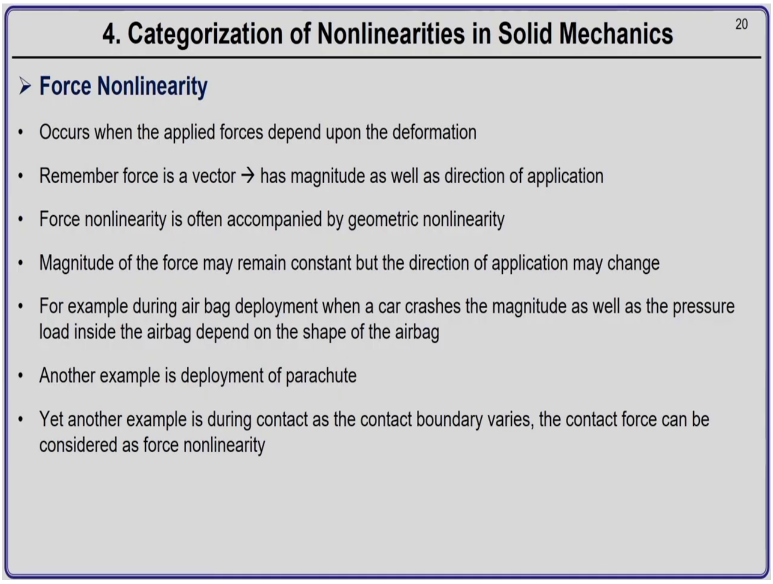
Computational Continuum Mechanics
Dr. Sachin Singh Gautam
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Introduction – Origins of Nonlinearities
Lecture – 01 and 02
Origin of nonlinearities - 2

So, I welcome you all in today's lecture it will be the final lecture on the Introduction in of this course. And if you remember we were discussing different types of nonlinearities in last lecture.

(Refer Slide Time: 00:50)



So, we had already discussed what is meant by geometric nonlinearity and what is meant by material nonlinearity. So, today we are going to look into two more kinds of nonlinearities which are present in any system non-linear system. The first one is a forced nonlinearity and the other is the contact or the boundary nonlinearity. And after this we are going to discuss several examples.

And so today we start with this forced nonlinearity. So, what is force nonlinearity? So, force nonlinearity occurs when the applied forces depend on the deformation ok. As you can see here the first point; the applied forces the external forces will actually depend on the deformation of the continuum or the body.

So, as the body deforms the applied forces the effect of the applied forces externally applied forces will change ok. So, now, you have to remember that force is a vector quantity. So, it has a magnitude as well as it has a direction.

So, in many cases it might happen that the magnitude of the force remains constant, but the direction of the force changes according to the deformation of the body ok. So, you would guess that forced nonlinearity is often accompanied by geometric nonlinearity is because of geometric nonlinearity the initial and the final configuration they are very different from each other so the shape of the body changes. So, the affect of the external force which are dependent on the deformation also changes ok.

So, this fourth point is what we say that the magnitude of force may remain constant, but the direction of the application of force may change ok. So, we will look into some examples the first example is the deployment of airbag. So, you all would have seen that in your car there is a air bag which is present ok.

So, what is the purpose of an airbag? So, when the vehicle is impacted ok. So, it may happen that you are somebody is driving a vehicle and the vehicle gets impacted against another vehicle or against some rigid obstacle or any other reason.

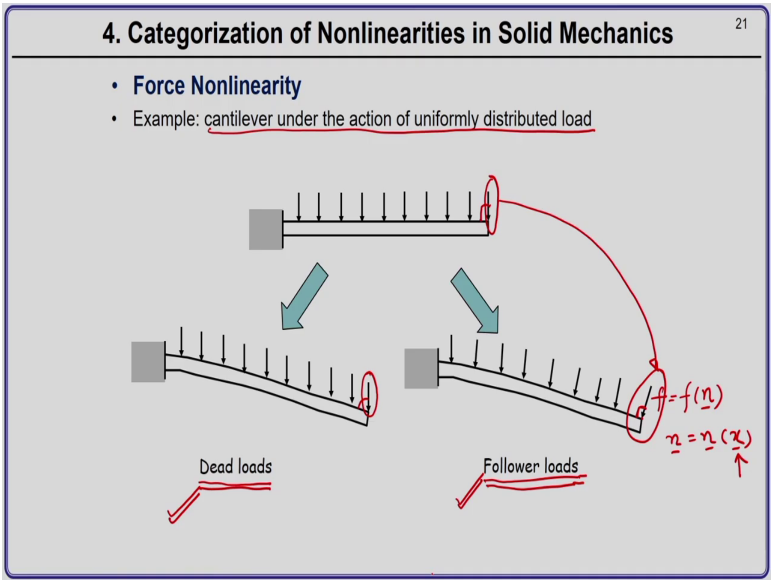
So, the purpose of the air bag is to deploy quickly enough so that it can protect the life of the passenger ok. So, when this airbag gets deployed ok. So, the air is blown inside the airbag and it expands ok. So, the pressure load inside the airbag will depend on the shape of the airbag that is first example. Another example is the deployment of a parachute ok. So, when somebody jumps from an airplane and the parachute is deployed ok. So, you would have seen that the parachute is initially a very compressed inside the bag ok.

So, when somebody jumps out of the airplane and he pulls up a lever so the strings go out and the air the parachute is deployed and it expands and becomes a huge structure. So, there also the shape the effect of the air pressure on the inner surfaces of the parachute has an effect ok. So, when you are designing a parachute you have to take this kind of forced nonlinearity into account.

So, another example is your contact problems ok. So, you know in the last class I have shown you a simulation of a contact problem. You had a small block it was kept pressed against a bigger block and it was dragged along the surface of the block bottom block. So, you would have seen that the contact boundary for the top block was always constant ok, but for the lower block the contact boundary change according to time ok; so the contact boundary was changing. Hence this contact forces also change ok.

On the lower block the contact forces will change a part of a surface which is in contact for a certain amount of time after a certain amount of time when the upper block has passed away their contact forces will become 0 and the non zero contact force will occur at some other point ok. So, these are few examples of force nonlinearity ok.

(Refer Slide Time: 05:50)



Now, coming to the example so; we will start off with a very simple example of the cantilever which is under the action of the uniformly distributed load ok. So, as you can see here you have a cantilever beam ok; it is fixed on the left hand side and on the top surface you have a uniformly distributed load which is acting over the entire surface ok. Now depending on the type of this load there can be two cases and we can classify the loads according to the way they act on the surface.

So, the first case is you have what is called the dead load ok. So, as you can see the direction in case you have dead loads the direction of the forces you can see here; the direction of the forces they are always in the same direction as they were in the un deformed configuration ok. So, neither the magnitude of the load has changed nor has the direction of the forces has changed ok.

