Computational Continuum Mechanics
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Kinematics – 1
Lecture – 10-12
Deformation gradient, Polar decomposition, area and volume change

So, welcome to the topic on Kinematics ok. So, they will be total of 6 lectures on kinematics, out of 6 the first 3 we will cover mostly Deformation gradient, Polar decomposition and Area and volume change. So, these topics will be covered in these three lectures. And we will try to do some numerical examples ok. So, that you get conversant with the idea you know how to work with some numerical problem ok.

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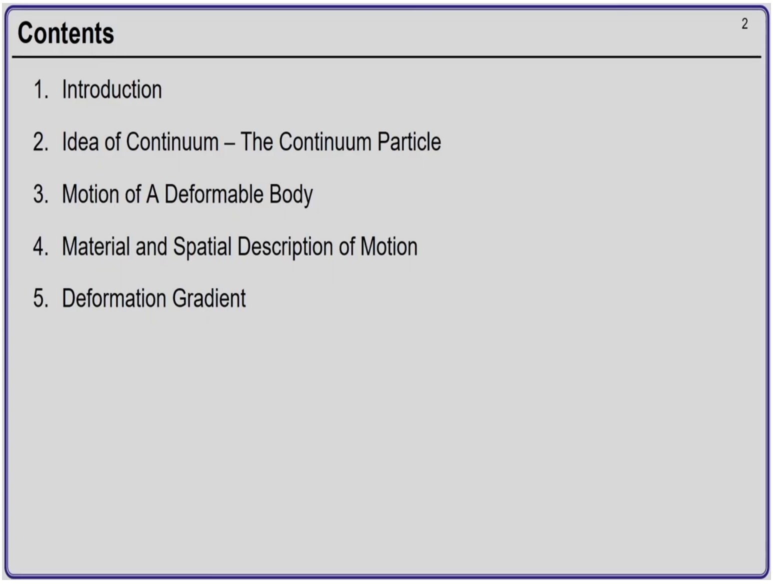

So, following are the contents of today's lecture we will start by introduction, followed by the idea of a continuum; that is what is meant by a continuum particle, this will be followed by description of motion of a deformable body ok. And then we will look into two different descriptions use to describe the motion of the body, that is material and spatial configuration spatial descriptions ok. Finally, we will see by what is T; meaning of the term deformation gradient and we will try to derive an expression for the deformation gradient ok.

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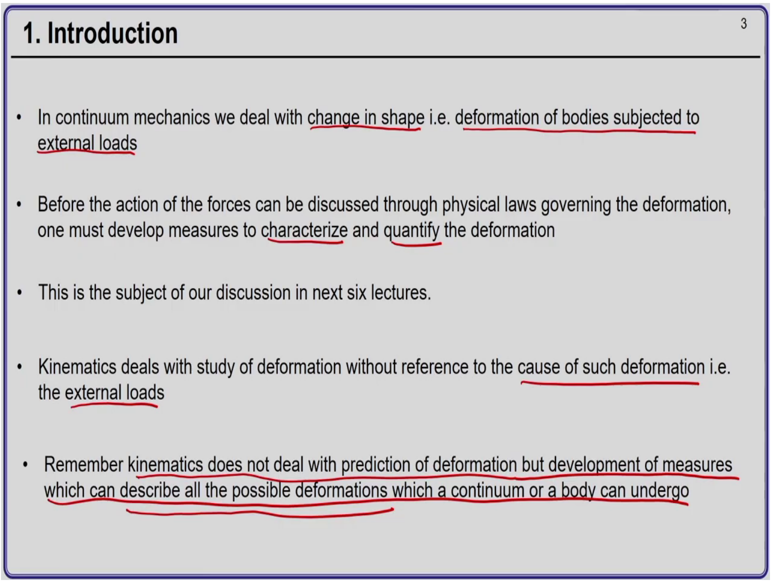

Now so, in continuum mechanics we are mostly interested in change in shape that is deformation of bodies subjected to external loads ok. So, this external loads can be mechanical, they can be thermal, they can be some other kind of loads ok, but in this course we are only interested in mechanical loads ok.

Now, before we can discuss the action of the forces which cause these deformation, it is necessary that we develop some understanding on how to characterize the deformation and how to quantify the deformation ok. So, characterize and quantify the deformation. So, the whole idea of next six lectures on kinematics is to develop measures which can characterize and quantify the deformation ok, before we actually move on to the physical laws which govern the deformation.

So, one of the very basic definition of kinematics is study of deformation without reference to the cause of such deformation ok. So, the cause of deformation is always some external load ok. Now here in kinematics we want to study the deformation, but without actually referring to the cause of the deformation that is the external loads ok.

Now, you remember that kinematics does not deal with prediction of deformation, but it deals with the development of measures; which can describe all possible deformations which are continuum or a body can undergo ok. So, kinematics does not mean ok, that you will be able to predict the deformation ok. So, here we are rather interested in development of certain measures, measures like measure for example, strains strain measures which can describe all the possible deformations that a body can undergo ok.

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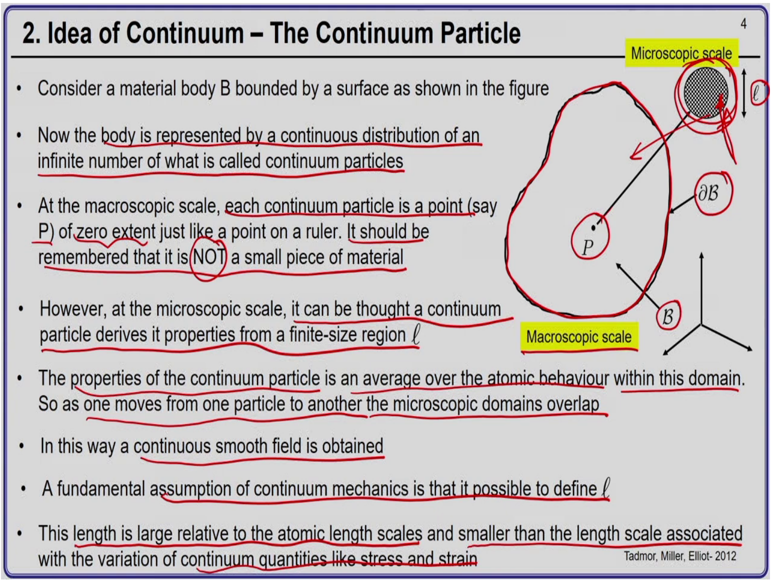

So, now next we look into, what is meant by a continuum particle; Ok. So, consider that you have this body ok. So, there is this body yes. So, this body consists of volume B ok. So, now, slowly we will introduce certain standard symbol which are often use in the terminology of continuum mechanics, they are pretty standard so we will be using the same as it is available in other book. So, this B this calligraphic B actually denotes the volume of the body and then partial B denotes the surface which bounds this body ok.

