Nonlinear Vibration
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Lecture - 15
Bifurcation analysis of fixed-point response

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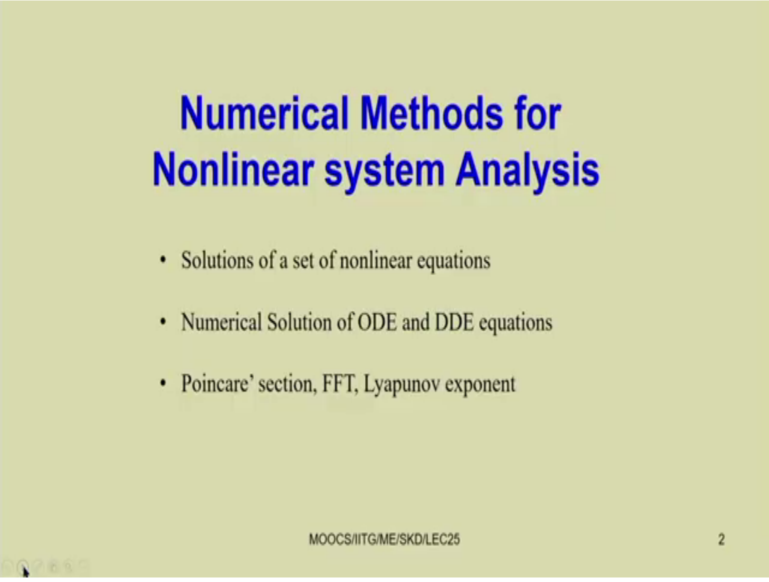

Welcome to today class of Non-linear Vibration. So, we are studying a new module on this numerical methods for non-linear system analysis. So, particularly in this module, we are going to study how we can solve a set of non-linear equations, then numerical solutions for this ODE and DDE equations, then we can find this Poincare' section, FFT and Lyapunov exponents.

Also, we will study different type of chaos and how we can find this chaotic systems for a different systems. So, in the previous modules, so you know regarding the development of equation of motion. So, in developing this equation of motions, you have used either the D'Alembert's principle or Newton second law or you have used this energy based technique like; Lagrange equation or this Hamilton principle for all these cases to derive this equation of motions.

Already I told you, you can use this symbolic software's to develop this equation of motion, instead of deriving those manually. So, when the complexity increases, you must use the symbolic software's and you must have to solve or derive this equation of motion. So, in the previous modules, also you have studied regarding the perturbation techniques; so, where you have reduced these equation, the second order differential equation to its a set of first order differential equation.

And for steady state conditions, so when they are not function of time. So, you got a set of algebraic or algebraic with trigonometric equations or transcendental equations. To solve these equations, so you may have a set of equations, sometimes they are not very easy or sometimes they are not giving rise to a closed form solutions. So, if you have only two equations, so in that case sometimes you can have the closed form solution.

But many times when the number of equations are more, so that time you must be going for this numerical methods to solve those equations. In today class, we are going to see how we can solve these equations also and also, several numerical techniques for example, how you can do these numerical differentiation, numerical integrations, all those things also you want to find.

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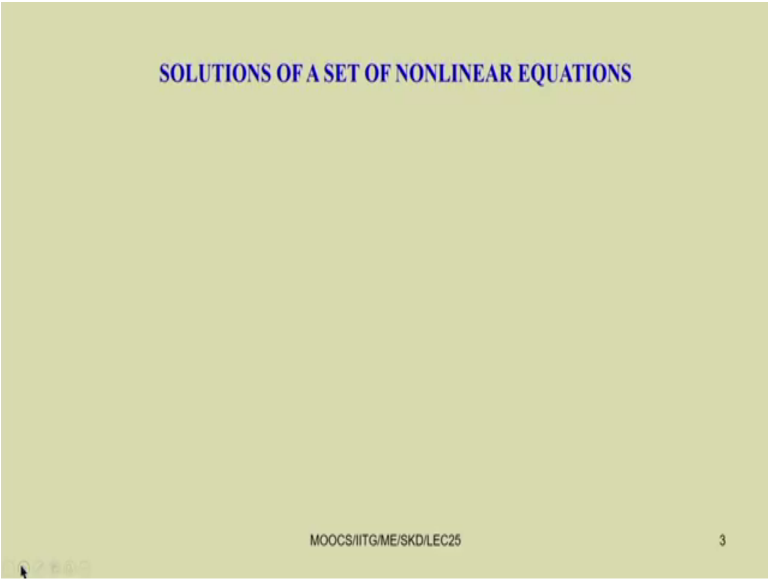

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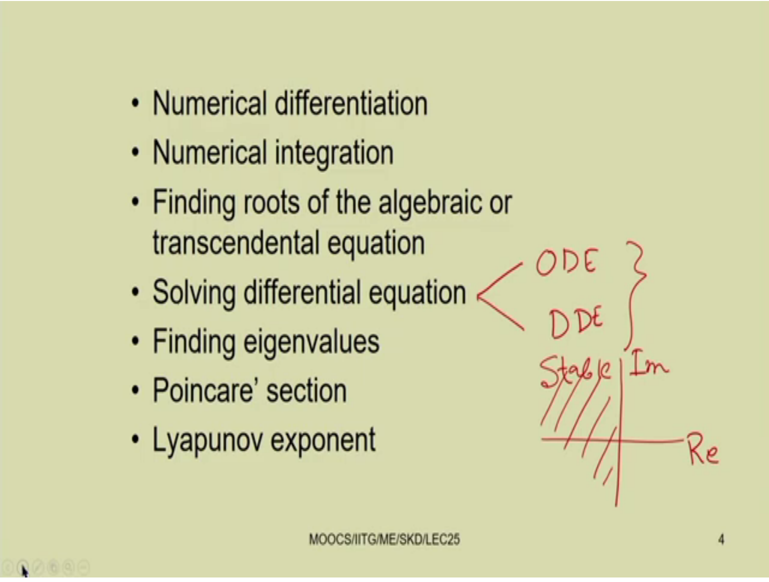

For example, so we will see how we can do this numerical differentiation, numerical integration, finding roots of the algebraic or transcendental equations, solving differential equations. So, in this differential equation, we may take this Ordinary Differential Equation, ODE; Ordinary Differential Equation or we may take this Delay Differential Equations also.

In the application parts of this course, so we will see. So, we can have a system with delay, that time we the governing equation will be that of a delay differential equation and for the delay differential equation, we must know how we can solve this delay differential equation.

But, if you know this how to solve this ordinary differential equation, then by taking the history of this ordinary differential equation, so you can develop a method of your own also to solve this delay differential equations. Then, how to find this eigen values. So, already you know these eigen values are useful or will be required when to study the stability of the system.