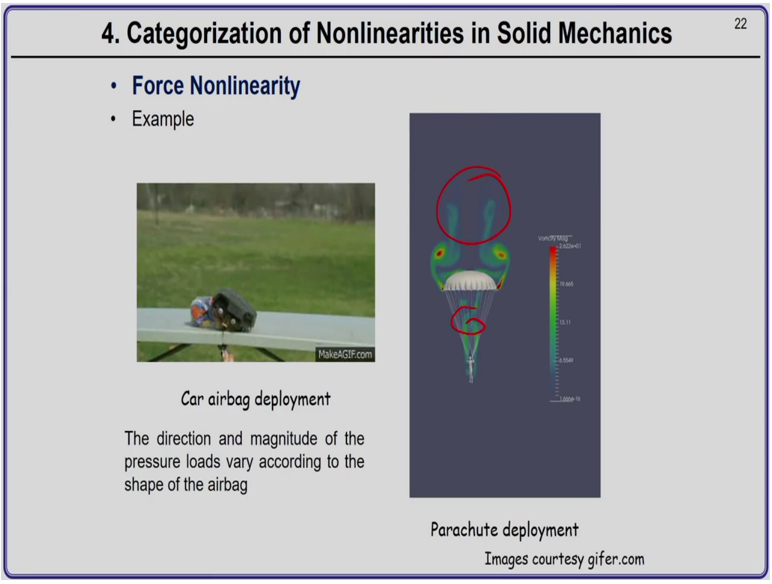
Next we come to see what are called the follower loads ok. Why the name follower? You can see from this picture over here ok. As you can see this load over here you can see here the direction the magnitude of the force for example, has remain constant. But you can see the direction of the force has changed and this force always remains perpendicular to the surface on which it acts ok.

So, in the first case you would see initially the force was acting the perpendicular direction, but later on it was not acting perpendicularly to the surface. In the second case what is called the follower loads you can see the load still acts perpendicular to the surface. So, these loads are called follower load follower because the loads follow the deformation ok. So, these are two different kinds of loads that we encounter usually.

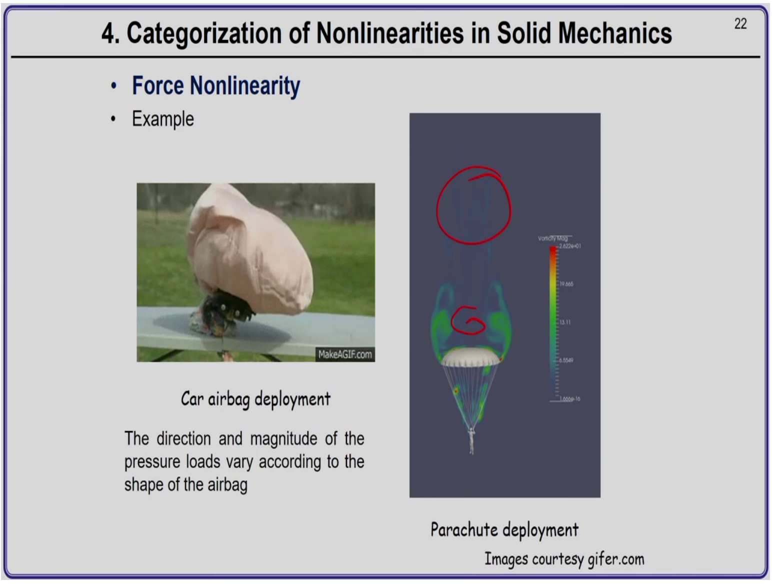
So, as you can guess dead loads are very simple to incorporate in any computational finite element code ok, but follower loads since they depend on the normal to the surface ok. So, this force over here if we write f it is a function of the normal to the surface at that point ok. So, where n the normal to the surface depending on the point ok; so this x is the coordinate of the point on which the force acts ok.

So, now, you could guess that this force will have a corresponding stiffness coming into the stiffness ok. So, we will come to it later ok. So, incorporation of follower loads is a little challenging ok.

(Refer Slide Time: 09:06)



(Refer Slide Time: 09:09)



Now, we will see some pictures of forced nonlinearity where forced nonlinearity actually takes place. So, the first picture that you see here is a deployment of car airbag ok. So, you can see suppose there is an impact and this car airbag comes out of a small enclosure and just fills up the space ok. So, and the objective of anybody who is designing this car air bag has to be to protect the passenger say the driver.

So, usually this car airbag is present on the steering wheel of the car ok. So, when the car gets impacted. So, this airbag has to be deployed and it has to deploy in a manner that before the passenger head hits the steering wheel this airbag should deploy. And instead of the passengers head striking the steering wheel the passenger head should strike the airbag ok. And he should not have the impact of the full impact of the steering wheel so that his life can be saved.

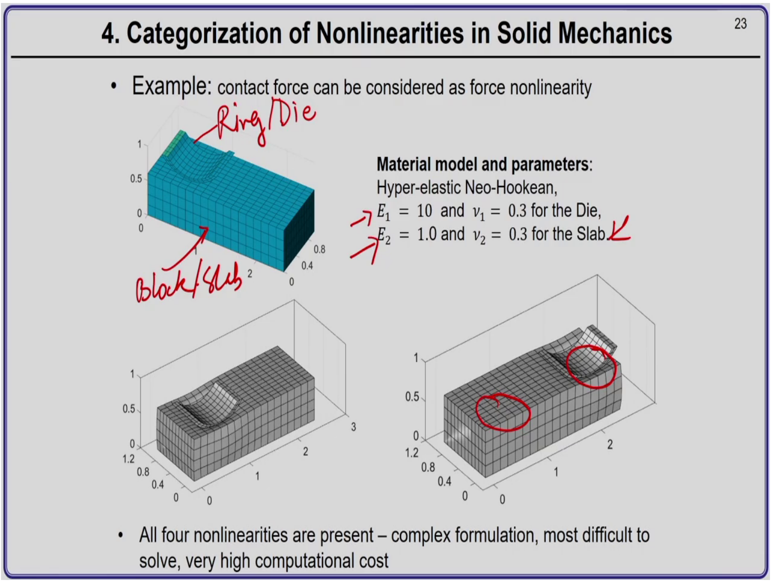
Also the airbags should not get deployed so fast that when it is in the total deployed situation when the head of the passenger gets impacted it should not get rebound. And then he may have so that he does not suffer the injury to it is his or her neck ok. So, this design of an airbag is a very challenging process. And you can see the pressure that is being the pressure of the air that is being blown inside this airbag is always normal.

So, the surface you can treat the surface at a very thin shell kind of structure ok. It is purely elastic there is no material non-linearity here, but there is a huge geometrical nonlinearity which is present and also there is this force nonlinearity. Because pressure always acts normal to the surface and you can see there is a surface changes over time ok.

