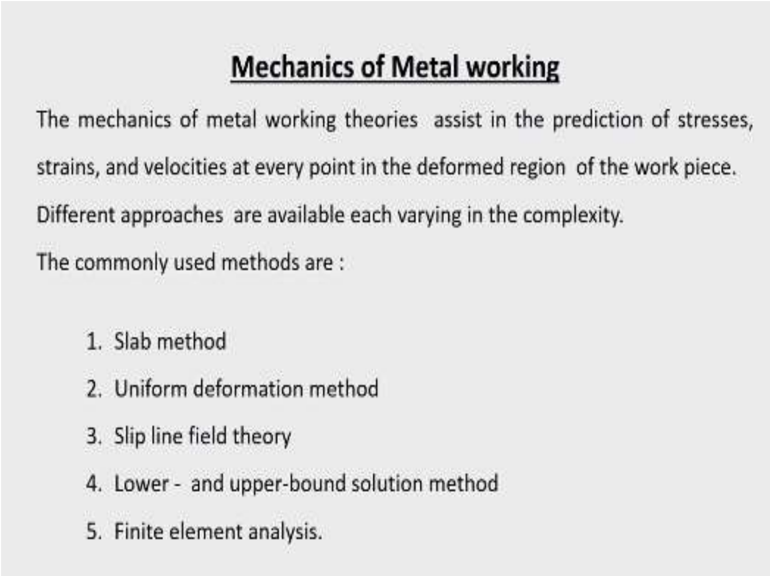
Plastic Working of Metallic Materials

Prof. Dr. P. S. Robi
Department of Mechanical Engineering
Indian Institute of Technology – Guwahati

Module 2
Lecture - 2
Mechanics of Metal Working

So today, we will start with mechanics of metal working. This mechanics of metal working is very important when you talk about the metal working operations.

(Refer Slide Time: 00:42)



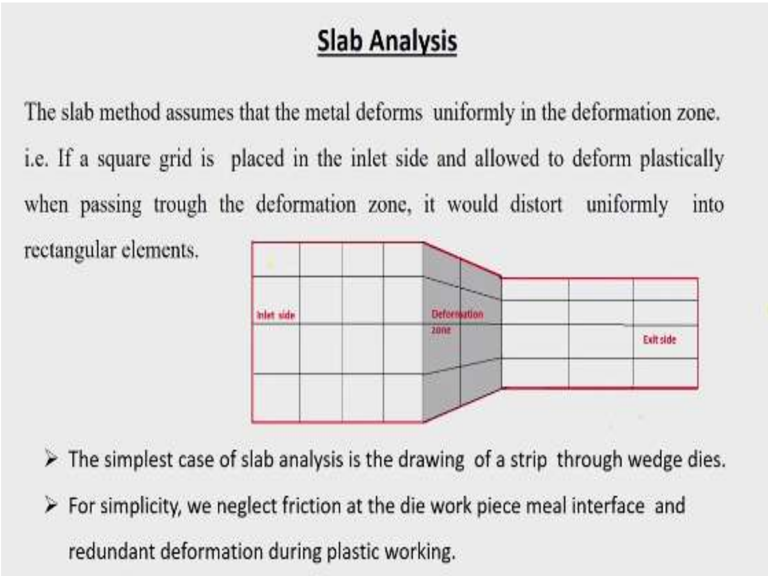
The theories in the mechanics of metal working theories, it assists in the prediction of stresses, strain, and velocities at every point in the deformation zone of the work piece. When the metal is subjected to plastic deformation, inside a deformation zone, so we can predict the stresses, strains and velocities of the metal flow, plastic flow and you will find a large number of approaches are available for describing the mechanics of metal workings and each one is having its own limitations and advantages.

Depending upon the complexity of the technique also it varies, but you will find that the commonly used methods are basically the simplest of, that is the slab method and the uniform deformation method. These two are the most simplest things which we can arrive at, to start with, that will be the best thing, and then followed by slip line field theory. So, slip line field theory also I will be explaining, but may not be doing much tutorial in the slip line field theory because it involves some drawing also.

Next is the lower and upper bound solution methods and finite element methods. So, all these techniques describing in this course will be beyond the scope of this course, so I will not be discussing about that. I will be discussing about the slab method and the uniform deformation method, taking a particular example of wire drawing operation, so and we will tell that what are the assumptions on that and what is the basis of the slab method and how these theories are evolved and may be that we will extend it to other processes like forging and explosion and other plastic working techniques also.