So, particularly, we know the Jacobian matrix and we find the eigen values of the Jacobian matrix to find the stability of the system. So, for example, you may recall that if you plot the eigen values in these s plot that is real part and imaginary part, the system is stable, if the roots are in the left hand side of the s plane.

So, if the roots are in the left hand side of the s plane, the system is stable and if it is if the real part of the eigen value is in the left hand side, so it is stable; otherwise, it is stable and if it is in the right hand side, it is unstable and also, we know regarding the marginally stable or unstable system.

Also, we can discuss regarding how we can reduce the order of the systems by using this Poincare section, then we will study to characterize the chaotic system by using this Lyapunov exponent. So, all these things, we will study in this next two classes in. So, total three classes, we will take for this numerical methods including today class.

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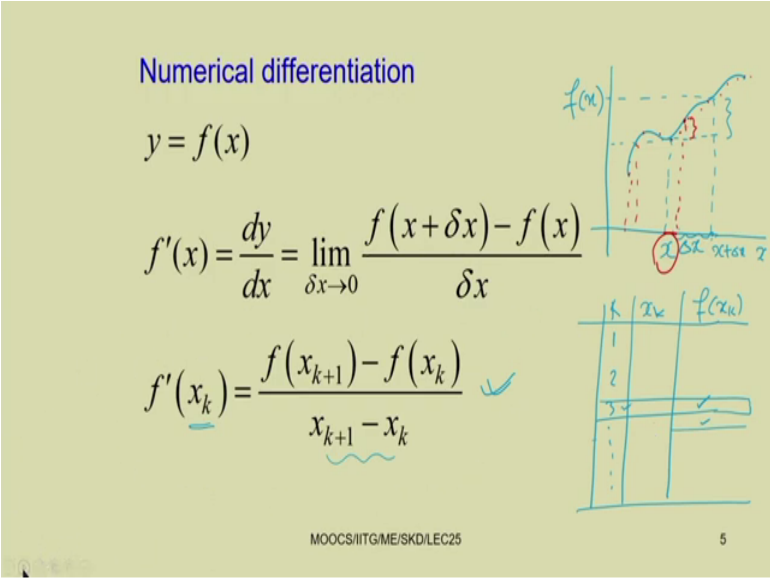

Today class, let us see how we can find this numerical differentiation. For example, in case of numerical differentiation; for example, so we have a function, so for. So, this is a function. This is x and let we have this function this is f x or y. So, these function, so we want to find what is the differentiation.

So, the definition of differentiation that is f dash x equal to d y by d x. It is limit delta x tends to 0, if x plus delta x for example, let us take at x equal to this is x at a particular time interval t ok. So, let this is x. So, then this is x plus delta x. So, if we take this interval to be delta x, then this is x plus delta x. These point is x plus delta x.

So, we can find the value of f x at this point. So, this is f x. So, this is f x plus delta x and we can find this derivative d y by d x equal to f x plus delta x minus f x by delta x. You just see, so depending on the value of delta x, so this value will differ. So, if I am taking delta x is this, so this is f x plus delta x minus f x. So, this is delta x.

So, if I will take for example, point this and this, let me take some other point with the different colour, let me take. So, if I am taking x at x at this point and delta x here. So, it can be seen that this f dash x will be these difference. So, this part divided by this part. As delta x tends to 0, so that means, if I will take this interval very small and small, then I can get more accurate solution or more accurate differentiation of this thing.

So, numerically, when we are solving, so we must know, so the different value of f x for particular value of x. f dash x k, so this is for example, k equal to 1, k equal to 2 so that way, you can find all the value. So, we can find x k so, f dash x k. So, at a point x, what you are writing here.

So, numerically when you are doing, so we can write this f x k. So, this f x f dash x k will be equal to f x k plus 1 minus x f x k by x k plus 1 minus x k. So, this is nothing but this is our delta x and this part is x, this f x plus delta x minus f x. That way, numerically, we can first we can find all the values and then, for a particular position, when we want to find this f dash x, we can find it.

So that means, so first we must have to generate, generate the value for example k. So, let us take k, k equal to 1, 2, 3. So, that way you go on increasing. So, then we can find what is this x k for k. So, then we can find x k and the corresponding value of x f x k we can find. So, preparing this table first, then we can find this x f dash x k.

So, let us take the third position for example, we know this value, so we can find this f dash x k will be equal to f x k plus 1. So, we will take this value, this is x k plus 1. If this is x k, then this will be x k plus 1. So, we will find the value at x k plus 1, then subtracting this value x k value.

So, that will give the difference between f x k plus 1 minus f x k by x k plus 1 minus x k. So, this is the interval what you are taking. So, that way it can be defined and from this thing, so we can numerically, we can find the differentiation of this thing.

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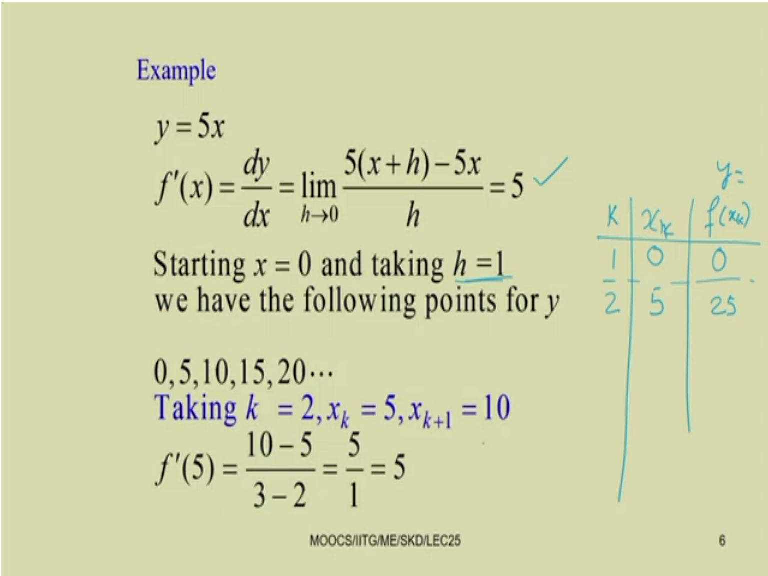

If we can take a linear function for example, let us take y equal to 5 x. So, all of you know this d y by d x will be 5. So, let us find it using this numerical method. So, in this case, we can write this f dash x equal to d y by d x equal to limit h tends to 0. So, there we have written delta x.

So, delta x is replaced by is here. So, limit h tends to 0, 5 x 5 into x plus h minus 5 x 5 h. So, this is gives rise to 5. All of us know that this is from the first principle we have derived; but numerically, when we want to derive, so starting x equal to 0. So, we can start and create the table.