The next example that we consider is the deployment of a parachute. So, in this figure you see it is a simulation of a person who is jumping with a parachute. And now you can see as the parachutes get deployed ok. So, suddenly you see all of a sudden you have a very huge shape change which takes place. And if you have to monitoring closely you can see the wrinkles ok.

So, the small wrinkles which are present at the edge of the parachute. So for a realistic simulation of a parachute ok; so that you can deploy you can design a very good parachute. So, you have to take into consideration the force nonlinearity ok.

(Refer Slide Time: 12:13)



The next example that we considered is the contact ok. So, this problem is similar to what we discussed in the previous lecture. So, this is a problem where you have a block and on top of that block you have a ring ok. So, you have a ring here you have a ring here and you have a lower block ok. So, the Young's modulus and the Poisson's ratio for this ring or as we call it here die is 10 times as much as the.

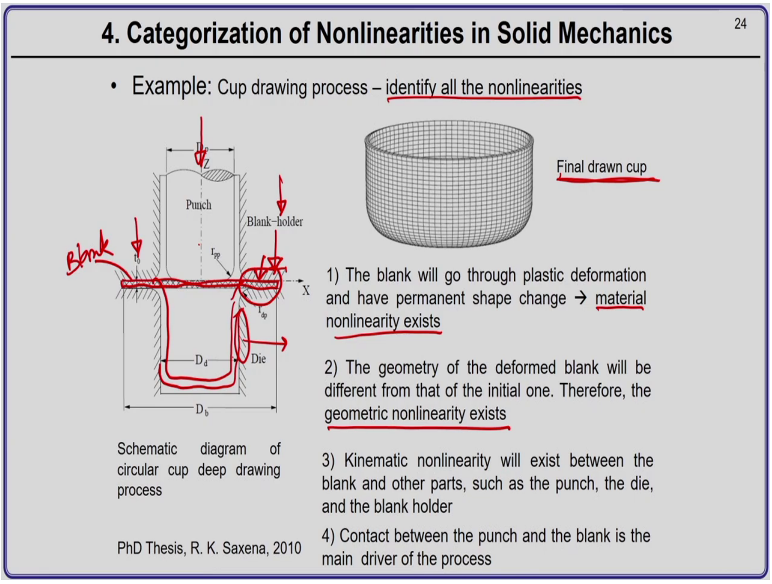
Young's modulus of the slab or the lever block we call it slab here. Now what we do here is we press these die against the slab by applying a vertical force and then keeping the vertical displacement fix we apply the horizontal displacement. So, that the die gets dragged along the surface of the slab ok.

So, in the first so as you can see in the first simulation here the die is pushed against the slab and you can see really appreciable chain in the deformation of the lower slab the slab deforms a lot and as you can see the contact boundary changes. So, initially there was a line contact and as you applied the forces the contact area changed ok.

Now once you have applied that the second simulation shows the die getting dragged along the surface of the slab. And you can see here that the contact area of the die more or less remains constant during the simulation. While the contact area on the slab changes over time so initially you can see here ok.

I will run the simulation again; so initially you can see for the low for the slab initially the contact area was present on the lower left end ok. Next at the end of the simulation the contact area for the slab has shifted to the right hand side ok. So, the contact area is changing and also the forces on the block are changing at from time to time. So, this is a case of force nonlinearity ok.

(Refer Slide Time: 15:00)



So, the next example that we consider is the cup drawings process ok. So, you can see you have a blank ok. So, this is a blank which is pressed against the die ok. And there is a blank holder which is putting a force on the blank to keep it keep the blank in it is position ok.

So, it does not allow the blank to move vertically it can allow the blank to move in the horizontal direction ok. And there is a die this is the shape of the die is say cylinder ok. And then what you do is you have a punch and you apply a vertical displacement at a certain rate.

So, that this blank goes inside the die and takes the shape of a cylindrical cuff ok. So, this somebody asks you to identify all the different kind of nonlinearities which are present in this particular problem. So, what are different kinds of nonlinearities that are present? So, the first is the blank ok. So, the blank that you see here this portion we will go large plastic deformation and there will be a permanent shape change of the blank. So, this is nothing, but material nonlinearity ok.

The next is the geometry of this blank so initially the blank is the cylinder with a very small thickness the radius is large, but the thickness is very small so, but it is a plain blank ok. So, as you put the pressure of the punch as you push the punch downwards the shape of the this blank will change ok. And it will change drastically so that from initially say a 2D surface or 2D blank you will get most a fully 3D cup ok. So, you have geometric nonlinearity which exists here ok.

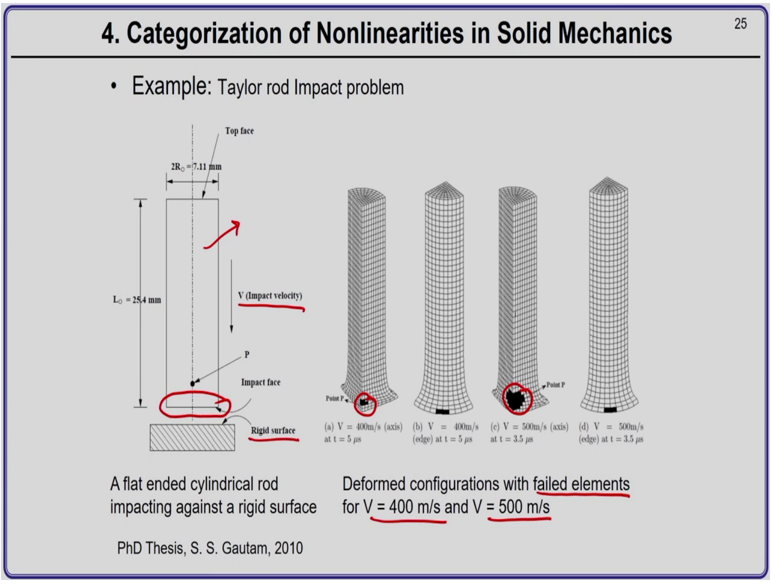
Now apart from that you have kinematic nonlinearity which is present between the blank and the other parts. For example, you have contact between the blank and the blank holder in this region ok. And as you push the punch downwards the blank will come in contact with the die ok. So, some this kind of shape you will get you will get some this kind of shape. So, you will have contact between the blank and the die at this portion ok.

So, you will have kinematic nonlinearity which will exist between the blank and the other parts; such as punch, die and the blank holder ok. So, between punch and the blank you will have contact between the blank and the die you will have contacts and between blank and the blank holder you will have contact. So, this is a kind of contact column where there is a contact between four different bodies.

The firm body is blank the other three are punch the blank holder and the die ok. And the contact between the punch and the blank is the main driver of this process. So, what do you achieve once you do it carry out this process so you get this kind of drawn cup. So, this is a finite element model final result from a finite element model of a blank which was initially offered cylindrical shape with very small thickness.

