Nonlinear Vibration
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Lecture - 01
Introduction to mechanical systems

Welcome to today class of Non-Linear Vibration. I am Professor Santosha Kumar Dwivedy from Mechanical Engineering Department Indian Institute of Technology or IIT Guwahati. I taught this course several times and already I have developed this NPTEL course on non-linear vibration both web course and video course and I welcome you to attend this MOOCs course on non-linear vibration.

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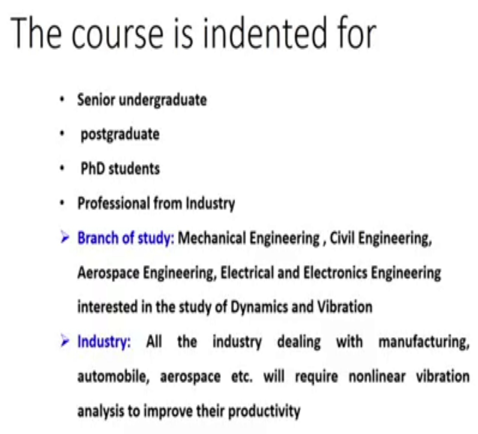

So, this course is intended for senior undergraduate students, post graduate students, PhD students and professional from industry also particularly the students from Mechanical

Engineering, Civil Engineering, Aerospace Engineering, Electrical and Electronics Engineering interested in the study of Dynamics and Vibrations are welcome to attend this course. So, many industry also will be benefited so, from the study of this non-linear vibration. So, particularly this manufacturing industry, automobile industry, aerospace will require non-linear vibration analysis to improve their productivity.

So, the professionals from these industry are welcome to participate in this course. For example, in case of this manufacturing the chattering in the turning operation is a common phenomenon. So, also in the case of other manufacturing like this milling, shipping and other manufacturing processes so, many times we observe the vibration in the system.

So, to mitigate or to observe or to reduce the vibration, so, this course will be useful. So, initially you should know how to recognize this vibration and then after knowing to recognize this vibration then the study how to mitigate or how to control those vibrations things will come into picture. So, we are going to study a detailed analysis on this non-linear vibration in this course.

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Module	Module Name	Week	Lecture No	Title of the lecture	Assignments
1	Introduction to Nonlinear Mechanical Systems	1	1	Introduction to mechanical systems	Online: MCQ, Program, Problem solving
			2	Equilibrium points: potential function Time response, phase portraits	
			3	Simulation of Phase portraits from potential function	
2	Development of Nonlinear Equation of Motion (Use of Symbolic Software to derive equation of Motion)	2	1	Force and moment based Approach	
			2	Lagrange Principle	
			3	Extended Hamilton's principle	
3	Solution of Nonlinear Equation of Motion	3	1	Numerical solution method	
			2	Harmonic Balance method	
			3	Lindstd-Poincare' method	
		4	4	Method of Averaging	
			5	Method of multiple scales	
			6	Recent advanced method	
4	Vibration Analysis of Nonlinear SDOF	5	1	Application of Duffing Equation, frequency and	

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	system with weak excitation			forced response plots	
			2	Application of Duffing Equation with simple resonance	
			3	Practical Applications of simple resonance condition	
5	Vibration Analysis of Nonlinear SDOF system with Hard Excitation	6	1	Nonlinear system with hard excitations	
			2	Super and sub harmonic resonance conditions	
			3	Bifurcation analysis of fixed-point response	
6	Vibration Analysis of Parametrically Excited system	7	1	Parametric instability region	
			2	Floquet Theory	
			3	Perturbation method to study parametrically excited system	
7	Analysis of systems with Periodic, quasiperiodic and Chaotic responses	8	1	Bifurcation analysis of periodic response	
			2	Analysis of quasi-periodic system	
			3	Analysis of chaotic System	

So, the course content. So, the course content will be covered in 10 to 12 classes. So, will be 9 modules; last 3 modules are last 3 last modules is on these application and the first module is on the non-linear introduction. So, which I am going to cover in the first three classes.

So, the introduction to non-linear mechanical systems particularly I will tell in the first three classes. So, it will be in this first week. So, in the first lecture we are going to discuss regarding the introduction to mechanical systems and particularly I will tell you the vibration of the linear system and non-linear systems.

So, I will take the examples of single degree of freedom system and two degrees of freedom system and we will see how the analysis of a single degree of freedom system in a linear case and in non-linear case are different. So, in the next class we are going to study regarding the equilibrium points. So, what do you mean by this fixed point and how the response of a non-linear system will behave. So, those things we are going to study particularly on the fixed point response in the second class.

So, then we will study about the potential function. So, then how the potential energy term is related to the equilibrium points that thing we will study, then time response, phase portrait using this time response and phase portrait we will study how to analyze a system particularly how will characterize a linear system how a non-linear systems so, that thing we are going to study in the second class.

In the third class we are going to study about the simulation of phase portraits from potential function and we will study or we will know regarding many functions. So, which are related to non-linear vibration. So, here also I will tell you about the superposition rule.

So, which differentiate between the linear system and non-linear systems and also we will study so, different type of non-linear commonly used non-linear systems or non-linear equations for example, this Duffing equation, Van der Pol equation, Mathieu equation, Mathieu Hill equations, Lorentz equation. So, many other equations which are commonly used in the non-linear cases we are going to introduce in this first three classes. So, in the second week will be devote for the development of non-linear equation of motion.

