Nonlinear Vibration

Prof. Santosha Kumar Dwivedy
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Lecture - 02
Superposition rule, Commonly used nonlinear equations

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Lecture 2
Introduction to linear and Nonlinear Equation of motion

Welcome to today class of Nonlinear Vibration. So, today class we are going to introduce the linear and nonlinear equation of motion. And last class I told you regarding the superposition rule which separates or which distinguish between the linear and nonlinear equations. So, we will briefly see what is this equations, what is the superposition rule, application of superposition rule, then we are going to study about some linear system and nonlinear systems.

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Text/References

1. Nayfeh, A. H., and Mook, D. T., *Nonlinear Oscillations*, Wiley-Interscience, 1979.
2. Hayashi, C. *Nonlinear Oscillations in Physical Systems*, McGraw-Hill, 1964.
3. Evan-Ivanowski, R. M., *Resonance Oscillations in Mechanical Systems*, Elsevier, 1976.
4. Nayfeh, A. H., and Balachandran, B., *Applied Nonlinear Dynamics*, Wiley, 1995.
5. Seydel, R., *From Equilibrium to Chaos: Practical Bifurcation and Stability Analysis*, Elsevier, 1988.
6. Moon, F. C., *Chaotic & Fractal Dynamics: An Introduction for Applied Scientists and Engineers*, Wiley, 1992.
7. Rao, J. S., *Advanced Theory of Vibration: Nonlinear Vibration and One-dimensional Structures*, New Age International, 1992.

So, these are the text and references which are widely available in the Nonlinear Vibration course, particularly you may refer the book by A. H. Nayfeh and D. T. Mook, Nonlinear Oscillations so by Wiley Interscience. Then there are many books by Nayfeh. So, for example, this is the, this is one book this the other book by Nayfeh that is A. H. Nayfeh and B. Balachandran Applied Nonlinear Dynamics by Wiley.

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8. A. H. Nayfeh Perturbation Methods, Wiley, 1973

9. A. H. Nayfeh, Introduction to Perturbation Techniques, Wiley, 1981

10.Wanda Szemplinska-Stupnicka, The Behavior of Nonlinear Vibrating Systems, Vol 1 &2, Kluwer Academic Publishers, 1990

11. Matthew Cartmell, Introduction to Linear, Parametric and Nonlinear Vibrations, Chapman and Hall, 1990.

12. T. S. Parker and L. O. Chua: Practical Numerical Algorithms for Chaotic Systems, Springer-Verlag, 1989

13. A. H. Nayfeh, Method of Normal forms, Wiley, 1993.

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Similarly, there are some more books by Nayfeh also, A. H. Nayfeh Perturbation Methods; A. H. Nayfeh, Introduction to Perturbation Techniques, and A. H. Nayfeh Method of Normal Forms. So, there are some other authors are also there. For example, this Hayashi, Nonlinear Oscillations, then Evan-Ivanowski Resonance in Oscillation in Mechanical Systems. Then Seydel From Equilibrium to Chaos; F. C. Moon Chaotic and Fractal Dynamics; J. S. Rao Advanced Theory of Vibration Nonlinear Vibration and One-dimensional Structure.

So, these are very important books which you may refer. So, some other books also by this Wanda Szemplinska-Stupnicka, The Behavior of Nonlinear Vibrating Systems. So, two volumes of these books are available. So, by then other book by this Matthew Cartmell, so Introduction to Linear Parametric and Nonlinear Vibration, then this Parker and Chua –

Practical Numerical Algorithms for Chaotic Systems. So, these are very good books which you can refer.

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14. D.W. Jordan and P. Smith, Nonlinear Ordinary Differential Equations, OxfordUniversity Press, 4th Ed. 2009

15. H. K. Khalil, Nonlinear Systems, Prentice Hall, 2002

16.M. Vidyasagar, Nonlinear Systems Analysis, SIAM 2002

Then this by D. W. Jordan and P. Smith Nonlinear Ordinary Differential Equations; H. K. Khalil, Nonlinear Systems; M. Vidyasagar, Nonlinear System Analysis. So, these are, there are several other books are also good books are available recently published many books are also there.

So, you can see all those books or some of the books you can refer. Particularly for this course I am going to refer the first book that is by A. H. Nayfeh and D. T. Mook, Nonlinear Oscillations; and in the book by A. H. Nayfeh and B Balachandran Applied Nonlinear Dynamics. And further we will refer several journals.

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Journals

- International Journal of Non-linear Mechanics (ELSEVIER)
- Nonlinear Dynamics (SPRINGER)
- Journal of Sound and Vibration (ELSEVIER)

JVC

- Journal of Vibration and Acoustics (ASME)
- Journal of Dynamical Systems, Measurements and Control (ASME)
- Physics D: Nonlinear Phenomena (ELSEVIER)
- Chaos, Solitons and Fractals (ELSEVIER)
- International Journal of Nonlinear Sciences and Numerical Simulation, (Freund Publishing House)
- Journal of Computational and Nonlinear Dynamics (ASME)

So, these are some of the journals which you may refer for this course. International Journal of Non-linear Mechanics ELSEVIER, Nonlinear Dynamics, Journal of Sound and Vibration, Journal of Vibration and Acoustics, Journal of Dynamical Systems Measurement and Control, Physica D Physics D Nonlinear Phenomena; Chaos, Solitons and Fractal; International Journal of Nonlinear Sciences and Numerical Simulations.

Then Journal of Computational Nonlinear Dynamics that is in ASME also Journal of Vibration and Control. So, another journal is there Journal of Vibration and Control. So, this is a very good journals.

And there are several other journals related to vibrations which also publishes the paper related to nonlinear dynamics. For example, one more journal is there Mechanism and

Machine Theory. And several other journals are there. But particularly we will be interested to see these list of journals, and refer some papers from the journals. So, let us see.

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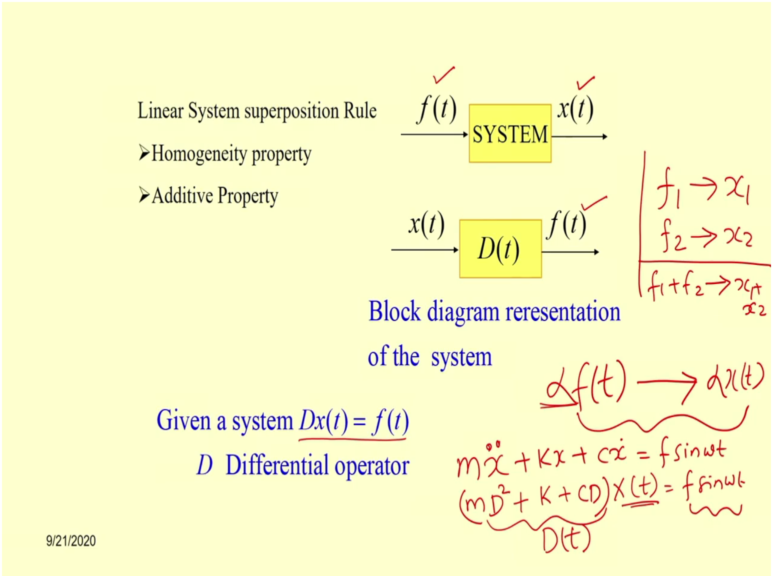

So, already I told you regarding the superposition rule. So, let us we have a system. So, if we are applying force ft. So, when we are applying a force ft, we have if the response is x t, so by applying the superposition rule, so it has to satisfy both the homogeneity property and additive property.

So, in homogeneity property, so for applied force ft, so if we have the response x t, then when you apply another force alpha f. So, if we are applying a force alpha ft, so our response, so the response must be alpha x t. So, if the response is alpha x t, then only we can tell that it satisfy the homogeneity rule.

So, in block diagram form, you can write the system using a differential equation D t. So, in this system, so if we are for the system you can write D x t or x t is the output of the system, and f t is the input of the system. So, if you are applying a force f t, then your response must be x t. So, in block diagram form it can be written x t D t equal to f t. So, given a system D x t, so in operator form, so we can write the equation of motion of the system in this form that is D x t equal to f t. So, D is the differential operator.

For example, let us take the spring mass system. So, in this case our differential equation is m x double dot plus K x so plus c x dot equal to f sin omega t. So, here we can write. So, our D x t will be equal to. So, in this we can write this in this form m D square plus k plus C D, so this into X t will be equal to F sin omega t.

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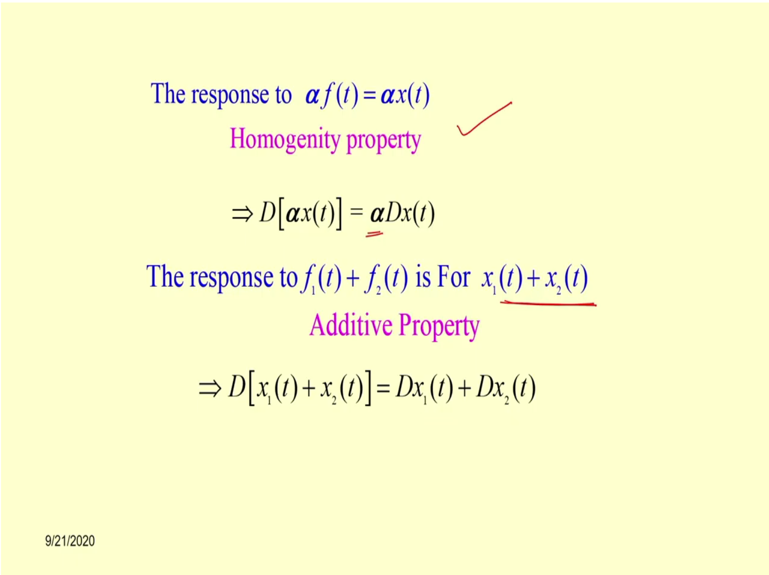

So, in this case, so this part, so the previous part, so this part is our D x t. So, D x t is in or D t is nothing but, so this is this part is D t operator. So, this is the x t that is the response of the system. And this is the forcing applied to the system. So, this way we can write the any given equation. And now we have to check whether it is satisfying the differential this superposition rule or not.