At the end of the simulation you will get something like this so this is with purely isotropic material ok. Obviously, if you change material to an isotropic you will get what is called wrinkling, but that is not part of this course. I will just wanted to show what kind of different nonlinearities which can come into this problem ok.

(Refer Slide Time: 19:10)



The last example of we consider is a Taylor rod impact problem. So, I can as you can see here Taylor rod impact problem is used for dynamic material property characterization. So, what is Taylor rod impact problem? So, what you have is a cylindrical rod or certain length and diameter and this cylindrical rod is impacted against a rigid surface ok.

So, this rod maybe made of a material that you want to characterize and then what you do is you fire this rod against the rigid surface at a certain impact velocity. Now so you remember now we have shifted from static to dynamic problem here velocity is come into picture. So, here what are the different kind of nonlinearities which are present?.

The first kind of non-linearity which is present is you will have plastic deformation of the cylindrical rod. There will be very severe plastic deformation which will occur in the cylindrical rod as the rod impacts the rigid surface see the impacted end. So, this end of the rod will be what will happen? Initially you will have a radius of R 0 and slowly it will mushroom out ok.

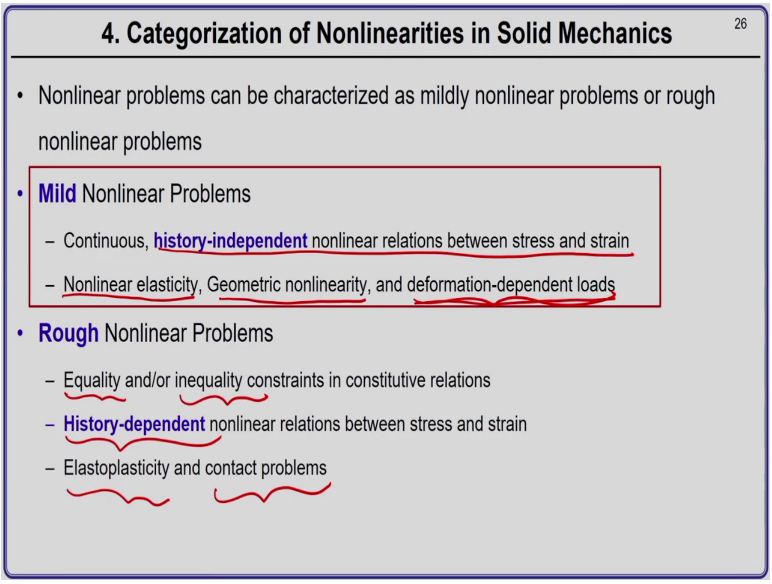
It will mushroom out and then you will have large plastic deformation that will occur. You will have geometric non-linearity because the initial shape of the rod and the final shape of the road will be very different. So, you have to take into account geometric nonlinearity. Also there is contact between the rod and the rigid surface. So, there is contact nonlinearity which is present ok. And the other kind of physics that is present over here is if you impact the rod against the rigid surface at sufficiently high impact velocity.

What will happen is; you can see you will see that the impacted end will fracture ok. You will see fracture of the specimen and you will get fragments ok. So, this kind of problems to simulate you have to take into account all kind of nonlinearities. So, here there are some finite element simulation results which I have shown and these are the deformed configuration ok.

And at two different impact velocities of 400 meter per second and 500 meter per second and you can see the failure of the rod as it happens at different impact velocity. So, this black zones that you are seeing here are nothing, but the failed elements. These are elements which have lost because of certain criteria they have lost their load bearing capacity. And we can imagine that there is a crack that has come at these places.

So, this is next level of complex problem where all different kinds of nonlinearities will be present. So, from the previous problem where we had a static contact this problem we apart from those four nonlinearities we also have the fracture and then the dynamic effects that come into picture. So, these kinds of problems are very computationally challenging ok.

(Refer Slide Time: 22:41)



So, next we come to the characterization of non-linear problems as either mildly non-linear problems or rough non-linear problem. So, what are mildly non-linear problems? The mildly non-linear problems are where the nonlinearity is continuous ok. And there is a history independent non-linear relation between the stresses and strain ok.

So, the relation between stress and strain only it depends on the current state and it does not depend on the history ok. If you have such a case then there is a case for mildly non-linear problem. So, you will have non-linear elasticity, you will have geometric nonlinearity and you also may have deformation dependent loads put into these kinds of non-linear problems. So, mildly non-linear problems will have these kinds of characteristics.

Next comes the rough non-linear problems ok. So, rough non-linear problem will have all these characteristics and on top of that you will have constraints ok. There are certain constraints which will be either equality constraints or inequality constraints on the constitutive relations ok. So, in the previous problem on the contact problem you would you will see that there is a constraint that one body cannot penetrate inside the another body ok.

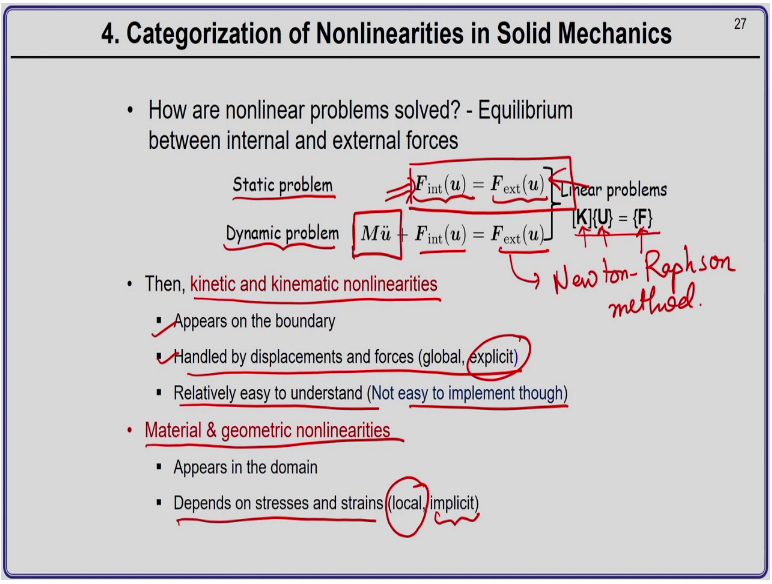
So, this constraint has to be when you are solving the system this constraint has to be satisfied. And to satisfy these kinds of constraints you have to device certain kind of constraint handling algorithm which are taught as a part of computational contact mechanics course.

So, here is this suffices to know that for roughly non-linear problem you will have either equality or non equality constraints coming into the picture. On top of that you may have history dependent non-linear relation between stresses and strain ok.

History dependent mean for example, plasticity. So, plasticity problems you have the relation between stress and strain which not only it depends on the current state, but also on the history of deformation ok.

