xy}^2 = l_3$$

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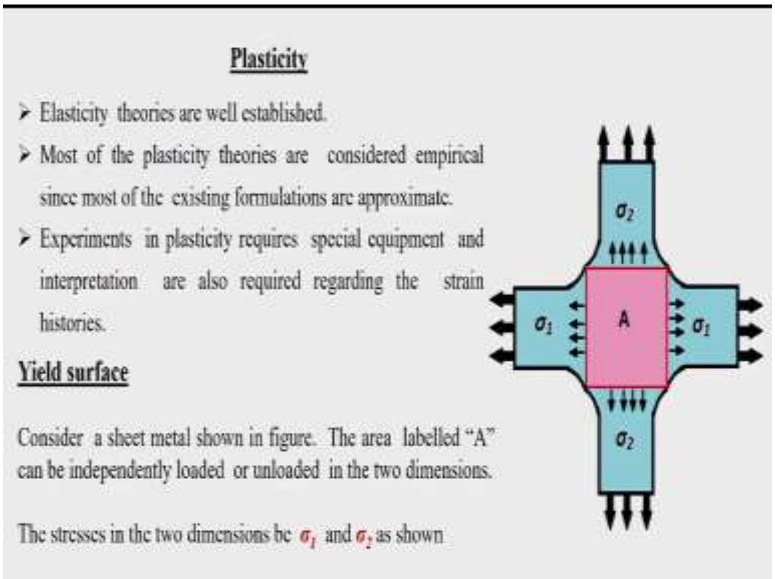

So in the plasticity theory, see unlike elasticity theory, it is well established, you can find out the relationship between the elastic deformation of a material when it is being stressed but it is not true with when you think about the plasticity theory because metals may be deforming in a different condition, different environment. You may have to know the history of the deformation, prior history of the deformation you may have to know. So let us see what are the theories which are there?

So most of the plasticity theories are considered empirical, most of them are empirical in nature and because most of them are from the existing formulations and these are all approximate conditions okay, but you do a lot of experiments and from the experimental data, you arrive at some empirical relationships and for determining the stresses at any point along any direction in an elastic case it is very easy.

You have to just connect the strain gauges and then you get it because the strains are very less and then you can find out the stresses when it is loading, but in case of plasticity, this is not true because the deformations are many times very large okay and you may need special equipment for getting this data like a strain during the deformation and the stress, stress of course, you can find out basically the strain in plastic deformation is very difficult to obtain, especially online and more than that the interpretation of the results are also required regarding the strain histories, you should also know about the earlier strain history also.

If you are dealing with a material which has been deformed earlier, then that is also very important. Many times, when you buy a material from the market, this may be subject to hot working or working cold working cold deformation, so what was the amount of deformation which has been given so that we can find out okay what will be yield strength of the material or if it is too high, the strain is too high, then you may have to do a heat treatment so that no your amount of strain which can be obtained during deformation can be increased okay.

So, these things are very much necessary. So we have to look at study something about the yield surface. If you just consider a sample which is shown like this okay in a cross way like this if it is there. So this is the case of a two dimensional state of stress. So we can just by applying some external load here, we can find out the stresses along the one direction and along the axis two direction we can find out, we can do that if you connect load cell on these two sides, then we can always find out what are the stresses in these directions.

If you can independently change the stresses by your good equality setup, then you can always vary this stresses which is may be if you take for the case, which is here which is shown by A, a square plate is here, what are the stresses in that direction, you can always change in a biaxial state of stress by changing this load and this load, so it is σ_2 and σ_1 is there and these are two mutually perpendicular directions.

So if in such a case, you are loading the material, and so you can find out this loading condition for this area A can be determined okay. So in this, let us say that the two cases of loads are σ_1 and σ_2 along axis 1 and axis 2.

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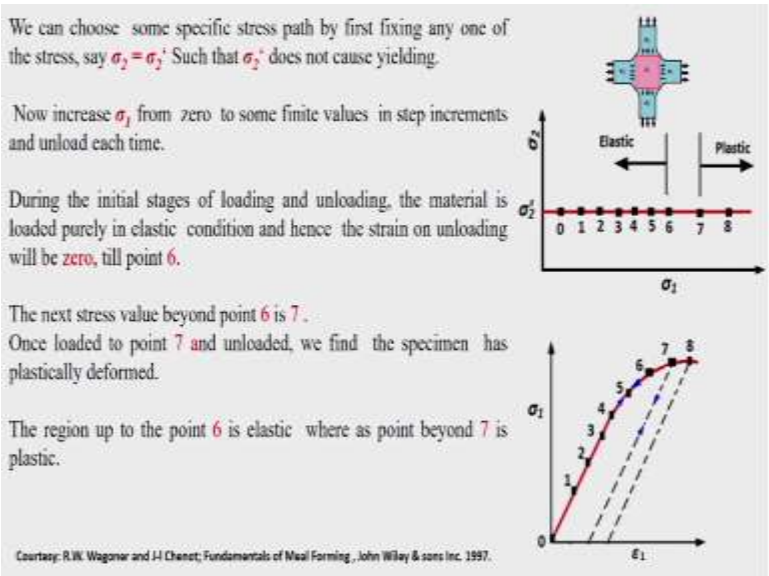

In a tensile specimen okay, you are having a uniaxial load, but here it is a biaxial load, so something let us just take arbitrarily along the axis 2, you fix load σ_2' okay. Once you fix that load, $\sigma_2'$and then keep on loading along the σ_1 direction, keep on increasing the load along the σ_1 direction, keeping the σ_2 fixed which is at σ_2' and then you keep on increasing the load.

Now, what happens is that when the say the load the stress versus strain curve will look like this what is shown here okay, it will look like this, may be after a small amount of strain, you unload it, it will come back to your 0 position. Again, you reload it to 2 and then unload it, it will again come back to initial stage and say 3, if you again unload it, it will come to the initial stage. These are the elastic regions.