Now any body is represented by a continuous distribution of an infinite number of what is called a continuum particles ok. So, we assume in continuum mechanics that this body over here is made up of continuous distribution of infinite number of continuum particles ok.

So, say we pick one point P so, at the macroscopic scale at the scale where we can see the entire body ok. So, each continuum particle is a point ok, say point P as it is shown in the figure. So, each continuum particle is a point of zero extent ok, just like a point on a scale or a ruler when you draw a line and use the scale to mark a point ok.

So, point has no dimension. So, just like that this continuum particle is a point of zero extent it has no dimension ok. However, you should remember that a continuum particle is not a piece of material it is not a piece of material ok. So, you can take say N number of these particles and you can assemble to get your volume B ok, it is not a piece of particle. So, there are an infinite such continuum particles which are present in the body at the macroscopic scale.

However, when we actually zoom in this area; so, when we go at the microscopic scale. So, at the microscopic scale the continuum particle can be thought of as something which derives its properties from a finite size region of dimension l ok. So, you have this region. So, the dimension of this region it may be a sphere of diameter l ok. So, continuum particle derives its property as a average of all the properties of the atoms which are inside this region of say length l ok.

So, the properties of the continuum particle is an average over the atomic behaviour within this domain ok. So, anyway all the objects are all the bodies are made up of atoms ok. So, you could guess that if I can somehow get the position and velocity of all the particle all the atoms that are there in the body then I can basically solve my system; that is all I want to solve, but then even a centimetre cube of a body may contain somewhere close to 10 is power 27 atoms ok.

And it is not possible to solve for position and velocity of each such atom. So, there are for each atom there will be 6 unknowns. So, there will be 6 times the number of atom that many unknowns which are there and it is nearly impossible. So, what we assume now is that the continuum particle ok, shown here we will derive its property from the average of the atomic behaviour of all the atoms which are there inside this domain ok.

So, when you take this domain of length l there will be many atoms present inside and then any continuum property will be the average of the atomic behaviour of all the atoms present inside this domain. So, as one moves from one particle to another continuum particle these domains will overlap. So, once you move from one point P to another just neighbouring point Q so, this domain will shift slightly and most of the atoms which are there in one domain will be present in the another domain as such these domains will overlap ok.

So, in this way you will get a continuous smooth feel of continuum quantity for example, stress and strain ok, you will get a continuous smooth feel. So, one of the fundamental assumption of continuum mechanics is that it is possible to define this length l ok. So, in continuum mechanics we assume that there is such length l which exists and then it is also assume that the length l is large relative to the atomic length scale ok. So, this length l is much larger than the atomics lengths scale which means the size of an atom this is much larger than the size of an atom; however, length l is smaller than the length scale associated with the variation of continuum quantities like stress and strain ok.

So, the when you see the scale at which the stress and strain are varying for example, this length l will be much smaller than that length ok. So, this l will is actually much bigger than the atomic length scale, but it is much smaller than the length scale at with the stress and strains will vary. So, our when we talk about in the coming lectures about a point P, we are basically referring to a continuum particle P and this continuum particle P will actually derive all its property from a region of length l ok.

As a average over the atomic behaviour of all the atoms which are present inside this domain l ok. So, once we have clarified. So, later on when we are saying a point P we actually will mean the continuum particle P.

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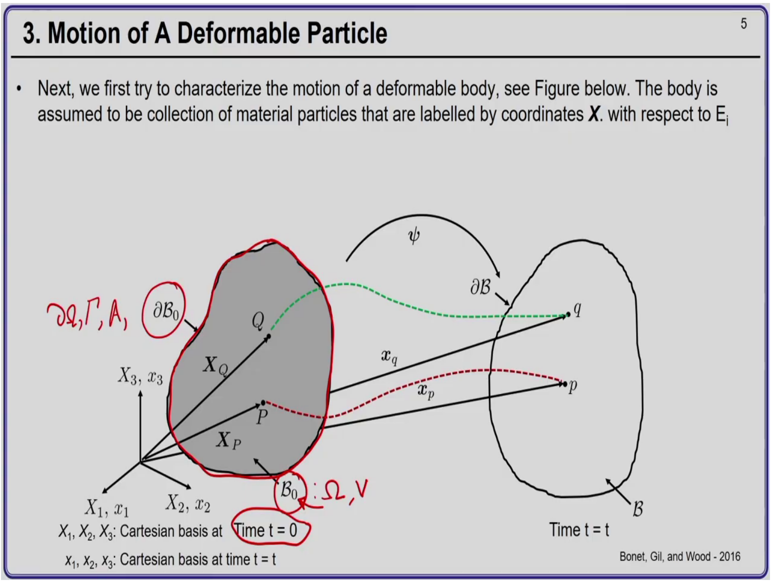

So, now, let us consider to how to define or characterize the motion of a deformable particle ok. Now in continuum mechanics we are dealing with bodies which are deformable ok. So, consider that you have a body ok. So, this is your body at time t equal to 0, remember in static problem time t is a pseudo quantity. There is no such thing like actual time in a static problem in dynamic problem there is indeed a time, but in static problem time is a pseudo quantity.

So, let us say at time t equal to 0, that is when you start your observation of the system ok, under the action of external forces, at time t the body is occupying a volume B 0 ok, notice that we have put a subscript 0 ok. So, that is a standard notation for volume in the configuration at time t equal to 0 and this volume is bounded by surface del B 0 ok. So, there are some notations people I will also make it clear that people also use; for example, omega they may use V for denoting the volume ok.

Similarly for the surface they might use tau, they might use A ok, like this and they might use del sigma like this one may use ok, but most commonly B and del B is usually use. Now you have this body at time t equal to 0 let me rub this ok. And now consider that the body is made of collection of material particles ok.