So, elastoplasticity problems and contact problems are classified into the roughly non-linear problem. So, in this course we focus on the mildly non-linear problems and also we do not discuss the deformation dependent loads ok. So, we just concentrate ourselves into material nonlinearity and geometric nonlinearity ok.

(Refer Slide Time: 25:20)



So, how are non-linear problems actually solved? So, before we begin into seeing some examples of problems that two simple problems we will solve ok. We should have a note on how non-linear problems are actually solved ok. So, you establish the equilibrium between the external and the internal forces.

Whatever maybe the problem static or dynamic you establish the equilibrium between the internal forces which are generated because you are applying external forces on the body and then the body resists the deformation and at a certain point of time there is a equilibrium which happens between the body and the external forces ok.

So, for static problem; problem where you can neglect the effect of inertia forces, you have following relation the internal forces written here as F subscript int and they are a function of displacement ok. So, internal forces usually depend on stresses and stresses depend on strain and strain depends on displacement. So, these internal forces are a function of displacements.

So, these internal forces should be equal to the external forces and these external forces may also have a dependency on the displacement of the body. For example, in follower loads you had that dependency ok. In case of dead loads you only have external forces without any dependency on the displacement ok. So, in static problem what you do is you try to balance the internal and the external forces ok.

So, you can remember connect this to your statics in your first year engineering mechanics. There are also in statics what you used to do was balance the internal and external forces to get the reactions at various joints ok. In dynamic problem we have these two terms ok; the internal and the external forces which are already present and we have an additional term of the inertia forces ok.

So, inertia forces is; mass into acceleration u double dot ok. So, u double dot denotes the double derivative in time of the displacement ok. So, in a non-linear problem you actually either balance the forces for static in case of static problem or you try to solve the Newton's second law in for the dynamic problem.

In case of usually on linear finite element these equation boil down to K U equal to F this is what in usual introduction to linear finite element is taught; K is your stiffness matrix, U is the vector of your unknown displacement and F is the external forces ok.

So, for the static or the dynamic problem this equation that you get are non-linear equations. So, what you do; you have to solve these using some method which can solve non-linear equations. And one of the most popular way to solve this non-linear equations is through Newton Raphson method ok. So, Newton Raphson method is usually employed to solve this system of non-linear equation which I shown here ok. How do these equations come we will see it in the course of our discussion ok.

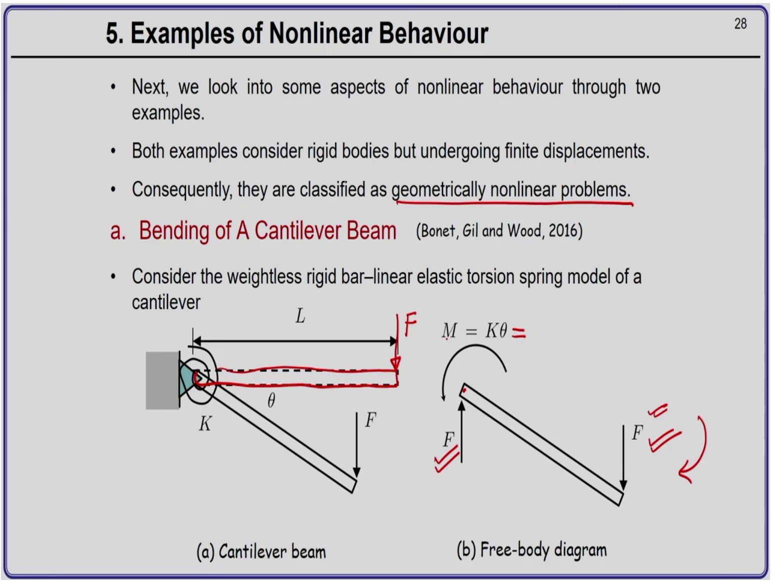
Later on we will come across how you get these different force vector and what are the explicit expression for these force vectors ok. Now again the kinematic and the kinetic nonlinearity they occur only on the boundary and they are handled by displacements and forces ok.

So, if you have kinetic or the kinematic nonlinearities they will usually occur on the boundary of the body and they are handled by displacements and forces in the global sense and you have to handle them explicitly ok.

And these are very relatively easy to understand and they are not very difficult to implement inside any code ok. On the other hand the material in a geometric nonlinearity they are usually present inside the entire domain and they will depend on the stresses and strain.

So, which means if they are present inside the entire domain which means you have local effect ok. And to handle that you have to have some implicit kind of strategy to solve the problem ok; which have these material and geometric nonlinearity ok.

(Refer Slide Time: 30:32)



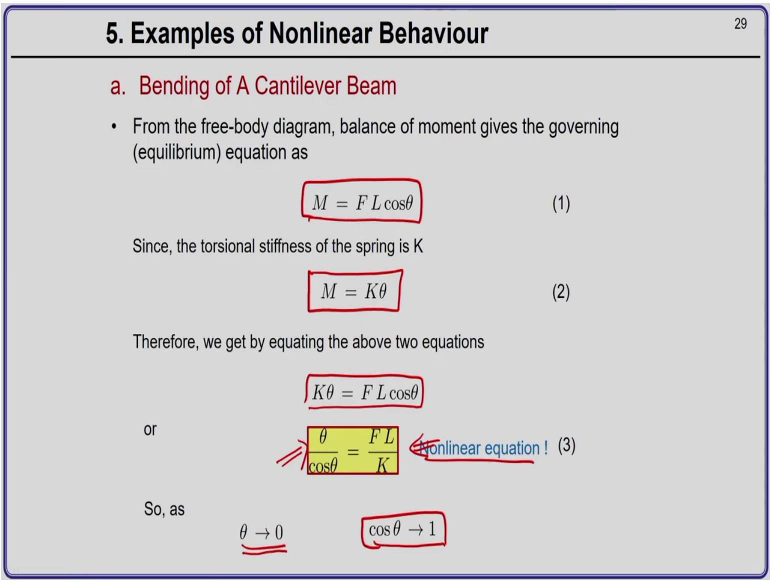
So, now we look into two different problems ok. So, some aspect of this non-linear behaviours; we look through two examples ok. So, these two examples have finite displacements and they do not involve any material nonlinearity neither they have force non-linearity ok. So, they are only geometrically non-linear problem ok. So, three through these two problems we will show you some aspect of non-linear behaviour.

So, the first example is bending over cantilever beam. So, as you can see here in figure a; so you have a weightless rigid bar ok. So, you had a weightless rigid bar over here shown by this dotted line and you apply a force at the tip ok. And there were a torsion spring present at the fixed end of the beam hinged end of the beam or the bar ok.

Now as you applied the force F the bar will rotate and so the rigid bar. So, we neglect the deformation of the bar ok. And the bar will rotate till a angle theta at which point the equilibrium is achieved ok. Now to derive the equilibrium equation what you do is you have to get the free body diagram first you have to draw the free body diagram of the bar. So, figure b; shows here the free body diagram of the bar.