The initial part you will see that Hooks law is applicable, but after certain stage, may be it is not linear, but nonlinear elastic comes, so you again increase the load, may be at 5, though there is a nonlinearity, but still it may come back to the 0 position and again you increase up to 6, there also it is unloading it, it is again coming back to the 0 position. So after 6, the material is elastic, initially it is linear elastic, then nonlinear elastic, but now you have some more increment in strain in σ_1, along the sigma along the one direction and in such a case, you reach up to load 7.

When you reach up to load 7 and you unload it, then you will find that it is not reaching the 0 position, rather a permanent amount of deformation has taken place, it is reaching at some value, so your strain along the axis 1 it has permanently deformed, so even if you unload it also, you will find that a permanent deformation has taken place. So, you don't know from 6 to 7 where exactly the plastic deformation has started, that you are not sure about it, but somewhere between 6 and 7.

So 6 was the last point of elastic deformation, but if your strain increment was very less, the increment in the strain was very less, may be you may be able to find out exactly where it was, but that is a very cumbersome work because taking very small interval and carrying out that it will take lot of time. So, but in that case no, if you just put a limit, between say 6 and 7, the strain was there. So up to 6, it was elastic and when reach 7, beyond 7, it is plastic, so in between 6 and 7, somewhere it was, there is a transition from elastic to plastic behaviour. The material will not return back, so you don't know the exact position of that.

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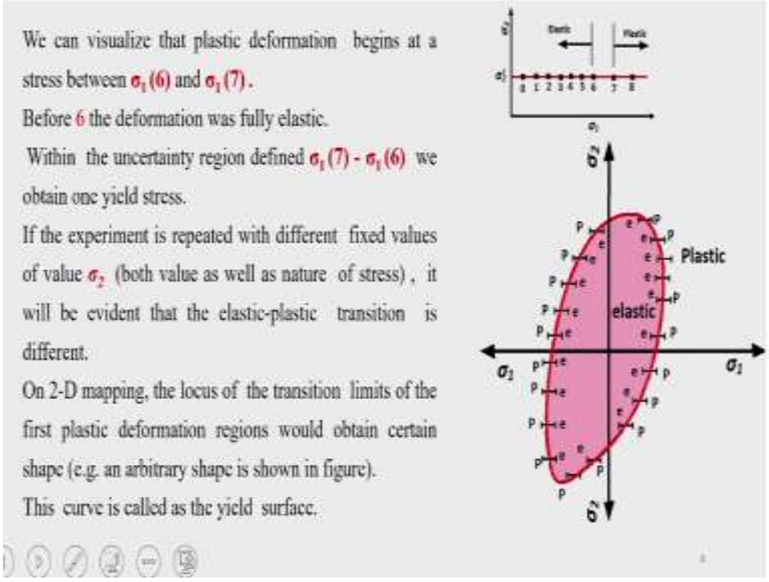

So in such a case, we can take that case here. See let us say, this was the σ_2', and this was the elastic region, this was the plastic region, somewhere in between was there. So now you just increase this load along the axis 2 σ_2' some other value, you take it somewhere here and carryout the same experiment, may be at say different value of σ_2, find out where the transition between elastic and plastic was taking place. So you can get a different value here, so that is what here you can get it.

Similarly, you keep on changing that σ_2 value and again keep on increasing the σ_1 and then you can get like this. So for different values of σ_2' in the tensile region and increasing the load along the 1 axis, you will get this one and now if you reverse the stress, σ_1, may be make it compressive here, then you will get different values. So there are 2 cases, so if you just spot that where the elastic last part of the elastic region was there, the elastic deformation was taking place and the first part or plastic deformation was taking place.

So you can either join these points which I have been labeled as e or may be by p or in between I say midway between that also you can take because you don't know the exact value, so that and then plot with a difference, in this case it is tensile, this case it is compressive σ_2 along the vertical direction, it is positive here and this is negative direction, so you keep on changing with all these types of stresses and then joint that point, the locus of that points at which it is shift from elastic to plastic region.

You will join all those points, the locus of the points, you will get a curve, arbitrary curve like this, this is an arbitrary curve, we will come to that exact shape what it will be, that will be coming in the subsequent lectures okay. So, this line which you join is called the yield surface or yield locus you call it as and what is important in this is as long as the material is inside this curve, the material is deforming elastically.

So, it has nothing to do with your history of loading or strain history has nothing to do with it, but the moment, the boundary of that, when you reach that boundary, that is the point that which it is going to shift from the elastic to the plastic region okay. The transition is going to take place. So beyond that, whatever you are doing it is the plastic deformation which is taking place. So, this is that called as yield locus. Now, we wanted to find out what is the exact shape under different condition. So there are different conditions of yield criteria where the metal yields or deforms plastically, what are those criteria, let us look at that.

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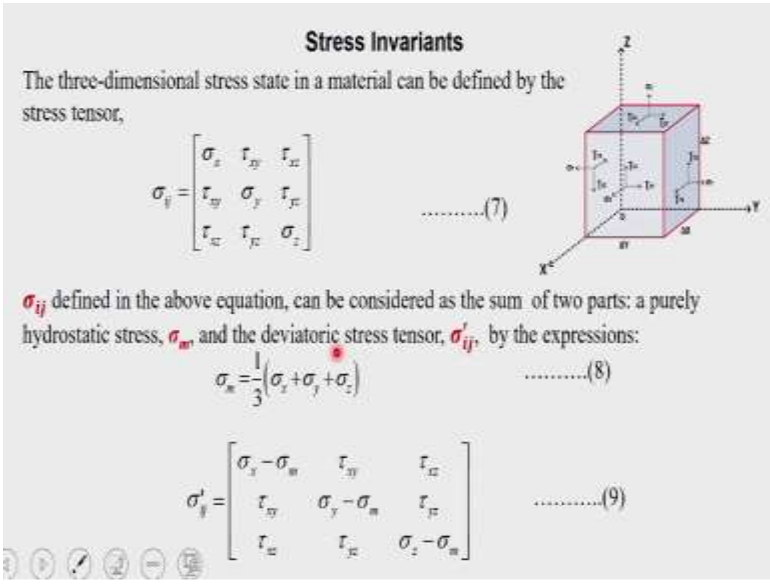