So, how to derive the equation of motion using different approach will be told briefly. So, as this is only a 10 weeks course. So, for 30 lectures will be delivered 30 to 35 lectures will be delivered in this course. So, briefly we are going to study how the equation motions will be derived. So, initially I will tell regarding the force and moments based approach.

Then we will see the energy based approach where I will tell regarding this Lagrange principle, Lagrange principle and then this extended Hamilton principle and we will solve some problems or we will derive some equation of motion using this Lagrange principle and extended Hamilton principle.

So, we will take very starting with a simple spring mass system and the simple pendulum system. We will take off different many different continuous systems also to derive this equation of motion. So, then how to solve these? So, after getting these equations.

So, we will see how we are going to order these non-linear equation so after getting the order of this non-linear equations. So, we will solve them these non-linear equations by different methods. So, we will take several class actually to solve these non-linear equations. So, particularly we may take around 9 class to solve or 9 6 class to solve these non-linear equation motions.

So, initially we will use this numerical solution method. So, directly how by using this Runge Kutta method. So, to derive or to find the solution of the equations we will first see. Then we will use some other methods like this harmonic balance method. Lindstd Poincare method, method of averaging, method of multiple scales and many recent advance methods which are coming up nowadays. So, or in last decade what are the new methods have been there. So, we are going to study all these things.

So, we will devote six class to study the solution of this non-linear equation motion. So, then we will study the vibration analysis of the single degree of freedom system. Particularly this duffing equation will be taken and we will try to solve this duffing equation using different methods. So, in duffing equation initially we will take that of a free vibration system, then we can take the force vibration system in which two different type of force vibration things we are going to take.

So, one is weak nonlinearity and or weak nonlinearity and weak forcing and second one the hard nonlinearity or the hard forcing term hard excitation term. So, by taking this hard excitation term. So, in module 4 so, we are going to solve many problems. So, in hard excitation particularly the if the order of the excitation is that of the order of the linear term then or it is known as hard excitation. So, we will devote three classes to solve these hard excitation

So, particularly we will see the super harmonic and sub harmonic resonance conditions and bifurcations and stability analysis and bifurcations of all these things we are going to study. Then in module 6 we are going to study regarding the vibration analysis of parametrically excited system. So, before that the systems what we have studied those are forced vibration system that is when the external excitation frequency is nearly the natural frequency of the system resonance occur.

But in case of a parametrically excited system. So, if the external frequency is twice the natural frequency or combination of the natural frequency, then only resonance will occur. So, we are going to study regarding the principle parametric resonance condition or combination parametric resonance conditions in this module. So, we will start with what is known as parametric instability region.

Then, we will apply the Floquet theory and different perturbation methods to solve the parametrically excited system then. So, till these thing we are going to study regarding the fixed point response particularly we will be interested to know the fixed point response up to module 6. In module 7, we will see other different type of solutions for example, the solutions may be periodic it may be quasi periodic or it may be chaotic response also.

So, these periodic, quasi periodic and chaotic response we will study in module seven and we will study their bifurcations also. So, bifurcation of periodic response. So, we will study in the first class of module 7, then we will study regarding the quasi periodic response and the chaotic response. So, when the response is called to be periodic. So, that thing will know in this case.

So, particularly if the response can be written as a time bearing term where the periods. So, it has same response at a particular interval of time then it is periodic. So, it may be single frequency or it may be multi frequency. So, in case of the periodic response, but in quasi periodic response the frequencies there is relation between the frequencies particularly if the frequency ratios are in the so, in case of the quasi periodic system the frequency ratios are commensurable that means in the form of.

So, route 2, route 3 if that ratio are in the irrational number that is route 2, route 3, route 5. So, then we can have these quasi periodic type of response for example, you may be knowing the beating phenomena. So, the beating phenomenon can be represented by a quasi-periodic response. Similarly, we can study the chaotic response also. So, in chaotic response which is a deterministic response.

So, we will study different methods or a different types of chaotic response for example, this period doubling route to chaos, quasi periodic route to chaos or torus break down route to chaos and then crisis and intermittency. So, all those things we are going to study in this chapter or in this module of module related to these periodic, quasi periodic and chaotic response.

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8	Numerical Methods for Nonlinear system Analysis	9	1	Solutions of a set of nonlinear equations	
			2	Numerical Solution of ODE and DDE equations	
			3	Poincare' section, FFT, Lyapunov exponent	
9	Practical Application 1: Nonlinear Vibration Absorber	10	1	Development of Equation of motion: symbolic software	
			2	Solution of EOM: Use of Harmonic Balance method	
			3	Program to obtain time and frequency response	
9	Application 2: Nonlinear Energy Harvester	11	1	Development of Equation of motion and its solution: symbolic software	
			2	Solution of EOM: Use of method of Multiple Scales	
			3	Program to obtain time and frequency response	
9	Practical Application 3: Analysis of electro-mechanical system	12	1	Development of Equation of motion and its solution	
			2	Use of Floquet theory	
			3	Parametric instability regions, Study of periodic, quasiperiodic and chaotic response	

So, then we are going to study several numerical methods for the non-linear system. So, how to solve a set of equations? So, particularly when our degrees of freedom will be large then no longer by a single equation we can write the equation motion. So, that time a number of equations will be required to write the equation of motion and in such cases we should know several non-linear methods or numerical methods to solve these non-linear systems.