So, in superposition rule, so we have to check the homogeneity property and then additive property. So, in homogeneity property, so we have to check, so given alpha x t force, so what will be the response? If the response is alpha x t, then we can tell that it satisfy homogeneity property, where this alpha is a scalar quantity scalar number.

Similarly, in case of additive property, so we have to take we have to check, so for example, let us apply this force f 1, so the response is x 2 x 1. When we apply force f 2, the response is x 2.

So, if we apply this force f 1 plus f 2 simultaneously, our response must be x 1 plus x 2. So, in that case, so it must this x 1 plus x 2 must satisfy the governing equation in the or the differential operator it should satisfy take care that part. So, if it is not satisfying that equation, then we should tell that the equation is no longer linear. So, for a linear system, it must satisfy the superposition rule.

So, let us see one example. So, already I told so the response alpha f t must be equal to alpha x t. So, this is homogeneity property, or D alpha x t must be alpha x t. So, you just see this alpha has come out of this operator.

So, if it is not satisfying, then we can tell that it is not satisfying homogeneity property. Similarly, the response to f 1 t plus f 2 t for the system must be x 1 t plus x 2 t. So, this is additive property in differential equation form or a differential form. So, D x 1 t plus x 2 t must be D x 1 t plus D x 2 t. So, let us see one example, then it will be clear.

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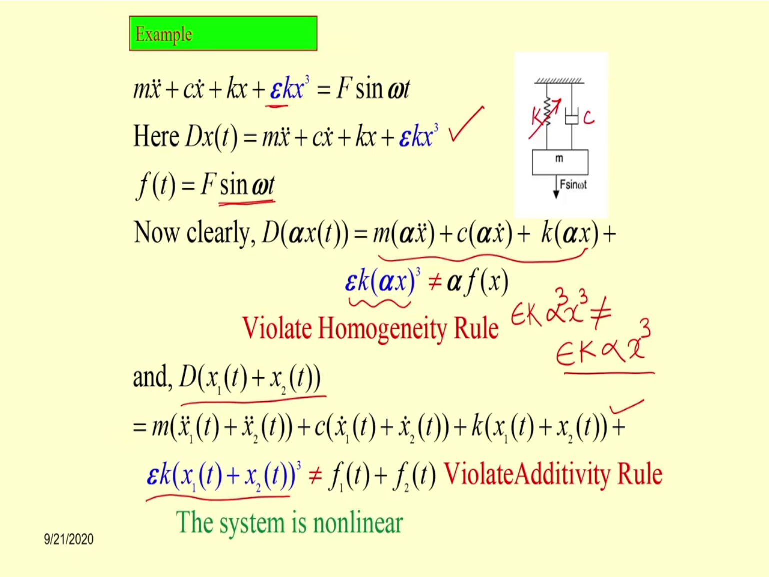

So, let us take this example. So, m x double dot plus c x dot plus k x plus epsilon k x cube equal to F sin omega t. So, this is particularly, so if you have a linear system the equation is m x double dot plus c x dot plus k x equal to F sin omega t. So, linear vibrating system with a spring constant k and damping c, mass m, subjected to a force F sin omega t can be written in this form.

But if this spring is nonlinear, so let us take a cubic nonlinear term. So, then in that case, it can be written as m x double dot plus c x dot plus k x plus epsilon k x cube. So, this epsilon k is a small term, epsilon is known as the book-keeping parameter. So, by using a book-keeping parameter, we can write this equation.

So, this is equal to F sin omega t. So, here our D x t is nothing but m x double dot plus c x dot plus k x plus epsilon k x cube and f t equal to F sin omega t. The forcing is written in terms of a harmonically excited system sin omega t due to the presence of term sin omega t, it is harmonically excited.

So, now, you can easily see if you are applying a force alpha f t, then the response if it is a linear system, then the response must have been D alpha x t equal to alpha D x t. But here you can see that D alpha x t equal to, that means, in this D x t let us replace this x t by alpha x t. So, in that case, m into alpha x double dot plus c into alpha x dot plus k into alpha x plus epsilon k into alpha x cube.

So, you just see, if I will expand this thing, so this is this will be equal to actually, so m, so I can take common this alpha up to this term alpha can be taken common. But for this part, this becomes epsilon alpha cube k x cube epsilon alpha x cube. So, this is epsilon k alpha cube x cube.

So, if I will expand this, epsilon k alpha x whole cube this is epsilon k ok. So, we can write this thing. So, it is epsilon, then k alpha cube x cube, epsilon k alpha cube x cube this is not equal to clearly this is not equal to epsilon. So, epsilon k alpha cube x cube not equal to epsilon k alpha x cube. So, as this is not equal to epsilon k alpha x cube. So, it is not satisfying the homogeneity rule.

Similarly, let us see the additive rule. So, in case of additive rule, so when you are applying a force of f1 plus f2, the response must be x 1 plus x 2. So, in the operator D operator let us substitute this D x 1 plus x 2. So, when you are substituting this D x 1 plus x 2, you can note up to this term, so it is satisfying the homogeneity this homogeneity rule. But this part that is epsilon k x 1 plus x 2 whole cube is not equal to actually epsilon x 1 cube plus x 2 cube.

So, this is not satisfying the additive rule, because this x 1 plus x 2 whole cube will be equal to x 1 cube plus 2 x 2 cube plus 3 x 1 square x 2 plus 3 x 1 x 2 square. Due to the presence of the term 3 x 1 square x 2 plus 3 x 2 square x 1. While expanding this cubic term, so it is not satisfying the homogeneity rule. So, this equation should not be a linear equation that is why this is a nonlinear equation. So, this equation is a nonlinear equation.

So, this way, by applying the superposition rule, so you can distinguish between the linear and nonlinear equation. So, now, initially we will briefly see some of the or review some of the linear system. Then we will see some commonly available nonlinear equations and just I will give a brief introduction to qualitative analysis of a nonlinear equation.

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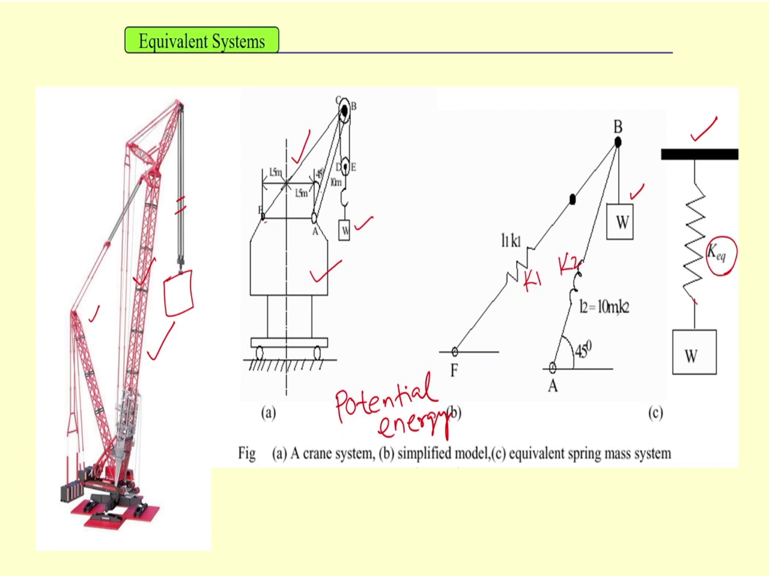

So, let us see. So, for example, now let us take a linear system or a system any system if you take and do the analysis. So, from a physical system, initially let this is a physical system. So, where this is the crane. So, this crane can be replaced or can be written in a schematic form like this.

So, here, so we have this is the boom, this is the wire and this is the weight acting on this thing. So, this part is replaced by a, it can be replaced by a pulley and rope system. And finally, a weight is acting here. So, we can take let us assume that a weight is acting here.

So, this physical system is now replaced by an equivalent system. And further this equivalent system or schematic diagram can be represented by the spring-to-spring system. For example, this rope can be or the string can be replaced by a spring with stiffness k 1.

Similarly, this beam which is subjected to this axial load can also be replaced by another spring of stiffness k 2, then this is the weight acting this. So, these two forces further can be replaced by a single force K equivalent force in the direction of the application of load. So, finally, so one can write this equation or one can have this equivalent system like this.

So, this complicated physical system can be replaced by a simple spring mass system by using this equivalent system. But one can complicate the analysis now this beam instead of a simple beam and a spring, so one can consider this as a continuous system also or one can consider this with a multi-degree of freedom system.

With by taking several degrees of freedom of the system, so one can consider this as a multi-degree of freedom system, in that case one may use this finite element method to write down this equation of motion. But for a simpler analysis, so one can convert this system to an equivalent single degree of freedom system.

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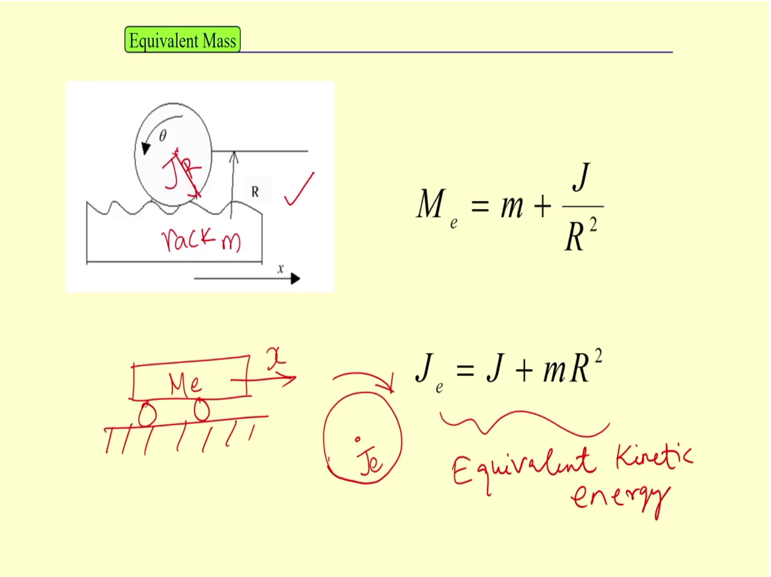

So, similarly let us see what do you mean by equivalent mass. For example, let us take the take the rack and pinion. So, this is the rack, and this is the pinion, so rack and pinion. So, this rack and pinion, so let the mass of the rack is m and the moment of inertia of the pinion is J.