So for that, we will have to again come to the stress and strain okay, the state of stress at any point. So as discussed earlier, suppose we are considering cuboid like this and then you are applying certain loads okay, may be depending upon a 3 dimensional state of stress if it is there, you can have a stress tensor defined by σ_{ij} because that based on the normal stresses on each face and the shear stresses which are there, so you can say that the normal stresses are represented by σ_x, σ_y and σ_z.

The shear stresses on the respective planes x, y, and z you can say thatτ_{xy},τ_{xz}, these symmetric tensor, so yz and τ_{xy}, and τ_{yz}, and $\tau_{xz}$and τ_{yz}. So, you can write in this second row of a tensor format, so you can write it. Now, later we will find that this σ_{ij} defined in above equation depending upon any condition, we can have consider it as the sum of 2 parts depending upon what are the loads, what are the stresses in a material, may be you may be applying stresses from different directions and other things and then at any point depending upon a particular coordinate system, you may get these things okay.

Now the thing is that this particular stress σ_{ij} you can consider it as a sum of 2 components, one is a hydrostatic component σ_m, defined by say 1 by 3 into the average of the normal stresses and another is called the deviatoric stress tensor so $\sigma_{ij}^{'}$ okay. So$\sigma_{ij}^{'}$ is defined, so that means σ_{ij} is a sum of $\sigma_{ij}^{'}$ + σ_m where σ_m is the hydrostatic stress defined by $\sigma_x + \sigma_y + \sigma_z$ by 3 and so if you subtract from that you will get the deviatoric stress.

$$\sigma_m = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)$$

So here, this hydrostatic stress, it does not contribute to the deformation okay or change in the shape of the material during plastic deformation, but it only contributes to the change in the volume of the material, so it has nothing to do with the deformation, whereas the deviatoric stress component or stress tensor $\sigma_{ij}^{'}$ that is the component which contributes to the shape change during plastic deformation when you are applying a load okay. So, we can just consider this.

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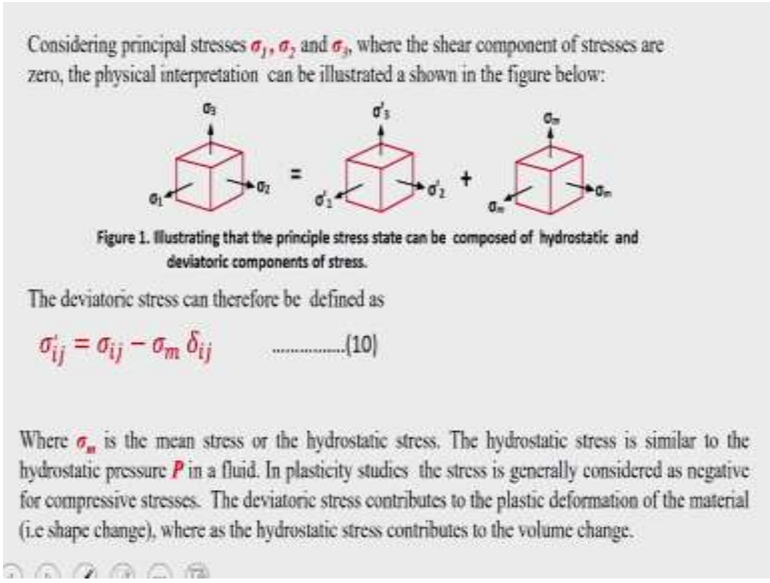

If you consider instead of the general Cartesian coordinate, you reorient the coordinate system so that the $x1, x2, x3$ are oriented along the principal stress directions, $\sigma_1, \sigma_2 and\ \sigma_3$ and then that stress tensor, so that stat of stress at any point can be, **s**ay what I was mentioning is that it is the sum of the hydrostatic component along this direction, which is equal here sigma m and the deviatoric component defined by σ_1', σ_2', and σ_3'okay.

So, in that case, this deviatoric stress which contributes to the plastic deformation can be written as sigma ij – sigma m x your conical delta that is delta ij okay, so that way we can, or comparing the if you are looking at the component σ_1 or $\sigma_1' = \sigma_1 - \sigma_m$ and = σ_2 dash = $\sigma_2 - \sigma_m$ or say $\sigma_3' = \sigma_3 - \sigma_m$, that way also we can write it. So individual components if you look at it, that way we can write it, but in the general tensor format, it can be written like this.

$$\sigma'_{ij} = \sigma_{ij} - \sigma_m \delta_{ij}$$

So, the deviatoric stress contributes to the plastic deformation of the material whereas the hydrostatic stress contributes to the volume change, not to the change in the shape.