So, particularly we will be interested to know how to solve a set of non-linear equation, then what are the numerical methods to solve this ordinary differential equations, numerical solution for the ordinary differential equation and sometimes we may have the delay differential equations also. So, we will study regarding these ordinary differential equation and delayed differential equation, then we will study regarding the Poincare section.

Then this FFT and Lyapunov exponents to characterize the periodic, quasi periodic and chaotic responses. So, we will learn how to differentiate different type of responses based on Poincare section. So, based on this Lyapunov exponent and by the FFT or by plotting the time and frequency responses of the system. Particularly, we will be interested in the previous modules regarding the fixed point response and there we can plot the frequency response of the system or the force response of the system.

So, in last module that is module 9. So, we will be interested to see different applications of this non-linear vibration course. So, we will take three physical examples and we will start with the development of equation of motion to see how to solve those equations and also how to analyze those results and we will discussion we will do the discussion on all of those things.

So, the first system we are going to study will be on the vibration absorber, the second system we are we will be studying on a non-linear energy harvester and the third system some electromechanical system. So, we are going to study where the applications may be related to parametrically excited system. So, in this course. So, the software also hand on practice session on the software also will be given and assignments will be given to solve these problems using software like MATLAB or open source software.

So, you must learn all these open source software or MATLAB software to solve these problems. So, we will cover all these things mostly in 12 weeks and so, you will get lot of applications and assignments and you have to do the self study. So, the journals and books also will be referred and by using those things slowly we learn.

So, how to solve all these methods particularly we will be using the book by Naphy and MOOC non-linear oscillation by Naphy and MOOC and another book by Naphy and Balachandran also we are going to study. So, next class I will tell you regarding the books and the journals available in this field.

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APPLICATIONS

So, today let us see. So, or start with some applications of vibration and let us see some applications of vibration and how this vibration whether this vibration is useful or not useful and how we are going to analyze the system when the equations are linear. So, initially we will start with a linear system then later we will go for the non-linear system.

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So, many household applications you can find vibration is there for example, you just take the hair dryer or the you just take a pump or this compressor, sprinkler, micro oven or there are several precision or this instrument. So, for example, all these musical instruments you just see. So, where you can have the vibration and noise are coming into picture. So, in some cases the vibration may not be useful and in some cases the vibration is very much useful.

For example, in case of the musical instrument the vibration of the string or the membrane is giving the pleasant notes and everyone enjoy those music vibration musical notes. So, many cases this vibration will be very much useful and many cases the vibration may not be useful for example, if there is lot of vibration in the pump. So, we may think that there is some failure in the bearing.

So, that time the bearing has to be changed. So, similarly if there is some misalignment in the bearing or there is some bending in shaft or some this gear failure. So, many other things may happen which may lead to the vibration of the system. So, by knowing the vibration signature of different instruments, we can tell the condition of the system. So, the condition monitoring of any systems can be done by taking the vibration signature of those instruments.

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Acceleration (m/s^2) magnitude and frequency of fundamental vibration mode for various sources

Vibration source	A (m/s^2)	F_{peak}
Car engine compartment	12	200
Base of 3-axis machine tool	10	70
Blender casing	6.4	121
Clothes dryer	3.5	121
Person nervously tapping their heel	3	1
Car instrument panel	3	13
Door frame just after door closes	3	125
Small microwave oven	2.5	121
HVAC vents in office building	0.2–1.5	60
Windows next to a busy road	0.7	100
CD on notebook computer	0.6	75
Second story floor of busy office	0.2	100

Roundy et al. (2003)

Car Engine Compartment

https://www.detailingspot.com/wp-content/uploads/2010/03/Engine_Initial.jpg

CD-ROM

https://en.wikipedia.org/wiki/CD-ROM#CD-ROM_drives

Hard Disk Drive

https://en.wikipedia.org/wiki/Hard_disk_drive

So, let us see some other instruments also for example, this automobile, car engine. So, sometimes you might have heard lot of vibration in the car. So, due to many malfunctionings of different parts of the engine or the transmission system or the other systems used in this automobile. So, by knowing their vibration. So, you can easily distinguish which part has to be repaired and also the ground undulation in the ground sometimes its transmitted to the vehicle.

So, you have to make your suspension system in such a way that these ground vibrations will not transmit to the driver. So, while designing this automobile. So, all this vibration aspect has been taken care and due to wear and tear in the tire or in different parts of the automobile sometimes this vibration will be severe. So, that time. So, your analysis no longer be in linear ranges no longer be valid and you must have to go for the analysis by taking the system to be non-linear similarly you can take the example of a CD-ROM, hard disk drive.

