Nonlinear Vibration

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Lecture - 25
Cantilever beam with tip mass for principal parametric resonance

Welcome to today class of Non-Linear Vibration. We are going to start the last module of this course. So, here we are going to present different applications. So, this module will cover in 9 classes, where we can have 3 major different type of applications.

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Particularly, we will be interested to study the flexible non-linear systems. So, we will take 3 class for studying these type of systems. Then non-linear vibration absorbers, then some electromechanical systems.

So, today class we are going to study these non-linear flexible systems.

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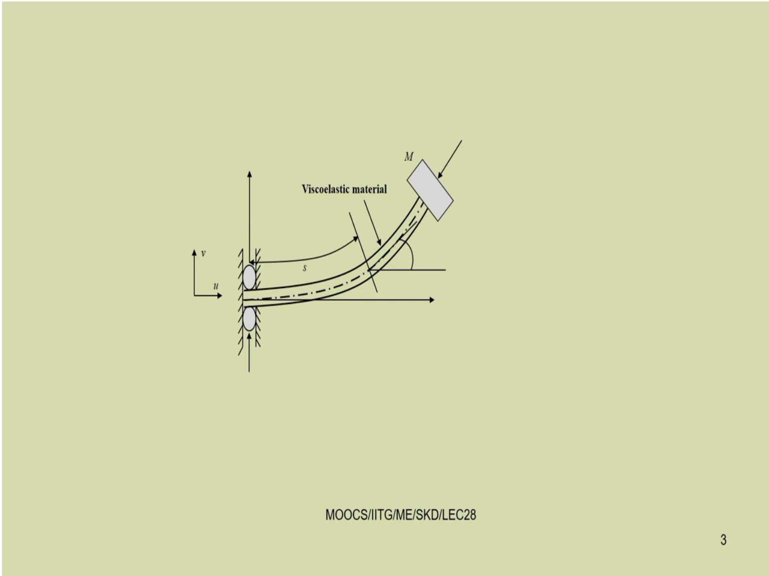

So, in actual case all the systems are flexible. So, if you are taking some rigid structure, the rigid structure actually will have bulky, so it has heavy inertia force. And due to that you may required large torque or force to activate those structure. They are not easy to transport. And though they will have less vibration, but they required more energy to operate. To make the system more and more efficient, so people are going to use or researchers are going to develop flexible structures structure.

The structures can be flexible in many different ways. For example, so we can change the material property, we can change the structure or we may change the shape of the structure, shape and size of those structure. And we may develop some topologically, optimized structure and that will give the same strength, but it will be flexible.

Today class, we are going to study many different type of flexible structure. For example, we may study 3 different type of beams, one with the elastic beam, second one it is visco elastic beam and third one with electro or this Ferro magnetic beam. So, these are 3 different type of materials we will take.

Also, we will study some structure for example, this base excited structure where the beam can be in horizontal position or the beam can be in vertical positions. In this structure, you can see so it is retain a viscoelastic material beam it is a base excited visco elastic material beam. The beam is moving in up and down direction and at the same time the beam is also subjected to an axial loading. We will study how to derive these equation of motion and also how we can solve these equation of motion. So, the basic structure of all these things we have already studied in the previous lectures.

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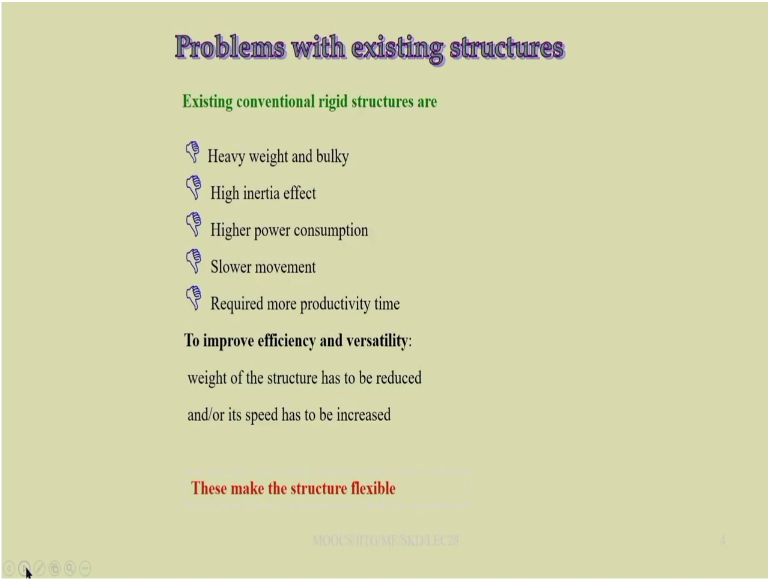

So, let us see why we want to make the structure flexible or what are the problems with the existing structure. So, existing conventional rigid structures are heavy weight and bulky, high inertia effect, higher power consumptions, slower movement, required more productivity time. To improve efficiency and versatility, weight of the structure has to be reduced or it speed has to be increased. So, for that purpose we have to make the structure flexible.

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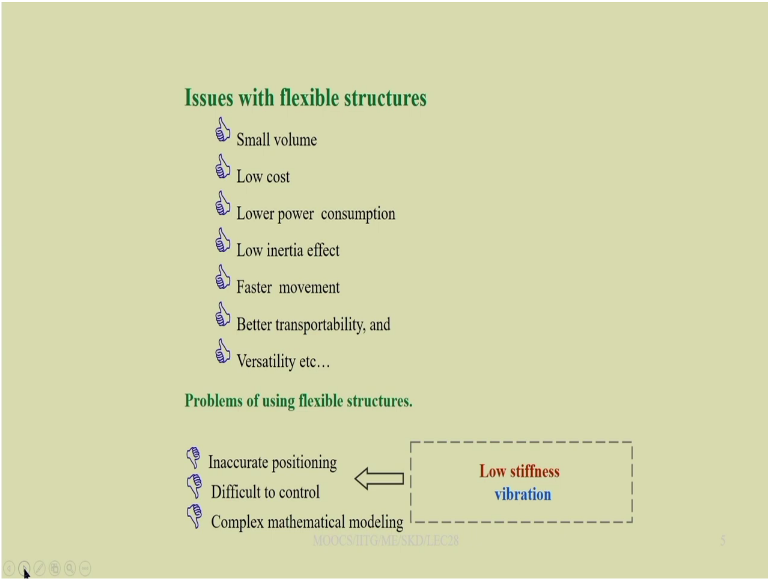

But what are the issues with this flexible structure? They are small volume, low cost, lower power consumption, low inertia effect, faster movement. So, better transportability and versatility and many other advantages are there associated with flexible structure. But it has a large disadvantage. So, the problem with the flexible structure will be inaccurate positioning, difficult to control, complex mathematical modeling because of the low stiffness. So, it is subjected to vibration.

So, this system is subjected to vibration. So, due to that thing, so it is not possible to have this accurate positioning of the structure. Then difficult to control also and the mathematical modeling is also in this case very very complex.

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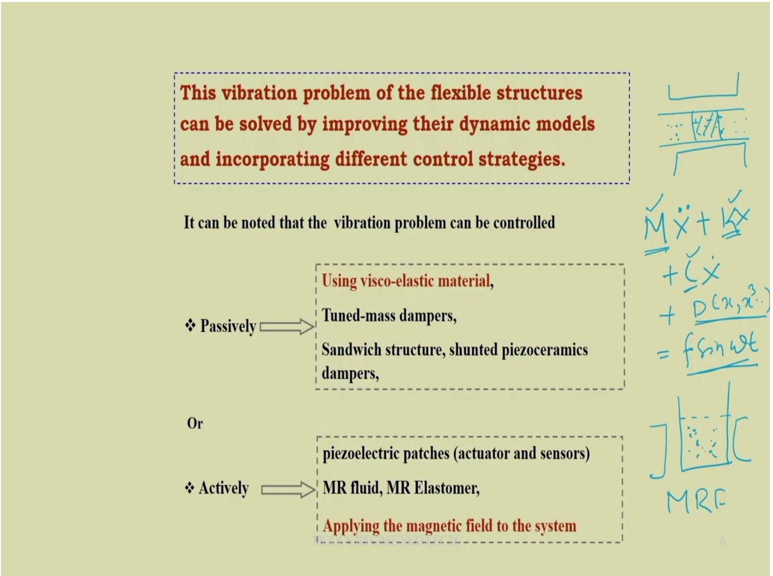

So, this vibration problem or the flexible structures can be solved by improving their dynamic models and incorporating different control strategy. So, it can be noted that the vibration problem can be controlled passively or actively. So, we can control this vibration passively by using for example, this viscoelastic material, tuned mass damper, sandwich structure, shunted piezoelectric materials and dampers. So, we will take all these examples how passively we can control the structure.

So, particularly we can control the structure. For example, you know all the systems can be written in this form that is Mx double dot plus Kx plus Cx dot equal to f sin omega t; Kx plus Cx dot plus, I can put some other matrix. Let it is D will be equal to f forcing, f sin omega t, if we are taking the periodic forcing. So, these D matrix contains the non-linear, nonlinearities.

So, I can write for example, this is x, x cube, all these things non-linear terms will be there, then this is the forcing.

So, here we can change the property of structure by changing these mass matrix. So, mass is a function of this rho that is density. Also, the stiffness matrix, so stiffness matrix maybe due to elasticity of the system or due to this gravity force acting on the system. Then, this damping matrix is also there. So, damping, so different type of dampers we can use. Similarly, this non-linearity matrix. So, these are also depend on the system parameters, and this f that the external forcing acting on the system.

Here, we can change the property or this matrix M, K and C either actively or passively. Actively mean, so we can apply some external stimuli. For example, by applying this magnetic field or electric field or gravity, where we can control these externally. So, we can change these matrix K and C.

For example, if we take for example, this damper MRF, magnetorheological fluid, let us take this magnetorheological fluid in this, so where in this fluid magnetic particles or these iron particles are embedded in this. Now, by applying this magnetic field we can orient these iron particles, so that the viscosity of these damper is changing. So, due to this change in viscosity the damping property of the system will change.

So, we can make the system or the damping in such a way that it may be linear or it may be non-linear also. Similarly, by using this magnetorheological elastomer, so this is rubber like material, so in this rubber like material. So, we are putting or embedding these iron particles.

And when we are applying this magnetic field, so due to this magnetic field these iron particles will tends to orient themselves there by changing its stiffness and damping property. That way we can actively change the property of K and C that is stiffness and damping of the structure.