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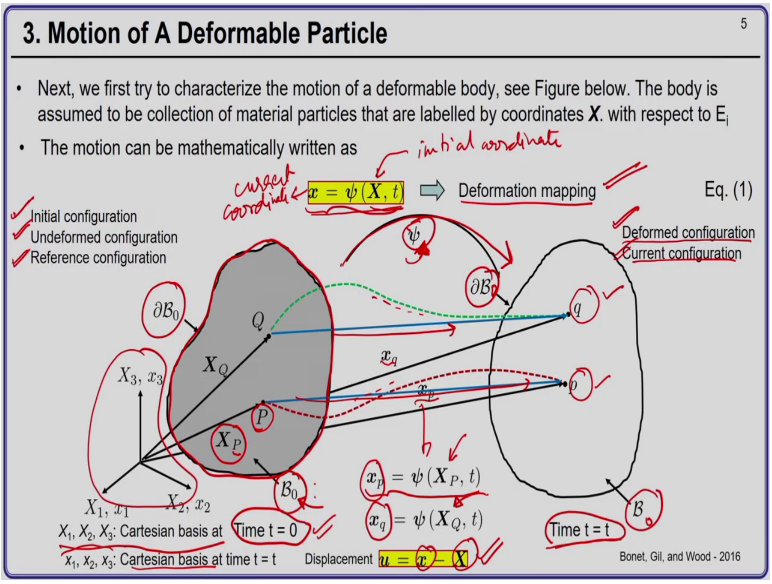

So, these are material particles P is a material particle and the coordinate of each material particle is given by capital X. So, we have a subscript P here to denote that this coordinate X corresponds to point P or continuum particle at point P. Similarly you have another continuum particle at Q a material particle at Q and the coordinate of that is X Q capital X Q at time t equal to 0 and these coordinates are with respect to basis Cartesian basis X 1, X 2, X 3 ok.

So, let capital X 1, X 2, X 3 is the Cartesian basis at time t equal to 0 ok, now once you apply the external forces what will happen, the body will undergo shape change, volume change and will occupy and it may actually move and it may occupy some other configuration in space ok. Let us say at time t the body is occupying a volume B bounded by del B notice I have we have not put any subscript ok. So, any if there is no subscript it means we are referring to the configuration at time t equal to t ok.

Now, when the body deforms the point P will move and will occupy say small p in the configuration at time t, similarly Q will move say along the green line and occupy position q ok. So, let us say x small x p and small x q I will come to it what is small x and small x p and small x q. So, let us say the coordinate of points p and q are x p and x q at time t and say this is with respect to Cartesian basis small x 1, small x 2, small x 3 at time t.

Now, what is usually assume there is no restriction that. So, what we have shown here in this figure that both capital X and small x Cartesian basis they coincide, but it is not necessary that they always have to coincide, but for all practical purpose in this course we take those two to be coincident in this course ok, but for notational distinction we consider them apart we use different notation for Cartesian basis at time t equal to 0 and Cartesian basis at time t ok.

Now, the configuration at time t equal to 0 there are certain terms which are used you can refer to the configuration at time t equal to 0 as initial configuration or the undeformed configuration or the reference configuration ok. So, these are some of the terms which are used for the configuration of the body at time t equal to 0 ok, at time t you refer to the configuration as either deform configuration or current configuration ok.

So, deformed means after the force is applied the body has deformed and as occupied this particular position in space ok. Now the motion so, the motion of particle P and Q are represented here by these dotted red lines and green lines ok. So, the motion can be mathematically written as x equal to psi is the function which is not known now which is psi is a function of capital X comma t. So, it is a function of the initial position of the point and the current time t ok. So, this small x is called the current coordinate ok.

So, the current coordinate is related to the initial coordinate X ok. So, capital X is called the initial coordinate and it is also a function of time current time t. So, this relation x equal to psi of X comma t is called the deformation mapping or the it characterizes the motion of the body ok. So, if you see this red line and green line these are representative of the motion of the particles at point P and Q ok.

So, the way it is shown is ok. So, you are do like this and write psi write psi here. So, this means from time t equal to 0 to time t equal to t the body is undergone a deformation given by mapping psi ok. So, our whole objective in continuum mechanics or finite element to be very precise is to get this mapping ok, we want to get this deformation mapping if we know this mapping explicitly for any time t and for any particle initially at capital X at time t equal to 0 we can know its current position small x ok. So, that is our objective ok.

So, we can write for example, the position current position of particle p which is given by small x p as psi of the initial position X P comma t and the current position of point q is psi of initial position X Q comma t ok. So, you can just plug in the time t and the initial position of the particle and you can get its current position at time t ok. So, in finite deformation analysis the displacement; so, these blue lines actually denote the displacement ok. So, the displacement in finite deformation analysis is given by u equal to final position x minus initial position capital X ok. So, this is the displacement ok. So, our whole objective as I said is to determine this deformation mapping ok.

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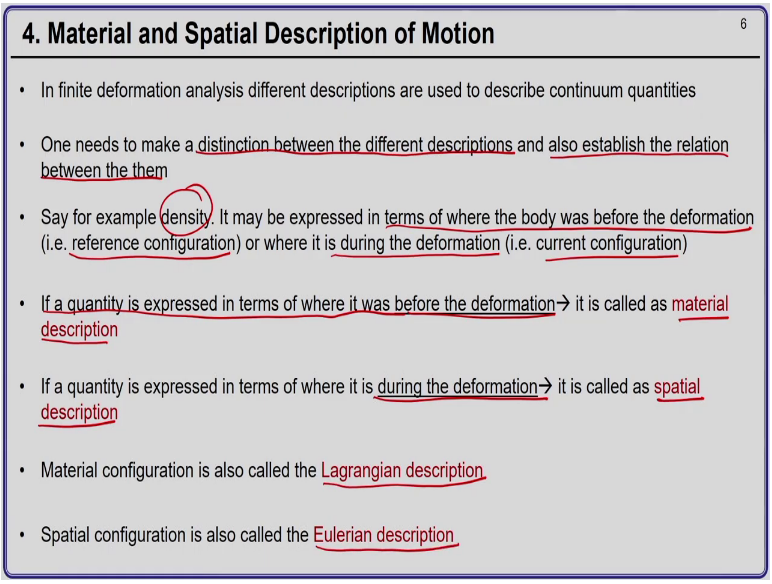

Now, in finite deformation analysis we can use different descriptions to describe the continuum quantities. So, we will come across various continuum quantities like strains and stresses and you can use different descriptions ok. So, one needs to make a distinction between the different descriptions and also establish the relation between them ok.