So, we can replace these two by a single translating mass translating system single translating system so with mass M e. So, this M e will be equal to in that case it will be equal to m plus J by R square, where R is the radius of the pinion. So, actually we can do this by equating the kinetic energy of the original system.

So, kinetic energy of the original system contains, the kinetic energy of the pinion plus the kinetic energy of the rack by its single mass translating mass which is moving with x. So, in this case, we can write the equation of motion by equating the kinetic energy of both the system. Otherwise we can replace the system by an equivalent rotating system also. So, let this is the equivalent rotating system.

So, in this equivalent rotating system, let J e, e the equivalent J equivalent moment of inertia. So, equivalent moment of inertia can be found out by equivalent by equating the kinetic energies also. So, by equating the kinetic energy of the original system and the system, so we can write the J equivalent will be equal to J plus m R square.

So, in the previous case, so by equating the potential energy, so here we have to find the equivalent spring or equivalent spring. We have to equate the potential energy, here we have to equate the potential energy. And in the next case, we have to equate for finding the equivalent mass we have to equate the equivalent kinetic energy. Similarly, to find the damping, so we have to equate the dissipation energy. So, those equivalent systems we will study in a later stage.

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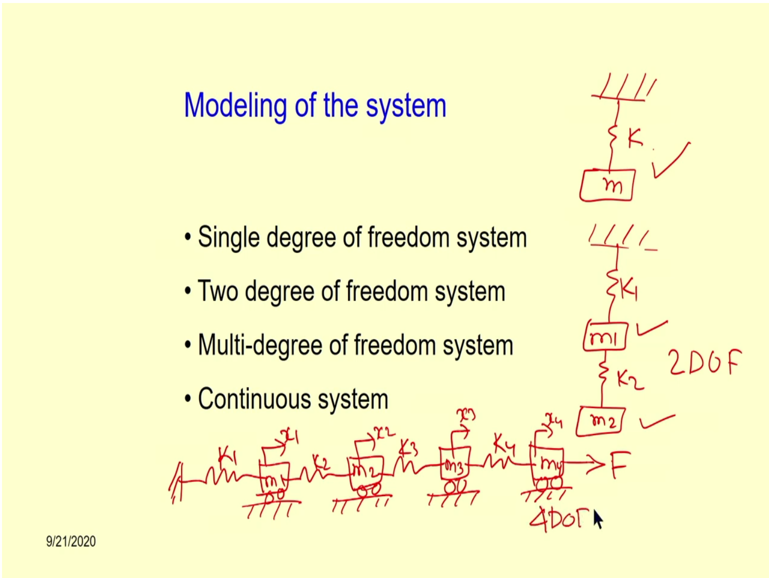

So, now, let us see so modeling of the system. So, the system can be modeled as a single degree of freedom system like the crane system I have shown. It can be model as a two degrees of freedom system. So, in case of two degrees of freedom system, so we can have two mass or two and two springs also. So, it can be model by a two degrees of freedom system. Similarly, we can have a system with multi-degree of freedom system or we can have a continuous system.

For example, in a single degree of freedom system, so the spring mass system, so this is a single degree of freedom system. In two degrees of freedom system, so we can have two mass for example, let us take. So, this is one mass, so this is another mass. So, here we have m 1, m 2. So, this is k 1, k 2. Similarly, we can have a multi-degree of freedom system multi degree of freedom system. For example, let us take several mass and springs we can attach. So, several mass and spring can be put to have a multi-degree of freedom system.

So, here we can put, so let this is the force acting on the system. So, it is constrained to move in the smooth surface or we may consider the friction also. So, by considering different type of conditions, so we can our equation motions will be different. So, here you just see, so this is k 1, mass m 1, spring k 2, mass m 2, spring k 3. So, we have a mass m 3 spring k 4, we have mass m 4. So, here four masses are there.

So, we can write the for example, the mass it is moving with x 1. So, this is x 2. So, this is x 3 and this is x 4. So, this is a four degree of freedom system. So, this is a four degree of freedom system. So, with single mass, we have single degree of freedom. So, here we have two mass, this is 2 degrees of freedom system. So, this is a 4 degrees of freedom system, 4 degree of freedom system.

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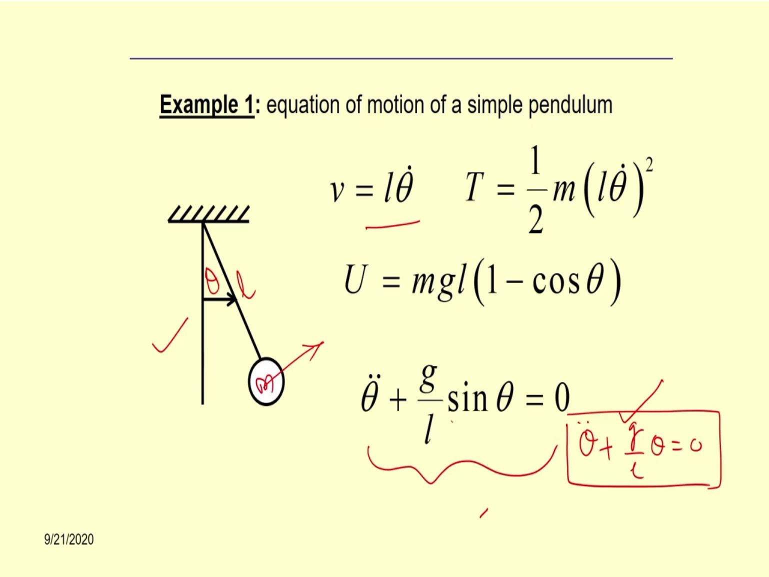

So, for example, let us take the take the case of a building multi-storey building. So, that thing can be considered as a multi-degrees of freedom system. We can model that thing also as a continuous system. So, another example also you can take. So, for example, this is simple pendulum. So, in this case of simple pendulum, already you are familiar with the equation of motion. So, the equation of motion can be written as theta double dot plus g by l sin theta equal to 0, so that equation motion you can find by using this energy method.

So, in the second module, we are going to study how to derive this equation of motion. So, that time I will tell how we are deriving this thing. But for the time being, so if you are interested you know the velocity of this mass can or the bob can be written by l theta dot where this is theta. Theta is the angular rotation of this thing. So, l is the length of the string. So, this is l theta dot.

So, kinetic energy if the mass is m, so the kinetic energy can be written as half m l theta dot square. And potential energy which is due to the change in the position of the bob can be written as m g l into 1 minus cos theta. By either using the Lagrange principle or using this Hamilton principle, you can derive this equation of motion.

So, the equation of motion can be written in this form theta double dot plus g by l sin theta equal to 0. Also you may use this Newton second law or D Alembert principle by drawing the free body diagram of this bob or the mass and derive the same equation of motion that is theta double dot plus g by l sin theta equal to 0. But you know for small angle theta, so the sin theta can be written as theta.

So, this equation will reduce to theta double dot plus g by l theta equal to 0. But if this is not small, if the angle is not small, so we can expand the sin theta and can write sin theta equal to theta minus theta q by factorial 3 plus theta 5th by factorial 5. And we can go on expanding up to higher order and in that case the equation of motion no longer will be linear.

So, in this case, you can note that this equation of motion is linear because this g theta double dot plus g by l theta equal to 0. So, it satisfy both the homogeneity and the additive property or the superposition rule. And this thing conveniently you can tell this is a linear system. But if we have this cubic nonlinear or quintic nonlinear as we have seen by expanding the sin theta, then that equation will no longer be linear.

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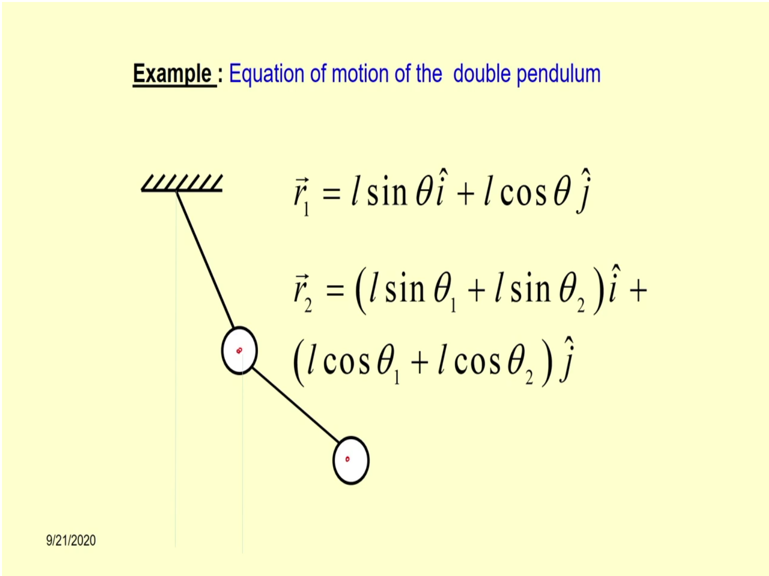

Similarly, for a double pendulum, so for double pendulum, you can initially write the position vector of this position vector of the second mass. So, the first mass and second mass after writing the position vectors by differentiating it, you can write the velocity, then you can write the kinetic energy and the potential energy also.

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$$\begin{bmatrix} (m_1 + m_2) l^2 & m_2 l^2 \\ m_2 l^2 & m_2 l^2 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{Bmatrix} +$$

$$\begin{bmatrix} (m_1 + m_2) gl & 0 \\ 0 & m_2 lg \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

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Then if you write down the equation of motion, so you can see this is the equation motion of the system. So, we will study how to derive this equation of motion in second module. And you can see if this theta 1 and theta 2 are very small, then you can write this equation in this form. So, it can be written as ok. So, it will be written as theta 1 double dot not theta square, similarly theta 2 double dot. So, m 1 plus m 2 l square m 2 l square theta 1 double dot this is theta 2 double dot, and then m 1 plus m 2 gl 0, 0 m 2 l g theta, see this is theta 1, theta 2 will be equal to 0, 0.