And passively, so in case of passive thing, so we have to replace the whole structure itself the structure has to be replaced or part of the structure has to be replaced. For example, in this sandwich or these rubber type material. So we can put only one path or single paths can be put on a single paths with magnetic material and there itself we can apply the magnetic field.

This is part of the active one. But for the passive one, so we have to replace this whole viscoelastic structure or we can replace the spring part by using another spring. So, that way we can change the stiffness or the damping of the structure.

Passively by using for example, these viscoelastic material then tune mass damper, ok. So, we know the tune mass damper or centrifugal type of dampers are also available which can reduce the or which can absorb the vibration for a wide range of frequency. Similarly, by using the sandwich structure or these piezoelectric material or by using these other smart material like these shape memory alloy, we can change the property of the system. Similarly, actively these piezoelectric material, actuator and sensors, MR fluid, MR elastomer applying magnetic field to the system, so we can control these vibration.

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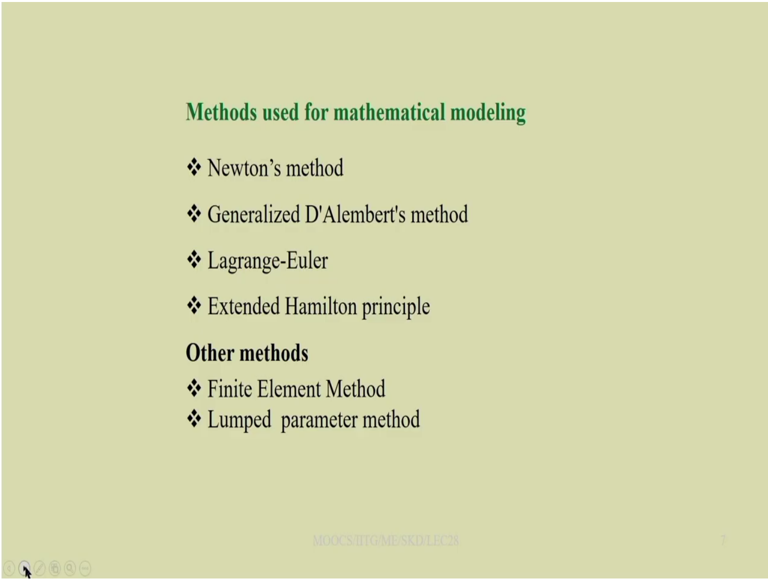

So, in mathematical modeling, so already in our previous classes we know how we can use these Newton's method, generalized D'Alembert's rule. Then this gal Lagrange Euler formulation, extended Hamilton principles, and this finite element method, lumped parameter models. So, all these things or all these methods can be employed to derive the equation of motion.

Depending on the complexity of the problem, so you have to choose which method you have to use. Particularly, this Newton's method is useful for systems with less degrees of freedom and this for continuous systems. Then generally Hamilton principle or extended Hamilton principle can be used and for multi-degree of freedom system Lagrange principle will be used. So, these are for systems with fixed coordinate frames.

So, if the coordinate frames are moving, for example, in case of the robotic system we have to go for this Lagrange Euler formulation or this Newton Euler formulation, and we can derive this equation of motion. And for a large structure generally these finite element methods are used, where it can be reduced to its lumped parameter models also. We will see all these applications.

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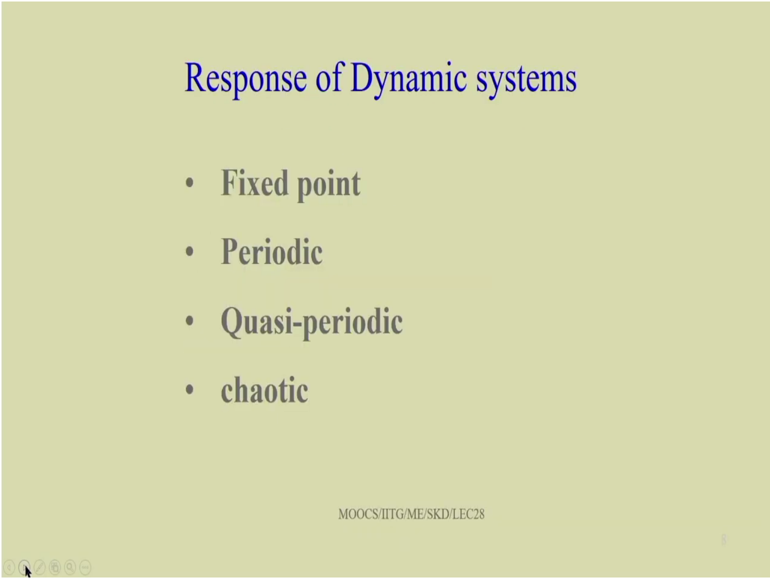

So, in the models to what we are going to study. So, already you are familiar with this fixed point response, period, quasi-periodic and chaotic responses which will be appearing in the dynamic analysis of the system.

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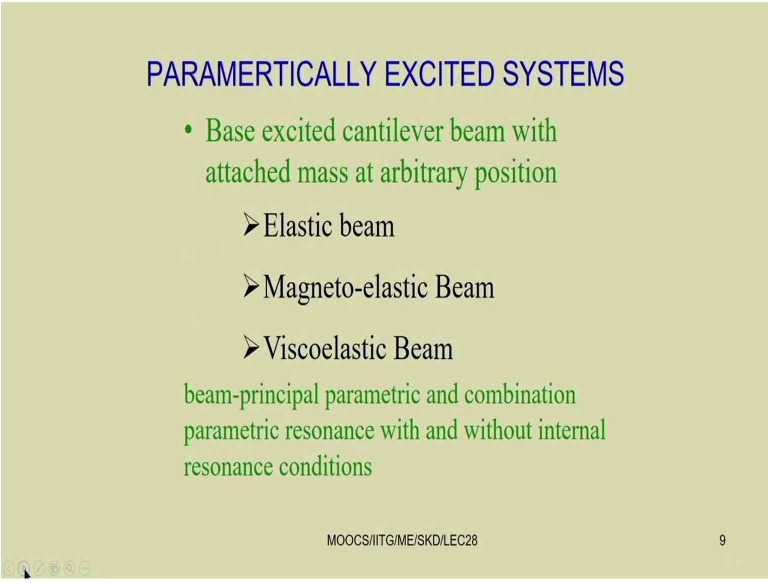

So, let us take these 3 different type of system base excited cantilever beam with the attached mass at arbitrary position. So, we will see this or we all will take this elastic beam, magneto-elastic beam and viscoelastic beam. So, these 3 different types of beams will take, and the beam will be subjected to principal parametric and combination parametric vigilance with and without internal vigilance. Today class, we will briefly study about all these things.

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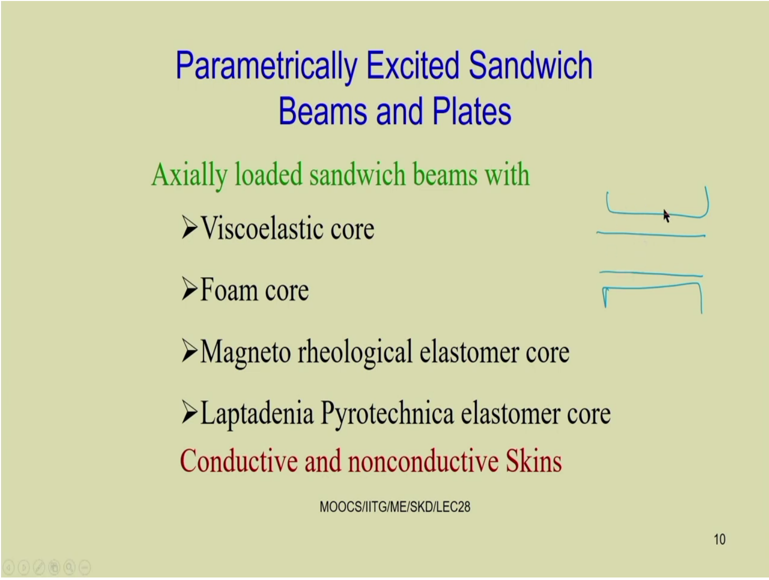

In case of the parametrically excited sandwich beam and plates, we will study the axially loaded and sandwich beam with viscoelastic core, we can take the foam core also, and this magneto rheological elastomer core, and Laptadenia Pyrotechnica elastomer core. The skin material may be conductive or non-conductive skin.

So, for examples who when we are taking these non-conductive type of skin and apply this magnetic field the skin layer will not experience any force or movement. But when you are taking a conductive skin and applying this magnetic field to activate these sandwich core material that time the skin also we will be subjected to this axial loading, so due to the Lorentz force and movement.

So, taking those force and movement in the skin layer, so the equation of motion will be different. So, your equation motion will be different for these conductive skin and on non-conductive skin.

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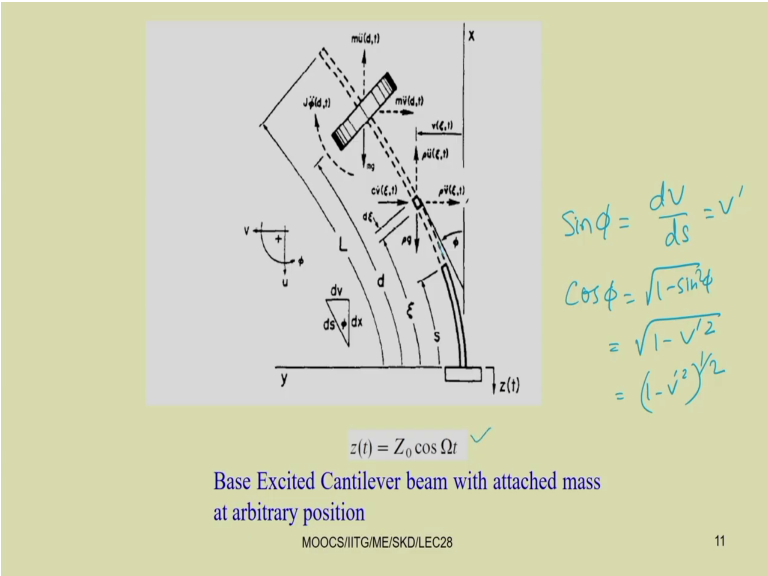

So, already we are familiar with these base excited system. So, here we have taken this transverse vibration to be very large. So, here we have taken these phi angle to be very large. So, we can write from here the sin phi, sin phi will be equal to dv by ds or it is written as v dash. Cos phi will be equal to; so, cos phi will be equal to root over 1 minus sin square phi, sin square phi. So, this will give rise to 1 minus v dash square or this is equal to 1 minus v dash square to the power half.