So, we want to make a distinction between different descriptions that are used to describe the kinematic quantities or the continuum quantities in general and also we want to relate these two, I mean these different descriptions ok. Say for example, density you can express density in terms of where the body was before the deformation. So, the current density say for example, we can express in terms of the body at time t equal to 0 or in the reference configuration.

Or we can also describe the density in terms of its current configuration that is where it is during the deformation ok. So, these are two different descriptions ok. So, the first one ok; so, if the quantity is expressed in terms of where it was before the deformation it is called as material description ok. So, what it means is, if you describe the current density in the configuration before the loads for applied say at time t equal to 0 then this kind of description will be called the material description.

If a quantity is expressed in terms of where it is during the deformation, that is current configuration say current density is described in terms of current configuration then it is called as this kind of description is called as spatial description ok. So, you have two different kind of descriptions, you have material description and you have the spatial description to describe the continuum quantities ok. So, this material configuration is also sometimes called as the Lagrangian description ok. And the spatial configuration is also called the Eulerian description ok.

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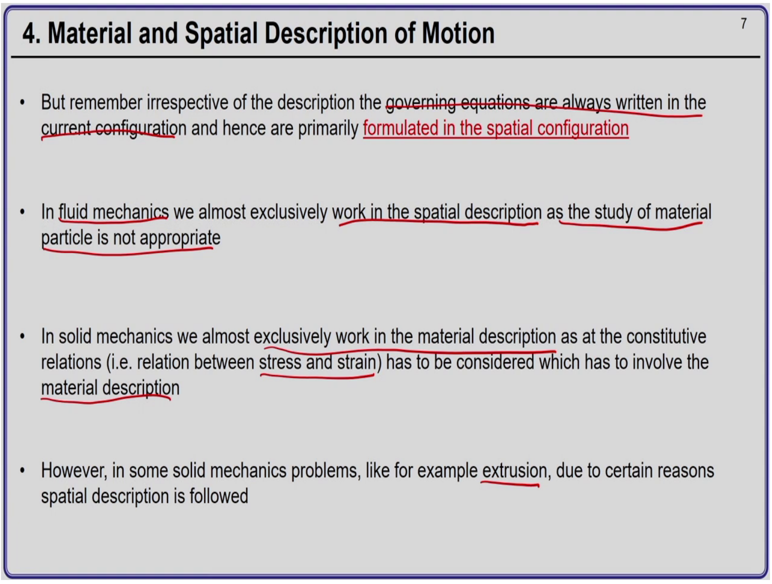

Now, but you should remember that the governing equations are always written in the current configuration that is at time t equal to t because that is where your equilibrium is achieved once you apply the forces the body will deform and will be equilibrium in equilibrium with the external forces at time t. So, governing equations are always written in the current configuration therefore, they are primarily formulated in the spatial configuration ok.

So, in fluid mechanics for example, one will work extensively in the spatial description as the study of material particle is not of interest ok. So, in fluid mechanics we will rather like to study what is happening at that particular point rather than following the particle itself ok. In solid mechanics we almost exclusively work in the material description ok, because in solid mechanics we have at some point of time we have to write the constitutive relation that is relation between stresses and strain and this will involve material description ok.

But it is not always that in solid mechanics problem he use material description there is some problem like extrusion, where for certain reasons it is better to use the spatial description you will rather like to point out your attention to a point in space ok.

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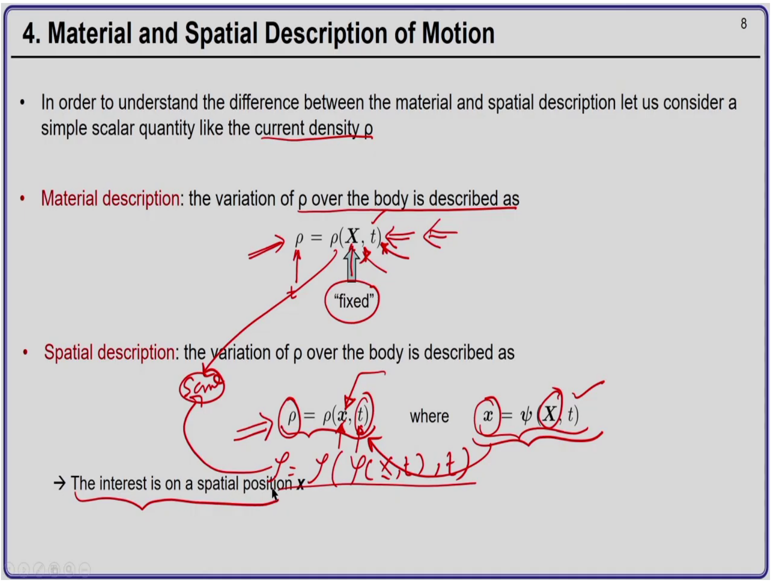

Now in order to understand the difference between the material in spatial description let us considers a simple scalar quantity like the current density rho ok. So, in material description the variation of the density over the body can be described in terms of its position at time t equal to 0 ok. So, the current density this is density at time t at time t at time t the body is occupying the configuration at t ok.

So, this density is at time t, but I can express this density in terms of its initial position ok. So, this initial position is fixed it does not change with time. So, what it means is what does statement actually means is on the left hand side you have density at current time, on the right hand side you have a function of material coordinates ok. So, capital X is also called the material coordinate, small x is also called the spatial coordinate. So, density is function of material coordinates and current time.

Since the material coordinate is fixed say you fix the material coordinate you fix the material coordinate ok. So, you have fixed the point at time t equal to 0. And as you vary the time t what do you get? You get the density of the variation of density of the particle which was originally at capital X at the material location capital X ok. So, you will always get the density this relation will always give you the density of the material particle which was at capital X at t equal to 0 ok.