So, this is when you are considering this theta 1 and theta 2 small. So, here you just see for this two degrees of freedom system. So, we have two equations. And for a single degree of freedom system, we have single equation. Similarly, for n degrees of freedom system, so we can have n differential equations.

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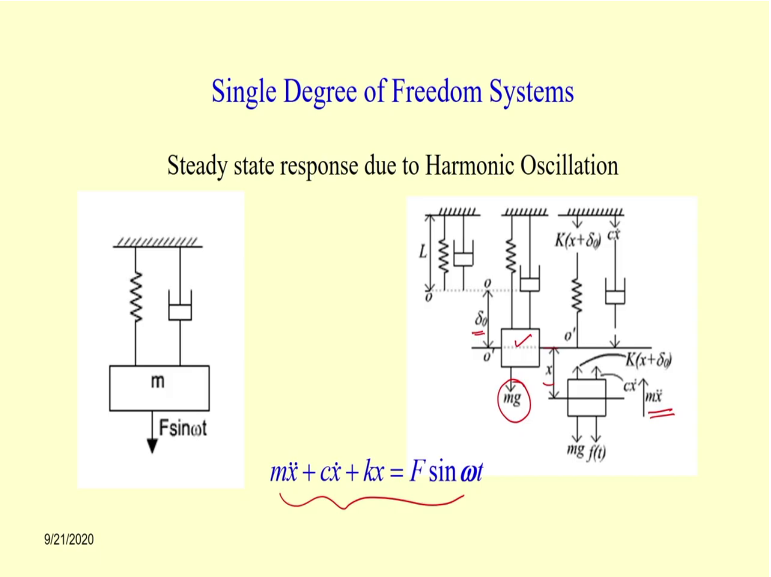

So, let us see some of the response of the how to get the response of a single degree of freedom system. So, just review the linear system response of a linear system, and here I will tell how you can find the response briefly. And I wish you can write a small code to find the response of the system. So, already you know by using the Newton second law, so we can derive this equation of motion.

So, for example, initially you just take the spring or this damper. So, this is the original length when the mass is not attached. So, when you are attaching the mass, so there will be a static deflection of the mass. So, let delta 0 is the static deflection of the mass.

So, if you draw the free body diagram with the static deflection, then you can write this mg that is the weight of the mass will be balanced by this k delta 0 plus c delta 0 dot. So, now, from the equilibrium position, so it is the equilibrium position. So, from this equilibrium position, so if you pull it by an amount x, so from this equilibrium position if you pull by an amount x then so the mass will be at this position.

So, in this case, the forces acting on this thing will be equal to m g force downward f t force you have applied plus the force due to the spring. Spring will as it is pulled down, the spring will exert a force in upward direction, so that force will be equal to K x plus delta 0. And this damping force will be as delta 0 you can take constant the damping force will be equal to c x dot. And in addition to that, you can have this in addition to that you can have the inertia force m x double dot.

So, if you are applying this D Alembert principle, then your equation motion will be equal to your external force plus the inertia force will be equal to 0; or if you are applying this Newton second law, it will be external force will be equal to mass into acceleration that is m x double dot. So, the equation of motion from the free body diagram so you can find equal to m x double dot plus c x dot plus k x equal to F sin omega t.

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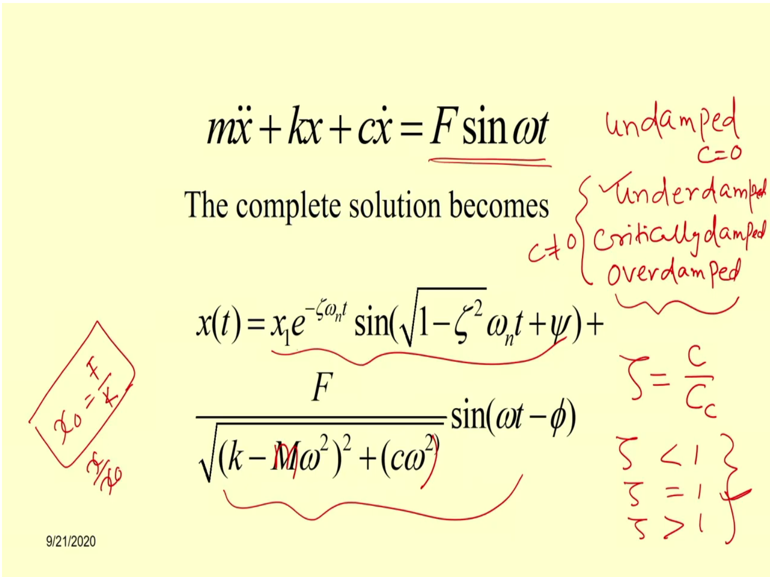

So, the complete solution of this equation can be written using two terms. So, one is the complementary part, so that is the free vibration part of the system when you are considering the force acting equal to 0. So, that is m x double dot plus k x plus c x dot equal to 0. So, then the solution will be equal to x 1 e to the power minus zeta omega n t sin root over 1 minus zeta square omega n t plus psi. So, this is the case when the system is under damped.

So, we have three conditions actually under damped. So, the system may be under damped. It may be critically damped or it may be over damped or the system maybe so this is if damping is there, then this, these are the things. If totally damping is not there that is c equal to 0 the equation is known as undamped, so it may be undamped also. The system is undamped if c equal to 0.

So, if c not equal to 0, so we have this; either we have under damped critically damped and over damped system. So, in linear vibration case, we studied all these things and that is why you may revise these types of different types of damping.

So, if the if this is the case of a under damped, so then the equation can be the complementary part can be written in this form. Where zeta is nothing but zeta is known as the damping ratio, so which is equal to c by c 0, c by c c, c c is the critical damping factor. c is the damping factor of the system; c c is the critical damping factor. So, for under damped system, so zeta less than 1; for critically damped system, so zeta equal to 1; and for over damped system, zeta greater than 1.

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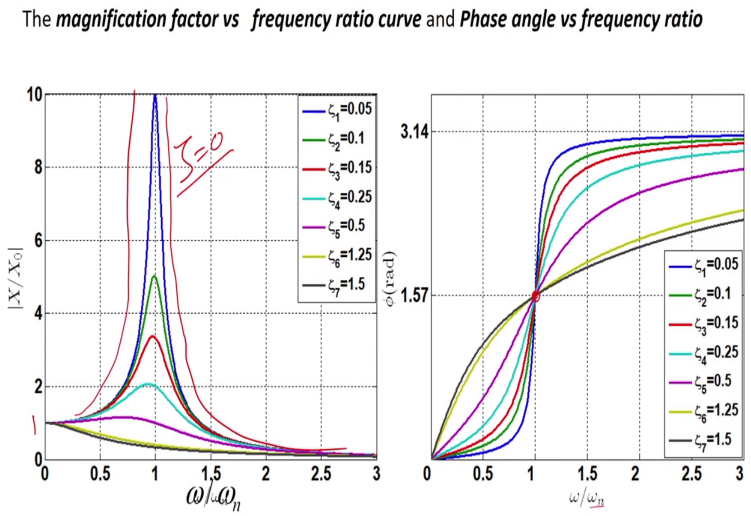

So, for these, three different equations, so this equation will be different, this part of the equations will be different. And so this part represent the particular integral or steady state response of the system, where the steady state part equal to F by k minus m omega square, so this is small m, so k minus m omega square whole square plus c omega square into sin omega t minus phi. So, this is the particular integral part of the system.

So, now, if you plot this x by x 0 that is x is the response of the system, and x 0 is nothing but your static deflection that is F by k. If you take x 0 equal to F by k. then this x by x 0 is known as the magnification factor.

So, if you plot this magnification factor versus the omega by omega n, so for a different value of damping, you can clearly see the response of the system. So, it starts with 1, the response start with 1. So, as you go on decreasing the damping, the response amplitude goes on increasing. And for an undamped system, so it will go to infinite. For an undamped system, you can see it is going to infinite. So, this is for zeta equal to 0.

So, with presence of zeta, the response is decreasing. And for higher value of zeta, so you can see the response is further decreased. So, for over damped system, so we can see the response is less than this x by x 0 is less than 1. So, this is the phase diagram. So, this is known as magnification factor versus omega by omega n. So, this is phase diagram that is phi versus omega by omega n.

So, irrespective of damping, so you can see irrespective of damping, so the system as a, so at pi by 2, so when omega by omega n equal to 1, so this is equal to phi equal to phi by 2. Similarly, for higher very high value of omega by omega n, the response approaches pi value.

And similarly here so different amplitude, so the amplitude of the response can be found, the maximum value of the response can be found which is equal to 1 by 2 zeta. So, all these things you must revise before starting the nonlinear vibration response. So, the linear vibration response part must be clear before going for the nonlinear vibration.

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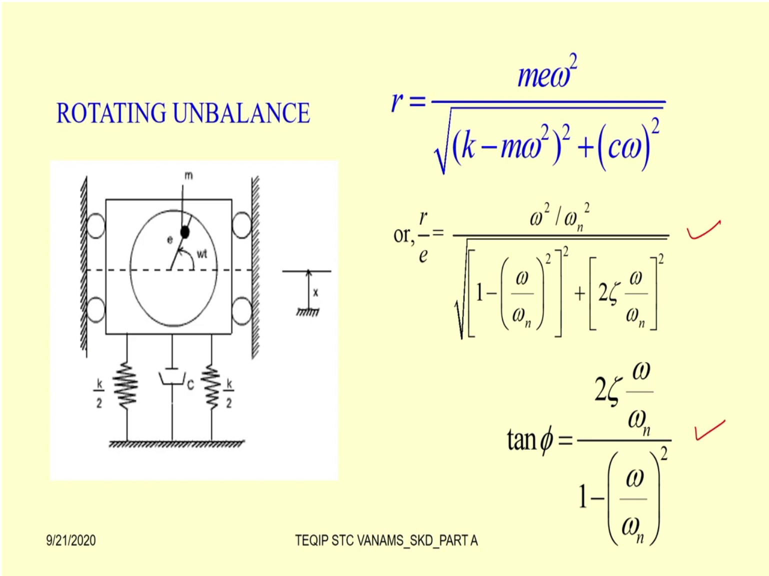

Similarly, if I have a rotating unbalance system the equation of motion can be written and the response you can find. So, if e is the eccentricity, m is the eccentric, this is the unbalanced mass, and capital M is the, so you can have the M as the mass of the rotating parts. Then you can write the equation of motion and from which you can find the relation between r by e. So, you can get this response. So, this is the phase equation.