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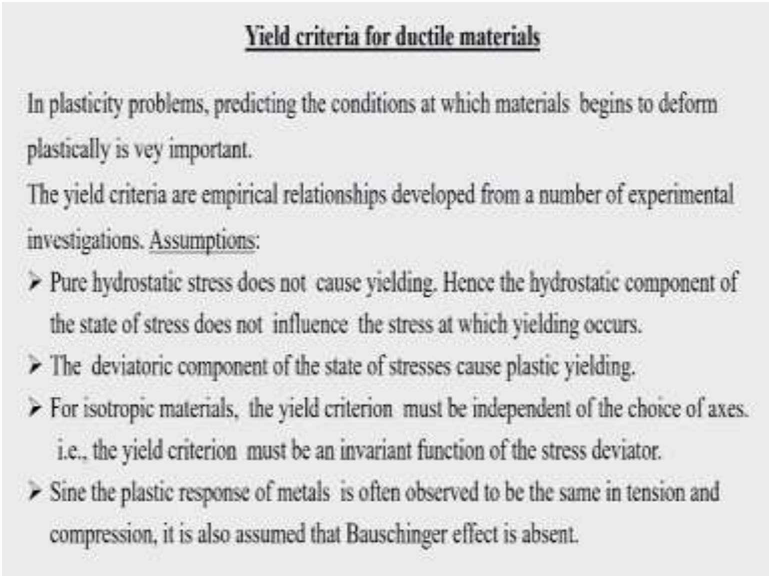

Now, so we have to look at various criteria for yielding, how the metal will deform when you are causing the yield locus okay, when you are outside the yield locus. So you have to, so what is the exact condition the boundary between the elastic and the plastic region, that is what we have to find out, so that is what the yield criteria for ductile material. We will be discussing with the ductile material, not brittle material okay. So in plasticity problems, predicting the condition at which materials begins to deform plastically is very important okay.

The yield criteria are empirical relations developed from a number of experimental investigations and some of the assumptions while developing or certain things which are going to assume are that pure hydrostatic stress does not cause yielding, so as we have mentioned. Hence the hydrostatic component of the state of stress does not influence the stress at which yielding occurs. So whether you have the hydrostatic component higher value or lower value that is not going to contribute the yielding okay.

Second the deviatoric component of the state of stress cause the plastic yielding and for isotropic materials, the yield criteria must be independent of the choice of the axis okay, so that is also another thing, that is the yield criteria must be an invariant function of the stress deviator. So these are very important point we have to do. So, it will not, it is independent of the choice of the axis and that means it is one of the invariant function of the stress deviator.

Since the plastic response of the metals is often observed to be the same in tension and in compression in metallic material, so one of the assumptions which we are having is that

Bauschinger effect is almost absent in the material, so that is the one we have to consider this. So under these conditions only the yield criteria has been developed. So let us do the two main important yield criteria for defining the plastic deformation are the von Mises strain energy criteria and the Tresca criteria okay. So, let us see that what are they.

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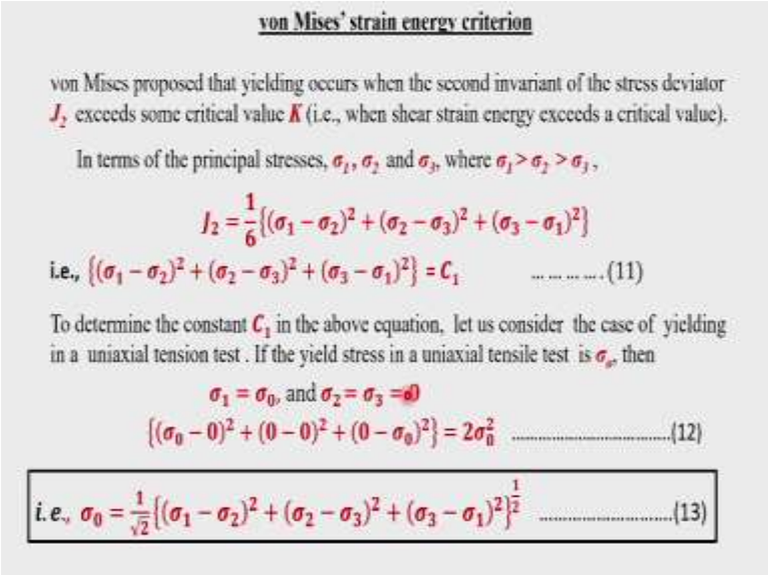

Von Mises proposed that yielding occurs when the second invariant of the stress deviator J2 exceeds some critical value K, says it is a stress deviator of the, second invariant of the stress deviator, earlier we were talking about the σ_{ij}, but here it is about the σ_{ij}', it exceeds some critical value, that means so he saying another way it can be when the shear strain energy exceed a critical value, the plastic deformation, plastic yielding of the material takes place. So that is a different way of telling it.

So, in terms of the principal stresses, σ_1, σ_2, and σ_3 where σ_1 is greater than σ_2 and σ_3 is the lowest principal stresses. Then this J2 can be written with the derivation we are not going here, it can be written as 1 by 6 into sigma 1– sigma 2 square + sigma 2 – sigma 3 square + sigma 3 – sigma 1 square, the whole square, so that is the J2. When this right hand side reaches a critical value K or some constant value, then we can say that okay it is plastic deformation or plastic yielding of the material thing that is the boundary point.

$$J_2 = \frac{1}{6}\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\}$$

$$\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\} = C_1$$

So determine the consent C1 in the above equation, we have to first consider the case of yielding in a uniaxial tension, so we can find out in a uniaxial tension, putting up the conditions and then we can find out what how we can get this value of C1 okay. So, in a uniaxial tension test, what happens, when you are deforming, say for example if you take a wire, a long wire and deform it, then the material will deform only and you are considering the tensile axis as your principal stresses, other 2 principal axis, the stresses are almost negligible we can say.