So but we will be going in depth in the slab method, may not be, not require it among the analysis altogether a full course and other thing that we cannot do. So here, we will just see what is to be done.

(Refer Slide Time: 02:50)

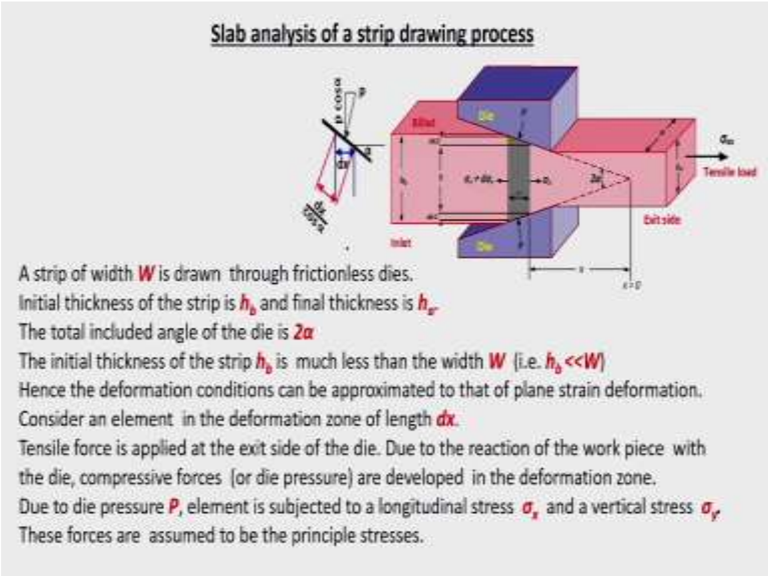


So in the slab analysis, we assume that metal deforms uniformly in the deformation zone. So, suppose this is the deformation zone, and occur that we are just may be pulling along this direction and the metal is trying to deform, this is the inlet side and this is the outlet side. So at the inlet side, if we keep a square grid and while passing through the deformation zone and when it comes out, this square grid will be distorted to obtain a rectangular element, so that is what is basically it is just shown by this figure in this case okay.

So that is what the slab analysis is assumed and the simplest case for the slab analysis which generally people describe is the drawing of a strip, a strip drawing in between through wedge dies okay. So in this case for simplicity, let us neglect the friction at the die work piece metal interface and also the redundant deformation which is taking place inside the die during the plastic deformation.

These few things we can neglect it and we will assume, we will predict the draw stress, what is the stress required for drawing of this strip when it is passing through a wedge die and later we will assume the friction also and then come out, find out, derive the equation for obtaining the relationships.

(Refer Slide Time: 04:24)



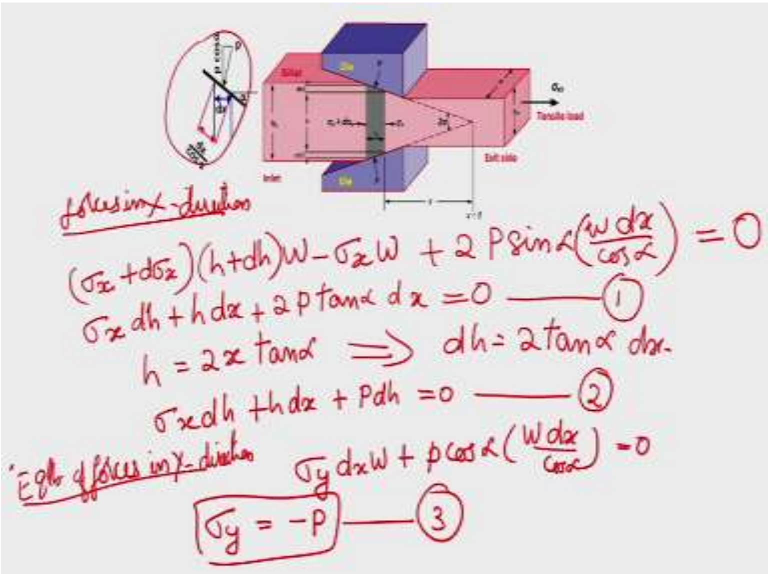
So in the slab analysis, the typical case is this where you are assuming a metal is being pulled by means of a tensile load here okay. This is an inlet and then this is a converging dye and having an included angle of 2α and then you are pulling it through this end, this end you are pulling it. The width of the strip is W and the initial height of the billet is say h_b and when it is passing through the die, it gets deformed between the deformation zone and then when it comes out, the exit side, you will find out the height of the billet is h_a which is given here okay.