So, in this beam, particularly in this vertically base excited beam, we can take a very small element. So, the small element is subjected to what are the forces acting, let us first see. So, this is the weight rho G and then the inertia force that is rho u double dot and in longitudinal direction. So in transverse direction this rho v w dot and in this direction; so, damping also we can take either beam is moving towards left, a damping force will act towards right, the dc v dot.

Similarly, for the attached mass we can take different forces and moments acting. So, the forces will be mv w dot, mu double dot, mg and j phi double dot when we are taking a small element. So, we can see that along this x direction or let us take this distance to be ds.

So, ds is along the beam, so that is ds. And dv is the displacement in transverse direction. So, from these things we can write the sin phi, sin phi can be written as dv by ds and from that the power v dash. So, we can write this way. So, the beam is subjected to a vertical direction motion that is z t equal to Z 0 cos omega t. So, these systems already we have seen several times.

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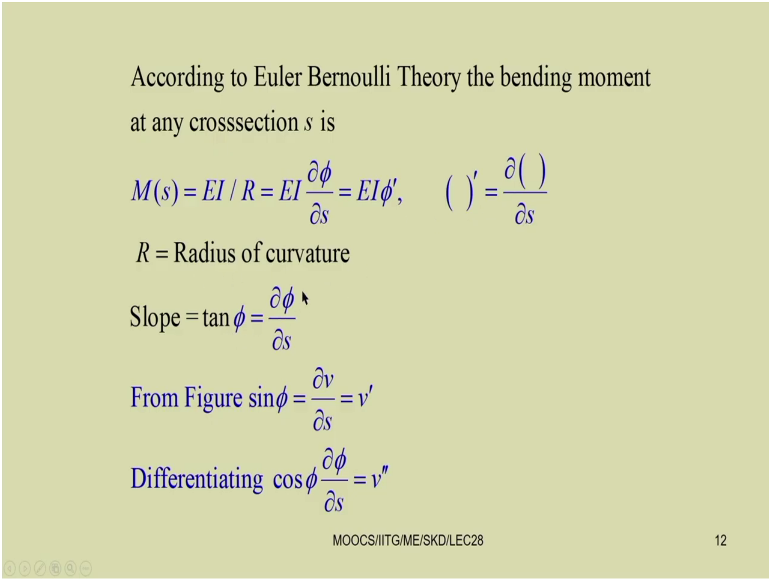

And just to revise the system, so let me tell it again. So, that we can extends similar analysis for the other elastic viscoelastic and magneto elastic beams.

So, here this image that is the moment about s to derive this equation motion. So, we can take the moment about this point. So, taking the moment about s we can write this M s equal to EI by R that is equal to EI del phi by del s is equal to EI phi dash, R is the radius of curvature.

So, slope equal to tan phi equal to del phi del s. So, from this figure already we have written sin phi equal to v dash. So, differentiating these sin phi, so we can write this cos phi into del phi by del s will be equal to v double dash.

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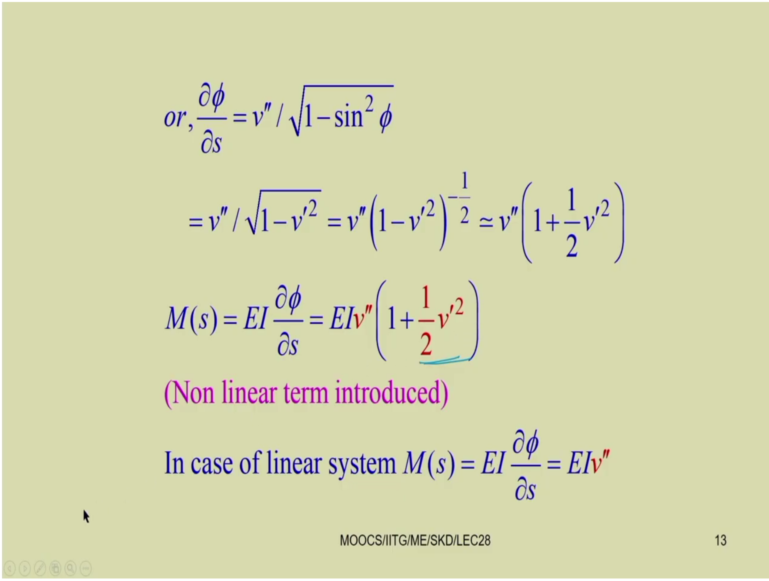

We can write this del phi by del s is equal to v double dash. So, that is tan phi equal to del phi by del s is equal to v double dash by root over 1 minus sin square phi, tan phi equal to sin phi by cos phi. So, sin phi already we know we can write these as v double dash that is del square v by del s square.

And for the cos part or the cos phi we can write this is equal to 1 minus sin square phi, so this becomes v double dash by root over 1 minus v del square. So, this is equal to v double dash into 1 minus v del square to the power minus half. So, this is equal to v double dash into 1 plus half v dash square. So, we have expanded this thing by the value. So, you can see this M s can be written as EI v double dash into 1 plus half v dash square. So, here this is the non-linear term introduced in this case.

In case of linear system, so it is simply EI del phi by del s is equal to EI del square v by del s square. So, particularly these equations we have used in the strength of material for finding the deflection of the beams. If it is non-linear, so we have added these term. So, to make the system further non-linear. So, you can or one may add more higher order terms also.

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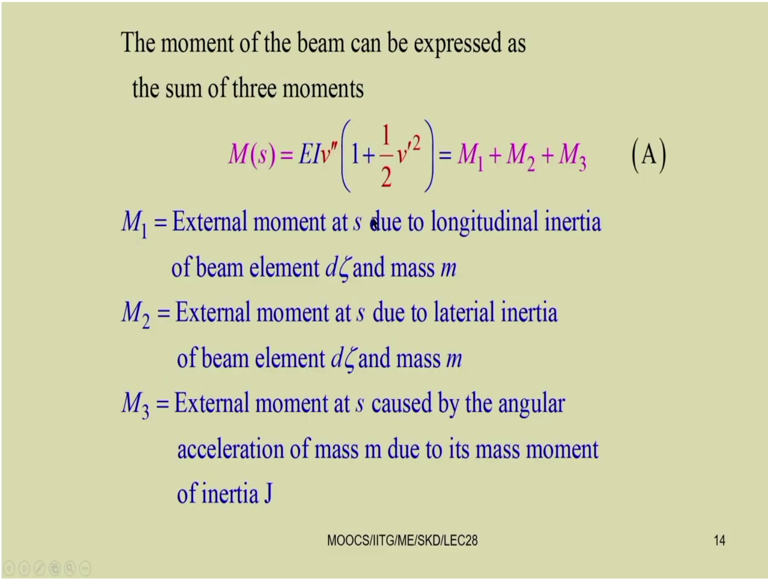

These moment, now taking the moment of all the forces. So, we know the forces in longitudinal direction or lateral direction and these inertia also moment due to inertia also we can find. That we have divided this into 3 parts that is M 1, M 2, M 3. External moment that s due to longitudinal inertia of the beam, external moment that s due to lateral inertia of the beam.

So, this is for the beam element d zeta we are taking. So, later we will integrate that thing to find for the whole beam and also for the mass m. So, external moment at s is caused by the angular acceleration of mass m due to its mass moment of inertia J.

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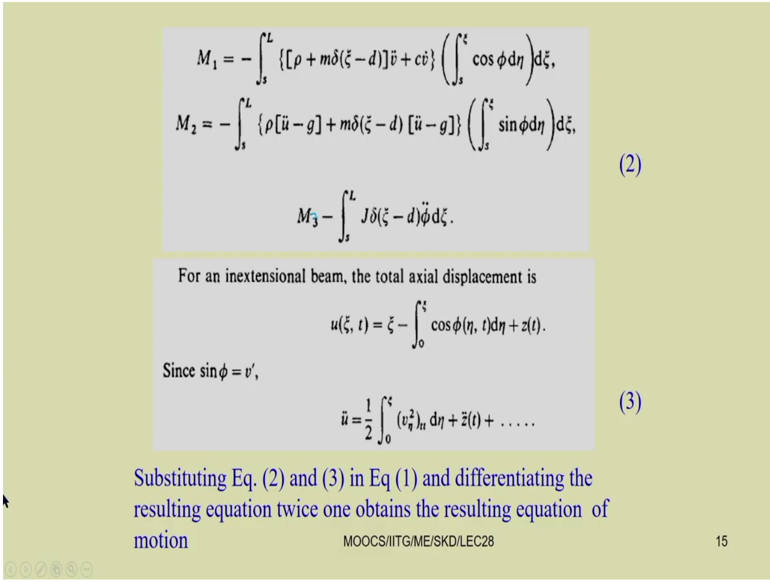

So, that way we can write this M 1, M 2, and M 3. So, these equations are known to you, so M 3 equal to this thing, M 1, M 2, M 3. Then, we can use this inextensibility condition that is the beam has length before and after deformation same and using that things. So we can have the relation between these axial deflection, axial and these lateral direction deflection.

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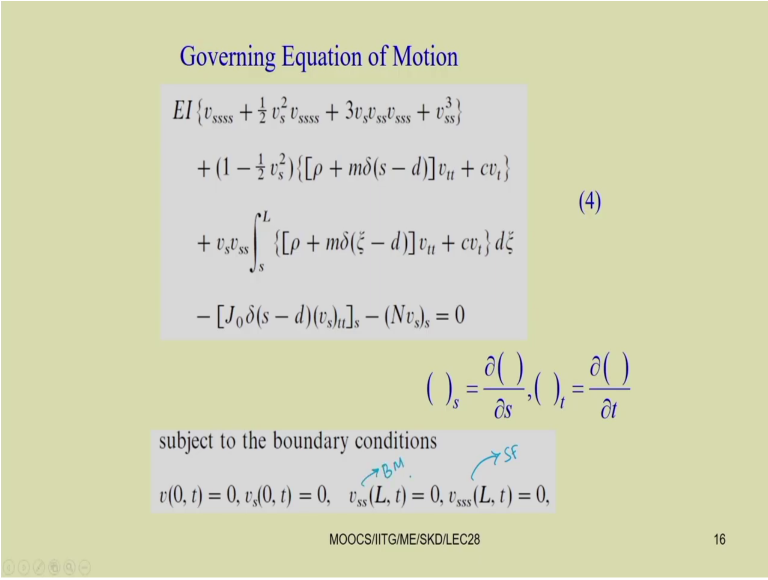

Using those things; that means, differentiating that equation twice, so moment if you differentiate twice that will leave the load loading condition, so one can obtain this equation. So, now, we just see if we are deriving so, we have used in this case these Newton's second law to derive these equation of motion. So, one can use energy method also to derive this similar equation motion.