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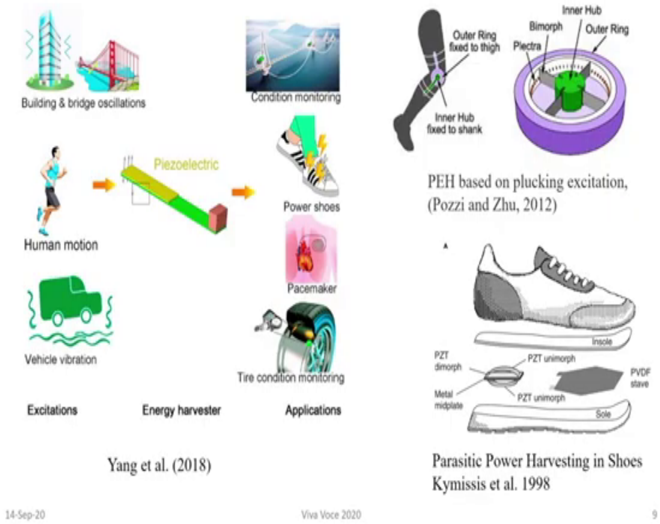

So, these are the common things what you have seen. So, where you can always see the vibration in the system you just take the example of the building bridge. So, these are the civil structure. So, building subjected to earthquake excitation, high rise building subjected to wind loading also similarly this bridge oscillation.

So, on the bridge when the vehicle is moving. So, due to this moving load you can get the vibration similarly. So, if it is a cable stayed bridge like this the bridge one here this is a cable stayed bridge. So, in this case the vibration of the cable. So, when the vehicle are moving on the bridge.

So, you may get vibration in the cable or you may get the vibration of the bridge similarly you may have many other structure. So, where this vibration are predominant. So, sometimes these wind vibration and the vibration due to this earthquake are not measured in a quantifiable manner. So, these are not a deterministic forces applied to the system many times the forces may be deterministic or without or within a range.

So, that time your force are deterministic or the application of excitation is deterministic when you know the particular amplitude and frequency of the excitation, but the excitation or in case of the earthquake or this wind loading. So, which are varying with time so, may not be known a priori. So, in that case it may be stochastic or probabilistic or the random type of vibration.

So, the system may be subjected to different type of loading. So, we will see what are the types of loading we generally consider in our case or in case of vibration those things will study slowly and so, so some other examples also you can see in this slide for example, this human motion. So, when he is moving. So, the motion of the human may cause the vibration also.

So, particularly when they are moving in a group on the bridge. So, you can find the due to the rhythmic motion of the persons. So, the force will be transmitted to the bridge. So, here are some examples actually given for the energy harvesting by using the shoes during this human motion. So, if we are putting some piezoelectric patch. So, in the shoe.

So, then the motion can be can be converted. So, the motion of the human being can be converted to electrical energy by using these piezoelectric patch similarly in the bridge or the building we can put the piezoelectric patches to convert actually the vibration energy into electrical energy.

Similarly, many other places we can use these piezoelectric patches or other different type of energy harvesting systems to generate energy. So, here in this slide several examples are given. So, when we will study the last module. So, that time we will study more regarding this energy harvesting system.

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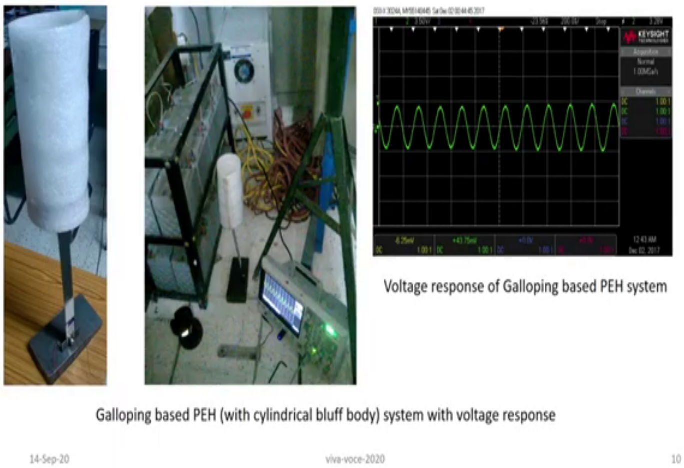

So, here also one more energy harvesting example is given. So, a galloping based energy harvesting that is piezoelectric energy harvester. So, here this is a bluff body. So, this bluff body for example, if it is placed in front of the UPS.

So, due to the small wind coming out of the UPS it get vibrated and that vibration energy by putting a piezoelectric patch. So, here we are put a piezoelectric patch by putting these piezoelectric patch you can see the voltage generated in the oscilloscope. So, the voltage generated in the piezoelectric patch can be seen by using oscilloscope.

So, you can generate a voltage of few milli Watt and in the order of several voltage volt also by using these type of energy harvester. So, you may have a vibration based energy harvester or this wind based energy harvester and there are several other way of generating these energy. So, those things we will study in the last module of this course.

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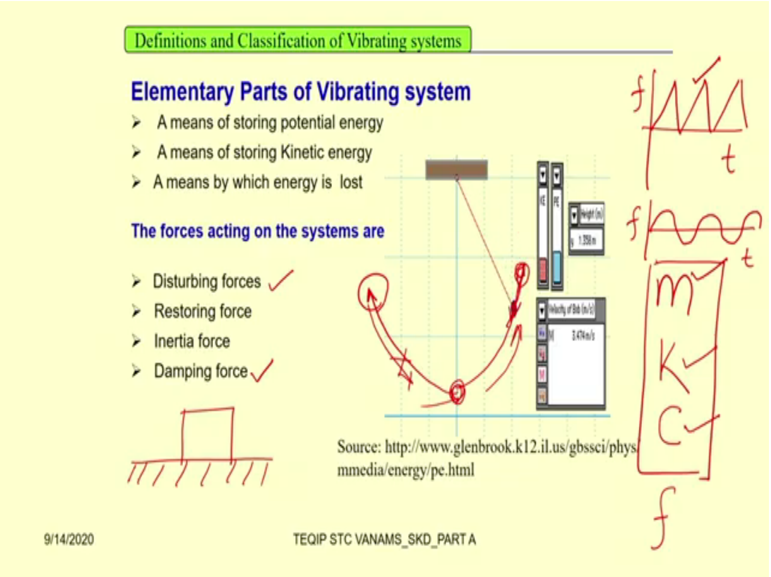

So, let us see. So, what is a vibrating system as we are going to study this vibration. So, this shows a simple pendulum. So, which has been given. So, initial disturbance. So, now, it has been given some initial disturbance, it has been taken to this place and it is released. So, if it is released.