Now in the spatial configuration that current density would be expressed in terms of the current coordinates small x comma t ok. Now if you fix x now, if you fix x you have focus your attention at a point in space and as you vary time what will happen? Because of this particular relation because x is fixed and time is varying what will happen? You will at particular point in space you will get different particles coming and occupying that position as a time varies ok. So, this density rho will give you the density at spatial position x as the time varies different particles will come and go, but the density at that point will be rho.

So, the in the spatial description this equation will give you how the density varies at a spatial position irrespective of what particle occupied that position ok. However, if you substitute the deformation mapping in this relation what do you get? You will get rho equal to rho of psi x comma t comma t if you use the deformation mapping. So, this relation will be in principle same as this relation they are both same then ok. So, this relation will actually give you the density at spatial position x small x of a material particle which was originally at capital X ok.

So, as you can see in the material description we are interested in say the density of a material particle ok, we are not interested in at a point in space we are rather interested in the density of a material particle how it varies for that particular particle. While in spatial description we are usually interested in how the density varies at a spatial location irrespective of which particle occupies that position ok. So, in spatial description the interest is on spatial; the interest is on spatial location x ok.

Now, coming to the topic of deformation gradient ok.

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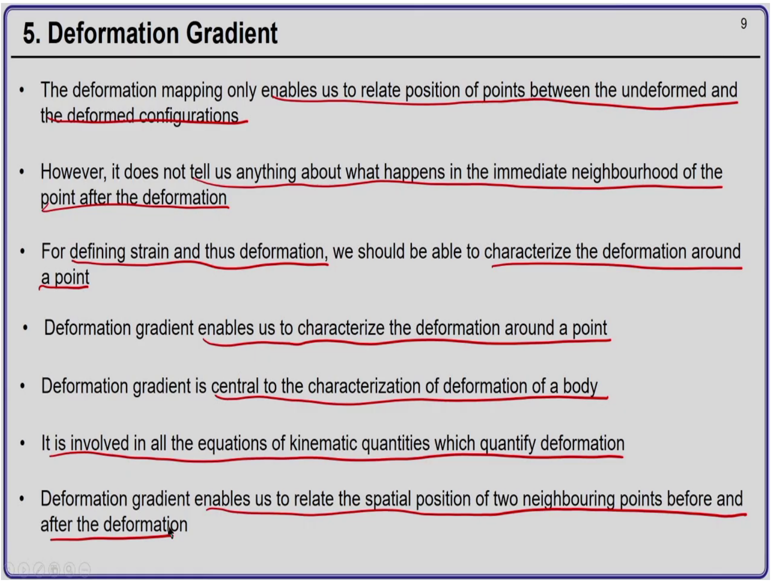

Now this deformation mapping that we had x equal to psi of capital X comma t it only enables us to relate the position of points between the deformed and the undeformed configurations. So, if you have the deformation mapping it will tell you how the current position is related to the its original position ok. The current position of a particle is related to the original position of the particle so, that much you can know.

However, the deformation mapping will not tell you anything about what happens in the immediate neighbourhood of the point after the deformation ok. So, all you are getting is in the current; if the current coordinates of point x is spatial position is x. So, its original position was capital X or if the original position was capital X at time t its spatial position is small x ok. That is what you get, but what you are not getting is, what is happening around the neighbourhood of that point basically you are not getting anything about the deformation.

So, for defining strain and therefore, deformation we should be able to characterize the deformation around a point ok. So, one if we want to define a strain we want to define what happens to say infinitesimal length d l around a particular point ok. So, strain is intuitively like change in length by original length ok. So, you want to. So, length means we have to have two points. So, you want to; so, if you want to define strain we want to define what happens to the length between two points which are in the immediate neighbourhood ok. So, if we can do that we will be able to characterize the deformation.

So, therefore, this we need what is called deformation gradient and this deformation gradient enables us to characterize the deformation around a point ok. So, if using deformation gradient we should we will be able to characterize what is happening around a particular point ok. So, how the say length is changing or how the angle between two material lengths is changing.

So, deformation gradient is central to the characterization of deformation of a body and it is involved in all equations of kinematic quantities which quantify deformation ok. So, all kinematic quantities that will come across in next 6 lectures we will have deformation gradient present somehow ok. And deformation gradient enables us to relate the spatial position of two neighbouring points before and after the deformation.

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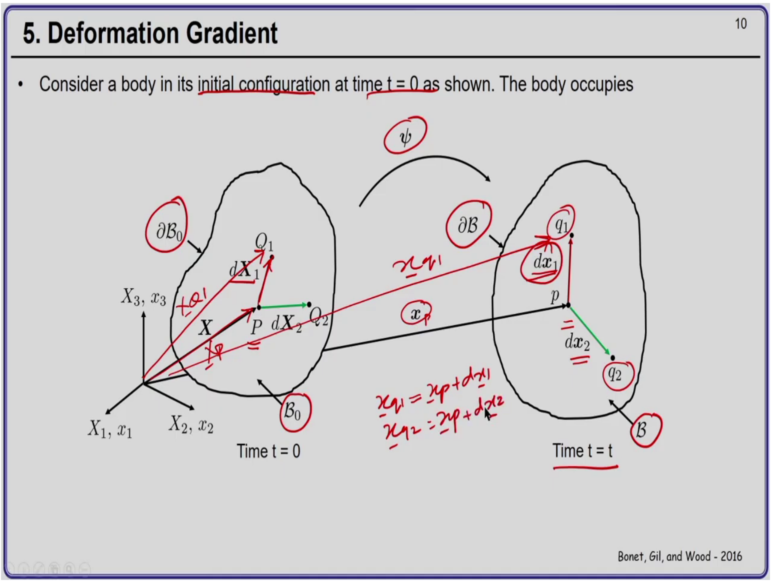

Now again consider a body which is in its initial configuration or say reference configuration or the undeformed configuration at time t equal to 0 ok. So, this is the configuration of the body at time t equal to 0. So, the volume is B 0 the and is bounded by surface del B 0 ok.

Now the body deforms so, deformation mapping let us say psi and the body occupies this configuration at time t volume B and surface del B. Let us say we pick up a material particle at point P the coordinate of this is capital X and now say in the immediate neighbourhood of P we pick two points Q 1 which is at the distance of d X 1. So, vector distance d X 1, another point Q 2 of vector distance d X 2.