So, F on the right hand side is a external force on the left hand side you will have a reaction force at the joint. And also this external force causes a clockwise moment to act on the hinged end of the bar ok. To overcome this moment the torsion spring will apply a moment equal to K times theta where K is the torsion stiffness of the spring. So, the moment M is applied by this spring will be K times theta.

(Refer Slide Time: 32:54)



Now, so with this using free body diagram and balance of moment you will get the governing equation given by equation 1 where; moment M equal to F into L into cos theta. Also you can see that the torsion stiffness of the spring is K so the moment by this spring will be equal to K into theta.

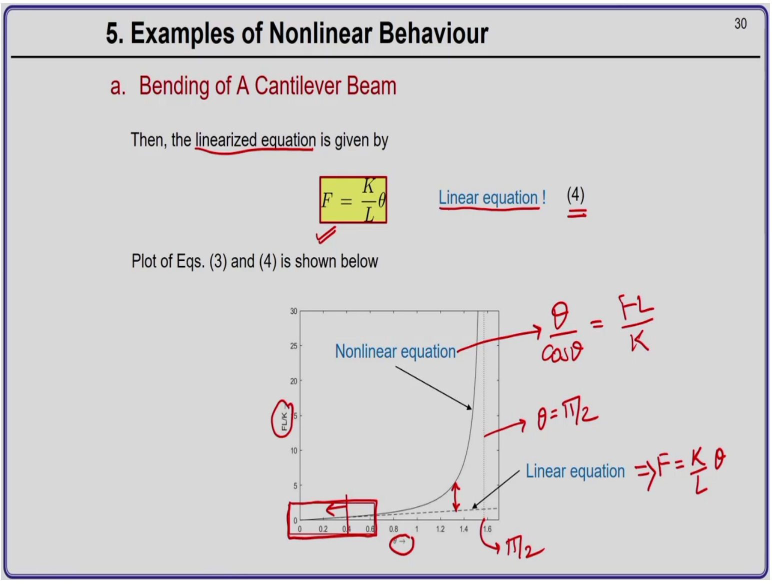
Now the moment is balanced the moment due to the external forces given by equation 1 and the moment due to the torsional spring given by question 2 should be balanced. This gives you K theta is equal to F L cos theta ok. So, this is the equilibrium equation or the governing equation ok. So, from this you get equation 3 which is a relation between the force F L by K ok.

So, give so we assume that K and L are given to us so it acts like a scalar multiple to the force. So, this is a relation between the force and the cos of this the effect of this force theta; once you apply the force you get theta. So, as you can see there is a non-linear relation which occurs between the force and the effect which is theta because cos theta is a non-linear term so there is a non-linear relation this equation ok.

You can see there is no material nonlinearity there is no deformation, there is no contact, nothing like this, but still for this very simple case you have this non-linear equation which comes into picture ok. Now this is a non-linear equation ok. So, as theta approaches 0 say; you apply a force which is very small is such that the deflection theta is very small ok.

So as this deflection approaches 0 what happens? The value of cos theta approaches 1 ok. So, as theta approaches 0 cos theta approaches 1 this we already know.

(Refer Slide Time: 35:22)



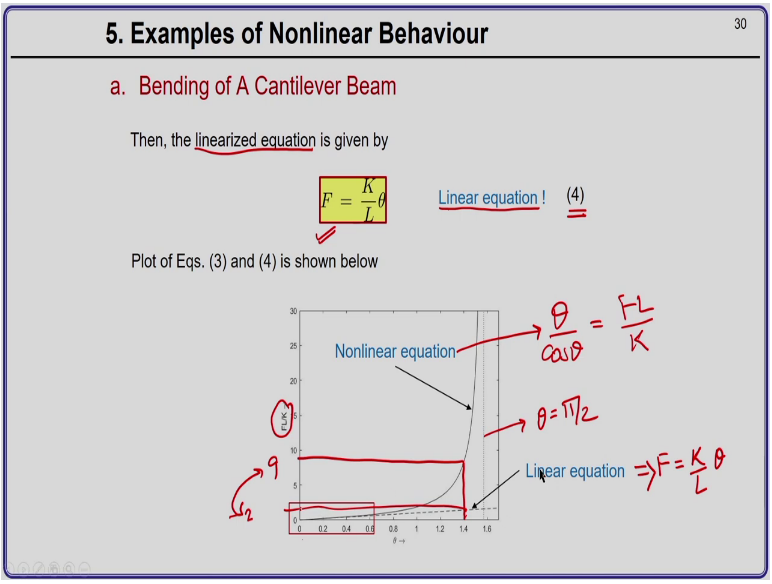
So, substituting these in equation 3 what we get is what we call the linearized equation ok. So, linearized equation means now our non-linear equation has been changed from a non-linear equation to a linear equation and this equation will be F equal to K L by theta. So, this equation is a linear relation between force and the angle theta ok; where K by L is a known quantity ok. You have been given K you have been given L and you know the value of K by L ok; so this is like y equal to M x ok. So, therefore, this is a linear equation ok.

Now we can plot to further show the effect of nonlinearity we just plot equation 3 ok. So, this is the non-linear equation theta by cos theta equal to F L by K. And this is a linear equation which is given by equation 4 which is F equal to K L by theta ok. So, on the x axis we have theta on the y axis we have F L by K ok. So, now, we just plot F L by K with the value of theta. On the y axis we can change theta from 0 to say pi by 2. So, this vertical line over here this vertical dotted line is for theta equal to pi by 2 ok.

So, you can see the non-linear equation how it behaves and you can see this dotted line over here is the linear equation. So, you can see when the value of theta is very small on the lower left in this red box over here which I am pointing out right now. You can see the difference between the two curves when theta is very small is marginal ok. I mean for values of theta which are less than 0.4 ok; you can see there is hardly any difference between the non-linear curve and the linear curve so there is no difference.

But as the value of theta increases as theta becomes significant you can clearly see there is a difference between the non-linear curve and the linear curve ok. So, if you are trying to solve this problem say; suppose you are given this problem and instead of using equation 3 you have used equation 4 for the analysis. And if your theta becomes very large what will happen you will report the value of force if you are using linear equation you will report ok. So, first I will rub this ok.