For example, so already I told you. So, k equal to 1. So, we can start this x k, then we can find f x k. So, x k here; k equal to so we have started with 0 so, then if this is equal to 0. So, this is y equal to f x k. So, this becomes 0. So, then next value k equal to 2. So, x 2, x 2 become now k by taking k equal to 2 here you substitute.

So, x equal to initially we have taken 0. So, now, next value we are taking. So, the difference equal to h equal to we have taken 1. So, these becomes 1, so, 5 into 1, so, this becomes 5; x equal to 5. So, x equal to 5, this becomes 25. So, y become so, we can find ok x equal to initially it was 0.

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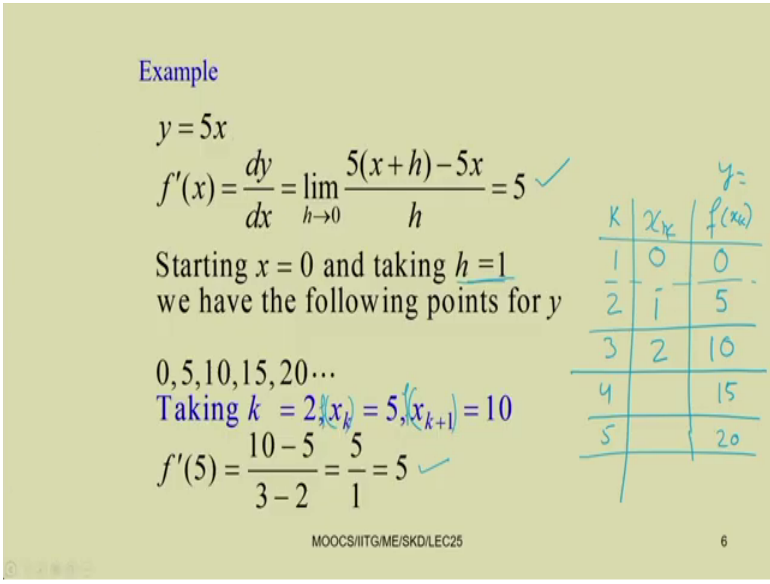

So, next we have this x equal to 1. So, our next value becomes 0 plus 1 equal to 1. So, 1 into 5. So, this becomes this is 1. So, this becomes 5. So, the next value, so this is 3. So, now, this becomes 1 plus 1, this is 2. So, this becomes 2, we have x k. So, x k 0, 1, then this becomes 2. So, if it is 2. So, then this value becomes 10. So, this way, one can find out different value of x k and f x k.

Here, we can have these y value 0, 5, 0, 5, 10, then this is 15, then this is 20. So, go on increasing this value and we can have different value of x k. So, taking k equal to 2, then x k equal to x k, taking k equal to 2 x k. So, f x k equal to 5 and f x k plus 1 equal to 10. We can have this f dash 5 equal to 10 minus 5 by 3 minus 2. So, this becomes 5 by 1. So, this is equal to 5. So, same things we are getting this is the linear equations; actually, we are getting the same thing what, when the equation is non-linear.

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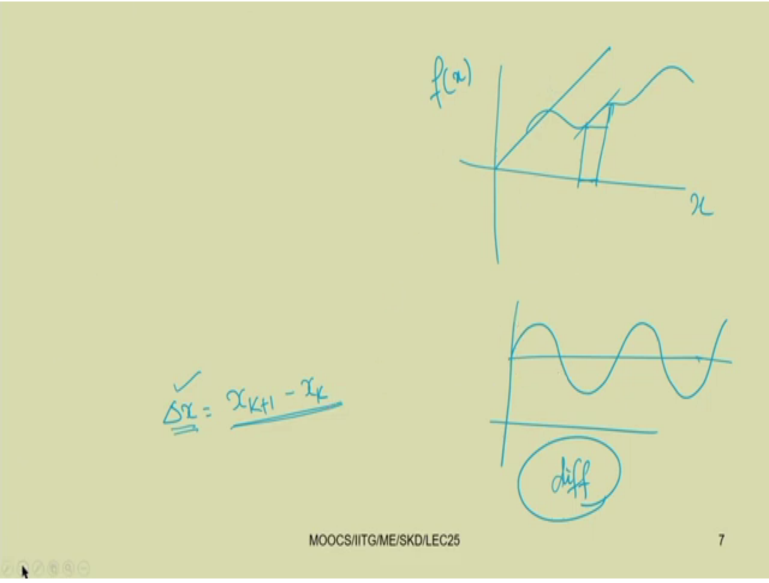

So, if we are taking for example, some other curve. So, that time, this interval takes that time this interval is very very important. These interval must to be taken very small to get the differentiation properly. So, by taking different value for example, let us have the sin curve also. So, in all these cases, we must take this interval x k plus 1 x k minus x x. So, this delta x is nothing but our x k plus 1 minus x k.

If we are taking this interval very very small, then we will get the accurate result. So, this is x and this is f x. To get the accurate result, we must take this value very small to get the correct result. So, this is a linear curve. So, instead of taking a linear curve, if you are taking some other curve, then in that case if we are not taking delta x very small; then, the result will differ.

So, this is nothing but this differentiation is nothing but the slope of slope at this point. So, here this d y by d x limit d y by d x tends to 0. So, this way you can find the numerical differentiation of any function. In this MATLAB also, you have a function, so you can use diff function in MATLAB to find the differentiation.

So, symbolically, you can do this differentiation, if you have a function definite function. So, otherwise you can find this value and use this formula, what is shown here to find the differentiation.

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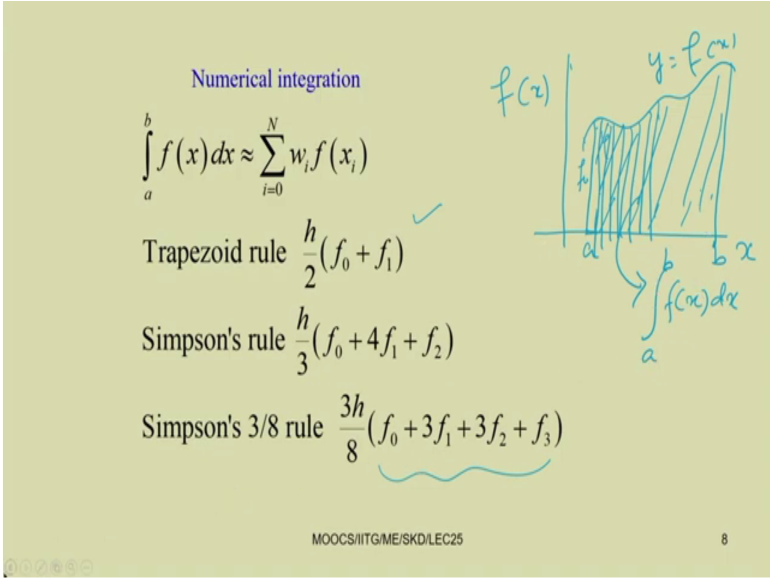

So, let us see the numerical integration so, similarly here also. So, let this is the function y equal to f x. So, if we want to find the integration of f x d x. So, this is x and this is f x. This integration is nothing but the area of the curve bounded by this curve and the x axis. So, this represent; so this area represent integration of; so, this area represent integration of f x d x from let x equal to a to b.