So, it has been taken to this position and it is released. So, you one required a disturbing force to start the vibration, then due to as it has reach this maximum position and due to this restoring force, it is it will be subjected to a restoring force due to this position.

So, due to this restoring force it will come back to this original position. So, this is the mean position. So, it can come back to this mean position and from this mean position. So, due to inertia force it will come back or it will go to this side. So, initially we have given some disturbance force then due to this restoring force, it is coming back it is coming to the mean position and in this mean position as it is moving.

So, it is subjected to these inertia force and due to this inertia force again it will go to upward position and reach the maximum position. So, here again reaching at this maximum position the velocity becomes 0 and again due to restoring force it come back to the mean position and due to inertia force it goes to the next this side. So, we have a disturbing force, we have this restoring force, then inertia force and if dumping is not there.

So, this motion will continue and we can have a simple harmonic motion. So, when it is at the extreme position the system has the maximum potential energy and this potential energy decreases and converted to kinetic energy when it come back to this mean position. So, at the mean position all the energy are converted to kinetic energy and the system has a maximum velocity and due to this at this position, the system has all the energy as kinetic energy again.

So, when it is going to the left side. So, this complete kinetic energy again converted to potential energy. So, in some intermediate position. So, the energy will be both kinetic and potential energy. So, we must for a vibrating system, we must have a means of storing this potential energy then we must have a means to storing the kinetic energy and a means to by which this energy is lost. So, if the energy is lost not lost due to dumping or due to some other means.

So, then the system will continue forever and it will vibrate forever, but generally the natural systems. So, the vibration stops after sometimes due to the presence of this damping force. So, the means of storing this energy. So, the device. So, or the parameter responsible for storing energy is generally known as stiffness of the system. So, that is generally represented by K and then means of storing this kinetic energy.

So, kinetic energy can be stored due to the inertia property of the system. So, that is the mass property of the system mass will be there and then these means of energy lost. So, that is another property that is the dumping parameter. So, we required mainly these three parameters that is mass of the system or inertia of the system, then stiffness of the system and dumping parameter of the system to represent a vibrating system.

So, for a vibrating system we must have a mass, then it must have some stiffness and there must be some damping and a disturbing force if must be acting on the system. So, this disturbing force if maybe it may be periodic, it may be periodic in case of periodic force also. So, the force may be periodic. So, for example. So, this is periodic and so, this is the force versus time. So, if you plot these force versus time. So, this may be periodic or it may be a.

So, these periodic response can also be represented by using the sine and cosine term. So, that time it is known as harmonic. So, if the periodic response is represented by using the sine and cosine function then it is known as harmonic function.

So, generally by using the Fourier series. So, we can convert any periodic function to its sine and cosine form. So, it may be the forcing maybe the initial disturbance or forcing maybe periodic or it may not be periodic also it may be may give a impulse force to the system or the force maybe a ram type or maybe the step type.

Many different type of force you may apply to the system, you may give a fixed loading fixed loading to the system for example, on the bridge a vehicle is standing. So, in the time. So, in the time. So, the load is a dead load. So, you were applying a dead load to the system. So, if the vehicle is moving on these things. So, you have a life load.

So, the load may be dead or the load may be life and in the life load. So, it may be deterministic or it may be random. So, in case of the deterministic loading. So, it may be periodic or it may be this impulse type, RAM type or step type. So, different types of loading conditions are available. So, we will study particularly the harmonically excited system and also we will study the generalized conditions of the systems.

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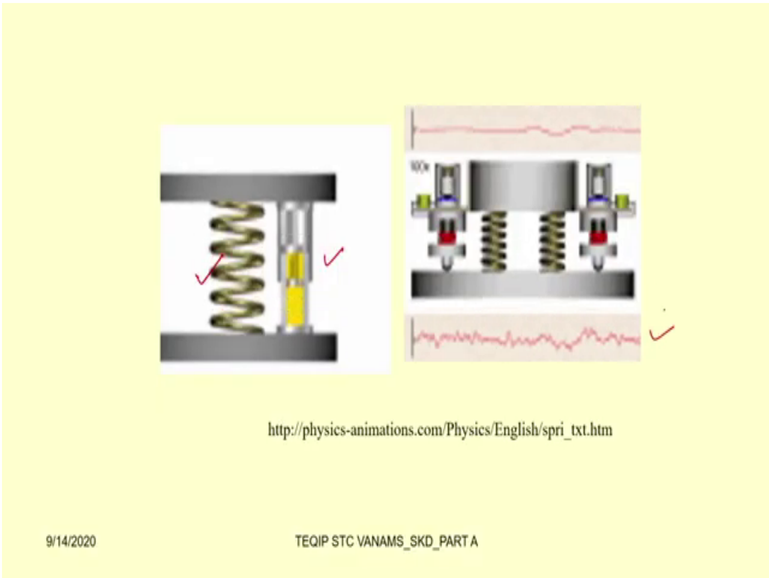

So, let us see one more example here. So, you just see this is a spring and damper system. So, the spring and damper. So, here we have a single spring and this damper. So, the vibration you can see here. Similarly, by putting two springs and the damper. So, we can see the response. So, the response. So, the response can be seen here.