Now, we want to see what happens to these two vectors d X 1 and d X 2 after the deformation once this deformation happens what happens to the these two vectors P Q 1 and P Q 2 ok. So, let us say P occupies position small p and its spatial coordinate is small x, let us say q 1 goes to small q 1 and the vector p q 1 is d x 1, q 2 goes to small q 2 and vector p q 2 is d x 2.

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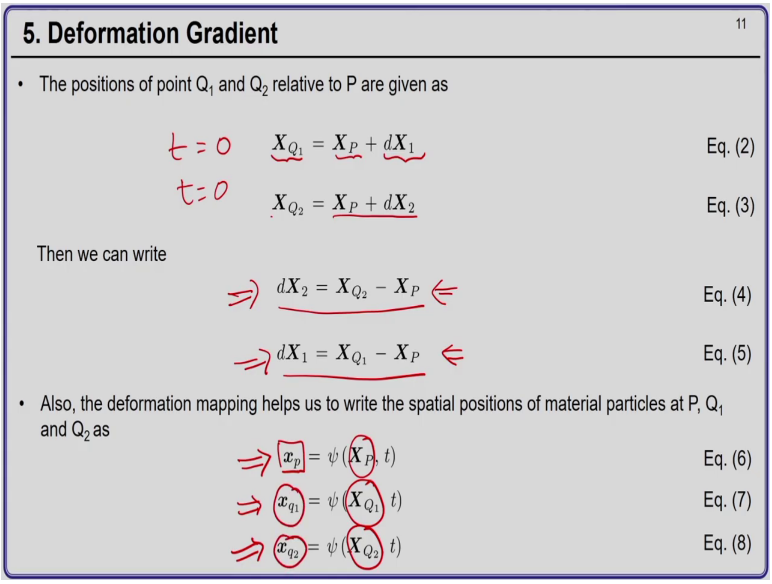

Now, using our usual expression for vectors we know that the initial coordinate of point Q 1 given by X Q 1 can be written as X P the coordinate of point P plus the vector from P to Q 1 ok. If you see this figure this is here X Q 1. So, X Q 1 is nothing, but X P plus X d X 1 ok. So, for purposefully we have not given the subscript P ok, but we can always write this as X P and this we can write as X P ok.

So, at time t equal to 0 at time t equal to 0 the position vector of point Q 1 is X P plus d X 1 at time t equal to 0 the position vector of point Q 2 is X P plus d X 2 ok. So, this is very simple. So, then we can express the vector d X 2 and d X 1 in terms of the position of the points P Q 2 and Q 1 ok. So, d X 2 will be X Q 2 minus X P and d X 1 will be X Q 1 minus X

P ok. So, what we are doing is just an equation 2 and 3 we are bringing X P on the other side ok.

Now because say we have the expression for deformation mapping we can know the current spatial coordinates of points P, Q 1 and Q 2. So, the current spatial coordinate of point P given by X P is nothing but in the deformation mapping you substitute the initial position of point P which is capital X P and you substitute time the current time you will get the current spatial coordinate of point P.

Similarly, you can substitute the initial coordinates at time t equal to 0 of point Q 1 and Q 2 which is X Q 1 and X Q 2 in the deformation mapping and you will be able to get the current coordinate or the spatial coordinates of point q 1 and q 2 given by x q 1 and x q 2 ok.

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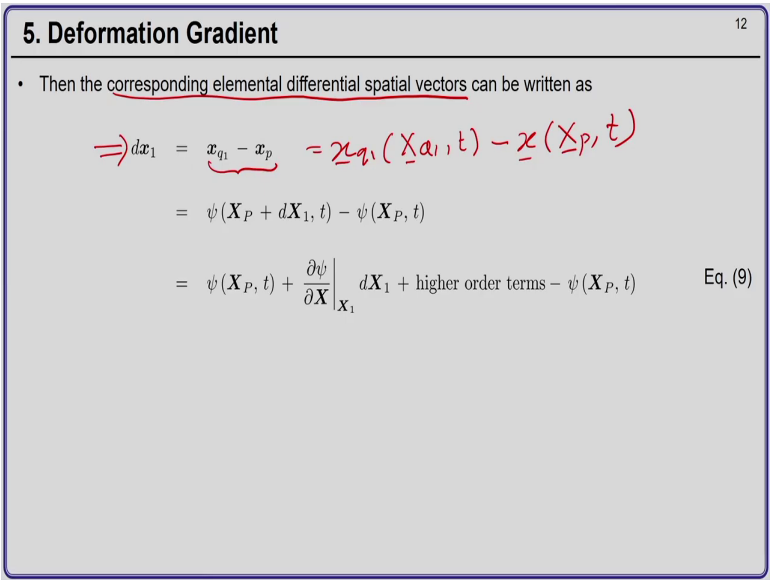

So, once we have this we can write the corresponding differential spatial vectors d x 1 small d x 1 if you see this figure over here. So, this was your d x 1. So, d x 1 would be your so, this is your x q 1 ok. So, d x 1 would be so x q 1 equal to x p plus d x 1 and x q 2 will be x p plus d x 2. So, you can write d x 1 as x q 1 minus x p ok. So, difference of the spatial coordinates of point q 1 in p.

Now the spatial coordinate of point q 1 is nothing, but x q 1 of X Q 1 comma t minus point x of X P comma t ok. So, sometimes we write x can also use psi here we can wrote psi here.

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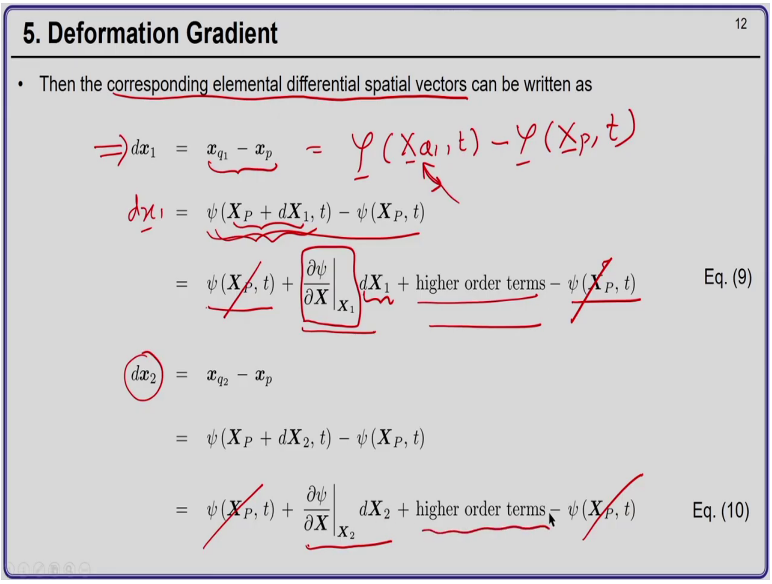

Now, X Q 1 is nothing, but X P plus d X 1. So, that is what we substitute so, that what we substitute here and then we get this expression; so, d X 1 the differential elemental spatial vector d x 1 is psi function of X P plus d X 1 comma t minus psi of X P comma t. Next what we do is, we use the Taylor series expansion and expand the first term on the right hand side.