If we take a case where the strip, the width of the strip W is far greater than the initial thickness of the strip, then end up case no the case of the deformation can be approximate to that of a plane strain deformation condition okay. So, let us now assume this particular case where the metal is being deformed. It is by the tensile load and we assume an element of thickness dx in the deformation zone or may be the length of the element is dx in the deformation zone.

So the tensile force is applied at the exit side of the die and due to that when you are pulling it, the reaction between the work piece and the die that will result in compressive stresses or may be a compressive forces which are developed at the interface of the die and the work piece where it comes in contact. So, it exerts a die pressure P, which is normal to the die surface okay and due to this die pressure, because this is the reaction when you are pulling with a tensile load.

So we will find an indirect compressive forces are generated, this is what you can find out, and due to this pressure P, the die pressure P, this element is subjected to two stresses, we can say these are the principal stress, one is the σ_x in the longitudinal direction or the longitudinal stress σ_x and another is a vertical stress which is along this direction, and the vertical stress σ_y okay. So, these forces are assumed to be the σ_x and σ_y, we are assuming it to be the principal stresses.

(Refer Slide Time: 06:51)



See if you take this element for equilibrium condition, we can say what are the longitudinal stresses okay. So equilibrium in the x direction, say this is the x direction, this is the origin where this die angle meets so that is the $x = 0$ and we are taking along the direction in the left side okay. So, the equilibrium forces in the x direction you can just assume there, there are 2 cases, one is due to the change in the longitudinal stress with x increasing positively towards the left this side, so that we can write it as the equation is sigma x + d sigma x into h + dh x W the width of the strip – sigma x into W.

This is one of the change in the longitudinal stress with x increasing towards the left side and next is the this uh the horizontal component of the force along the x direction when you resolve this force p so that will be you can say that it is a, so that also if you add it up, it will be 2 P sin alpha x, from this geometry, we can just derive that it is equal to Wdx by cos alpha, so for equilibrium this should be equal to 0.

$$(\sigma_x + d\sigma_x)(h + dh)W - \sigma_x W + 2PSin\alpha\left(\frac{Wdx}{Cos\alpha}\right) = 0$$

Now, the thing that under equilibrium, the sum of this total forces is equal, if you just simplify this and if you neglect the higher ordered terms, we can write that we will arrive at sigma x and say do a simple mathematical manipulation when you do W all get canceled off, so you will arrive at a sigma h dh + hd sigma x + 2 P sine tan alpha dx = 0. So, we are getting this equation, may be this is equation number 1. Now, the thing is at this figure, this geometry, we can also write that h = 2 x tan alpha so that which implies that the differential form dh = 2 tan alpha dx.

$$Equilibrium\ forces\ in\ X - direction$$

$$\sigma_x dh + hdx + 2Ptan\alpha dx = 0$$

$$h = 2xtan\alpha$$

$$dh = 2tan\alpha dx$$

So if you substitute this relationship in equation number 1, so we will get as sigma x dh + hdx two + 2 tan alpha dx, that is equal to dh, so that we can write it as Pdh = 0 okay, we can write this equation number 2 okay. So, similarly if you write the equation for the equilibrium forces in the y direction, now we were writing along the x direction, so forces in the x direction. Now, equilibrium forces in the y direction is you write that, we can just get the same way that is sigma y into dxW + P cos alpha into Wdx by cos alpha = 0 or we can write it as sigma y because this cos and cos alpha will go, is equal to minus = –P.