(Refer Slide Time: 38:20)

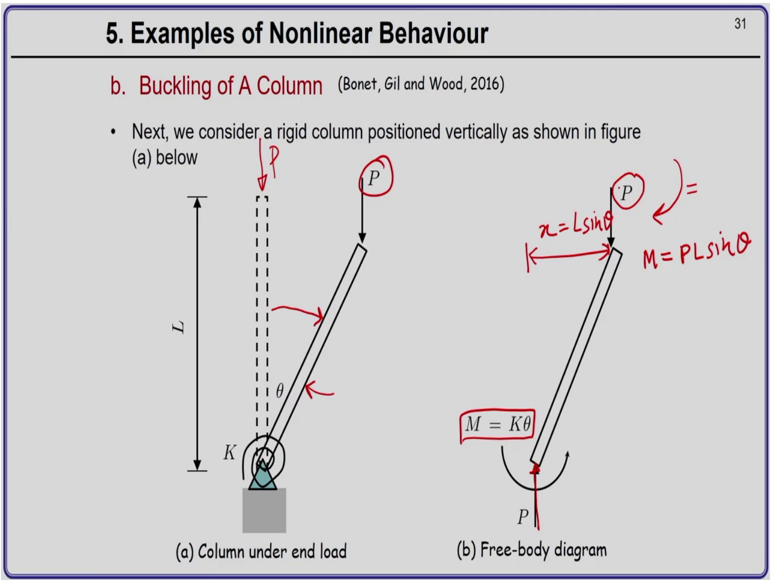


So, say theta equal to 1.4 ok; so if we are using linearly governing equation you will report a value of force let us say close to 2 ok. And if you are using non-linear equation you will have a value of force say let us say equal to 9 ok. So, now, a non-linear governing equation will give you a solution 9 while a linear equation will give you a solution 2. And so you can see already there is a huge difference between the value of the forces ok.

So, what this example shows is that you should be very careful when to use linear equations and when to use non-linear equations ok. So, if your problem characteristic is such that you expect only small values of theta. So, there is no point of going for non-linear equation you can just simply go with linear equations ok. And so you will remember from our last class that linear systems are always computationally faster ok; you can achieve the solution faster.

So, instead of going for non-linear you can get this solution using linear same accurate solution using linear system linear equation ok.

(Refer Slide Time: 39:46)



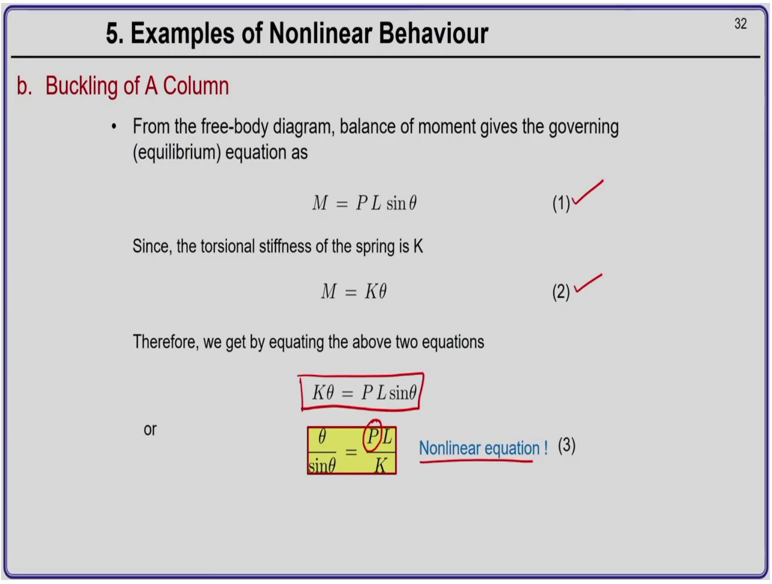
Next we discuss a final problem which is buckling of a column ok. So, this example is similar to the previous one where we had the bar which was a vertical I mean horizontal; now we have a bar which is vertical. The torsion spring is already present and now we apply a vertical force P here ok.

There also we applied a vertical force here also we are applying a vertical force. And what will happen so this the column. So, we call this as a rigid column it will rotate. And then there will be a position at which the column will come in equilibrium with the external force P ok. So, to get the governing equation what do we need to do first is we need to draw the free body diagram ok.

So, the figure b shows you the free body diagram ok. So, at the pin joint end you have a reaction force which will be equal to the applied external force P ok. You will have a reaction force equal to P. And at the fixed end there will be a moment because of this external force there will be a clockwise moment because of this external force and that moment will be P into this distance say x.

And this will be nothing, but L sin theta so this distance is L sin theta. So, the moment because of this external force P will be P into L sin theta ok. And this moment will be registered by the torsional spring which is present at the pin joint end and this moment will be equal to K times of theta K is the torsional stiffness ok. So, the reaction forces are equal ok.

(Refer Slide Time: 41:58)

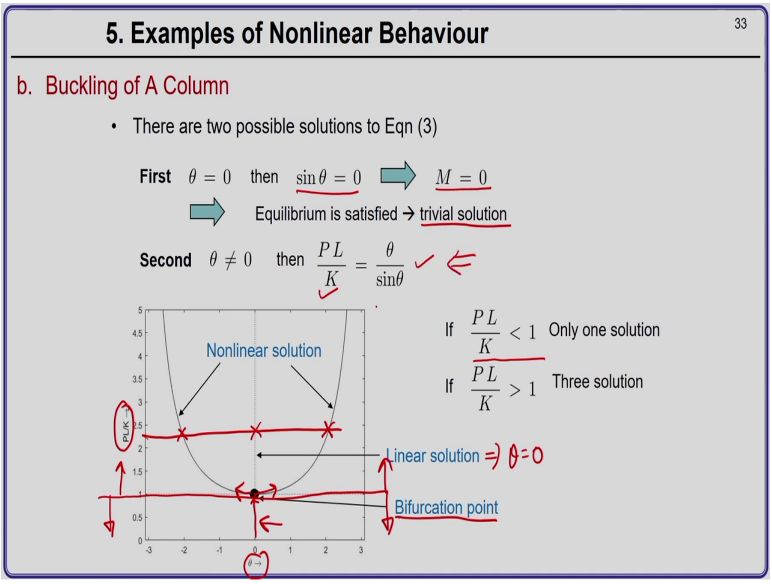


Now balancing of the moment will give you this equation K theta is P L sin theta ok. So, equation 1 is the moment external moment because of the external forces. And second equation is basically your resisting moment because of the torsional spring for equilibrium the moment because of the spring should be equal to the moment because of the external forces which is K theta should be equal to P L sin theta ok.

So, now, you can get a relation between the forces external force P and the angle theta ok. So, just like in our first example there where there was a non-linear relation between the force and the angle; here also we have a non-linear relation between the force and the angle ok. So, you can see here sin theta is a non-linear function.

So, theta by sin theta is a non-linear function. So, there is a non-linear relation between P and theta ok. So, here K and L are given to you and it is like a scalar multiple to P ok. So, this is a non-linear equation ok.

(Refer Slide Time: 43:13)



So, now there are two possible solutions to equation 3 ok. So, two possible solutions to equation 3 the first one is theta equal to 0 ok. If you have theta equal to 0 sin theta will be equal to 0 ok. And then this means the moment M will be equal to 0 and with M equal to 0 equilibrium will be satisfied, but then this is what we call is a trivial solution ok. So, this is of not much interest to us. The second is if you have a non 0 value of theta. So, then what happens?.