So, the first term will be if you use Taylor series expansion it will be psi of X P comma t plus del psi by del X evaluated at X 1 at the current at the initial position X 1 multiplied by original elemental differential material vector d X 1 plus. There will be higher order terms minus X psi of X P comma t. Now both of these will go away and then you will be left with the second term plus some higher order term.

Similarly you can get an expression for d x 2 you can get the expression of d x 2 again the first term and the last term will go away and you will have del psi by del X evaluated at X 2 d X 2 plus some higher order terms ok.

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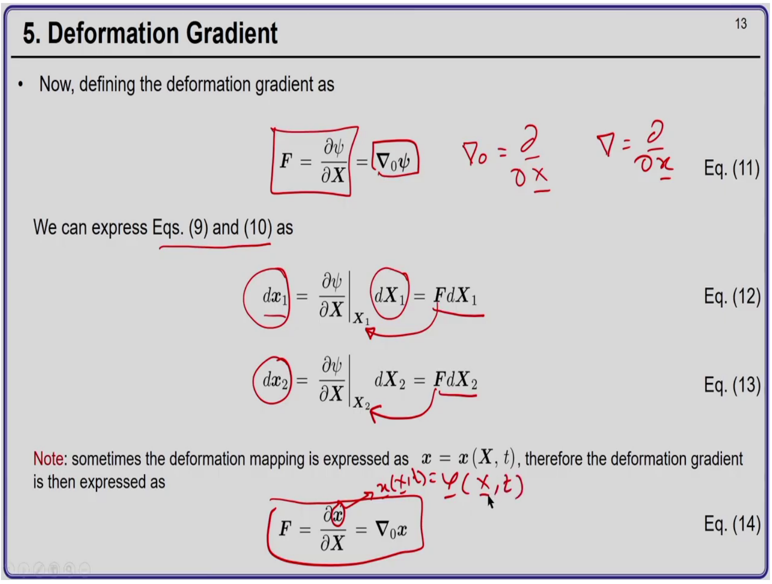

Now, we define our deformation gradient as del psi by del X that is how we define the deformation gradient that is how the deformation is varying at a particular location X. So, F is the symbol used for deformation gradient. So, capital F is more often than not is the symbol which is used for deformation gradient and it is given by del psi by del X or we can always write this as del 0 psi ok.

So, del 0 means gradient with respect to the material coordinates ok. So, del will actually mean gradient with respect to the spatial coordinate that is the standard convention ok. So, using this definition of deformation gradient we can express our equations 9 and 10 as and we neglect the higher order terms there will be negligible because d X 1 itself is very small. So, the square of all those will be even smaller quantity then we can just neglect and we can relate the elemental differential spatial vector d x 1 to the elemental differential material vector d X 1 through this relation, d x 1 is F of d X 1.

Similarly d x 2 will be F of d X 2, where this F here means the deformation gradient is evaluated at material position x 1 and this F means deformation gradient is evaluated at material position x 2. So, in general we can express the deformation gradient sometimes as F equal to del x by capital X ok. So, instead of writing del psi by del X given by equation 11 we will write del x by del capital X this is also sometimes used ok. So, here this x actually means a function ok, it is means this particular function which is thing, but same as our deformation mapping ok.

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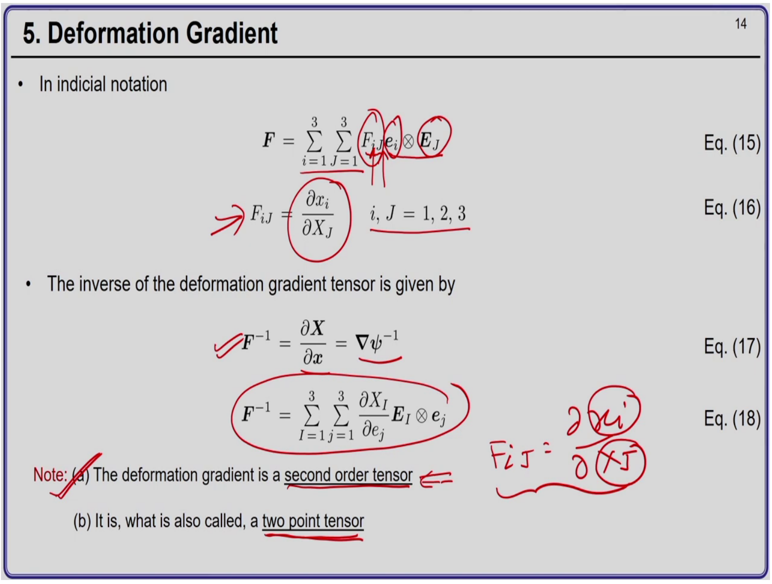

Now, so, in indicial notation the deformation gradient will be written as double summation F i comma J e i tensor product E J, where small e i is the Cartesian basis for the description at time t and capital E is the Cartesian basis for the material description that is configuration at time t equal to 0 ok.

An F i J is one component of the deformation gradient and i and J are sum from 1 to 3. So, you notice that one of the indices is in the lower case, the other indices is in the upper case, because we use lower case indices for quantities in the spatial configuration and uppercase indices for quantities in the material configuration or configuration at time t equal to 0. So, F i J is nothing, but del x i by del X J where both i and J go from 1 to 3. So, in total there are 9 components of deformation gradient.