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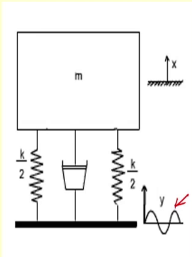
Support Motion



$$\left|\frac{X}{Y}\right| = \sqrt{\frac{1+\left(\frac{2\zeta\omega}{\omega_n}\right)^2}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(\frac{2\zeta\omega}{\omega_n}\right)^2}}$$

So, similarly you can have the equation for the support motion. Let the support is vibrating. For example, you have a vehicle moving on a uneven road, so that time you can model the system like this. So, you have a undulation, road undulation can be modeled by a harmonically excited system or harmonic term. So, this is your y, and then so the force is transmitted. So, this ground motion is transmitted to the vehicle of mass m. The equation of motion can be written using that. And the response this X by Y that is the steady state response can be found by using this equation.

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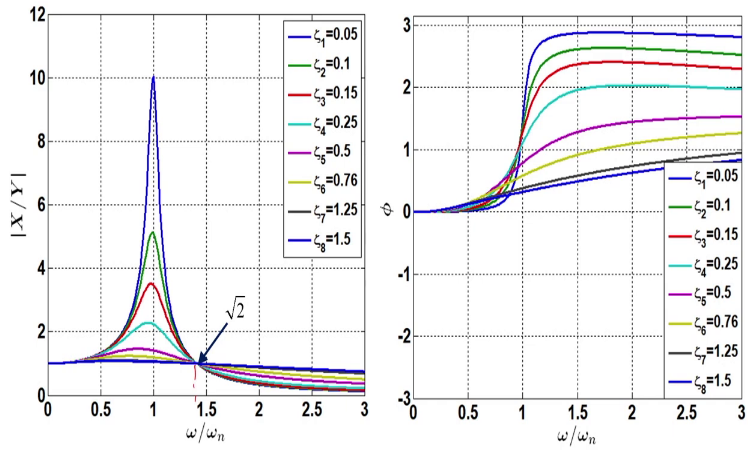

So, one can plot the X by Y versus omega by omega n. And in this case, it can be clearly seen that when omega by omega n is greater than root 2, the response the x by x 0 is less than 1. So, to isolate the vibration one must operate the system at a frequency greater than root 2 times the natural frequency of the system.

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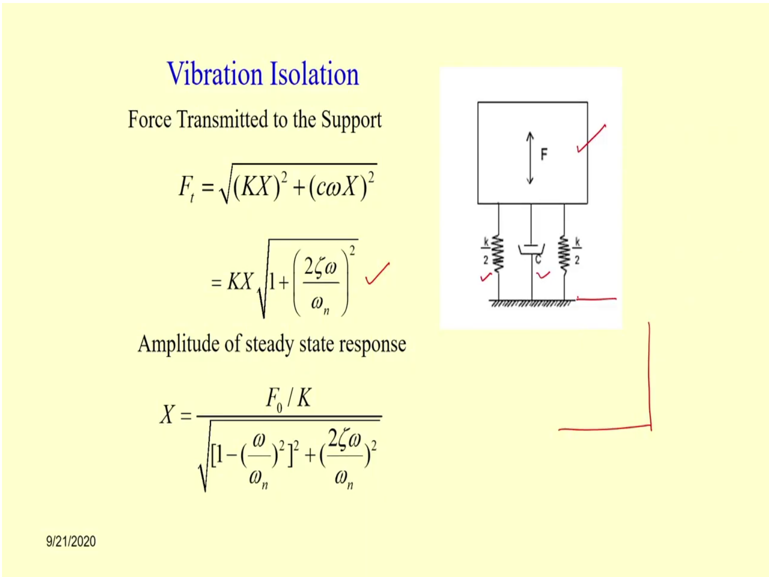

So, similarly one can draw the phase diagram and vibration for vibration isolation. So, two different things are there. So, one thing we have seen from the support motion. Similarly, so if the system is vibrating and we want to isolate the vibration of the system to the ground, so then also we can find what is the force transmitted to the ground.

So, from F t, we can found we can find that the force is transmitted through the spring and the damper. So, the force we can write spring force – maximum spring force will be KX. And similarly maximum damping force will be equal to c omega X, but this damping force and spring force as they are in quadrature that is perpendicular to each other.

So, the resultant force F t can be written as KX whole square plus c omega X whole square. And you can write this in this form. And then we can write this X equal to as we know X equal to F 0 by K by root over 1 minus omega by omega n whole square whole square plus 2 zeta omega by omega n square. So, substituting this equation in this above equation, so we can find so the amount of force transmitted to the ground.

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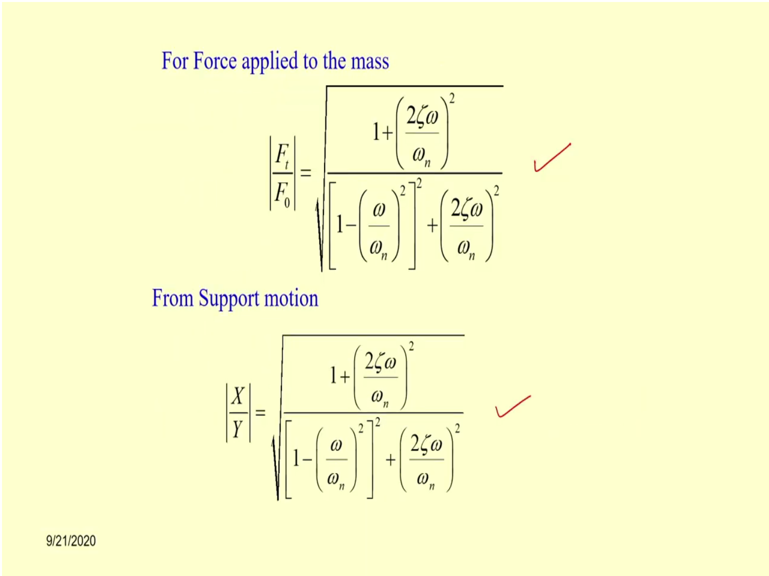

So, this F t by F 0 equal to this expression you can find. So, this expression you can see is similar or same as that in case of a support motion X by Y. So, in both the cases, so the plot what we have already seen, so can be used for F t by F 0 also. So, that means, so this force can be the force transmitted to the ground can be isolated. If the system is operating at a frequency greater than root 2 times the natural frequency of the system.

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vibration will be isolated when the system operates at a frequency ratio higher than $\sqrt{2}$

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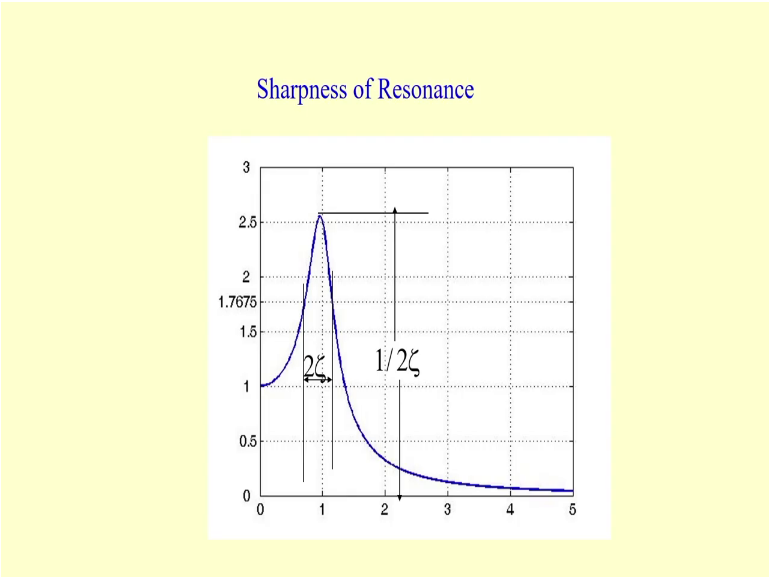

So, similarly one can study the sharpness of resonance.

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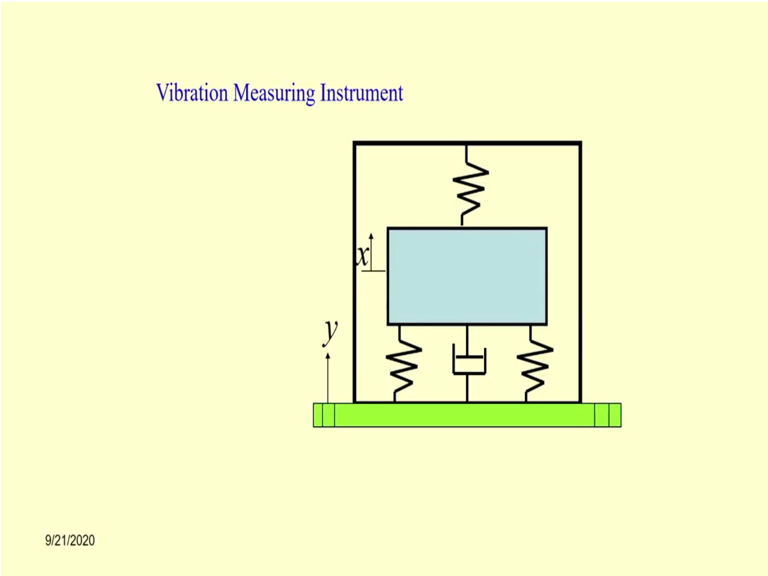

And vibration measuring instruments.