So, for a part next I will show you how we can use this energy method also to derive similar equations. For another system for example, we will take the viscoelastic beam and for that case we will find how it can be derived. In this case, as you are taking a cantilever beam, the boundary conditions will be at the peak strain the displacement and the slopes at 0 and at the pre-end the bending moment and shear force will be 0.

So, bending moment is proportional to del square v by del s square and shear force is proportional to del q, v by del s cube these are the 3 end that is at s is equal to L. So v s is L t equal to 0 and we replace L t equal to 0. So, this part is this here force, this proportional to shear force, and these proportional to this bending moment, bending moment and shear force, ok.

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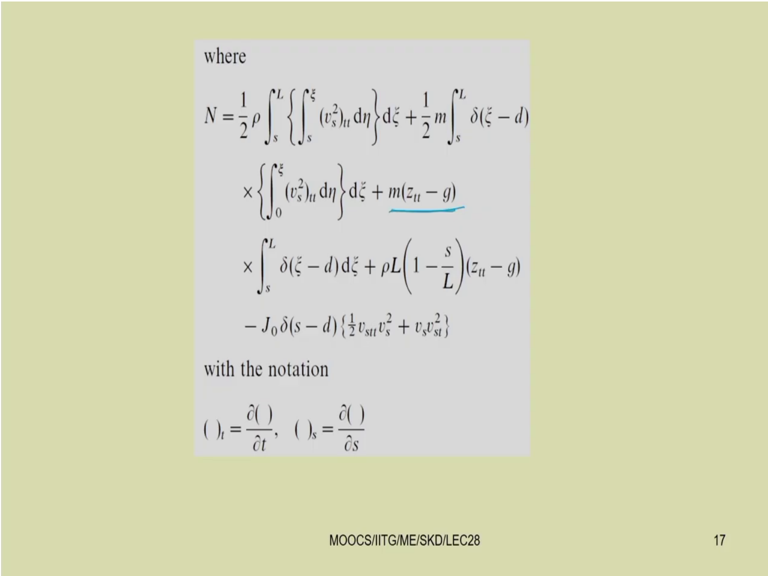

Where this N term, so we have a N term here. So, this is non-linear terms are many non-linear terms are there. So, these particular N term can be written in this way. So, here you can note that the base excitation is incorporated in the system in this form of the inertia force that is m z t double dot minus g, g due to this weight component.

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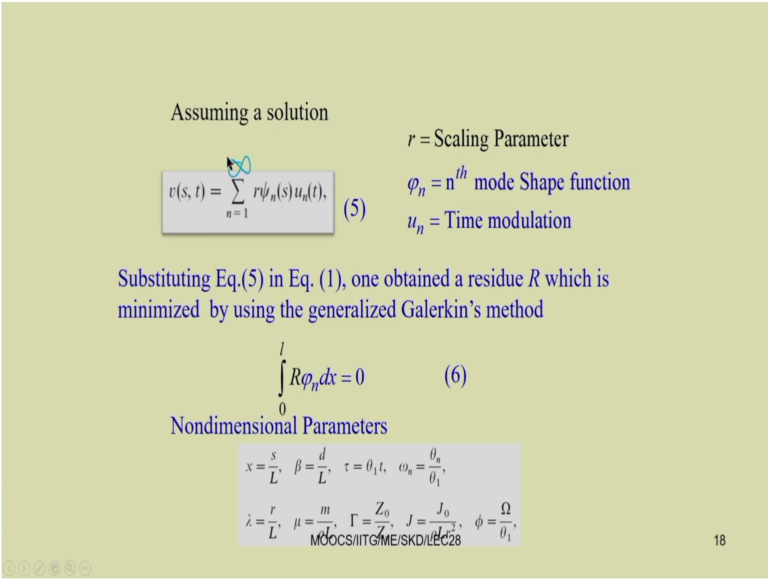

So, here let us see this part thoroughly. So, now, this is a continuous system, so in this continuous system this displacement we can v written by using both space and time. So, this displacement v is a function of both space that is s and time t. It can be written equal to n equal to 1 to infinite. So, we can write n equal to 1 to infinite.

So, by taking up to infinite modes R psi n s and u n t. Here you can take depending on the number of mode you want to study. For example, let it is subjected to an external excitation which is near the second mode. It is not required to study the higher mode for example, more than 4 or 5 it is not required to study.

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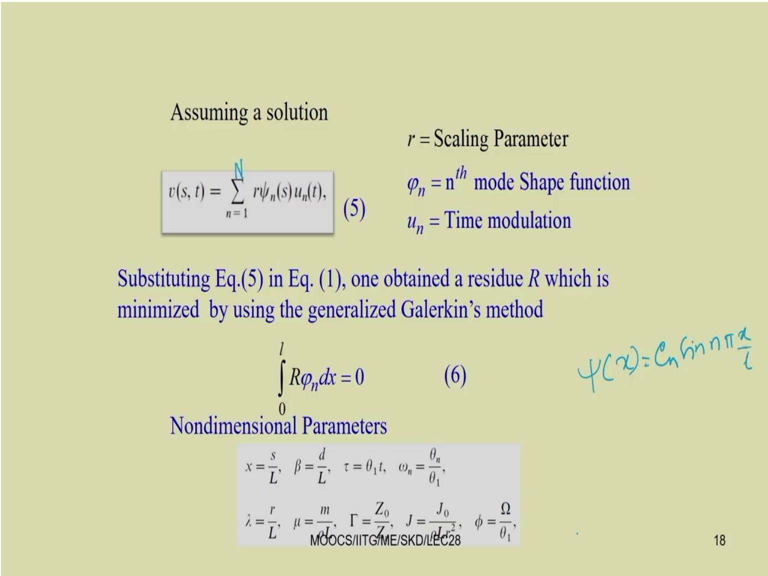

So, in that case, you can limit the or for end of this n equal to 1, 2; we can write n equal to 1 to capital N. So, capital N will be the number of modes we want to take in this analysis. In that way, so we can limit or truncate the number of modes.

We can see later that due to as the damping they are very small and as there will be no modal coupling between the lower and higher modes. If they are not connected by internal resonance, so when exciting this lower mode it will not be able to excite the higher modes. So, these higher modes will die down with time. So, for steady state always we can neglect these higher modes and we can keep our analysis limited to only if you will do your modes, ok.

So, in that way, so we can write n equal to 1 to N, r psi n s and u n t, where psi n is the shape function and u n t the time modulation. The psi n actually we are going to take the mode shapes assuming the mode shape. So, we are we can take the assumed mode shapes of a cantilever beam or that have a cantilever beam with some attached mass.

So, we can derive that part from the linear equation. So, by putting the non-linearity equal to 0 in the governing equation, so we have the linear equation of motion. So, in this linear equation motion by applying this variable separation method we can easily derive the shape functions of the system.

For example, in case of a simply supported beam already you are familiar that the psi n can be written, psi n can be written equal to c n sin n pi x by L, n pi x by L or n pi x sin n pi x you can write, s by L you can write, where s by L is equal to x. So, here the mode shapes is in the form of a sin curve.

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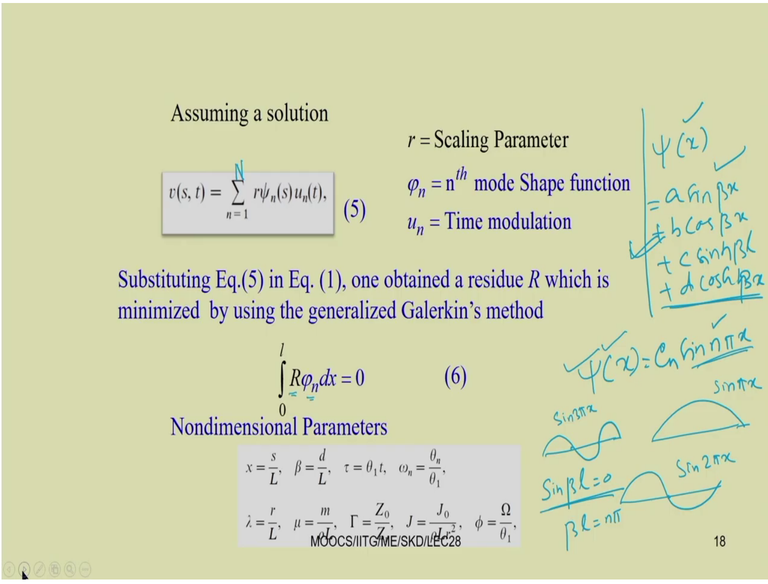

For example for the past mode in this case, so it will be just looking like a sin curve, part of the sin curve then for n equal to 2. So, you can have one node here. So, n equal to 3, so two nodes may be formed. These are nothing, but the sin n pi x. So, this is sin pi x, so this is sin 2 pi x, so this is you can plot sin 3 pi x. So, by changing different value of n, so you can get all these curves and you can plot the different mode shapes.

Similarly, for a cantilever beam, so you can get the generalized equation. So, generalized equation initially you can write. So, which is the solution for the Euler Bernoulli beam equation and that is written psi x equal to or psi n x equal to a sin beta x plus b cos beta x plus c sin hyperbolic beta x plus d cos hyperbolic beta x hyperbolic beta x.

So, now, by applying boundary conditions, so you just see we have 4 boundary conditions and here we have for unknown constants a, b, c, d. So, by applying these boundary conditions. So, we can have or we can find these coefficients. And after finding these coefficients, so we can know the mode shape of the system.

So, by when we are putting these boundary conditions, so we can get some characteristic equation. For example, in case of the simply supported beam the condition is sin beta l equal to 0. So, from which we know this beta l equal to n pi.

Similarly, in case of the cantilever beam you can have the equation for example it may be cos beta l into cos hyperbolic beta l equal to minus 1. By solving that equation numerically, so you can find the beta l value and after getting this beta l value. So, you can find the or you can draw the mode shapes. So, in case of the cantilever beam or the free end. So, you will have 0 displacement and 0 slope or the free end the shear force and bending moment will be 0.