When we solve this problem using numerical methods, we can divide this curve into several segments. So, for example, so, these are several segments I can take. It may be rectangular segments, I may assume. So, by taking very small value, so I can assume this area to be that of a rectangle or if I am taking slightly larger area, then I can consider this to be a trapezoid.

Depending on the area what I am taking the rules will be different. So, it may be rectangular, it may be trapezoid or I can feet a polynomial also here; polynomial with several plot points or this Gauss points. By taking different points, then these integration schemes will be different.

So, one can use this trapezoidal rule. So, where the trapezoidal rule is nothing but so if this value is f 0 and this value is f 1, so let us take first find what is the area of this thing? This area equal to, so this is h. So, area will be equal to h by 2, f 0 plus f 1. Similarly, one can use the Simpson's rule to find this area also.

So, it will be h by 3 f 0 plus 4 f 1 plus f 2. So, by taking three points, so we can find or use this Simpson's rule; otherwise, one can use the Simpson's three-eighth rule. So, this is Simpson's one-third rule. So, one can use Simpson's 3 by 8 rule also to find the area. So, this will be equal to 3 by 3 h by 8 into f 0 plus 3 f 1 plus 3 f 2 plus f 3.

So, at three different points, four different points, we can find the value and then, by using the Simpson's 3 by 8 rule, so we can also find the area of this entire curve. So, that will give the integration a to b f x d x.

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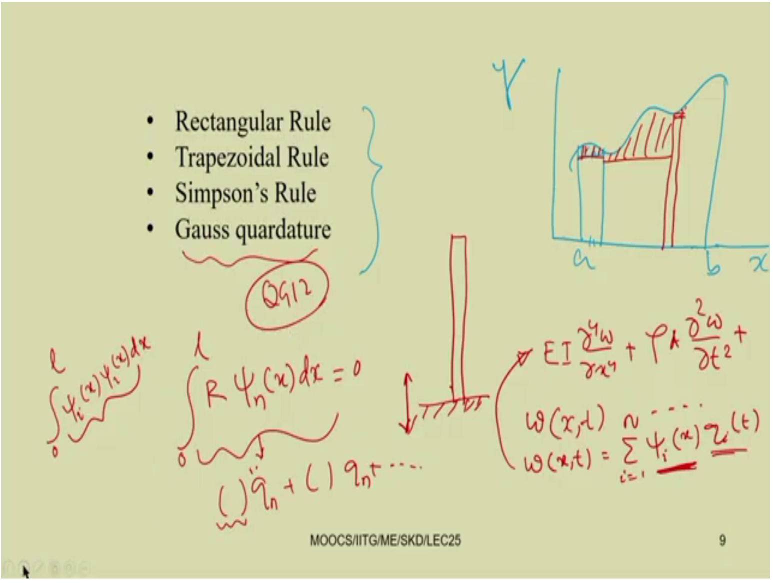

So, there are several other methods like this Gauss quadrature method also, this Rectangular rule, Trapezoidal rule, Simpson's rule, Gauss quadrature. So, all these methods, you can use to find this integration in MATLAB. So, you can use this int function to integrate a function symbolically.

As you will be given assignment in each of this method and you can use these methods to find the area of different curve bounded by this x axis and the curve. So, in between a to b. So, let this is a, this is b. So, you can find this area. You may note that when I am using these rectangle, you can find this is the. So, if I will draw a line parallel to this here.

So, this portion is not accounted. When I am taking this as a rectangle, so it can be observed, so this point is not taken into account. When I will take this interval to be large, then you can see this error is coming to be very very large. So, if I will take the interval to be very small, in that case this error also will be small. So, by taking proper interval size, one can use these methods numerical methods to have less and less error.

This way, one can perform this different numerical differentiation or integration and find the integrations. So, these integrations are particularly useful, when we are solving or we are using this spatio-temporal equations, where we have the spatio-temporal equation for a continuous system.

So, we have seen this continuous system. For example, we have seen the cantilever beam, base excited cantilever beam. So, in this base excited cantilever beams.

So, you have seen in this base excited cantilever beam, you have seen this spatio-temporal equation of motion. For example, the first two terms if you want to write for the Euler Bernoulli beam equation, you have this equation EI del 4th w by del x 4th plus rho A del square w by del t square.

So, this other non-linear terms are also present, when we consider the system to be non-linear. So, in that case, so this w is a function of both. So, here w is a function of both space and time. So, to reduce it to its temporal form, generally we use this w x t equal to summation psi i x into q i t. So, here we are taking n number of; so, i equal to 1 to n. So, taking n number of modes generally we take. So, psi i is the mode shapes and q i is the time modulation.

So, we substitute this equation in this original equation and then, integrate it, ask this equation this shape functions may or may not satisfy the all the boundary conditions and the differential equation. So, there will be residue when we substitute this w x t in this original equation. So, there will be some residue.

And to minimize that residue, so usually we integrate over the length of the beam these residue. In this residue, we multiply psi n x d x. So, we integrate and make it equal to 0. So, while performing this integration or to find out these things. Now, this term can be written something into q double dot q n double dot plus if I will take only these two terms, then it will be q n so, plus other terms will be there.

So, these part contains the integration which take care of or which contains these mode shapes of the system. For example, one term may be like this. So, it may be psi i x into psi i x d x integration 0 to l. So, this type of integration will be there. So, in that case, so you must be able to find this integration.

So, sometimes you may use this Gauss quadrature method to find this integration. So, here you may use this Gauss quadrature for example, 12 point; Q g 12 generally it is known as Q g 12, 12 point Gauss quadrature method you may use to find this integration. Depending on the complexity of your equations, you may have to take more number of points and you can find this integration accurately.