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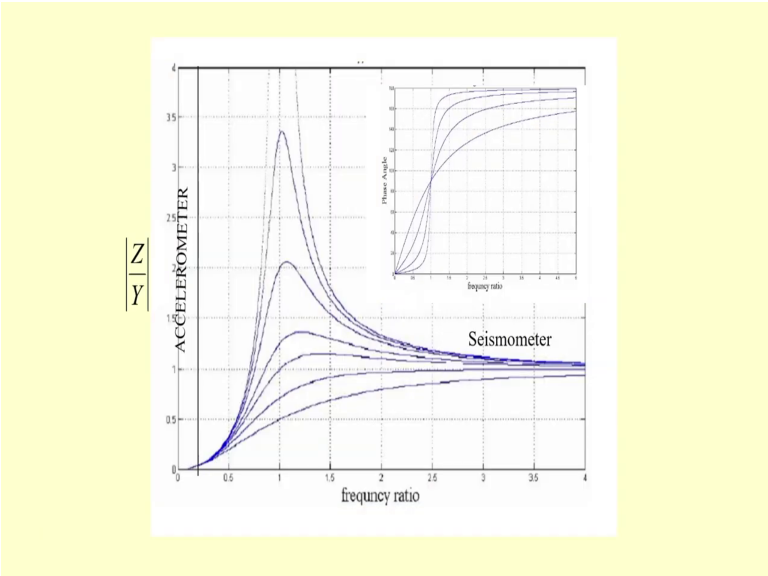

And revise all the things available for the single degree of freedom system.

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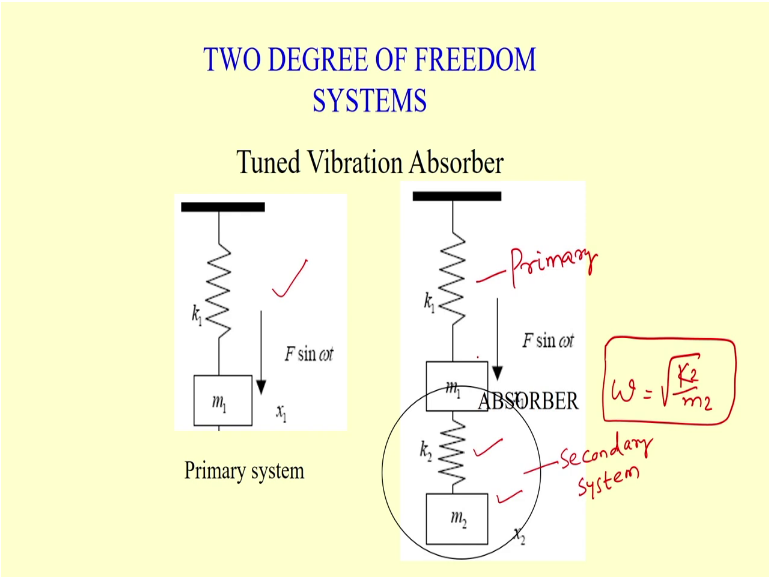

The single degree of freedom system instead of application of a harmonic force, it may be subjected to other type of force also. For example, it may be subjected to impulse force or some other periodic force. So, in that case, in case of other periodic force, it can be converted to an equivalent simple harmonic force and the response can be found out. In case of impulse force, so the system can be modeled as that of a freely vibrating system.

So, after the impulse force is applied as there is no force acting on the system, so then it can be considered as a single degree of freedom system with free vibration, but the conditions the final initial conditions will be different. So, taking proper initial conditions, so one can find the response of the system.

So, let us now take the two degrees of freedom system. So, this is a single degree of freedom system we have already seen. The amplification factor so that is it tends to infinite with omega tends to omega n that is the external frequency equal to the natural frequency of the system.

So, in that case, resonance will occur and the response tends to infinite. So, to absorb that response, so one can use a secondary spring and mass. So, in this case, this equation of motion will be reduced to that of a two degrees of freedom system. And so if we can properly tuned this spring and mass, then we can observe the complete vibration of the primary system. So, this is the primary system.

So, the vibration of the primary system can be completely absorbed by using this absorber or the secondary system. So, here the condition is, so the condition is. So, our omega can be taken root over k 2 by m 2. By taking omega equal to root over or k 2 m 2 in such a way that this root over k 2 by m 2 equal to omega, then it will absorb completely the vibration of the primary system, and the secondary system will vibrate.

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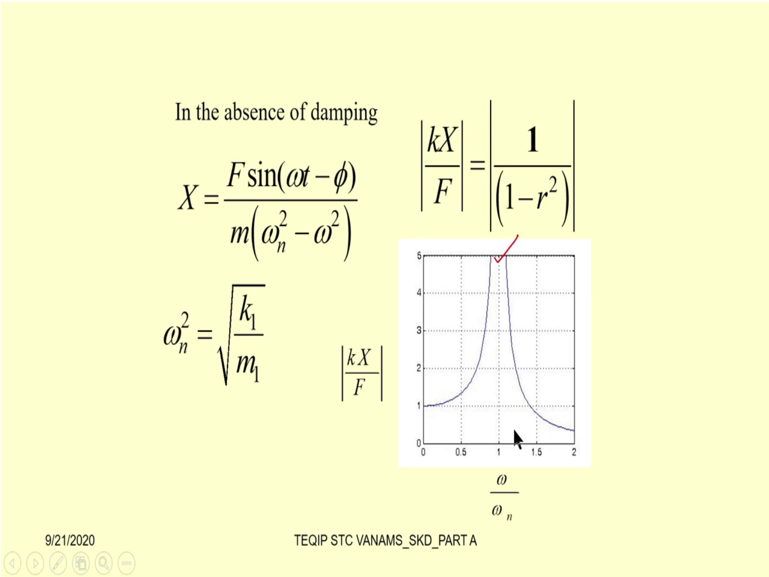

But the there will be no vibration in the primary system. So, you can see.

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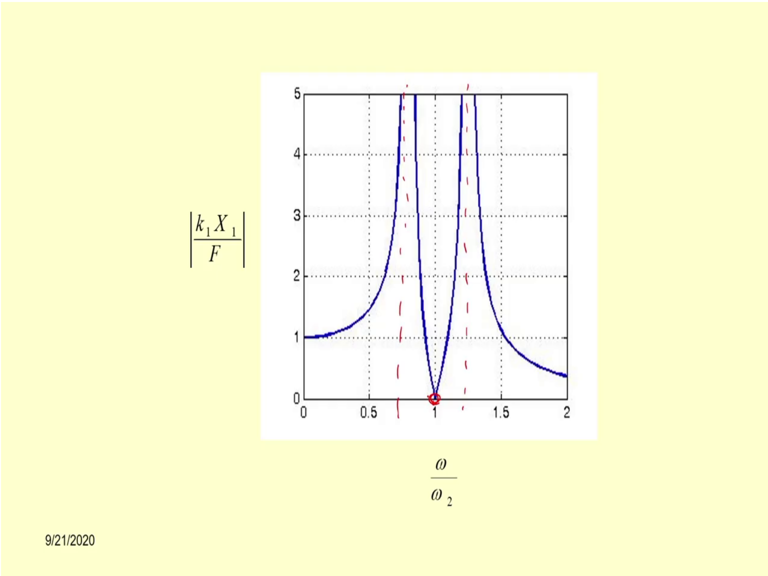

So, initially, we have initially the response is infinite. So, as at omega equal to omega n, the response is infinite. But by using the secondary mass, now you can see it has two natural frequency. So, the actually the resonance now are occurring at or shifting resonance condition is shifting to other two positions, but at omega equal to omega n the response becomes 0. So, the primary force the primary system vibration is completely absorbed by using the tuned vibration absorber.

Similarly, other things of the second of the two degrees of freedom system and multi-degrees of freedom system can be studied modal analysis method can also be applied to study the response of the system. And let us see the continuous system. So, in this multi degrees of freedom system, one can use the concept of eigenvalues and eigenvectors and normal modes to study the response of the system.

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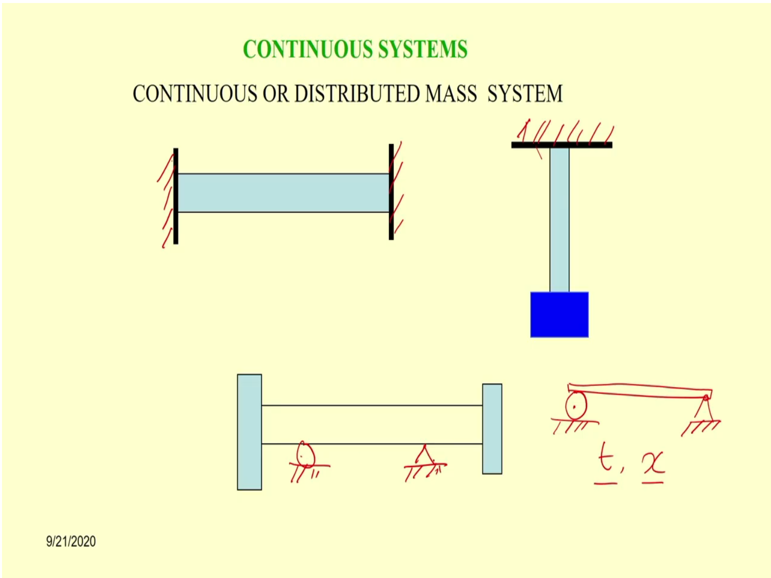

So, for a continuous system, so we tell a system to be continuous when it has it can be model as a infinite number of spring undamped system. For example, this is a beam. So, in this beam both end are fixed, or the beam we can take a beam. For example, the bridge, if you are modeling the bridge, so one side of this thing is roller supported and other side of the bridge is hinged.

So, this beam or this bridge can be modeled as a continuous system. So, here the, so in case of continuous system, so the response depends on both time and the position with time and position. So, this is a spatio temporal equations one can find. Previously we have only the equations only depend on the time.

But in this case, in continuous system, the response of the system depends both on space and time. So, we have to one may use this variable separation method to separate or to write down the equation of motion in terms of the temporal form from the spatio temporal equations. So, these are some of the example of the system.

So, here for example, so we can make so it depends on the initial, so it depends on the boundary conditions also. So, in this case the boundary in the boundary of the system, this is a beam in the boundary it is fixed both the ends are fixed. Similarly, here this is a cantilever type system. So, it is supported at the uppermost position. And here the support conditions are similar to that of a simply supported beam. So, in case, so we can have two different type of equations.