That means, the material will yield which σ_1 reaches the yield stress sigma not. So this sigma not is the yield stress of the material and in that condition, the other two principal stresses in a uniaxial tensile test where the gauge length is very large, the sigma 2 and sigma 3 are considered as 0, is equal to 0. So if you substitute these conditions into this equation number 11, so we can say that σ_1 is substituted by sigma 0 – 0 square + these two are 0 + 0 – sigma 0 square = 2 sigma square.

$$\{(\sigma_0 - 0)^2 + (0 - 0)^2 + (0 - \sigma_0)^2\} = 2\sigma_0{}^2$$

So from that, if you, that means the right hand side of this equation comes to say 2 sigma square for a uniaxial testing, so that means sigma 0 is equal to if we substitute that, the yield strength of the material is 1 by root 2 into sigma 1 – sigma 2 square + sigma 2 – sigma 3 square + sigma 3 – sigma 1 square of whole raise to 1 by 2, so that is what we will get that.

$$\sigma_0 = \frac{1}{\sqrt{2}}\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\}^{\frac{1}{2}}$$

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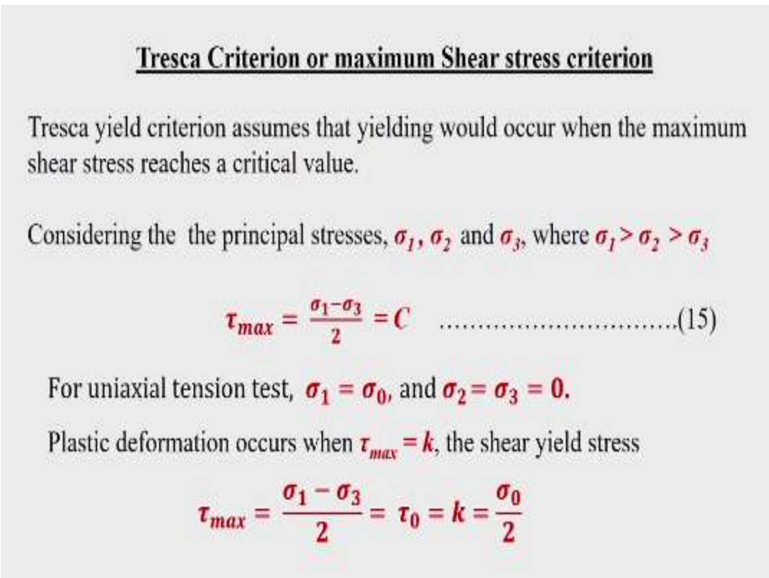

The other condition Tresca criteria or maximum Shear stress criteria is that it assumes that yielding would occur when the maximum shear stress reaches a critical value. If you look at the Mohr's circle value know that maximum shear stress in a 2D space no, you will find that sigma 1 is the largest and sigma 3 is the least and sigma 2 if you assume it as 0, then T max = sigma 1 – sigma 3/2, so that is a constant value. So when this shear stress, maximum shear stress, when it crosses certain value due to your state of stress, then yielding will deform.

$$T_{max} = \frac{\sigma_1 - \sigma_3}{2} = C$$

So for a uniaxial tensile test if you just consider the same condition, tau max = k where the plastic deformation occurs when tau max = k where k is the shear yield strength of the material okay. So, that is what we will get. So tau max in that case because sigma 3 = 0 in a uniaxial tensile test, it will be sigma 1/2, so sigma 1 = sigma 0. So you will find that k = sigma 0 by 2 under the Tresca criteria okay or under the maximum shear stress criteria, you will get this condition.

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \tau_0 = k = \frac{\sigma_0}{2}$$

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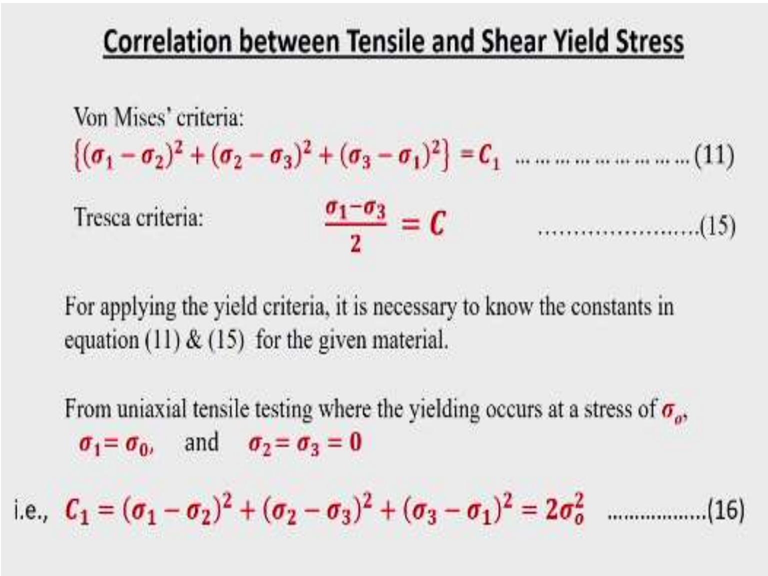

Now, we wanted to have a correlation between the tensile and shear yield stress, under these 2 conditions, so that is very important okay. So, the von Moses criteria, we have mentioned sigma 1– sigma 2 square + sigma 2 – sigma 3 square + sigma 3 – sigma 1 square = C1, so that means when it reaches a critical value, it deforms and Tresca criteria is sigma 1– sigma 3 by 2 = C, that means when that reaches a critical value, the deformation takes place okay.