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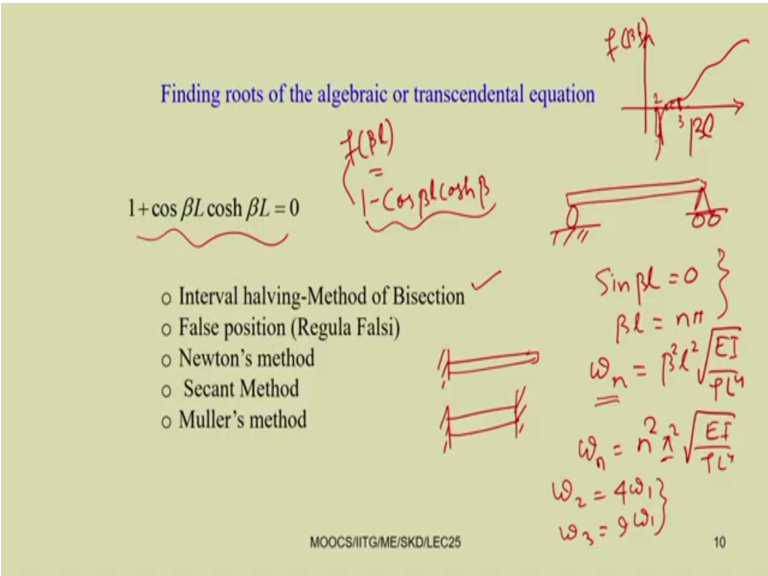

Continuing with the same example, so sometimes for example, when we are taking these Euler-Bernoulli beam equation with different boundary conditions; so, in case of the Euler Bernoulli beam equation, you know if it is simply supported. So, let us recall when it is simply supported in the Euler Bernoulli beam, we are taking and if it is simply supported, we know these characteristics. So, we got a characteristic equations, which contain the sin beta L equal to 0.

So, here to solve this equation, we can easily write beta L equal to n pi; where, this actually this natural frequency omega n is written equal to beta square L square root over EI by rho L 4th. For simply supported case, you can find the solution to be very easy. So, sin beta L equal to 0.

So, that is why you can write the sin beta L equal to sin n pi so that beta L equal to n pi. And you can find the us depending on the value of n; so, beta L value beta L will take the value pi, then 2 pi, then 3 pi. So, that way, it will take and you can observe that these omega n will be equal to n square pi square.

So, the simply supported case, it can be written as n square pi square root over EI by rho L 4th. And this EI rho L 4th will be same for all for different modes. For example, first mode it will be pi square; second mode, it will be 4 time the first mode so that means, omega 2 equal to 4 times omega 1 and omega 3 equal to 3 square that is 9 time omega 1.

For the simply supported case, this characteristic equation is very simple and you can get the solution easily. But, when for example, you just take the cantilever beam so, in that case, so it is not so simple or if you take the case of fixed the equations, the characteristics equation will not be simple. So, in those cases you may get this equation 1 plus cos beta L into cos hyperbolic beta L equal to 0 or you may get this equation also, 1 minus cos beta L into cos hyperbolic beta L equal to 0.

So, how to solve these equations? We should know the method to solve this equation. So, we can use these Brute force method. That means, we can plot so, we can plot this beta L. So, taking these equal to. So, this part equal to f beta L. So, taking f beta l equal to f beta l equal to 1 minus cos beta L into cos hyperbolic beta L.

So, we can solve this equation or we can plot this and find the point, where it is crossing the 0 line. So, this is beta L and this is f beta L. So, where it is crossing this 0 line? After getting this point, where it is crossing the 0 line, so we can find the beta L value. We have to find this numerically to obtain this value of beta L.

There are several methods of finding these roots. So, some methods are like this Interval halving - a method of Bisection; so then, False position or Regular Falsi, then Newton's method, Secant method, Muller method. So, there are several methods are there. The basic principle in all these methods for example, in method of bisection and false positions, you have to find two value of beta L for which this f beta L will have both positive and negative value.

So, for example, so if we can take this beta L value these and these. So, in between, so here, here it has a negative value and at this position it has a positive value. So, taking these two value; so, then we can find the next iteration, where we can take these positions this next i plus oneth position or beta this next value of this beta L can be taken. So, by taking the half of this thing for example, let this is this value is 2, this value is 3, the next value can be taken. So, it will be equal to 2.5. So, by taking the middle of this thing that is 3 plus 2 by 2. So, that is 2.5. So, we can find the next value, later 2 point value, again it has a positive value.

So, next iteration will be between 2 and 2.5. So, this time by taking 2 and 2.5. So, we can find the next value. Next value of beta will be equal to 2.25 we can take. So, that way, we can go on iterating and we can find a value for which this f beta L will be equal to 0. So, that way, we can find using this basic principle and we can find the root of this equation.

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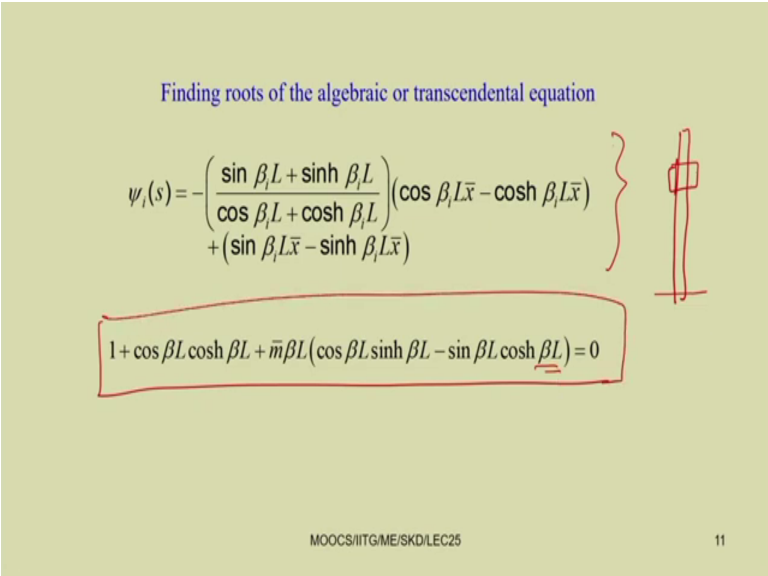

For example. So, in case of the cantilever beam what we have studied before with a attached mass. So, in that case that equation will not be so simple. So, in this case you just see we have a complex equation like this for psi i s; psi is contain these minus. So, sin beta i L plus sin hyperbolic beta i L divided by cos beta i L plus cos hyperbolic beta i L into cos beta i L x bar minus cos beta i L x bar plus the sin beta i L x bar minus sin hyperbolic beta i L x bar.

So, to find this beta L value, so we must have to solve this equation. So, which is known as the characteristic equation. So, this characteristic equation has to be solved numerically to find this beta L value. We can use this pulse position method or this method of bisection or the other methods; particularly, this gradient based method which is this Newton methods or Newton Raphson methods, so one can use to find more effectively the root of the equation.