$$\sigma_x dh + hdx + pdh = 0$$

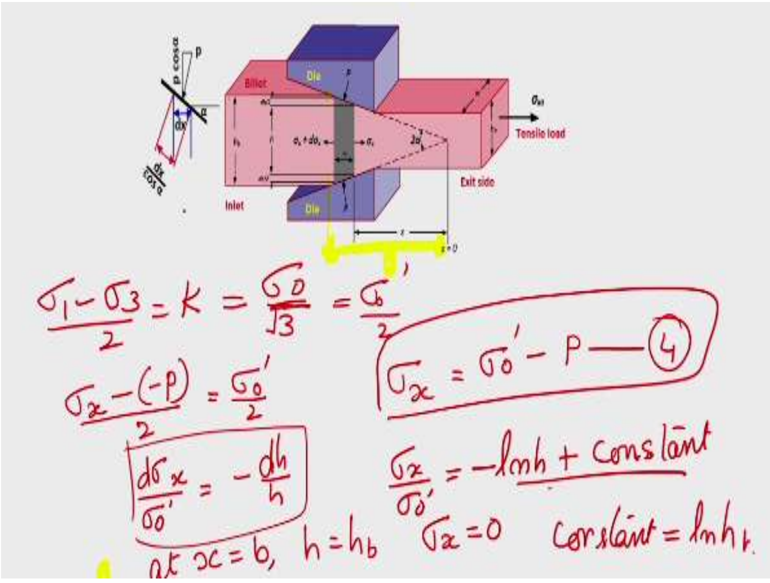
$$Equilibrium\ forces\ in\ Y - direction$$

$$\sigma_y dxW + pcos\alpha\left(\frac{Wdx}{cos\alpha}\right) = 0$$

$$\sigma_y = -P$$

So, so far we were not having any idea what is the nature of the forces in this, so from here, when you find that in the σ_y, when it is along this direction, the upward direction, the y direction, that stresses which are acting which is P, so that is in the compressive stresses, so you will find that. So, now we can say that the force P is the compressive force.

(Refer Slide Time: 11:46)



Width of the strip is much larger than h_b, the thickness of the strip we can assume because here we have drawn it in a way so that it will be for simplicity for understanding it is drawn like that, but in actual case, the strip, the width of the strip is very large compared to h_b; Then the plain strain conditions prevail. So under plain strain conditions, we have discussed earlier that von Mises criteria and Tresca criteria for yielding, it is the same, that is nothing but sigma 1 – sigma 3 by 2 = K so that we have derived it earlier, that is equal to sigma 0 by root 3.

So that is equal to we can write it as sigma dash by 2 where sigma 0, sigma 0 is the yield strength, uniaxial yield strength of the material and K is the shear yield strength of the material. So, this is the condition for plain strain deformation of, plastic deformation conditions okay. So if you apply that sigma 1 and sigma 3 as sigma y and sigma x, sigma 1 and sigma 3 as sigma x and sigma y, if you substitute it, we will get it as sigma x on this condition – -P by 2 = sigma 0 dash by 2 or we can write that from this sigma x = sigma 0 dash – P okay.

$$\frac{\sigma_1 - \sigma_3}{2} = k = \frac{\sigma_0}{\sqrt{3}} = \frac{{\sigma_0}'}{2}$$

$$\frac{\sigma_x - (-P)}{2} = \frac{{\sigma_0}'}{2}$$

$$\sigma_x = \sigma_0^{'} - P$$

So, now if you substitute this into equation 2 okay, equation 2 as that first differential equation, we can get it as say d sigma x d sigma x by sigma 0 = -dh by h by h and do the variable separation, we can get it in this form as simple differential equation we can get it. So this is integrated. So you will get it as sigma x by sigma 0 dash, this is equal to –log h + a constant. So how to find out we apply the boundary conditions, so that we can get this constant, what is the value. So that means, at the entry, entry into the die, at this point at the entry into the die, that means that h is equal to h_b okay.

$$\frac{d\sigma_x}{{\sigma_0}'} = -\frac{dh}{h}$$

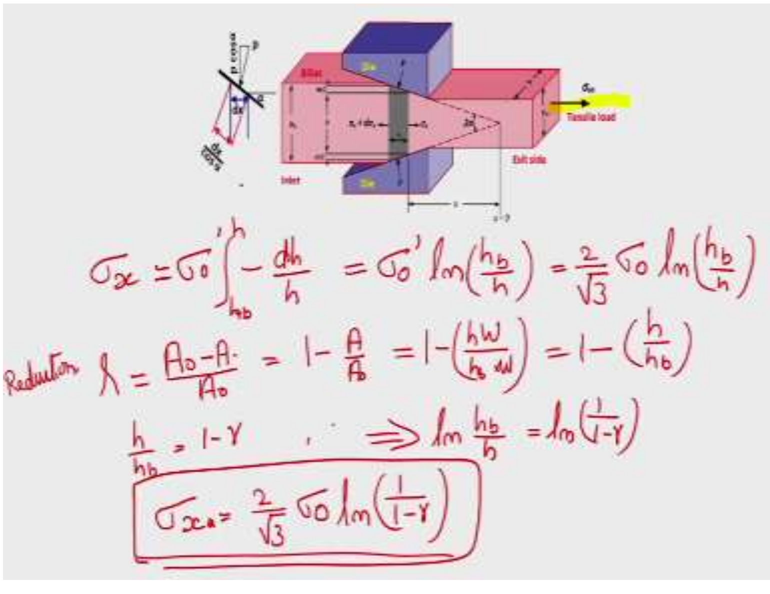
$$\frac{\sigma_x}{{\sigma_0}'} = -\ln h + constant$$