The shape functions after knowing the shape functions, sometimes while deriving these shape functions, it is not possible to satisfy the differential equation and all the boundary conditions. So, that is why one may use these assumed mode that is one can take only these geometric boundary conditions, take a functions, will satisfy only the geometric boundary conditions, it may not satisfy the differential equation or the post boundary condition.

So, in that case by substituting that mode shape in the governing equation will result in some residue. So, that means, so it will not be equal to 0 and so that residue we have to minimize that residue.

So, to minimize that residues we have to use some weird function. So, let that residue is R. So, we can multiply weight functions psi n and integrate it over the length of the beam to get equation, resulting equation in its temporal form. So, this is the Galerkin's procedure.

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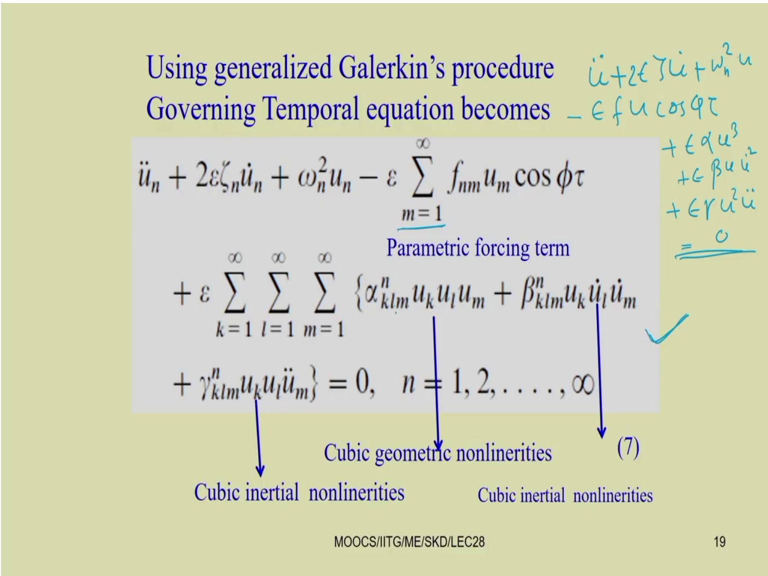

So, now, using these generalized Galerkin procedure, so we can derive the temporal equation of motion. For these case, we taking these infinite number of modes, so you can find the equation of motion this way and if this if we are taking only single mode then this equation can be reduced in this form that will be u double dot plus 2 epsilon zeta u dot plus omega n square u minus epsilon f, you just see this is f u cos phi tau and plus epsilon alpha u cube plus epsilon beta u u dot square plus epsilon gamma u square u double dot equal to 0.

So, this is the equation for n equal to 1. Or so, if one take only single mode, so you can get this equation. So, this equation is available in the paper by (Refer Time: 27:51). And here when we take number of modes, let for example, we take 2 modes or 3 modes or number of modes then in that case in the forcing term you can have a summation sign and in the non-linear term also you can have summation sign.

So, by taking two modes for example, in this non-linear case you can have 2 into 2 into 2 that is 8 terms, 8 into, so these 3 that is 24 terms will be there. So, similarly in this forcing, so you can have m equal to 1, 2, so 2 will be there. So, forcing term it will be n m. So, as n is taking the value 1 and 2, and m is also taking the value 1 and 2, we can have f 11, f 12, f 21, and f 22.

So, 4 forcing terms will be there and these non-linear terms for klm, so as we are taking each term that is k equal to 1 and 2, l equal to 1 and to and m equal to 1 and 2, we can have 8 variations of these thing and for a particular mode for a particular value of n, we will have 8 number of alpha, similarly, 8 number of beta and 8 number of gamma.

Again by taking different modes, for example, n equal to also vary from, if n is varying for example, 3 modes we are taking. So, total non-linear terms will be 24 into 3 that is 72 non-linear terms will be there in this case. To write in a comprehensive form, so you can use this summation sign and write this equation in this form.

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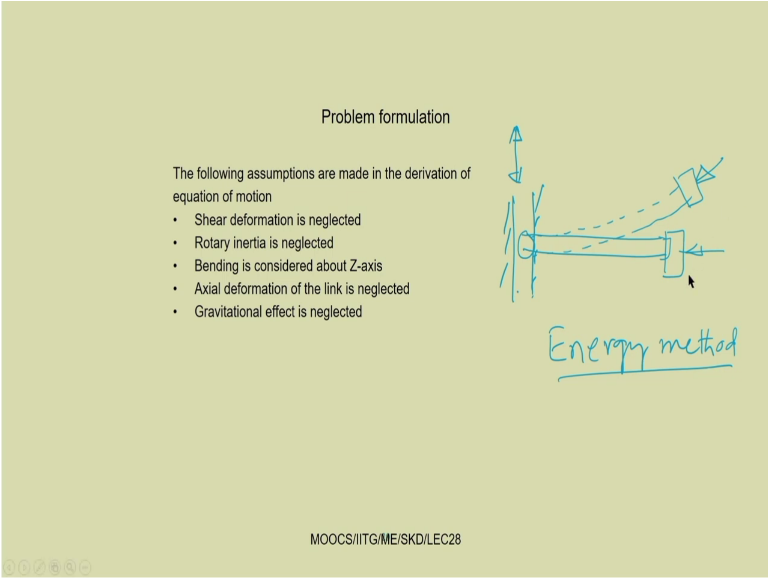

Let us see another example that is for the viscoelastic beam. So, how we can derive this equation motion? We have a viscoelastic beam. So, for example, this is the beam, and we have a support here, and the so it is moving up and down in the support. So, this is it may be subjected to, so it has a mass also let us consider and it is subjected to axial force.

When it is moving up and down, so already we have seen, when it is moving up and down it will be subjected to a force or due to that thing. So, it may have a tendency to bend. So, it will bend and so, this will be the after deformation and this is before the permission. We have to derive this equation motion. So, let us derive this equation motion by using these energy method.

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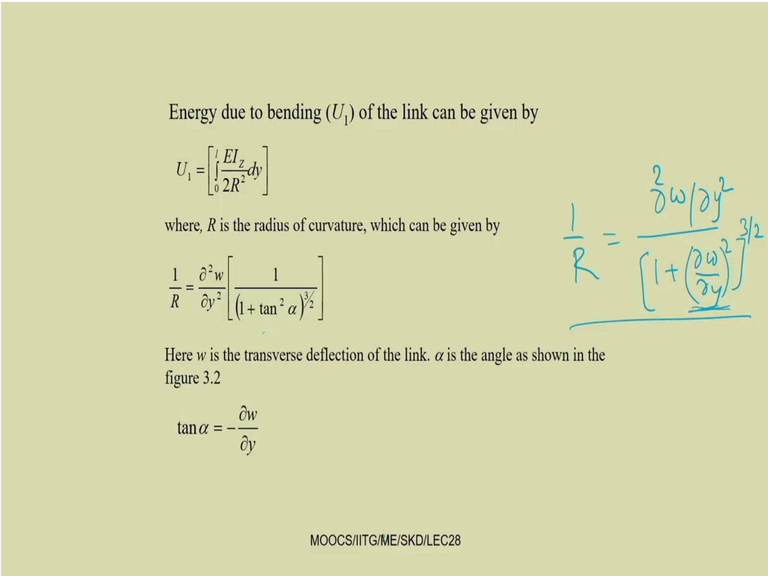

So, previously we have seen, so we have used the Newton's second law to find the equation of motion. So, now, we will derive by using energy method. So, energy method when you are using. So, we will have these potential energy or energy due to bending.

So, energy due to bending can be given by U 1 equal to integration 0 to l EI z, half EI z by R square dy. This way we can find 1 by R, already you know these 1 by R equal to del square w by del y square into 1 by 1 plus tan square alpha to the power 3 by 2 or you can recall these things 1 by R in case of a bending.

So 1 by R equal to, so del square w by del y square. So, w is the deflection and we are taking this along the y direction we are taking, so divided by 1 plus d del w by del y whole square to the power 3 by 2. Expression for curvature from calculus you have seen, so it can be written this way. So, 1 by R equal to del square w by del y square divided by 1 plus, del w by del y whole square to the power 3 by 2.

Here we can substitute actually due to the presence, so if we neglect this term many times in linear vibration these del w by del y can we neglected with respect to this one as it is very small. So, in that case the inner part becomes or the denominator become 1 and 1 by R equal to del square w by del y square.

Here w is the transverse deflection of the link, alpha is the angle as shown in the previous figure. I have shown the figure previously. This is the figure.

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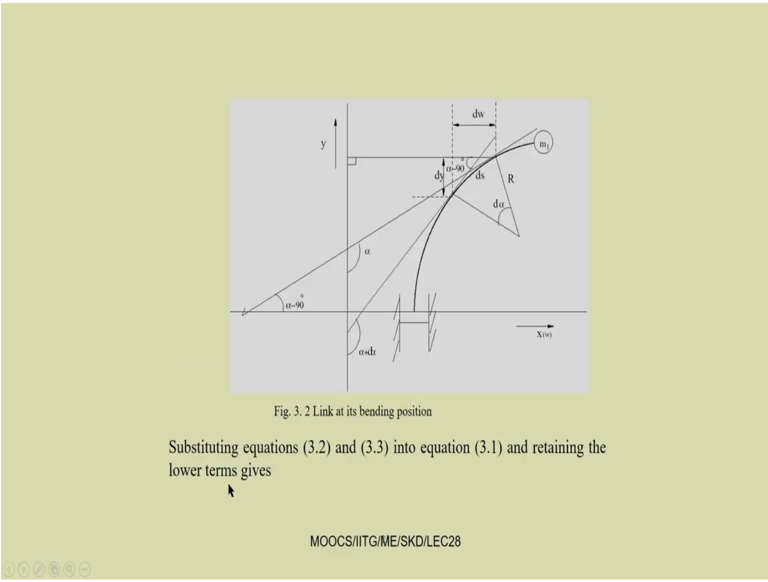

So, in this figure you just see this is the deformed beam. So, in case of the deformed beam, so we can take this is the y direction, so this is the x direction and this is del w d, ok. So, that the deflection of the beam.