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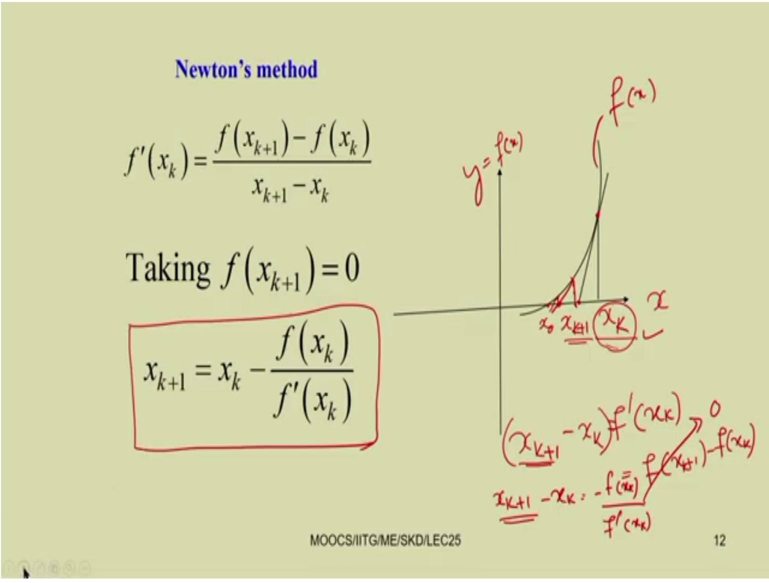

In Newton's method, so we can take this is x, this is x 0, x that is the actual value of x, that is x 0 and so, this is the function. So, let this is the function f x. So, this is we have plotted x verses y or y equal to f x. This is the x axis and y axis. So, initially we can take this point x k. Let x k point, we have taken x k points. So, at x k this is f x k. So, here if we draw a tangent.

So, you can see it cots this line x axis at this point. So, this point can be taken as x k plus 1 point. So, this can be x k plus 1. We can find this x k plus 1 from this equation that is we know this f dash x k equal to that is derivative at x k equal to derivative of the function at x k is nothing but x k plus 1 minus this f x k by x k plus 1 minus x k.

So, here as we are assuming that at x k plus 1, y equal to 0. We can take this f x plus 1 equal to 0, substitute in this equation. So, we can get this x k plus 1 will be. So, we can write this x k plus 1. So, in that case, we can write x k plus 1 minus x k will be equal to. So, already we put this value equal to 0. These into f dash x k is nothing but f x k plus 1 x k plus 1 minus f x k.

So, we have taken this value equal to 0, that is why this x k plus 1 can be written x k plus 1 minus x k is nothing but minus f x k by this f dash x k and you can find this x k plus 1 equal to x k minus f x k by f dash x k. In this way we can get the next step. So, taking this as the initial point; x k as the initial point, so we can find the next point x k plus 1.

So, it is equal to x k minus f x k by f dash x k by taking a point x k correctly or if the guess point is very accurate or this guess point is if we are taking a guess point properly, then we can get the or we can come to the root very quickly. For example, the next point will be; so, this is the next point.

So, here we have to draw another tangent. So, you just see. So, here this is the x k point. x k plus 1 point, next iteration. So, we are starting from this point, then this will be the next point. So, the next point is very closer to x 0. Similarly, from these, we can find this one and then, we can draw another tangent. So, this tangent is more closer to x 0. So, this way, we can get the convergent very first by using these Newton's method. So, this Newton method what is explained.

So, this is for a single equation. So, this method can be; so, for single equation, it is generally known as Newton Raphson method and for multi or number of equations, these equations also can be expanded or this equation can be used, where this differentiation can be written in the form of the Jacobean matrix and one can get this x k plus 1 by using similar equation.

So, here it will be replaced by the vector for a set of equations, one can easily find the solution by using this method.

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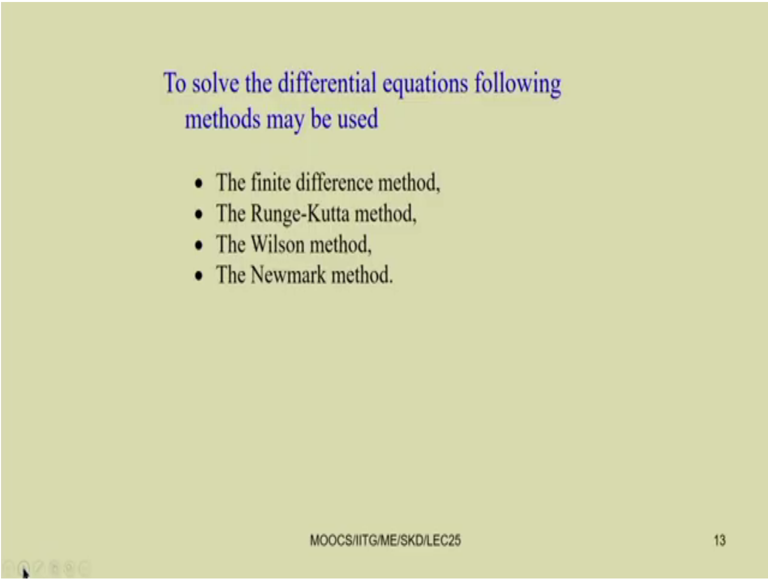

Now, to solve the differential equations, there are several methods are there. So, for example, one can use this Finite different method, one can use this Runge-Kutta method, this Wilson theta method and Newmark beta method. There are several methods are available. Already, we have discussed regarding this Runge-Kutta method.

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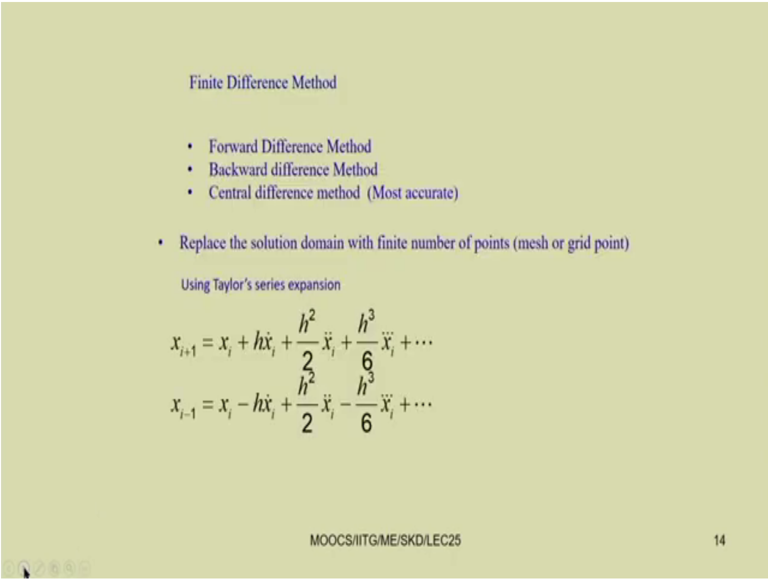

This method again let me tell. So, we can use for example, in finite difference method, one can use this forward difference method, backward difference method or central difference method. So, mostly, the central difference method is used. So, we can replace this solution domain with finite number of points; mesh or grid points.