When x = b, from here to here when you come that this is equal to b okay, you get it as b. So then, h is equal to, the height is equal to h_b. So at x = b, h = h_b and at that condition, the entry to that, before that, there is no stresses. So, the longitudinal stress sigma x = 0 here okay, that is the condition. So, if you substitute this condition at h = hb, the constant is equal to, in this equation if you substitute, constant = log h_b okay.

$$at\ x = b, h = h_b\ \ \sigma_x = 0$$

$$constant = \ln h_b$$

(Refer Slide Time: 15:38)



Now if you substitute that and do the integration, so we can get it as sigma h = sigma 0 dash from the previous equation, sigma 0 dash x integral from hb to h okay – dh by h okay. So that if you do that, that we will end up with sigma 0 dash log h_b by h, I will be getting this relationship. So sigma 0 dash is equal to nothing but it will be 2 by root 3, in the previous lecture we have discussed this with sigma 0 which are uniaxial yield strength of the material log h_b by h, we are getting this relationship.

Now, we can write this in terms of this relationship log h_b/h we can write it in terms of the reduction because most of the plastic working, people generally used to term is nothing but the reduction, the cross sectional area, reduction in area, so which is r, where r is equal to nothing but your initial cross sectional area A0, the fractional reduction in area – A1, A by A0 okay, so that we can write as 1 by A by A0, so A0 is the initial cross sectional area and A is the instantaneous cross sectional area that, we can write that it is equal to in this cross sectional area if you are writing, is equal to h into W by h_b into W, this is h_b into W.

So, so that is equal to 1/h/h_b okay or from this we can write it as h by h_b = 1-r or it implies that log hb by h from this equation = log 1 by 1-r. So, that means we finally end up with a relation sigma x, what is the drawer stress xa at the exit is nothing but 2 by root 3 the uniaxial yield strength sigma 0 log into 1 by 1-r, where r is the reduction. This is the axial stress at the die exit which you wanted for the drawing operation where the axial stress which is necessary at the die exit.

$$\sigma_x = \sigma_0' \int_{h_b}^{h} -\frac{dh}{h} = \sigma_0' \ln\left(\frac{h_b}{h}\right) = \frac{2}{\sqrt{3}}\sigma_0 \ln\left(\frac{h_b}{h}\right)$$

$$r = \frac{A_0 - A}{A_0} = 1 - \frac{A}{A_0} = 1 - \left(\frac{hW}{h_b \times W}\right) = 1 - \left(\frac{h}{h_b}\right)$$