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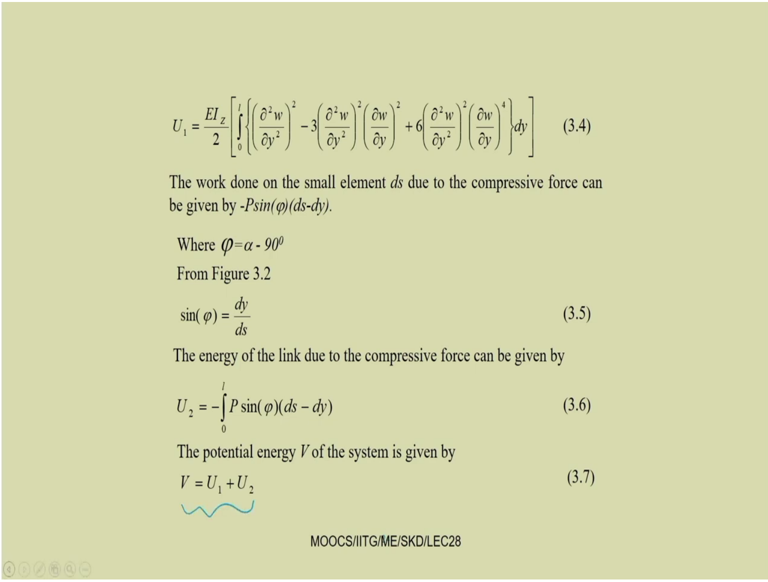

So, now, we can find these U equal to U equal to EI z by 2 integration del square w by del y square, minus 3 del square w by del y square whole square del w by del y whole square, plus 6 del squared w by del y square whole square into del w by del y to the 4 to the power into dy. So, the work done on the small element ds due to the compressive force can be given by, so p sin phi into ds minus dy. So, the deflection will be ds minus dy. Here psi equal to alpha minus 90 degrees. So, from this figure you can see, we have taken this the r as the beam is bending like this.

So, this is the center of curvature, this is the radius of curvature, this length we have taken ds length and this angle is d alpha. So, this angle is d alpha. These angle is alpha, as we are taking these angles equal to alpha. So, this is equal to alpha minus 90 degree, this is alpha minus 90 degree.

Similarly, the energy of the link due to the compressive force can be if we are applying a compressive force at the end of the beam a compressive force is acting. So, in that case it can be written let the P the force acting. So, it is equal to minus P sin psi into ds minus dy. The potential energy V of the system total potential energy will be equal to U 1 plus U 2.

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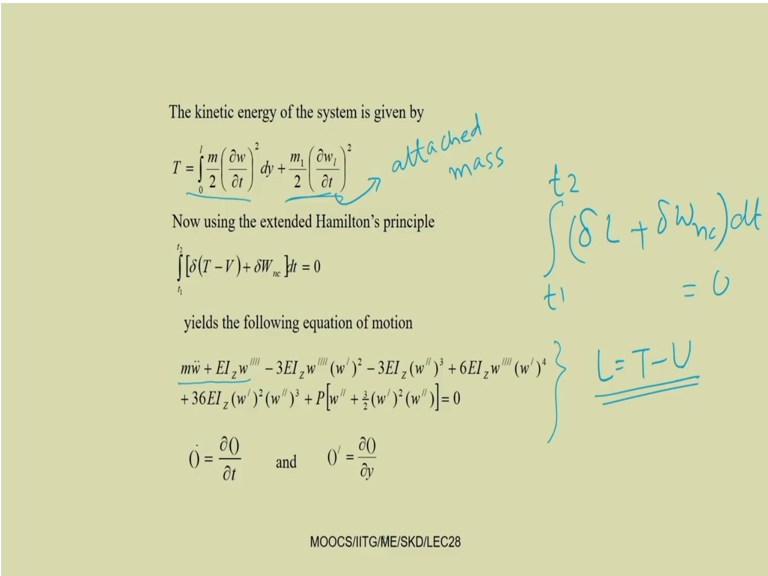

So, from this U 1 plus U 2, so we can write down. So, the kinetic energy of the systems can be given by T equal to half m into del w by del t square dy plus tip mass we can write also. So, this is due to the beam.

So, this is due to tip mass, this is due to attached mass. So, this is due to attached mass and due to attached mass it can be m 1 by 2. So, attached mass is m 1, so m 1 by 2 del w l by del t whole square. So, you just note that w is a function of both space and time, that is why this partial derivative is used.

Now, using extended Hamilton principle, so you can derive this equation of motion in extended Hamilton principle it is t 1 to t 2, integration t 1 to t 2 del L; this is t 1 to t 2 del L plus del W nc dt equal to 0. In this case, we can write these L is nothing but T minus U. So, that is why it will be equal to which yield the following equation similar to the equation what we have seen in the previous case.

So, here also we have this equation mw double dot plus EI del forth w by del s 4th minus 3 EI del forth w by del s forth into del w by del s whole square minus 3 EI del square w by del s square cube plus 6 EI del forth by del s forth into del w by del s to the power 4 plus 36 EI del w by del s whole square into del w by del s, del square w by del square q plus P into delta del square w by del s square plus 3 by 2, del w by del s whole square into del square w y del s square.

So, you just see we have so many non-linear terms. In case of the linear case only one can have these term and, but in the non-linear case, so we have the additional terms.

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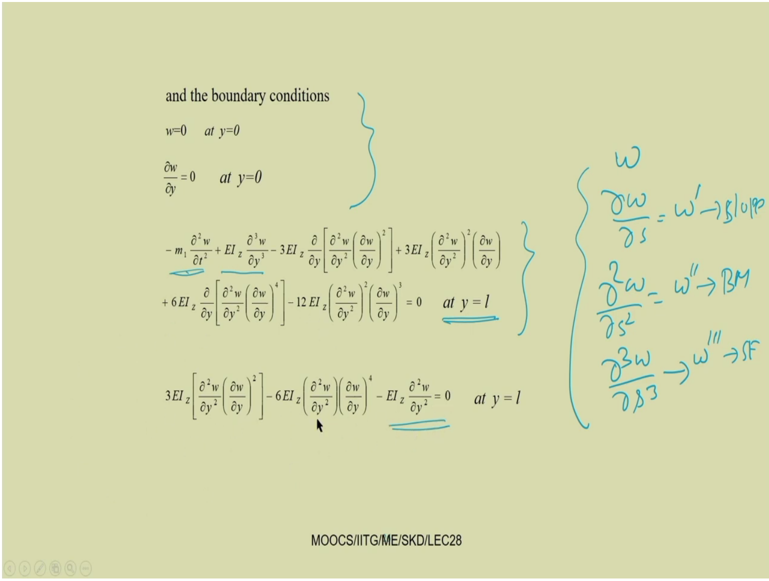

Now, similar to the previous case. Here also in when you are applying these extended Hamilton's principle, in addition to this governing equation, so you can get the boundary conditions also. So, these are the boundary conditions. So, at y equal to 0, so w equal to 0; at y equal to 0 also the slope equal to 0 that is the peaks end, and as we are keeping the mass at the end that is at the tip.

So, in that case, so the inertia force will come in the end that is why these boundary conditions are the pre-end can be written in this form that is minus m 1 del square w by del t square plus EI del q w by del y cube.

So, you will just see this is the shear force and this part is the inertia force, inertia force due to the attached mass at that end. Minus 3 EI z, so these are the non-linear terms associated with that thing, non-linear torque associated with the shear or this is shear del square w by del, this is this is bending moment, del square w proportional to bending moment del square w by del y square. So, at the pre-end either we have this bending moment or shear force will be equal to 0.

These are the combination of different things. For example, w is the deflection. So, del w y del s equal to w dash is the slope, del square w by del s square equal to w double dash. So, that corresponding to bending moment and del cube w by del s cube these correspond to w triple dash. So, that is the shear force. So, in the boundary conditions these 4 parameters will come.

In the non-linear terms, it will be product of these terms, it will be product of these terms and you can get these boundary conditions directly from the extended Hamilton principle. Similarly, one more boundary condition. So, this is also 3 EI del square w by del y square, del w by del y whole square minus 6 EI z del square w by del y square into del w by del y to the power 4 minus EI del square w by del y square equal to 0.

These correspond to the bending moment keeping the bending moment equal to 0 and taking these conditions, so we can have the boundary conditions. So, these are the boundary conditions.

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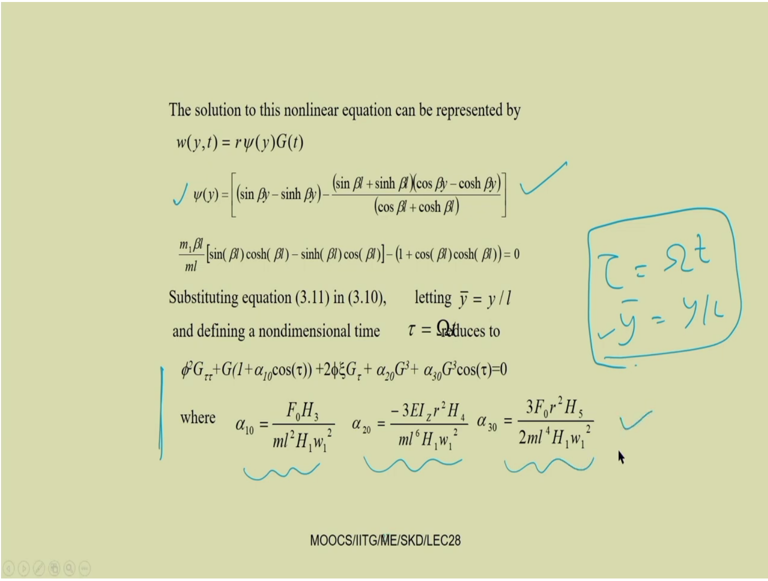

So, the solution of the non-linear equation can be represented why if we are taking only single mode approximation. So, then this equation can be written in this form w y t equal to r psi y into G t, where r is the scaling factor, psi is the shape functions and G t is the time modulation.

Here for a beam with tip mass you can use these shape functions. So, psi y equal to sin beta y minus sin hyperbolic beta y minus sin beta l plus sin hyperbolic beta l into cos beta y minus cos hyperbolic beta y divided by cos beta l plus cos hyperbolic beta l. m 1 beta l by m l into sin beta l cos beta l minus sin hyperbolic beta l into cos beta l, minus one plus cos beta l into cos hyperbolic beta l equal to 0.

Now, taking these non-dimensional time that is tau equal to omega t and y bar equal to y by l, so this is the non-dimensional time and non-dimensional time and these position. So, y is the along the length of the beam. So, then y bar is the non-dimensional positioning. So, it is equal to y by l. So, taking that thing, so this equation reduces to this form.

So, you have to apply the Galerkin method. So, substituting this mode shapes in the original equation, spatio temporal equation, and multiplying or finding the residue and then multiplying the weight function, so the weight function can be taken as the same as this one.