And then, by using the Taylor series, so we can expand this thing to or we can write this x i plus 1 equal to x i plus h x i dot plus h square by 2 x i double dot plus h cube by 6 x i triple dot and this way and neglecting this higher order terms, so we can write x i plus 1 equal to x i plus h x i dot plus h square by 2 x i double dot.

Similarly, the previous point can be written. Let x is the central point; so, x i plus 1 will be the forward point and x i minus 1 will be the backward point. So, this thing equal to x i minus h x i dot plus h square by 2 x i double dot minus h cube by 6 x i triple dot. So, this way one can expand this thing using Taylor series.

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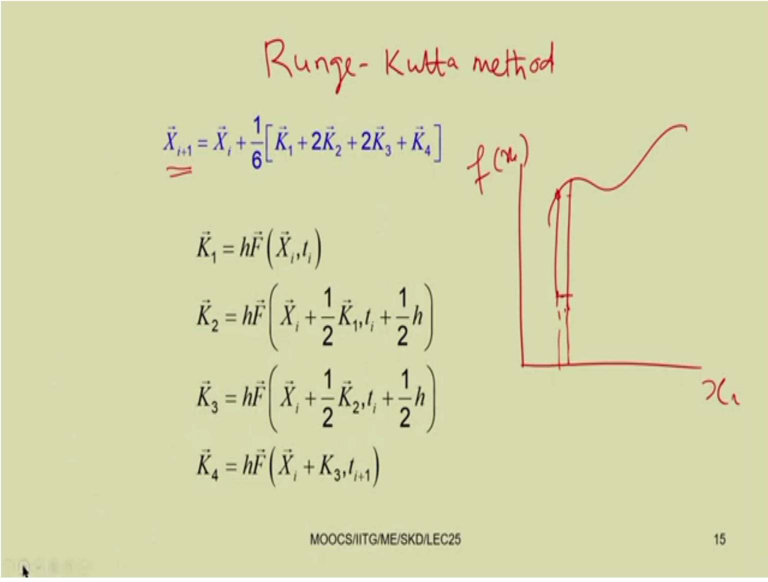

So, when we are going for this Runge-Kutta method, in this method, one may use this formula for the X i plus 1. So, X i plus 1 will be equal to X i. So, X i is the initial point is given to us. So, X i plus 1 by 6 K 1 plus 2K 2 plus 2K 3 plus K 4, where K 1 equal to. So, if we are taking this increment as. So, this is the value. So, we can take this increment h, time interval h.

So, then X K 1 can be written X h into F X i t i. Then, K 2 will be h F X i plus half K 1 t i plus half h. Then, K 3 will be equal to h F X i plus half K 2. So, this is time will be equal to t i plus half h. Similarly, K 4 can be written h F X i plus K 3. So, this X component will be X i plus K 3 and the time is t i plus 1.

Using these methods, we can find the next point given these initial condition, we can find out. So, what is the initial condition is X i and F X i. So, this is X. So, X i and F X i if it is known to us, then we can find the next point by X i plus 1. By using this thing, so already we have seen, we have solved already several systems to find this periodic, cos periodic and chaotic responses.

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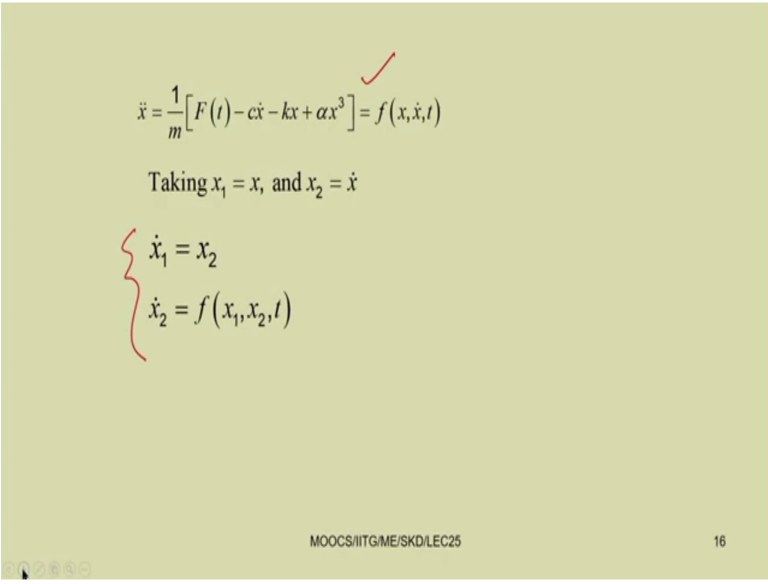

So, for example, so if you can use this Duffing equation. So, you can write this equation this way. In that case these x double dot can be written 1 by m F t minus c x dot minus k x. So, you can put either plus or minus depending on the value of alpha. Alpha x cube equal to f x x dot t. So, taking x 1 equal to x, so x 2 equal to x dot. So, we have a set of first order equation, where you can use the set of first order equation to find the solution using this Runge-Kutta method.

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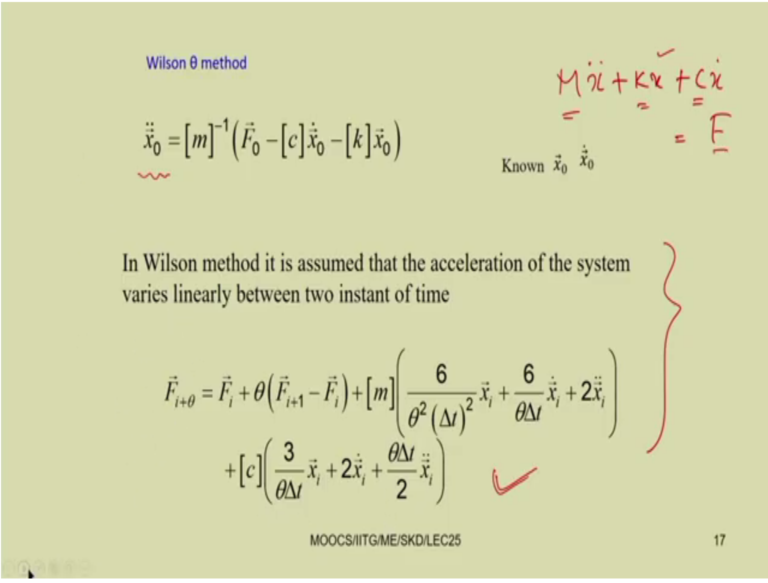

So, particularly for multi degrees of freedom system, it is not simple to use this method. So, they are; the equation is written in terms of this acceleration, particularly when we are using this finite element method, so we have multi degrees of freedom. So, in that case the acceleration component x 0 double dot can be written using this form. For example, we have this equation M x double dot plus K x plus C x dot equal to F.