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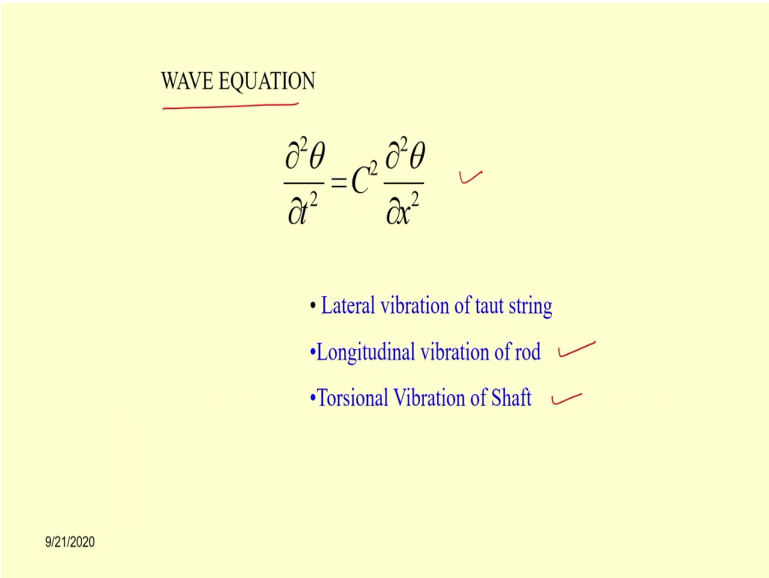

So, for example, we can have the wave equation. So, wave equation is particularly when it is subjected to axial loading, torsional vibration, so in that case, so we can write the equation in this form for example, the vibration of a string.

So, in case of the vibration of a string the equation can be written in the form of wave equation. Similarly, the vibration of longitudinal vibration of a rod or a torsional vibration of shaft, this equation can be used. So, here theta is a function of both x and t.

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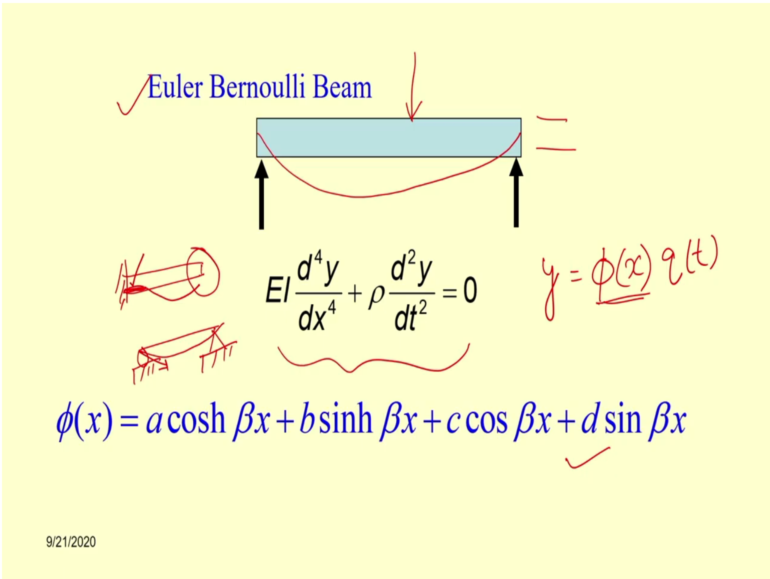

Similarly, one can see the transverse, for example, the transverse vibration of a beam by using this Euler Bernoulli beam equation. So, in this transverse vibration, the beam is subjected to bending.

So, it is subjected to, so it can bend the beam can bend. So, it is subjected to bending pure bending. So, if we are taking thin or this thickness of the beam is very small in comparison to the length of the beam, so in that case, we can consider pure bending. So, in case of pure bending, we may go for this Euler Bernoulli beam equations.

So, the Euler Bernoulli beam equation can be written by using a fourth order equation. So, previously you have seen the equation wave equation is a second order equation, where del square theta by del t square equal to c square del square theta by del x square. But this Euler Bernoulli beam equation is a fourth order equation E I del 4th y by del x 4th plus rho del square y by del t square equal to 0.

So, to find the response of the system, so or the we can use this variable separation method, so we can write this y equal to phi x into qt. And use this variable separation method and we can find the expression for the phi x by applying the boundary conditions. So, the general equation will be in this form. So, phi x will be equal to a cos hyperbolic beta x plus b sin hyperbolic beta x plus c cos beta x plus d sin beta x.

So, at, so this a, b, c, d, these are four constants which can be obtained from the boundary conditions. For example, in case of a cantilever beam, so cantilever beam. So, we have four boundary conditions. So, at the fixed end both the deflection and slope are 0; and in the free end, the bending moment and shear force are 0. So, by using these four conditions, so we can find these a, b, c, d.

Similarly, in case of a simply supported so in case of simply supported, so we have both hinge end and roller support. So, in both the cases, both the ends, we have deflection equal to 0. You just see slope is not 0; deflection is 0, bending moment is 0. So, slope is not 0. So, at this point, slope is not 0. So, it has this much slope.

But in case of a cantilever beam, you can see up to certain distance the slope is, so there is no slope. Here the both slope and displacement are 0, but in case of the simply supported only the displacement and bending moment are 0. So, by putting the boundary conditions, so we can get this a, b, c, d, and we can plot this phi x versus x. So, these are known as the mode shapes.

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$\beta^2 l^2$

Beam Conf	First mode	Second mode	Third mode
Simply supported	9.87	39.5	88.9
Cantilever	3.52	22.0	61.7
Free-free	22.4	61.7	121.0
Clamped-clamped	22.4	61.7	121.0
Clamped-hinged	15.4	50.0	104.0
Hinged-free	0	15.4	50.0

$$\omega_n = \beta^2 l^2 \sqrt{\frac{EI}{\rho l^4}}$$

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So, here actually we can find applying the boundary conditions. So, we can find the expression for beta and those are known as characteristic equation. So, from that thing, we can find this beta square l square. And after getting this beta square l square, so we can write this omega n equal to beta square l square root over E I by rho L 4th.

So, here the natural frequency of the system can be obtained from this expression. So, you just see as this is a continuous system. So, we can have large number of natural frequency. So, for up to first three modes, so this slide shows up to first three mode the natural frequency for simply supported, cantilever, free-free, clamped-clamped, clamped-hinged, and hinged-free boundary conditions.

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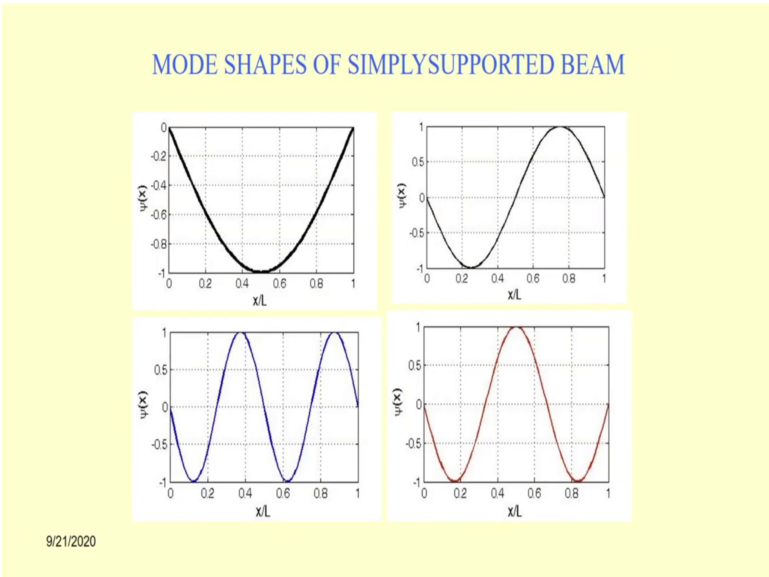

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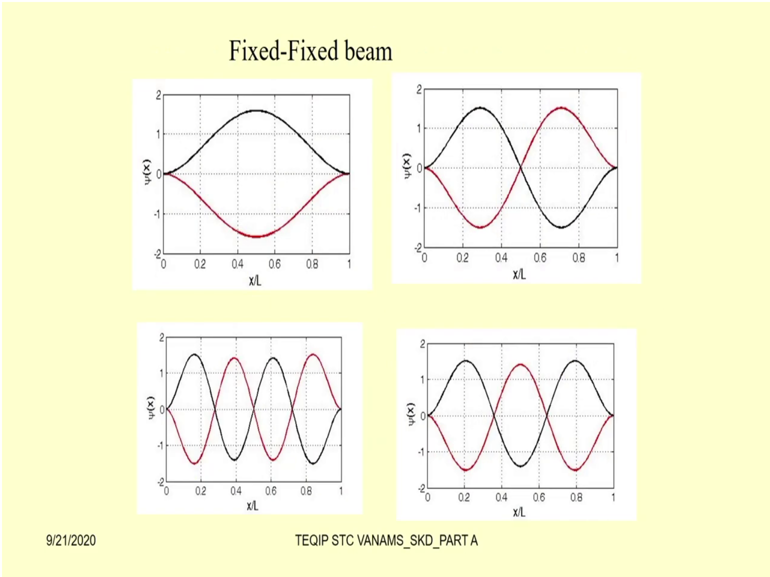

So, these are the mode shapes.

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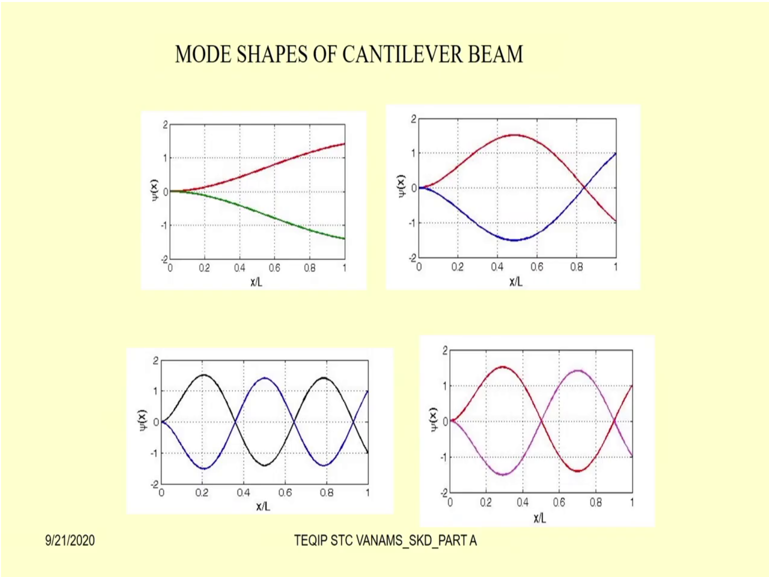

So, for different boundary conditions, you can see these are the mode shapes.