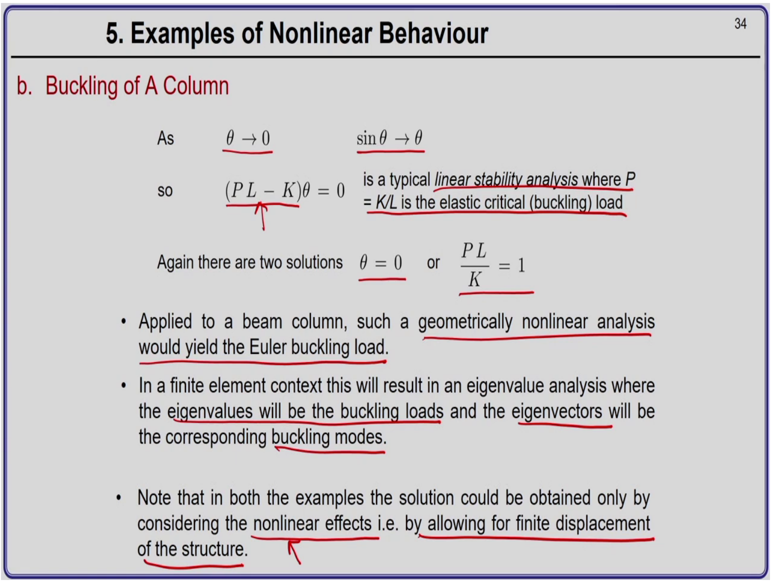
Then you have P L by K equal to theta by sin theta that is your solution. And now we can plot this equation P L by K equal to theta sin theta ok. So, on the x axis you have theta and on the y axis you have the value P L by K. And then we can plot the non-linear function P L by K equal to theta by sin theta and you get the parabolic kind of curve ok. And this dotted vertical line this vertical line at theta equal to 0 which is dotted is nothing, but your first solution which is theta equal to 0 ok.

So, these two solutions intersect at this point which is also called the bifurcation point. Why bifurcation point? Because if you approach from the lower end as you come to this point. At this point you have two different paths to solution ok; either you can go this half or you can go this half ok. So, your solution bifurcates that is why it is called bifurcation point. I do not discuss beyond this point here.

Now if value of P L by K is less than 1 so which means if this is your line P L by K equal to 1. So, below this line so below this line you only have one solution which is theta equal to 0 and now if P L by K is more than 1 which means on this side ok. So, you have three possible solutions; why three possible solution? If you draw a vertical line so you have one solution over here, you have one solution over here and you have one solution over here ok.

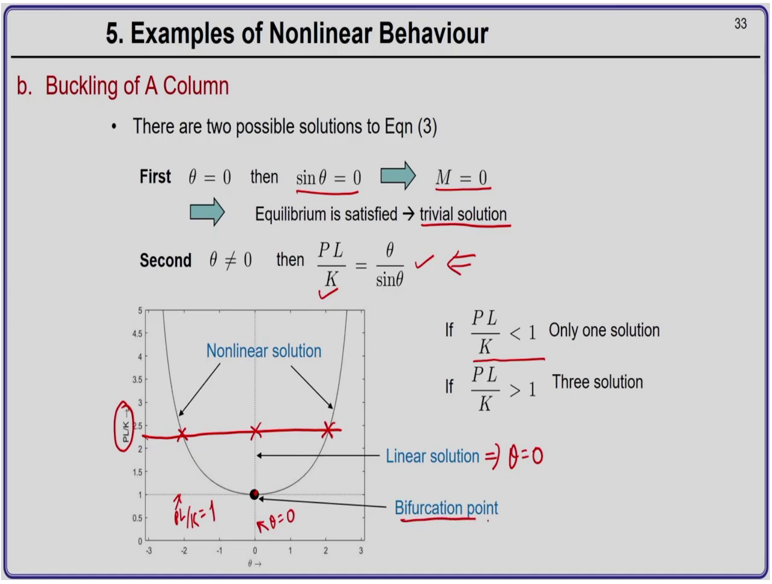
So, now, for the value of P L by K which is equal to greater than 1 you have three possible solutions which come into the picture ok.

(Refer Slide Time: 46:06)



So, next you can linearize. So, as theta approaches 0 what happens? Sin theta approaches theta ok. And then you substitute this in this equation what you get P L minus K into theta equal to 0. And one can recognize that this is a typical linear stability analysis where P equal to K by L is the critical buckling load for a column ok. So, again for this equation over here you have two possible solutions; one is theta equal to 0 and the other is P L by K equal to 1 ok.

(Refer Slide Time: 46:59)



So, you see here you can see here this is this dotted line over here is P L by K equal to 1 and this is your line theta equal to 0 ok. So, for a linearized system you only have one solution this is that solution and that is your bifurcation point ok. So, that after this point the column becomes unstable ok.

So, when applying to beam column such a geometric non-linear analysis will yield you the Euler buckling load ok. And this finite element context the eigen values analysis will give you eigen values which will be nothing, but the buckling loads. And the eigen vectors of that eigen value analysis will give you the buckling modes ok. So, eigen value we will give you the buckling loads and the eigen vectors will give you the buckling modes ok.

So, in both the examples that we have considered we can get the solution only when you consider the non-linear effects ok. That is when you allow the finite displacement of the structure to happen. If you did not allow for the non-linear displacement to happen non-linear effects to come into the picture you will not have got the solution ok. So, with this final slide I will end today’s lecture ok.

So, remember there are four different kinds of nonlinearities which are present in to the system; geometric nonlinearity, material nonlinearity, force nonlinearity and boundary nonlinearity. Each of this nonlinearity has to be identified in a system ok. And appropriate algorithms have to be devised or they had to be chosen. So, if you are writing your own code; you have to write a algorithm which can handle this kind of non-linearity.

Or if you are using a software you have to enable options which can take into account these nonlinearities. Remember that all these kinds of nonlinearities are some are difficult to solve other not that difficult to solve, but you need a significant amount of computational effort to solve these problems if nonlinearity is present. So, non-linear codes can solve linear problems ok.

So, linear problem can be solved by a code which has the non-linear capability ok, but solving a linear problem using a non-linear code would not be a wise choice ok. Because at the end you will spend a lot of computational effort which you could have avoided if you just knew that in my code there is no non-linear effects ok. And then you could simply use a linear code to get your solution very fast ok. So, it is very essential that you know how to identify these four kind of nonlinearities.

So, with this I end today’s lecture and in the next lecture we will start with the introduction to tensors and tensor calculus ok. And I hope you have some idea about the nonlinearities. So, we will meet you in the next class.

Thank you.