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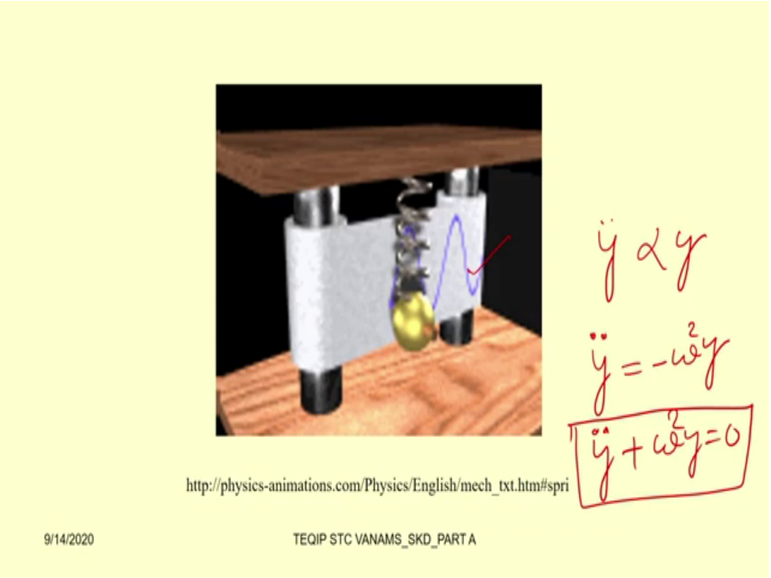

So, you can clearly visualize how the motion of a simple pendulum can be represented. So, here we have a spring and this is the mass. So, a pen is attached to the mass. So, as the spring is vibrating. So, you can see clearly it is stressing a sinusoidal curve the motion is sinusoidal. The motion generally becomes sinusoidal.

So, in case of the sinusoidal motion generally we write the acceleration that is acceleration is proportional to your displacement. If the acceleration is proportional to displacement and takes place in a direction opposite to that of displacement then, that type of motion is known as simple harmonic motion. So, in case of the simple harmonic motion.

So, we can always write these y double dot equal to minus omega square y or these y double dot that is acceleration plus omega square y equal to 0. So, here omega is nothing, but the frequency of the response. So, by using a spring and a mass. So, you can always generate a simple harmonic motion, but here you are assuming that spring that spring constant to be linear.

So, in that case only you can find this type of response. So, if the spring is not linear spring then the response no longer will be a simple harmonic motion and that is the study of this course. So, if the spring is non-linear then what is the equation of motion and if we know the equation of motion then how to solve that equation of motion. So, this is the thing we are going to study in this course.

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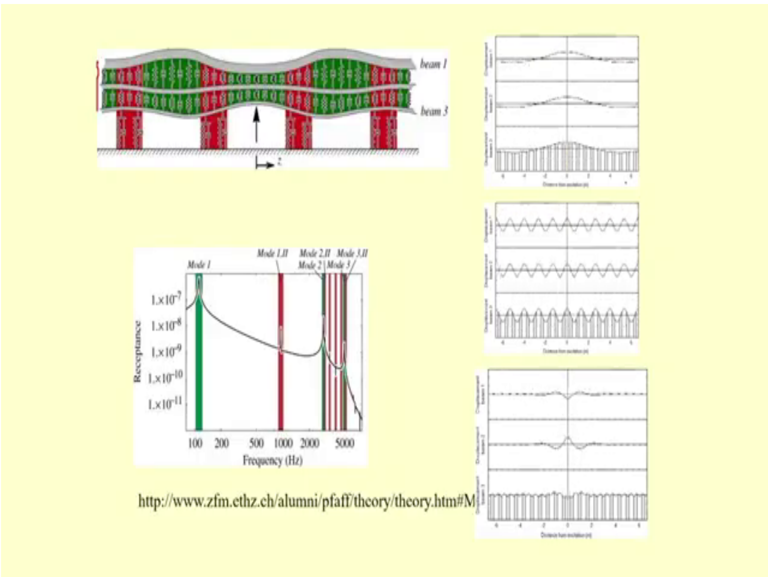

So, similarly for example, you just take the support in a railway line. So, this rail road you can see. So, it is subjected to. So, it has different layers. So, you can have the pebble layer, you can have the stones, pebble and then some this are sand layers will be there and then this railway track will be there.

So, the motion of the railway track you can visualize. So, different motion can be represented by different spring and mass spring and a damper system or it can be retained by a Pasternak foundation. So, you can see the response of different layer due to the motion of the train ok.