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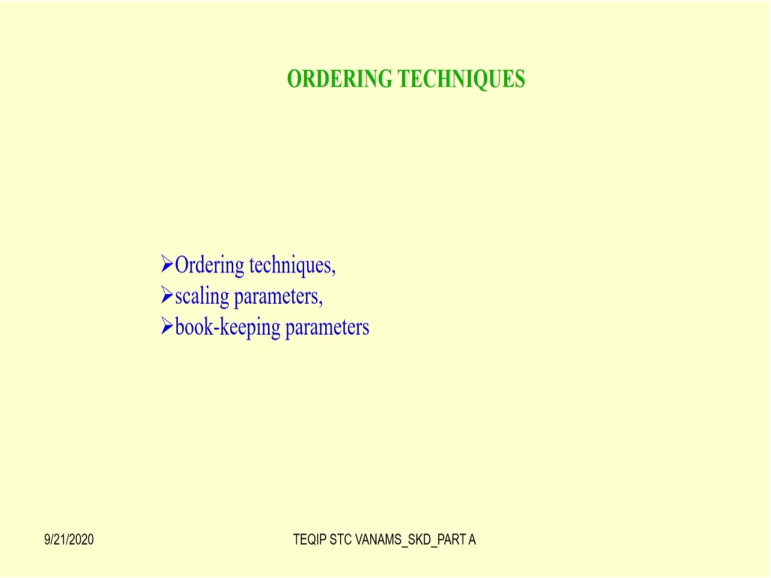

So, this way you can see how we can or we have revised these linear systems. Now, we have seen the difference between the linear and nonlinear systems by applying the superposition rule. So, after getting a nonlinear equation, actually we must know how to order that equation for our analysis purpose. So, this ordering technique using the scaling parameter or book-keeping parameter are very important in the analysis of nonlinear systems.

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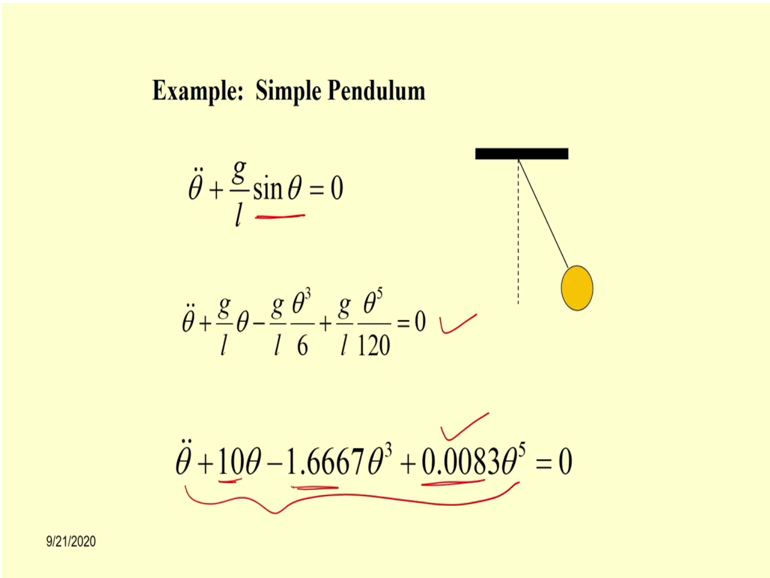

So, let us see the example of a simple pendulum. So, in this case of simple pendulum, already you know this equation equal to theta double dot plus g by l sin theta equal to 0. So, if we expand the sin theta, then we can write this equal to theta double dot plus g by l theta. The sin theta we have expanded here by writing this equal to this minus theta cube by 6 this 3 factorial is 6 plus g theta 5 by 5 factorial that is 120. So, thus equation is written in this form. So, theta double dot plus g by l theta minus g i g by l theta cube by 6 plus g by l theta 5th by 120 equal to 0.

So, if by taking this g equal to 9.8, so we can write this equation up to this fifth order and it can be written theta double dot plus 10 theta or if you are taking g equal to 10. l equal to for example, let us take l equal to 1 meter, then this equation will be theta double dot plus 10 theta minus 1.667 theta cube plus 0.0083 theta 5th equal to 0.

So, you can see the coefficient of theta 5th is very small in comparison to the coefficient of the linear term 10. So, one may have a tendency to neglect this 5th order for this case, because this can be neglected with respect to the linear coefficient of the linear term. Similarly, the coefficient of this cubic term also is very small in comparison to that of the linear term.

So, in that case, so one can use this ordering technique, so that one can write this equation where the coefficients are no longer of the order of looks like very small. But it will looks like similar to that of the linear term by using some scaling parameter and book-keeping parameter.

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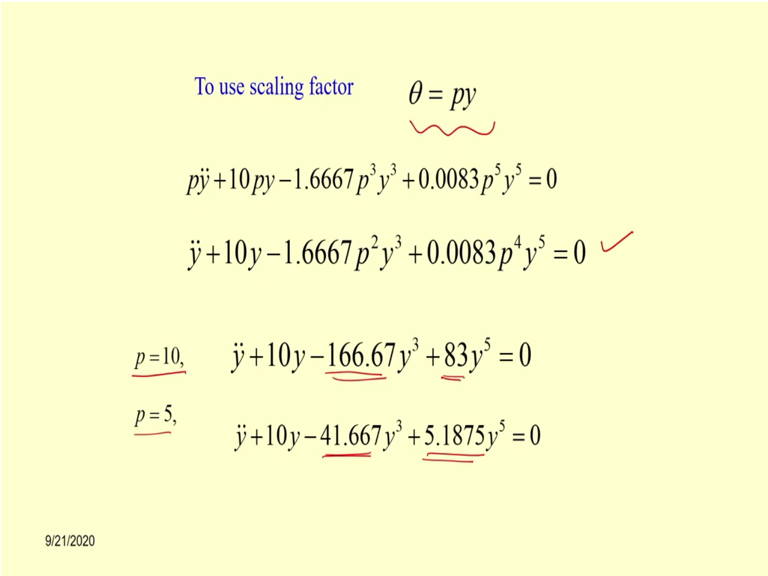

So, let us see the use of a scaling parameter scaling factor. If we are substituting this theta equal to p y, then in that case this equation can be written as p y double dot plus 10 p y minus

1.6667 p cube y cube plus 0.0083 p 5th y 5th equal to 0. So, if we take this p common from everywhere and as p not equal to 0, so we can write down this equation in this form.

So, here this becomes y double dot plus 10 y minus 1.6667 p square y cube plus 0.0083 p 4th y 5th equal to 0. So, by taking different value of p, you can see the equation can be written in different form. By taking p equal to 10, you just see the coefficient of the y cube can be written 166. So, here you need not have to or you will have a tendency not to delete this term or neglect this term.

Similarly, this y 5th has a coefficient of 83. By putting this p equal to 5, you can see the coefficient has come down to 41 from 166, it has come down to 41, and it has come down to 5. Similarly, by properly choosing a parameter p, you can write down the equation in a form, so where the linear and nonlinear terms coefficients will be same or of the similar order.

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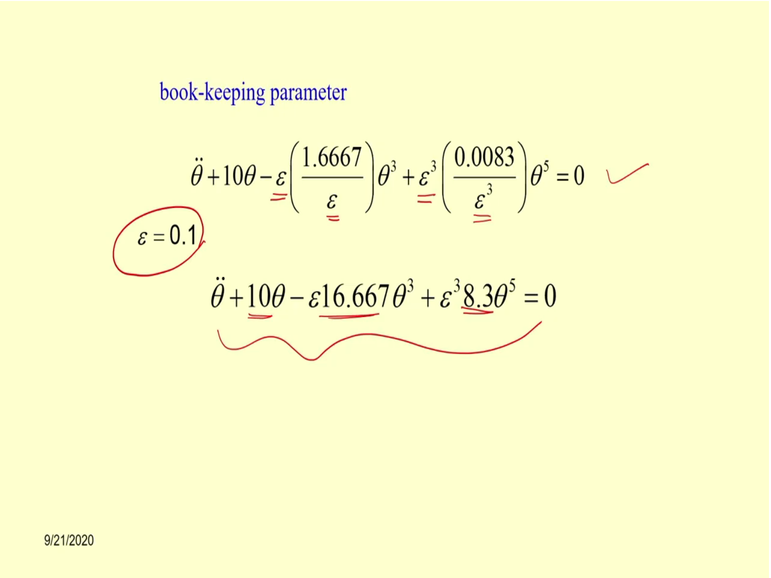

For example, so in this case ok, so now, we can use a term book-keeping parameter epsilon, also epsilon this is a book-keeping parameter. So, this book-keeping parameter is always less than 1. By using a term book-keeping parameter, for example, in this original equation let us multiply epsilon and divide epsilon.

So, here we are multiplying epsilon cube and dividing epsilon cube. So, we can write this equation in this form. So, where you can see the coefficient of theta cube is written in this form 16.667, which is comparable with respect to this term 10. Similarly, the coefficient of theta 5th is written in coefficient of theta 5th is 8.3 which cannot be neglected when you compare it with 10.

So, this way you can order the equation by using the scaling parameter and book-keeping parameter. So, next class, we will see this qualitative analysis of the nonlinear equations and the also we will briefly study about the type of response of the system. So, type of response already you know, it may be fixed point that is a fixed number or it may be periodic quasi-periodic and chaotic. So, regarding these four different type of responses, we will study in the next class.

Thank you.