So, in that case, so M is a matrix, K is a matrix, C is matrix and F is the vector also. So, in that case, initially by taking this K x and c x dot to the right hand side, so we have M x double dot equal to F minus C x dot minus K x. Now, pre-multiplying by M inverse we have this x dot equation. Taking these initial conditions of x and x 0 that is displacement and velocity, so we can use this Wilson theta method.

So, generally, it is known as Wilson theta method. So, by taking different value of theta. So, we can write down this equation in this form that is F i plus theta equal to F i plus theta into F i plus 1 minus F i plus m into 6 by theta square delta t square x i plus 6 by theta delta t x i dot plus 2 x i double dot plus c into 3 by theta delta t x i plus 2 x i dot plus theta into delta t by 2 x i double dot.

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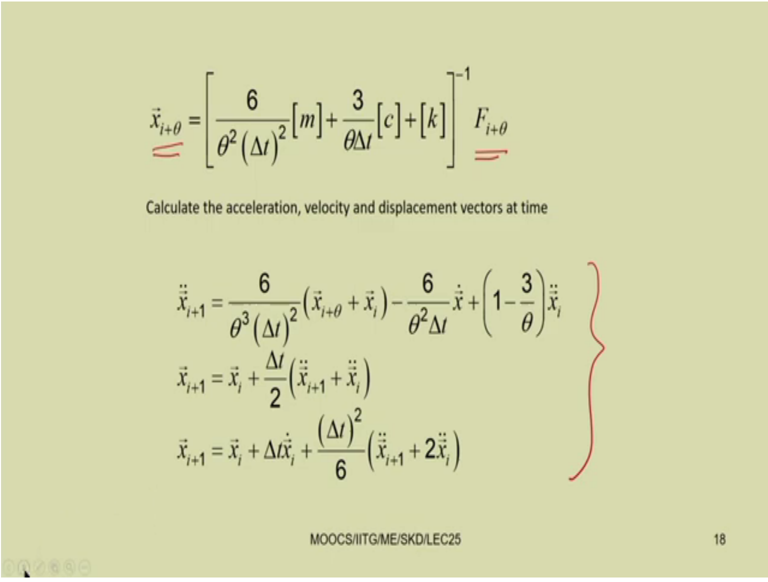

So, this way this next iteration. So, this four spill can be found or the load vector can be found using this equation and these x i plus theta can be written by using this equation 6 by theta square delta t square into m plus 3 by theta into delta t into c plus k this inverse into F i plus theta. So, already we got this i plus theta and using that thing, so we can have this expression for this acceleration and this x i dot and x i i plus 1 and x i plus 1. So, this way we can get it.

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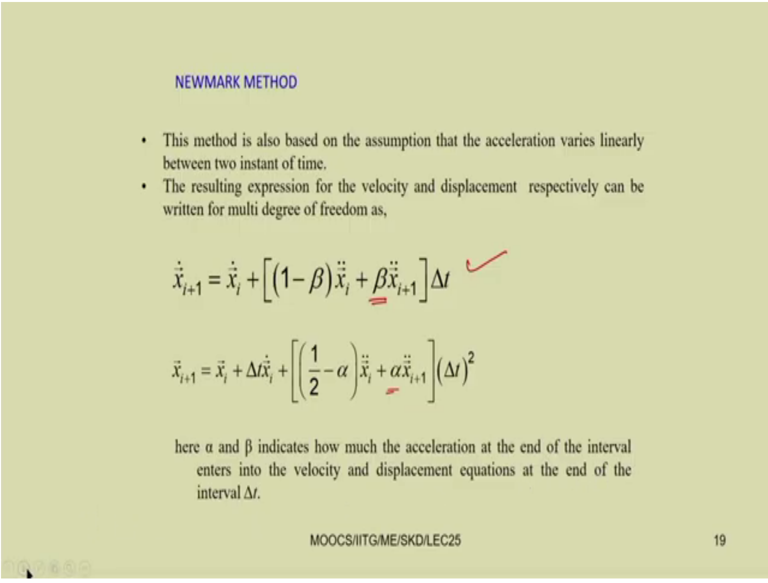

One can use this Newmark beta method also. So, you just note that this Wilson theta and Newmark beta both are empirical relation. Here it is assumed. So, in previous case also, it was as same thing was assumed. So, the assumption is that the acceleration varies linearly between two instant of time. So, here this velocity term can be written by using a term beta and these displacement can be written using a term alpha.

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$$[m]\ddot{\vec{x}}_{i+1}+[c]\dot{\vec{x}}_{i+1}+[k]\vec{x}_{i+1}=\vec{F}_{i+1}$$

$$\vec{x}_{i+1}=\left[\frac{1}{\alpha(\Delta t)^2}[m]+\frac{\beta}{\alpha\Delta t}[c]+[k]\right]^{-1}$$

$$\times\left\{\vec{F}_{i+1}+[m]\left(\frac{1}{\alpha(\Delta t)^2}\vec{x}_i+\frac{1}{\alpha\Delta t}\dot{\vec{x}}_i+\left(\frac{1}{2\alpha}-1\right)\ddot{\vec{x}}_i\right)\right.$$

$$\left.+[c]\left(\frac{\beta}{\alpha\Delta t}\vec{x}_i+\left(\frac{\beta}{\alpha}-1\right)\dot{\vec{x}}_i+\left(\frac{\beta}{\alpha}-2\right)\frac{\Delta t}{2}\ddot{\vec{x}}_i\right)\right\}$$

MOOCS/IITG/ME/SKD/LEC25 20

So, here alpha and beta indicates how much the acceleration at the end of interval enters into the velocity and displacement equation are the end of interval delta t. By taking this equation, for example, let us take this same single degree of freedom system, m x double dot plus k x plus c x dot equal to 0.

So, here this x i plus 1 can be obtained by using this equation. You can take several equations and using these formula, so you can find the next value of x i plus 1, that way you can solve these equations to get the response of the system.

Next class, we will see some more numerical methods; particularly, how to solve this ODE and Delay Differential Equation and using that thing, we will find several interesting phenomena or these fixed point response, then periodic response and chaotic response and then, in next two next class, we are going to study about these Lyapunov exponent method and other Poincare method and Lyapunov exponent method to study the response different type of response of the system.

Thank you very much.