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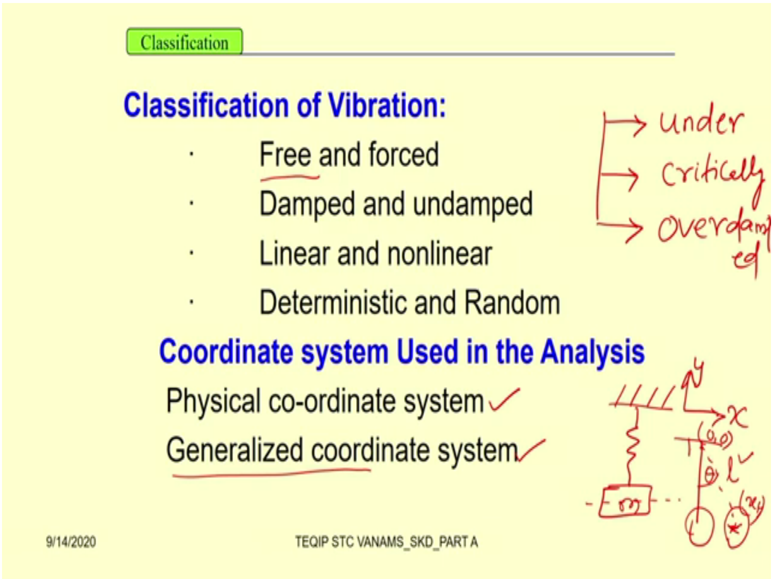

So, this basically we can classify the vibration as free vibration or force vibration if after the disturbing force no force is acting on the system, then the vibration is known as forced vibration.

So, if a continuous force is acting on the system then the vibration is known as force vibration. Similarly, we may have damped or undamped system. So, if damping is there then we have damped system if there is no damping then the system is undamped. Again this damped system can be divided into three category.

So, one will be under damped under damped one will be under damped system, one will be critically damped and the last one is over damped system. So, we may have so, we may have over damped system. So, we have under damped system critically damped system and over damped system similarly we can divide the system into a linear system or non-linear system.

So, we will see how we can distinguish between the linear and non-linear system particularly the superposition rule can be used to distinguish this linear and non-linear system. So, we may have a deterministic system and random system. So, generally while deriving the equation of motion of a system we are using physical coordinate or generalized coordinate system.

So, physical coordinate or generalized coordinate can be. So, while solving the or while writing the equation of motion of a system generally we use different coordinate system. So, one is the physical coordinate system and other one is the generalized coordinate system. So, in case of the physical system for example, let us take the spring and mass system.

So, this is a spring and mass system. So, we can write this equation of motion of the system by using a physical coordinate here at the we may write the coordinate system physical coordinate system at the base of the system or we may take a coordinate system here about the equilibrium position also. So, depending on where we are taking the physical coordinates. So, based on that thing we can write the equation of motion.

The other way of representing the coordinates of a system is the generalized coordinate. So, in case of the generalized coordinates. So, we may use minimum number of coordinates to represent the motion of the system for example, so, if we have a two degrees of freedom systems. So, let us take a simple pendulum. So, this is a simple pendulum the example what we have seen just before.

So, in case of the simple pendulum the position of this mass can be represented by writing the x coordinate and y coordinate. But this x coordinate and y coordinate are related by a constant equation which can be which is the length of the pendulum. So, we know if the coordinate of this point is 0, 0 and the coordinate of this point is xy then this x square plus y square is equal to l square.

So, we have one constraint equation and two physical coordinates two physical coordinates they are x and y and the constraint is l. So, knowing this constant equation and the physical number of physical parameter to represent this equation. So, we can find the number of generalized coordinate. So, the number of generalized coordinates will be number of physical parameter minus the number of constant equation. So, in this case of this simple pendulum.

So, we have two physical parameter that is x and y and also a constant equation that is l, l square equal to x square plus y square. So, the number of physical. So, number of the generalized coordinate will be only 1 so; that means, by using only one parameter. So, we can represent the motion of the pendulum. So, here we can use theta to represent the motion of the pendulum. So, or we may write only this x or y to represent the motion of the pendulum as the other coordinate can be known from the constant equation.

So, here if x is known. So, we can find y and if theta is known we can find x and y. So, by knowing only one parameter. So, we can write the equation of motion. So, those are the generalized coordinate system.

So, when we will derive the equation of motion so, that time we will know more about the generalized coordinates to derive the equation of motion particularly when applying this Lagrange principle or Hamilton principle. So, we are going to take the generalized coordinates and by taking this generalized coordinates easily. So, we can derive the equation of motion.

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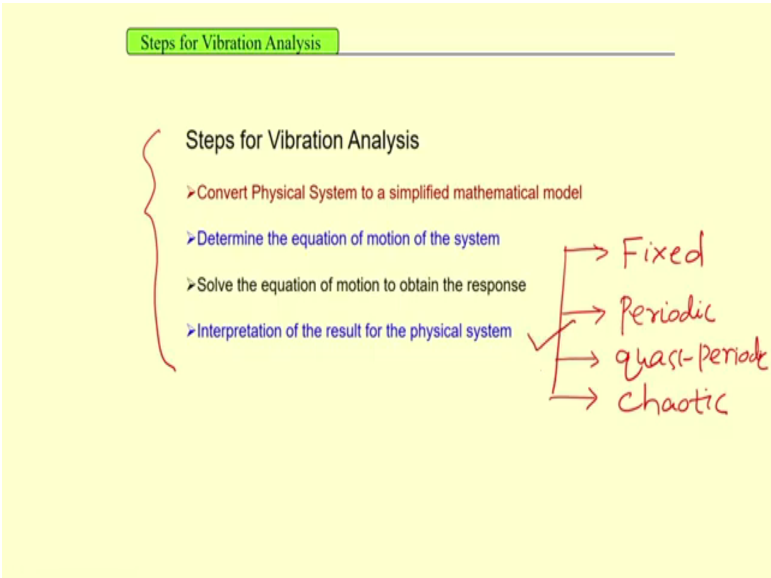

So, these are different steps to solve the equation of motion. So, for example, first we must convert a physical system to a simplified mathematical model. So, then we have to determine the equation of motion of the system, third we have to solve the equation of motion to obtain the response then after getting the response so, we may have to interpret the results for a physical system. So, when we are solving the equation of motion of the system. So, we may get.