So, the inverse of a deformation gradient is defined as del X by del x or del of psi inverse and this is the indicial notation of the inverse of the deformation gradient tensor. So, you can show and I will leave this as a proof you can do it very easily that the deformation gradient is a second order tensor it is also called the deformation gradient tensor and also deformation gradient tensor is a two point tensor. Two point means it has certain components from one description or one configuration there are certain components which from which are from another configuration ok.

So, you can see here in equation 16 F I J was del x i by del X J ok. So, this x i is from the spatial configuration and capital X J is from the material configuration. So, there are two different configuration hence the name two point tensor. So, a two point tensor relates quantities in two different configurations ok. So, therefore, F is not only a second order tensor it is also a two point tensor ok.

So, the first one I will leave as a exercise for you to do it yourself it is very easy to show you can start from any of the expression here given from 15 to 15 or 16 and you can use the fact that small x is a vector ok. So, you know the transformation relation for a vector and you can show that deformation gradient is a second order tensor ok.

Now, there are two other concepts which are used predominantly one is called the push forward operation ok.

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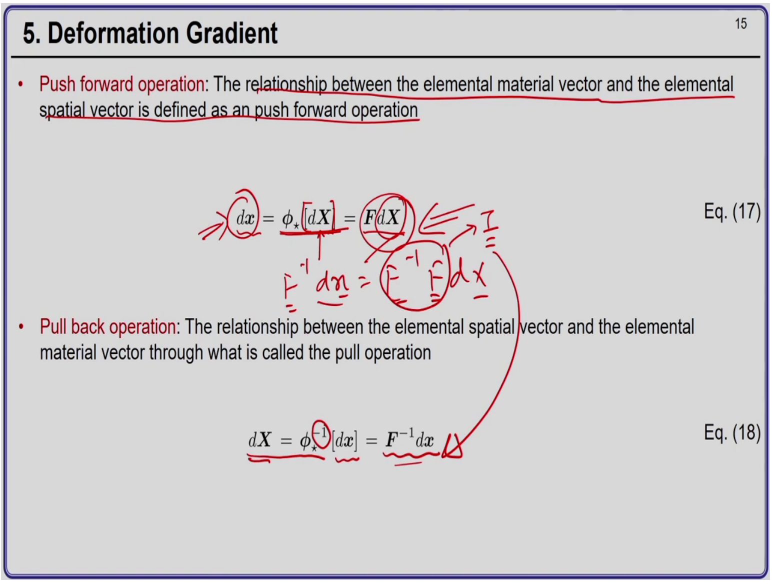

So, push forward operation when the relationship between the elemental material vector and the elemental spatial vector is defined by this particular relation d x equal to phi star d X or explicitly F d X. So, this is called the push forward operation you are by using this operation you are pushing the material vector in a way from time t equal to 0 to spatial vector at time t ok.

This is you have pushing d capital X 2 d small x ok. So, the notation used for push forward operation is phi star and then there are the square brackets d X. So, square bracket does not mean multiplication it means that the push forward takes the input d X and there are certain relation that relation is F d X and it will give you the push forward of d capital X which is nothing but d small x similar to as for example, there is push forward operation there is something called pull back operation ok.

So, if you take the inverse of push forward. So, given the spatial vector d x the pullback operation will give you the material vector d X ok. So, d X will be 5 star inverse. So, notice we use minus 1 to denote the pullback. So, d X is phi star inverse d x or F inverse d x ok. So, from equation 16 you can see that if you multiply both side by F inverse and using the fact that F inverse F is identity you can get this relation, that F inverse d x is d capital X ok.

So, this called the pullback. So, you have a spatial vector and you are pulling it back to the material configuration, push forward is you have a material vector and you pushing it to the spatial configuration.

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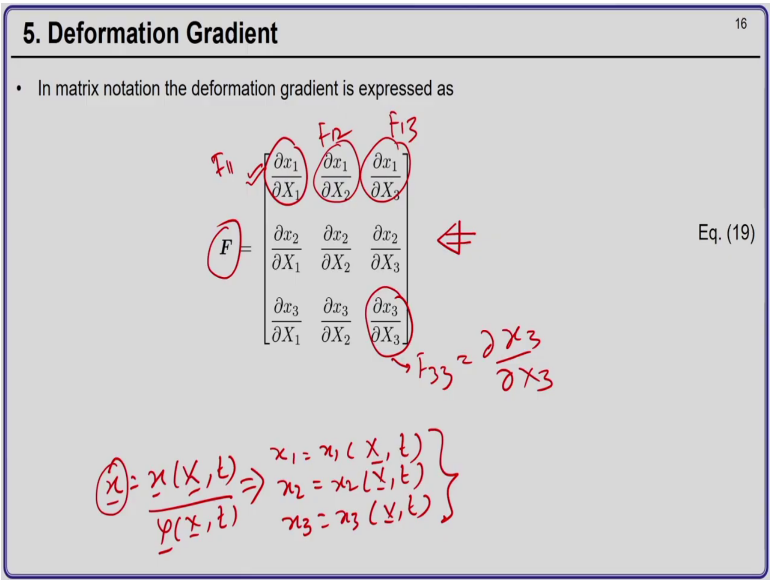

Now because the deformation gradient is a second order tensor it can be expressed in terms of in a matrix form and equation 19 shows you how it can be expressed in a matrix form ok. So, the first so, it will be a 3 by 3 matrix the first component will be F 1 1 which is nothing, but del x 1 by del capital X 1 F 1 2 is nothing, but del x 1 by del capital X 2 ok, similarly you have F 1 3 which is del x 1 by del capital X 3 so on and finally, you have F 3 3 ok. So, this is F 1 1, this is F 1 2, this is F 1 3, F 3 3 is del capital x 3 by del X 3 ok.

So, where do you get x 1 x 2 x 3 the spatial x 1 x 2 x 3? That is obtained from the deformation mapping. So, you will know from the deformation mapping ok. So, you will be. So, you basically have x equal to x of capital X comma t or this is same as psi of X comma t both means the same. So, this so, because x is a vector it has 3 components. So, it. So, there are 3 equations here you have X comma t x 3 is x 3 X comma t.

So, if you know these 3 equations you can take the derivative of spatial coordinates x 1, x 2, x 3 with respect to material coordinates capital X 1, capital X 2, capital X 3 to get the deformation gradient ok. So, this completes our discussion on deformation gradient ok. So, next we will move to the definition of strain.