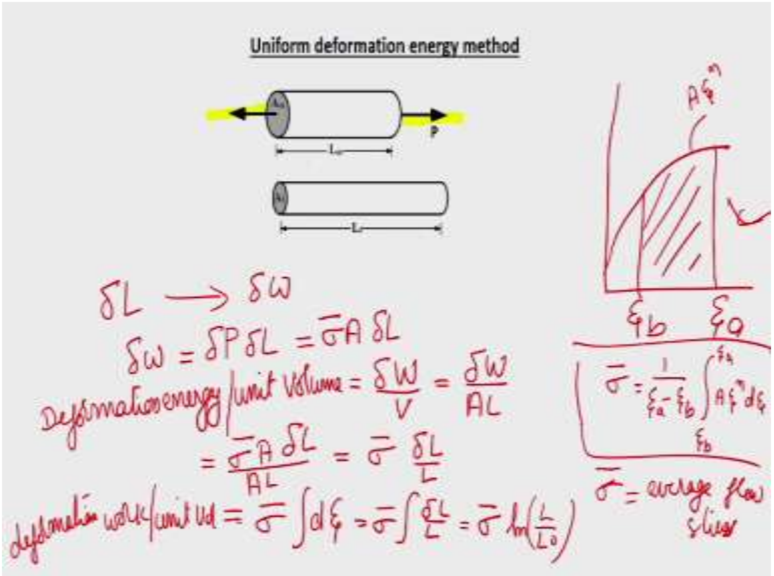
$$\frac{h}{h_b} = 1 - r$$

$$\ln\frac{h_b}{h} = \ln\left(\frac{1}{1-r}\right)$$

$$\sigma_x = \frac{2}{\sqrt{3}}\sigma_0 \ln\left(\frac{1}{1-r}\right)$$

If you just multiply by this cross sectional area, it will give you the draw load okay, draw force which is necessary. So, this gives the axial stress at the die exit needed to cause plastic deformation under conditions of zero friction and redundant deformation, so that means, it is nothing but the energy required for plane strain deformation that is what.

(Refer Slide Time: 19:39)



Now, the thing is that the same relationship we can arrive, a similar type of relationship we can arrive whey you use by the second method, so that is the uniform deformation energy method. So that is very simple case in which you are assuming a cylindrical piece okay, a cylinder which is loaded in tension, so like what we have written here okay. So like if you are loading it in tension here, applying a tension load P and it is deformed from an initial length L0 to L1 and during the time, their cross section area changes from A0 to A1.

So, that is a reduction in cross sectional area and there is an increment in length when you are applying a tension load, the simple very first lecture itself we have, taken this figure from that only okay. So, there is an increment in length during the plastic deformation. So when you just take that case, this incremental length which is delta L, which in the gauge length, due to this delta L, you will find that there it results in an incremental work delta W okay.

So, that delta W can be written as say in the differential form delta P into delta L, the force into distance, so that we can write it as in average flow stress sigma into the cross sectional area into your delta L okay. This average cross sectional area either you can find out the value from the flow stress if you know the history of plastic deformation of the material which you are going to do or if it un-yield an case, the epsilon from 0 to whatever strain you are going to do depending upon your reduction.

$$\delta W = \delta P \delta L = \bar{\sigma} A \delta L$$

So that is very simple like if you are just your tensile flow curve no if you are just drawing like this, and from maybe say ϵ_b to ϵ_a if you are deforming, what is the area under this okay.

So your average flow stress will be sigma bar will be, = 1 by epsilon a – epsilon b. So if this is of the form a epsilon raised to n okay. So integral from epsilon b to epsilon a, a epsilon raised to n d epsilon. So, this is how we can get the average flow stress.

$$\bar{\sigma} = \frac{1}{\epsilon_a - \epsilon_b} \int_{\epsilon_b}^{\epsilon_a} A\epsilon^n d\epsilon$$

So may be in this case, let us assume that when it is deformed from say b, height b to height a, that we can say, ϵ_b to ϵ_a, the average flow stress is sigma bar. In terms of that average flow stress, we can write this the incremental work in this form where this is the $\bar{\sigma}$ is the average flow stress okay. So in this case, the deformation energy per unit volume, so that is nothing but delta W/V, so per unit volume. Deformation energy per unit volume is equal to delta W by V or that = sigma bar A into delta L by so here it will be delta W by A into L okay, so that will be A into L.