So for applying the yield criteria, it is necessary to know the constants in equation 11 and 15 okay for the given material. So again if you look at this condition, say this we have arrived earlier itself.

$$\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\} = C_1$$

$$\frac{\sigma_1 - \sigma_3}{2} = C$$

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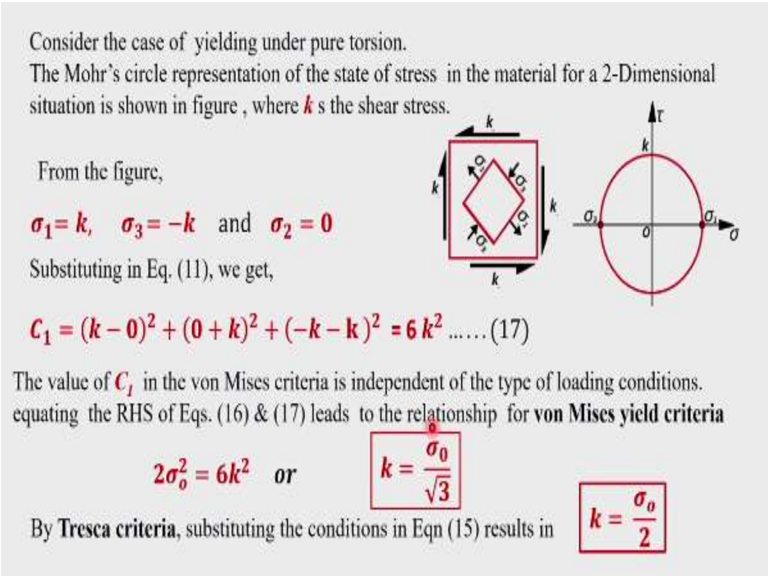

Now let us consider the case of yielding under pure tension, torsion sorry, yielding under poor torsion, and if you look at the Mohr's circle representation for a 2-dimensional k, this is a σ_1 and this is σ_3 and σ_2 is equal to 0. So, for the k if you look at it, these are the state of stress, here the k is there, shear stresses are there, but if you look at the principal stresses, σ_1 and σ_3, it will be looking like this.

So, the maximum shear stress k = sigma 1 – sigma 3 by 2, so that is what sigma 1 = k from this condition and sigma 3 = –k okay because this is the 0 value and sigma 2 = 0. If you substitute that in equation 11, you will get the C1 as k that is equal to 6 k square okay and the value of C1 in the von Mises criteria is independent of the type of loading conditions, so you get in the right hand side of equation 16 and 17, the other case say it was $2\sigma_0^2$ and here you are getting it as 6 k square.

$$k = \frac{\sigma_1 - \sigma_3}{2}$$

$$C_1 = (k-0)^2 + (0+k)^2 + (-k-k)^2 = 6k^2$$

$$2\sigma_0^2 = 6k^2$$

So 2 sigma square = 6k square or from that, you can find out that for von Mises criteria, the shear yield strength is equal to your uniaxial yield strength/root 3, so that means sigma 0/root 3 where sigma zero 0, the uniaxial shear strength of the material, tensile strength of the material, uniaxial yield strength of the material whereas if you substitute that sigma 1 – sigma 2 by 2 where sigma 1 = k and sigma 2 = 0, sorry sigma 3–k, then from that 2k = sigma 0 you will get or k = sigma 0 by 2.

$$k = \frac{\sigma_0}{\sqrt{3}}$$

So if you look at that for the von Mises criteria, the shear stress you get a relationship for sigma 0 by root 3 whereas for under Tresca criteria you get sigma 0 by 2 okay, so this is lower than this value. So, these two values you are getting.

$$k = \frac{\sigma_0}{2}$$

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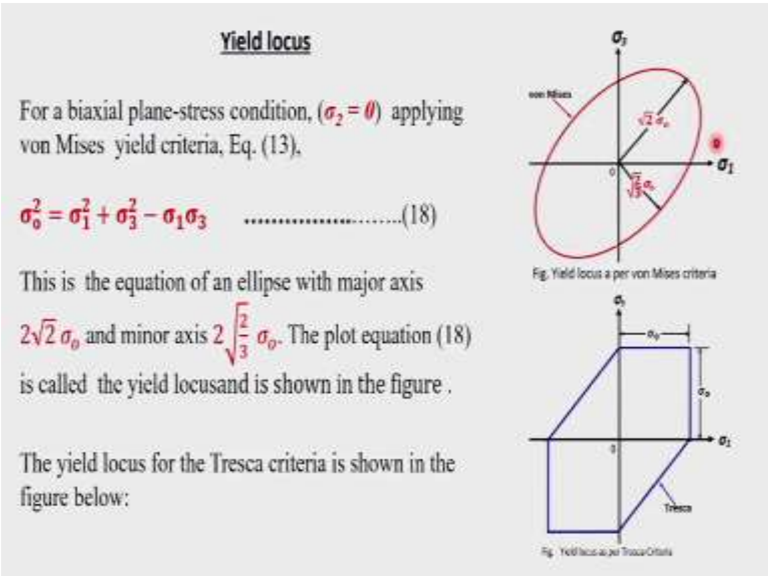

Now we have to come to the exact shape of the yield locus, so that is why we were discussing about all those things. So for a biaxial plane stress condition when you assume $\sigma_2 = 0$ and applying the von Mises criteria, you will get this relationship, sigma 0 square = sigma 1 square + sigma 3 square – sigma 1 sigma 3. So this relationship it is an equation of an ellipse okay on the $\sigma_1\sigma_3$ plane, so that is what you are getting that ellipse.

$$\sigma_0^2 = \sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3$$

It is having a major axis of 2 root 2 σ 0 and a minor axis of 2 root 2 by root 3 σ 0 or we can say the major semi axis is root 2 σ 0 and the minor semi axis is root of 2 by 3 into σ 0. So, you will get this one on the σ_1 σ_3 plane and σ_2 is perpendicular to that, normal to this plane, so that is what. The plot of this equation is called as yield locus and is shown in figure. So yield locus means, so inside it is exactly the elastic deformation taking place whereas outside it is the plastic deformation which is taking place.

So once you cross this boundary, that is the onset of plastic deformation, whether, however, you change it, σ_1 or σ_2 or 3, you give a different set of values and other things the moment it crosses that. So as long as you are inside it, there is no problem, you really unload it, it will again come back to that, but the moment you by any means if you cross this, this red line, this is the yield locus, so by von Mises criteria, this is the condition and now the yield locus of the Tresca criteria also you can get it in this form, this is like a hexagon type thing you are getting it.