And applying this orthogonality principle, so you can reduce this equation to this form. So, this is in temporal form. So, you just see in temporal form this equation can be written in this way that is pi square del square G by del t square plus G into 1 plus alpha 10 cos tau plus 2 phi zeta G tau plus alpha 20 G cube plus alpha 30 G cube cos tau equal to 0. So, where the expression for alpha 10, alpha 20, and alpha 30 are given here. This way you can derive this equation of motion in case of a beam using energy method.

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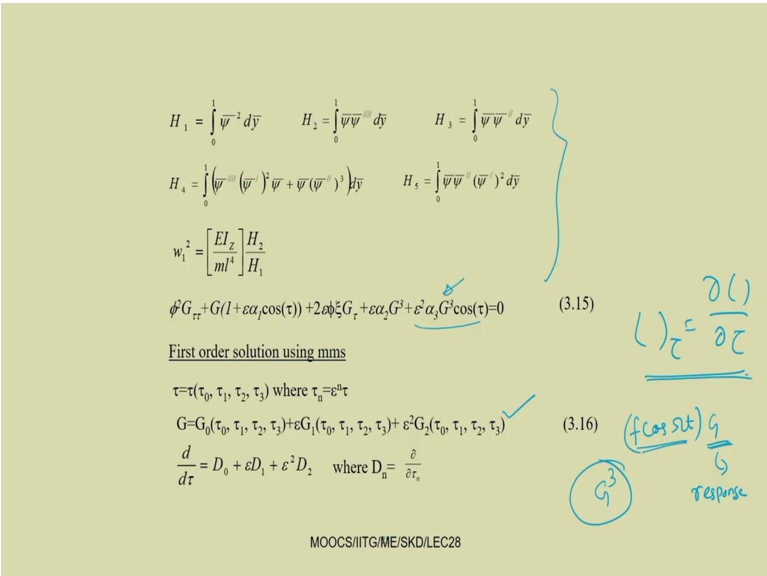

So, previously I told you how you can use the Newton's methods also to derive this equation of motion. So, here I told how you can use this energy method to derive this thing.

In energy method, first you find the potential energy, kinetic energy, then the Lagrangian of the systems. Either you these Lagrange principle or the Hamilton principle to derive this equation of motion. As you are taking a continuous system the derived equation of motion is in the form of partial derivative. So, this variable, state variable is a function of both space and time.

So, you can reduce it to its temporal form that is to it is time for by applying these generalized Galerkin's method. So, this part of the coefficient or when you want to do this find the coefficients all these terms will be there. So, now, this equation is reduced to this form that is phi squared G tau, tau, this tau equal to delta of by delta tau. This way you can write the first order solution, so you can apply this method on multiple scale.

So, previous case what I have shown you in the base excited cantilever beam, so we have taken multimode approximation. So, in this case, I have taken only a single mode approximation. So, where tau can be retained as tau, tau 0, tau 1, tau 2, tau 3, where tau n equal to epsilon to the n tau.

So, the G can be written also up to higher order. So, let us take up to these epsilon square. So, G can be written as G 0 plus epsilon G 1 plus epsilon square G 2 and here this d by d tau equal to D 0 plus epsilon D 1 plus epsilon square D 2 where D n equal to delta by delta tau n.

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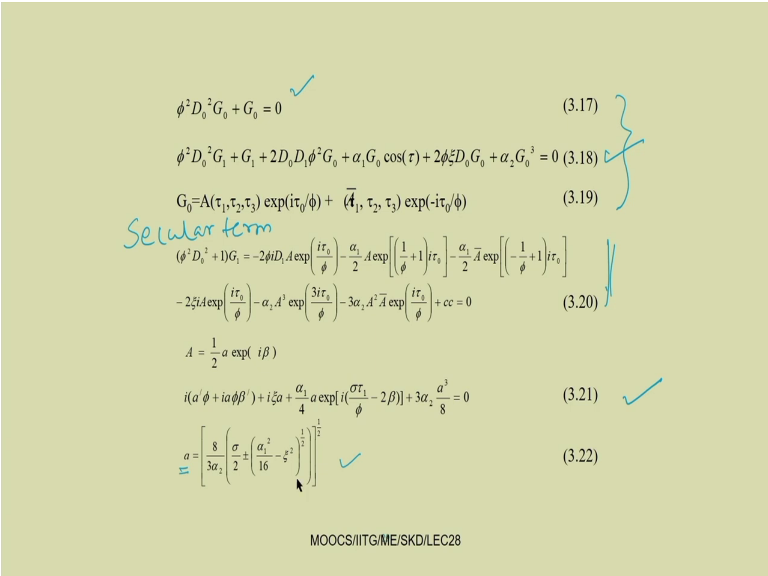

So, this way we can expand these or collect the term with coefficient of epsilon 0, epsilon 1, epsilon 2, and we get these 3 equations. So, here you just note the solution of the first equation you can find it easily. You can note the form of equation used in this case and the previous case are slightly different.

Here we are using phi square D square G 0 plus G 0 equal to 0. And the previous case we have written or we have taken the non-dimensional parameter in such a way that, so we have this G 0 square G 0 plus G 0 equal to 0. So, it is up to you how you want to derive that equation or how you can model. So, depending on your model, so you can have it.

So, now, by applying this or the solution of this first equation that is this equation can be written, we can find it, and substituting that solution we can substitute it in these second equation and from that thing we can eliminate the secular error term. So, eliminating secular error term. So, eliminating secular term, so we can get the equation. And in this equation substituting as age in the form of a complex number.

So, it can be written in its polar form that is half a to the power i beta, and substituting that there so we can get this equation. From these equations, so you can find these expression for a. So, a is equal to 8 by 3 alpha 2 into sigma by 2 plus minus alpha 1 square by 16 minus zeta squared to the power half whole to the power half.

So, here you just see the response amplitude a is a function of sigma alpha 2, alpha 1, and then zeta that is damping. So, all these terms are there.

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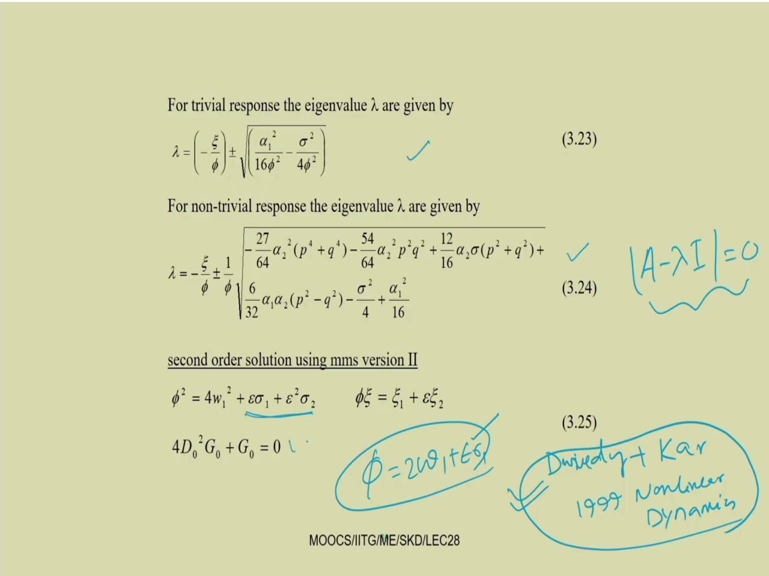

For trivial the response the eigen value we can find. So, we can find the Jacobian matrix by perturbing these two equations we can find the Jacobian matrix. After finding these Jacobian matrix, so we can check whether the system is stable or not, by finding the eigen value.

Here the eigen value can be written in this form, and for the non-trivial brands the eigenvalue can be retained in this form. So, actually we can find a minus lambda i determinant of a minus lambda i equal to 0, make determinant of a minus lambda i equal to 0 to find the eigen values. So, this way you can do the first orders.

So, if one to do the second order, so there are two different versions of method of multiple scale. So, if we are using the method of multiple scale version 2, which is proposed by Rehman (Refer Time: 47:11), and you can see a variation of that report. So, you can see the paper by Dwivedi and Kaur published in the, published 1999, so in non-linear dynamics.

So, there we have used this method of multiple scale, used this method of multiple scale for parametrically excited system. So, in this particular case, you just see if you loop this equation motion. So, the equation you just see this is a parametrically excited system with non-linear, so it is not similar to that of the Maxwell equation. You just see what is the difference between Maxwell equation and this equation.

So, in this case the coefficient this is not G, so this is G cube. So, you have a non-linear term. So, non-linear, so cubic, so you have a non-linear term. The coefficient of these non-linear response is G that is time modulation is G, coefficient of G cube is a time varying term that is alpha 3 cos tau. So, previously in case of these Mathieu, Hill type of equation the forcing term f cos omega t is multiplied with the G, G or u or x, whatever you want to write you write. So, this is the displacement or the response term. So, this is the response.

So, that is the coefficient of response was a time varying term. But in this particular case, so it is not the coefficient of G, but it is the coefficient of G cube that the non-linear term. So, this is the difference basic difference between this equation and the Mathieu, Hill equation what you have studied. But as you know the superposition rule cannot be applied to non-linear systems. So, in this case, so you cannot extend the idea of this Maxwell equation, but you must have to derive your own equations and find the response.

So, in that way, so you have found the response. Here you can use up to, so this is used to first order method on multiple scale, and we can use this higher order method on multiple scales.

So, in higher order method or multiple scale we have different versions, so one such version is this method of multiple scale version 2, and for parametrically excited system; so, you can refer this paper where it is done for a parametrically excited system. So, in case of method or multiple scale version 2, so these phi square you can take. So, you can expand these phi square equal to 4 omega 1 square.

So, previously, in previous case, we have taken phi equal to 2 omega 1 plus epsilon sigma one and we did it. So, this is external excitation. So, this is nearly equal to twice the omega 1 that is principal parameter vigilance conditions when we are studying plus epsilon sigma 1, where sigma 1 is the detuning parameter, but here we are taking in the form of phi square. So, pi square equal to, so 2 square that is 4 omega square and here we are taking up to epsilon square.

So, we are using two detuning parameter. So, for omega 1 square plus epsilon sigma 1 plus epsilon square sigma 2 and phi zeta equal to that phi zeta, then damping term phi zeta you were writing it equal to zeta 1 plus epsilon zeta 2. Then this 4, D 0 square G 0 plus D 0, now we got on the first equation. So, we got this this is equal to 0.