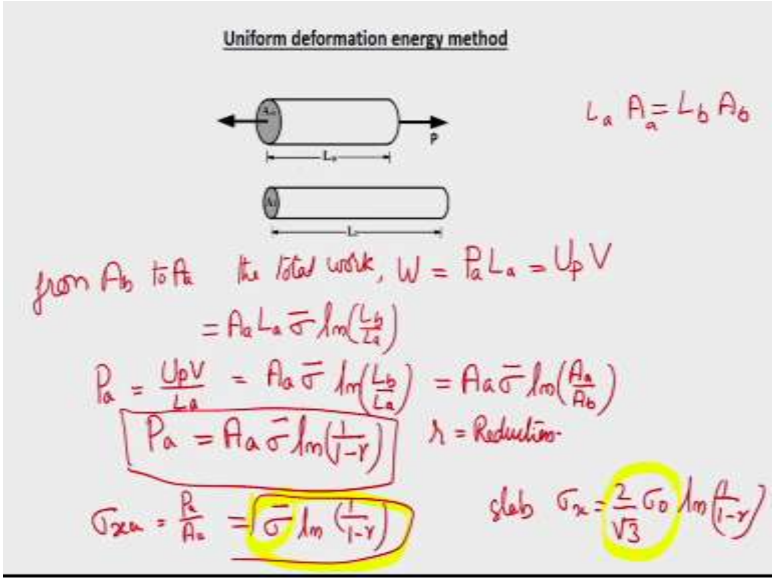
So we will end up with sigma bar into delta L by L we are getting this okay. So, the plastic deformation work per unit volume, so deformation work per unit volume that is from this relationship, now we can get that from this relationship, from this curve, if you do that, that it will be sigma bar x integral d epsilon, so this equal to sigma bar into integral delta L by L or we can write it in terms of sigma bar into log L by L0, where L is the instantaneous value of the length of this, here in this figure no if you look at this L1 and L0 is the initial length okay.

$$Deformation\ energy\ per\ volume = \frac{\delta W}{V} = \frac{\delta W}{AL}$$

$$= \frac{\bar{\sigma} A \delta L}{AL} = \bar{\sigma}\frac{\delta L}{L}$$

$$Deformation\ work\ pere\ unit\ volume = \bar{\sigma}\int d\epsilon = \bar{\sigma}\int \frac{\delta L}{L} = \bar{\sigma}\ln\left(\frac{L}{L_0}\right)$$

(Refer Slide Time: 24:44)



So for drawing a cylindrical specimen from an area A, from area Ab to Aa, the total work W = say Pa into La what is the length, so that is equal to your plastic work for, the work for plastic deformation, Up into V for unit volume okay, the total volume. So that we can write it as Aa into La into sigma bar the average flow stress into log Lb by La, this is the total work you are getting. So from this no, we can get the draw force Pa that is at the die exit = Up into V by La.

So that is equal to Aa the cross sectional area at the exit into sigma bar into log Lb by La where this Lb is the length at the exit site per unit volume and sorry inlet side and La is the exit side, Lb is at the inlet side and La is the length at the exit side. So from that now we can just write it as Aa because if you just assume that La into Aa, L constant volume relationship no, is equal to Lb into Ab. From this if you write it as Aa = sigma bar into log Aa by Ab, we will get it okay.

$$from\ A_b to\ A_a the\ total\ work, W = P_a L_a = U_p V$$

$$= A_a L_a \bar{\sigma} \ln\left(\frac{L_b}{L_a}\right)$$

$$P_a = \frac{U_p V}{L_a} = A_a \bar{\sigma} \ln\left(\frac{L_b}{L_a}\right) = A_a \bar{\sigma} \ln\left(\frac{A_a}{A_b}\right)$$

So this log Aa by Ab, the similar way we can write it as that is equal to Pa = Aa that is the cross sectional area at the exit into sigma bar into log 1 by 1-r where r is the reduction, so we can get this relationship. So if you just compare between the slab method, the sigma xa, so because sigma xa if you in this case no, sigma xa is equal to you will find that Pa by A okay, so that is equal to Pa by Aa, so that is nothing but sigma bar log 1 by 1-r, this is by the uniform deformation energy method and by slab method what we got is sigma xi = 2 by root 3 into sigma 0 into log 1 by 1-r okay.

$$P_a = A_a \bar{\sigma} \ln\left(\frac{1}{1-r}\right), r = Reduction$$

$$\sigma_{xa} = \frac{P_a}{A_a} = \bar{\sigma} \ln\left(\frac{1}{1-r}\right)$$

$$Slab\ \sigma_x = \frac{2}{\sqrt{3}} \sigma_0 \ln\left(\frac{1}{1-r}\right)$$

So, more or less, it is coming to the same. Here in the uniform deformation energy method, we are using this average flow stress okay. So, here we are using the average flow stress, by using this one, which we have derived, however, whereas in this case no we are using this 2 by root 3 into sigma 0 where sigma 0 is the uniaxial yield strength of the material okay at the beginning, so that is what.

Otherwise log 1 by 1-r, it is the same in both the case, only difference is this, but these two are almost similar cases, only may be a very slight difference in values only will be there depending upon what is the strain you are going to give or what is the reduction you are going to give, so that is the condition. So with this, today’s lecture we will stop it.