So, that is what you are getting, so say here you will find that for uniaxial case no when you are applying the load along this direction, it deforms at this place whereas along this direction it deforms at this case, the same thing is there, but in between you will get these two. When you are varying σ_1 and σ_3, so you will get in a different way.

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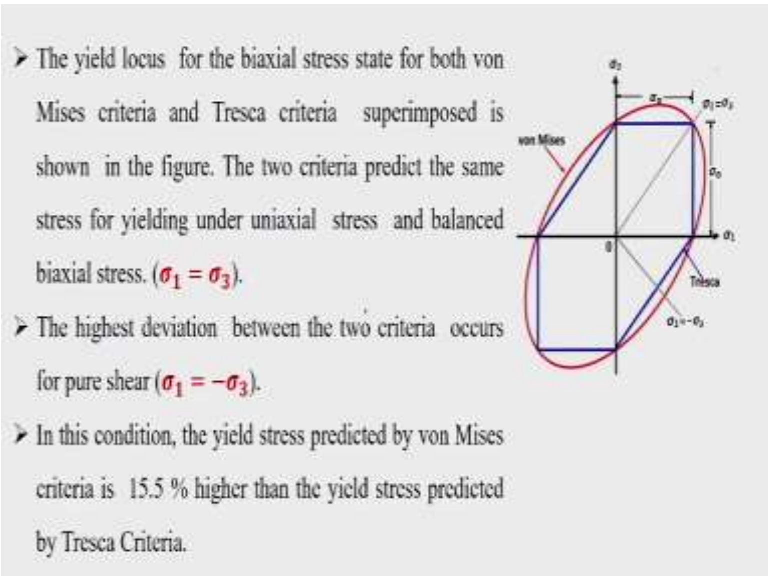

So here, the two conditions, the von Mises criteria and Tresca criteria we are just superimposed each other and it is shown here. The red line corresponds to von Mises criteria and the blue line corresponds to the Tresca criteria okay. So, yield locus for the biaxial stress state for both von Mises criteria and Tresca criteria superimposed is shown. The two criteria predict the same stress for yielding under certain conditions. So mainly, they have the same values for uniaxial loading, you see that if it is only along $\sigma_3 = 0$ and σ_1 is then.

For von Mises criteria and Tresca criteria, you will get the same value of the yield strength, similarly is the case here also, whether σ_1 is 0 and σ_3 is positive, you will find that okay this is the same value for the material to yield and these are for isotropic material. Similarly, if you are just reversing, in metallic material, you will find that the yield strength in tension and yield strength in compression is more or less same okay, so there is not much variation compared to some anisotropic material okay. So here, that is the case.

So, here you will find that whether it is σ_1 is the negative direction or just compressive in nature and σ_3 is 0, then you will find that for von Mises criteria and Tresca criteria, they are same. Similarly here also, it is considered same and there also same for the balanced biaxial stress, that means when σ_1 and σ_3, $\sigma_1 = \sigma_3$, then also it is same okay. So, same case is here also, say σ_1 and σ_3 when they both compressive in nature also, yielding will takes place, you will get as per von Mises criteria and Tresca criteria, you will find that they are same okay.

Now for pure shear, the deviation, deviation will be when $\sigma_1 = -\sigma_3$ for the pure shear stress, you will find that the deviation is the largest at this condition okay so and similarly here also, this, this is the largest deviation is at this point. So that means, $\sigma_1 = \sigma_3$. In this condition, the yield stress predicted by von Mises criteria is almost 15% higher than that predicted by Tresca criteria because this red line, it gives the locus of the yield criteria as per von Mises criteria and this red line gives that for the Tresca criteria.

The highest value you will get when it is $\sigma_1 = -\sigma_3$. So when you look at this, under these two different criteria, inner plane, we are not considering the deviatoric stress, so this you call it as pi plane also, this is also called pi plane okay or deviatoric plane you call it as. So, you will find that, see earlier we discussed that the hydrostatic component of the stress is not going to contribute to the change in the shape of the material or plastic deformation. Hydrostatic component contributes only to the change in the volume of the material, not in the change in the shape or it is not contributing the plastic deformation okay.

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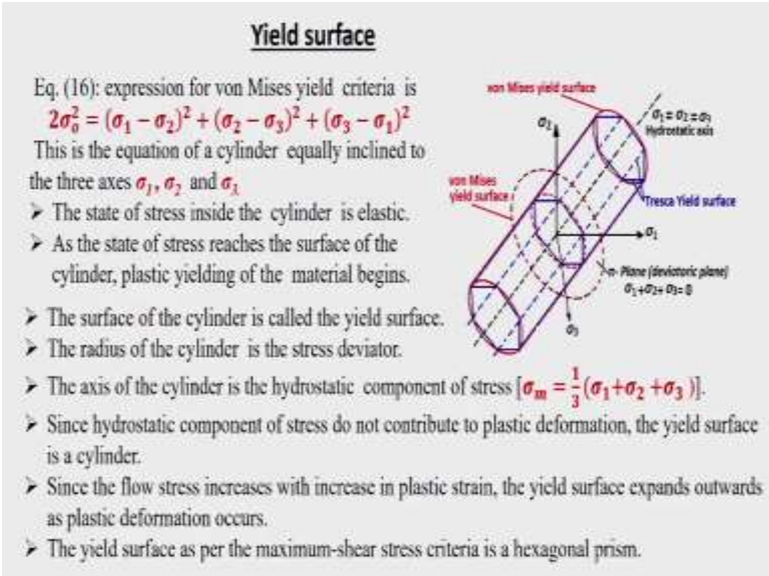

So if you look at this condition, this what you call it as, the von Mises yield criteria, you got this expression 2 sigma square = sigma 1 – sigma 2 square + sigma 2 – sigma 3 square + sigma 3 – sigma 1 square. This is an equation of a cylinder okay, so that is also there. Now if you just keep on an axis which are mutually inclined to all the 3 axis, so this will be that particular case, you call it as hydrostatic axis okay, so that is equal to sigma 1 + sigma 2 + sigma 3 by 3.

$$2\sigma_0^2 = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$$

$$\sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

So whether you are increasing it, only thing it will be shifting towards that thing. So, the entire thing for different values of hydrostatic stress sigma m, you will find that you are getting the case as a cylinder okay and the equation for a cylinder which is equally inclined to three axis σ_1, σ_2, σ_3 and this is your yield surface, this shown by the red line, corresponds to the one which is yield surface okay, so that is a cylinder.