So, in this non-linear system we may get four different type of solutions. So, the first type of solution is the fixed point solution. So, first type of solution is the fixed point solution, the second is the second type of solution is the periodic solution. So, the if response is periodic then this is periodic solution. Then, we may get quasi periodic solution quasi periodic or we may get this chaotic solution. So, we will study regarding all these types of solution in this course.

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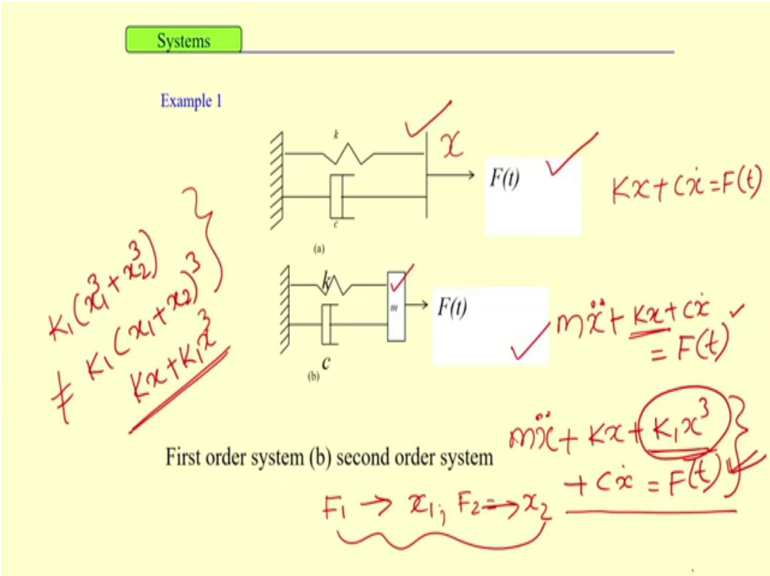

So, let us first see what do you mean by the order of a system for example, let us take this one. So, we have only a spring and a damper. So, a force F t is applied to this thing. So, we can write this force F t equal to.

So, if displace by an amount x, then we can write this K x plus C x dot equal to F t. So, here the equation is a first order equation, but if we are adding a mass to the system then now you have these inertia force and the equation of motion of the system can be written as m x double dot plus K x plus C x dot equal to F t.

So, this is a first order system and this is a second order system. So, if the spring force is written in terms of K x, then the equation is linear, but instead of writing the spring force as K x. If I will write this equation or this spring force as let me write the spring force as K x plus K 1 x cube that is a cubic order nonlinearity I have added to this spring then that equation can be written m x double dot plus K x plus K 1 x cube plus C x dot equal to F t. So, this equation what we have seen. So, is no longer linear. So, due to the presence of these term. So, because it will not satisfy the superposition rule.

So, what is superposition rule? So, for example, superposition rule have two parts one is the additive rule and second one is the homogeneity rule. So, in case of the additive rule. So, if we are applying a force of F 1. So, let the displacement is x 1. So, when we have applied a force of F 1 the displacement is x 1 and when you have applied another force F 2 the displacement is x 2 displacement is x 2.

So, now if we apply this force F 1 and F 1 plus F 2 simultaneously, then if the response is x 1 plus x 2 then it obeys the additive rule, but if it will not produce x 1 plus x 2, then the response will not be or it will not obey the additive rule. So, it will not obey the superposition rule for example, now you just apply a force x 1. So, the equation will be in second case the equation will be m x 1 double dot plus K x 1 plus K 1 x 1 cube plus C 1 x 1 dot equal to F 1 t.

In the second case let us apply force F 2, then the equation will be m 1 x 2 double dot plus K x 2 plus K 2 x 2 cube plus C x 2 dot will be equal to F 2 t. So, if we add these two then we will have for this F 1 plus F 2, the equation will be m x 1 double dot plus x 2 double dot plus K x 1 plus x 2, but here what we will have. So, in the one case we will get this is K 1 x 1 cube plus x 2 cube.

But in actual case this is not equal to K 1 x 1 plus x 2 cube. So, if the response is equal to x 1 plus x 2, then it must be a K 1 x 1 plus x 2 cube, but as this K 1 x 1 plus x 2 cube plus x 1 cube is not equal to this. So, it is not satisfying the superposition rule. So, as it is not satisfying the superposition rule. So, this equation cannot be a linear equation.

So, this is a non-linear equation. So, in this class you have seen. So, how the vibration can we applied to many systems and also we have seen. So, how the superposition rule we can apply the superposition rule to distinguish a linear and non-linear system. So, next class we are going to study regarding the linear single degree of freedom system two degrees of freedom system and the non-linear systems also.

Thank you.