Now that is the case when the material is going to deform, but after deformation what happens, you unload it, you will find there is a permanent deformation has set in and now since the positive strain has taken place, may be the work hardening has taken place, so your yield strength will increase. So, you will find that in such case are there when the material deforming, your deviatoric plane that also just expands outward during the plastic deformation.

So, the condition state of stress inside the cylinder, the von Mises criteria, inside the cylinder whether whatever be the value of your hydrostatic stress, you will find that your cylinder, its cross section is an ellipse under that thing if you take a section, it is elastic, the moment the surface of that you crosses, it is a plastic deformation, the yielding is initiated or it is the onset of yielding, so that is the case. So, as the state of stress reaches the surface of the cylinder, plastic yielding of the material begins.

The surface of the cylinder is called the yield surface and the radius of the cylinder is the stress deviator okay. So, all these combinations are being found out. The axis of the cylinder is the hydrostatic component of stress given by this relationship and since the hydrostatic component does not contribute to a plastic deformation, the yield surface is a cylinder. So when you are, when the plastic deformation takes place, you will find that your pi plane or the deviatoric plane, it expands outward okay.

So, then you may get, once a plastic deformation has taken place, then new set of plane you will get it with the different values okay, so that is the thing. Now since the yield surface as per the maximum shear stress criteria, you will find it is a hexagon. So, for von Mises criteria, it was a cylinder whereas for this particular case under Tresca criteria, you will find it as a hexagonal prism. So, immaterial of what is our hydrostatic axis, it is not going to change, only different is only in this okay.

Initially, we discussed about how the yield surface is looking, now we came to the shape of the yield surface, depending upon whether you are following the von Mises criteria or you are following the Tresca criteria, here also the blue line represents the case of Tresca and red line corresponds the von Mises criteria and then you will find that okay this is the hydrostatic axis which is equally inclined to the triple directions. So, now let us look at other conditions.

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Levy-Mises Flow Rule

- During elastic deformation, the strains are determined from the stresses using the Hook's law and does not consider how the particular state of stress was achieved. (i.e loading history is not important in elastic deformation).
- During plastic deformation, the strains are dependent on the entire loading history. The incremental plastic strains during the entire loading path is summed up to determine the total strain.
- The Levy-Mises equations or the **Flow rule** provides the relationship between stress and strain for an ideal plastic solid (the elastic strains are negligible) that deforms under constant volume.
- As the hydrostatic stress has no influence on the plastic deformation, it is only the deviatoric component of stress that causes the shape change.
- The Levy-Mises Flow Rule states that during plastic deformation of an ideal plastic material, the ratio of plastic strain increments to the current deviatoric stress remains constant. i.e.,

$$\frac{d\varepsilon_1}{\sigma_1'} = \frac{d\varepsilon_2}{\sigma_2'} = \frac{d\varepsilon_3}{\sigma_3'} = d\lambda$$

See if you look at during elastic deformation, we wanted to discuss also about the Levy-Mises rule. So during elastic deformation, the strains are determined from the stresses by using the Hook's Law. So you can find out if the stresses is this much, these are the strain, may be along this direction, that direction, so may be direction one or axis 1, axis 2, or axis 3 or principal strain directions, these are the strains which are there, we can always find out.

It is very well established, and in that case, there is no need for you consider the particular state of stress, how which was achieved, what was the strain path it is not important to know about that in elastic deformation okay. So that means, a loading history is not that important whereas during plastic deformation, the case is entirely different. The strains are dependent upon the entire loading history, how you reached it, the plastic strain.

So the incremental plastic strain, when you look at it, the plastic strain increment we have to look at, whether it was towards one direction, you elongated it along one direction and then applied a compressive stress and then compressed it, so these things are very important okay. So the incremental and the total strain will be the sum of the incremental plastic strains during the entire loading path, you have to consider that okay, so that is how you get the final total strain.

So this Levy-Mises equation or the Flow rule provides the relationship between stress and strain for an ideal plastic solid, that is the elastic strains we are considering it is negligible, and in that case, the Levy-Mises equation which is also called as the Flow rule, so it explains that it gives you the relationship between stress and strain for the ideal plastic solid that deforms under a constant volume. As the hydrostatic stress has no influence on the plastic deformation, it is only the deviatoric component of the stress that causes the shape change.

So that repeatedly I am telling it and the Levy-Mises Flow rule it states that during plastic deformation of an ideal plastic material, the ratio of plastic strain increments to the current deviatoric stress remains constant, so that is it, for the instantaneous value of the deviatoric stress, the ratio of the plastic strain increment d epsilon 1 by sigma 1 dash = d epsilon 2 by sigma 2 dash = d epsilon 3 by sigma 3 dash and that remains always a constant which is the d lambda. So this is the Levy-Mises Flow rule which is applicable for this. Ya, for today, this is.

$$\frac{d\varepsilon_1}{{\sigma_1}'} = \frac{d\varepsilon_2}{{\sigma_2}'} = \frac{d\varepsilon_3}{{\sigma_3}'} = dlamba$$