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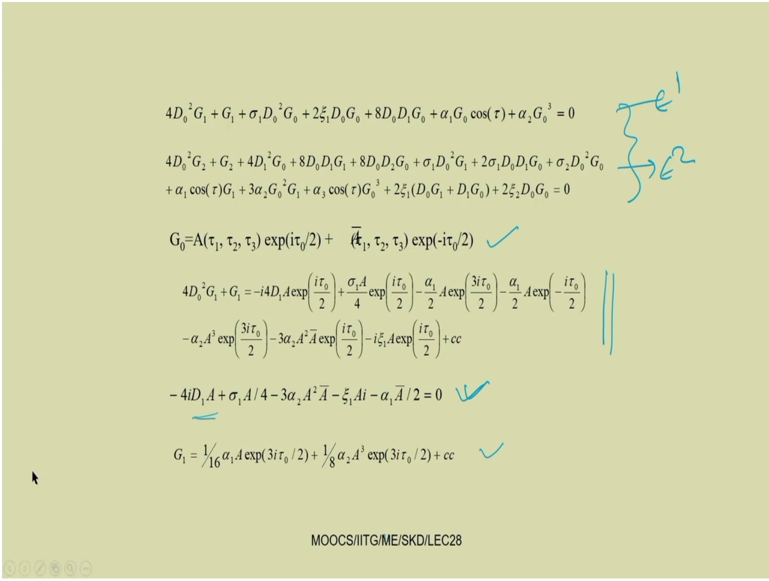

By substituting that thing, so similarly we can get the solution of G 0 in this form. Then substituting, so other two terms are epsilon to the power 1 epsilon to the power 2 equations are here. So, this is epsilon to the power 1 epsilon to the power 2.

Now, the solution of epsilon to the power of 0 it can be written as A e to the power i t 0 by 2 plus A bar, A to the power i 0 by minus t 0 by 2. So, these plus, it is complex conjugate. So, substituting this equation in this equation, so we can get the term and from here we can see this is the secular term.

So, here the secular terms can be obtained. And the secular term eliminating the secular term we can have the expression for D 1 A, then now from the second equation after the elimination of the secular term we can find the expression for G 1. And substituting that thing in the third equation, so we can get G 0 and G 1, and eliminating the secular terms, so we can get this expression.

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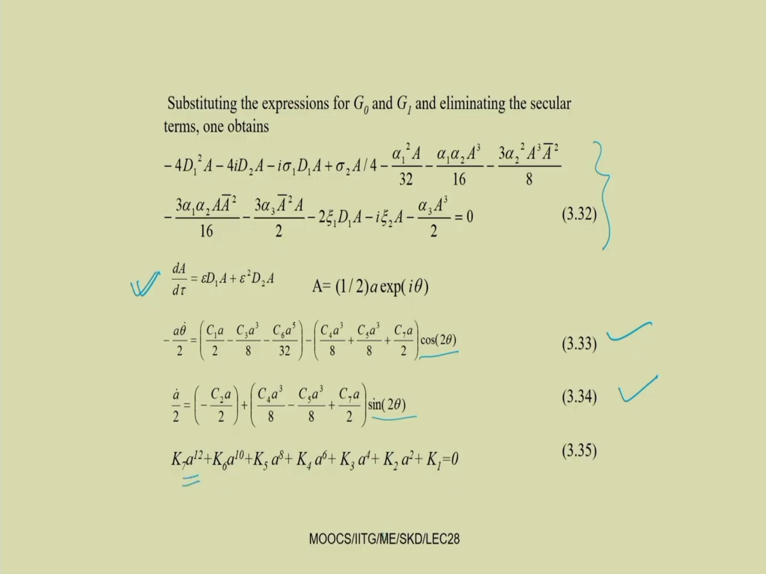

So, here now by substituting these dA by; so, you just note this part. So, here we are taking dA by d tau equal to epsilon D 1 A plus epsilon square D 2 A, and taking these A equal to half a to the power i theta and separating the real and imaginary part. So, we have these two equation.

And from these two equation, we have these cos 2 theta and sin 2 theta term are there. We can simplify this equation and you can see or observe that in this equation we have a to the power 12.

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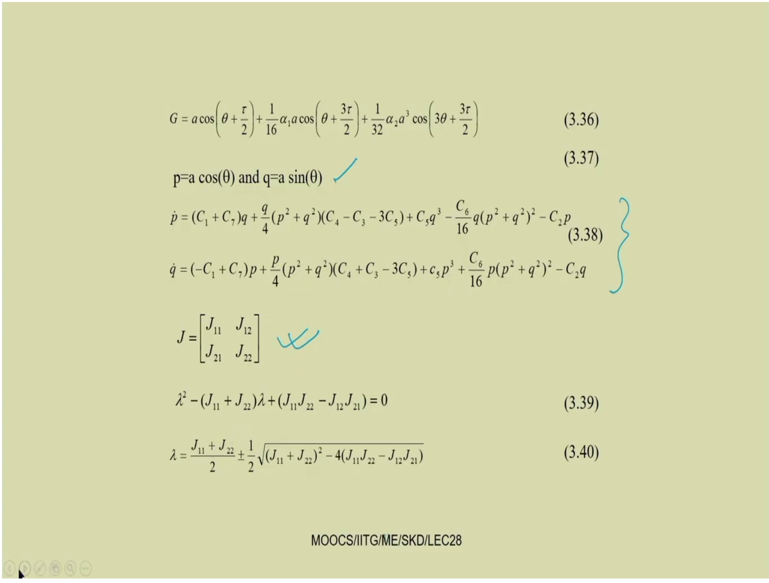

So, by taking a square equal to x, so it can reduce to a 6th order equation. So, you can solve the 6th order equation by numerically and you can find the solution. Here again you may replace this equation by using its secular form p and q, and you can get these two equation, then you can find the Jacobian matrix and from the Jacobian matrix by storing these eigen value, so you can study whether the system is stable or unstable.

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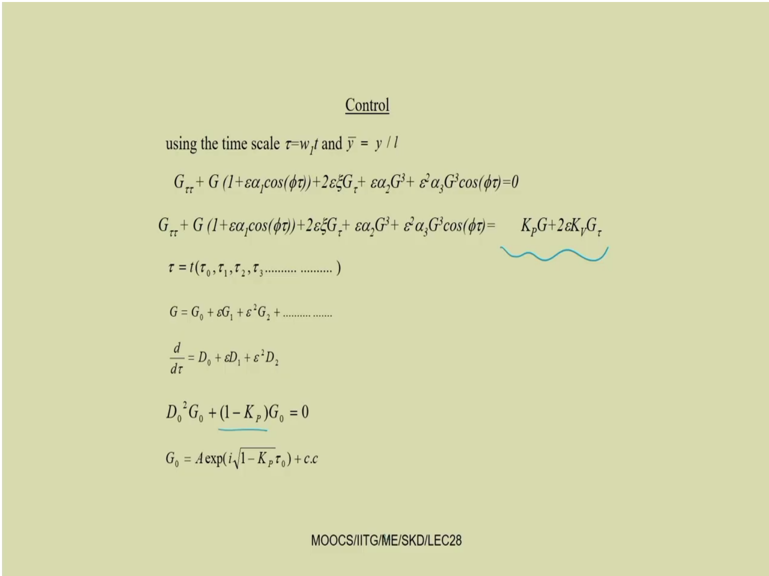

If you want you can use a controller here also to control the vibration. For example, so if you want to use a proportionate controller, so you can use a controller term. And using this controller for example, right hand side of this equation instead of putting 0 let us put this K p and K v. So, K p G plus 2 epsilon K p G, and now we can play with these parameters to find the response of the system and get the value of K p and K v for which the systems will be unstable.

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$$D_0^{\,2}G_1 + 2D_0D_1G_0 + (1-K_P)G_1 + \alpha_1\cos(\phi\tau)G_0 + 2(\xi - K_V)D_0G_0 + \alpha_2 G_0^{\,3} = 0$$

$$\sqrt{(1-K_P} = w_1 + \varepsilon\sigma_2 \qquad \phi = 2w_1 + \varepsilon\sigma_1$$

eliminating the secular terms

$$-2i\sqrt{1-K_P}\,A' - \frac{\alpha_1}{2}\bar{A}e\{(\sigma_1 - 2\sigma_2)\tau_1\} - 2(\xi - K_V)Ai\sqrt{1-K_P} - 3\alpha_2 A^2\bar{A} = 0$$

$$A = \frac{1}{2}a\exp(i\beta) \qquad 2\gamma = (\sigma_1 - 2\sigma_2)\tau_1 - 2\beta$$

$$a\gamma' = \frac{a(\sigma_1 - 2\sigma_2)}{2} - \frac{\alpha_1 a\cos(2\gamma)}{4\sqrt{1-K_P}} - \frac{3\alpha_2 a^3}{8\sqrt{1-K_P}}$$

$$a' = -\frac{\alpha_1 a\sin(2\gamma)}{4\sqrt{1-K_P}} - (\xi - K_V)a$$

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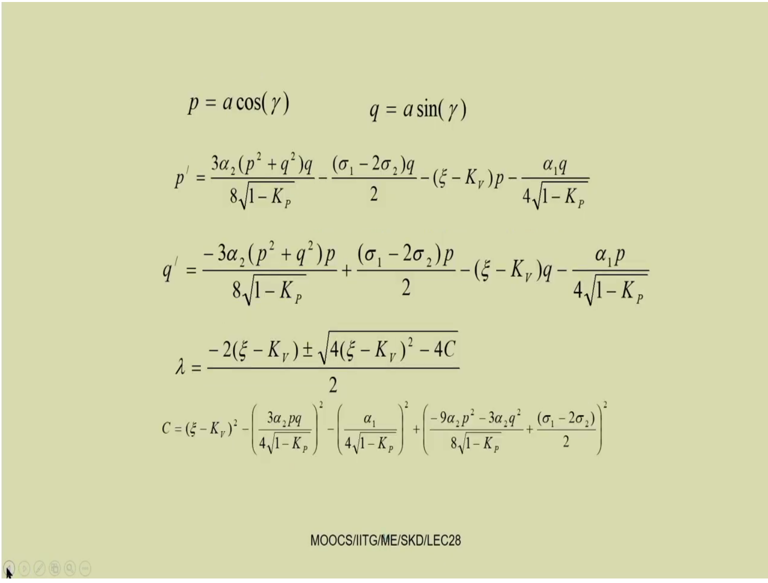

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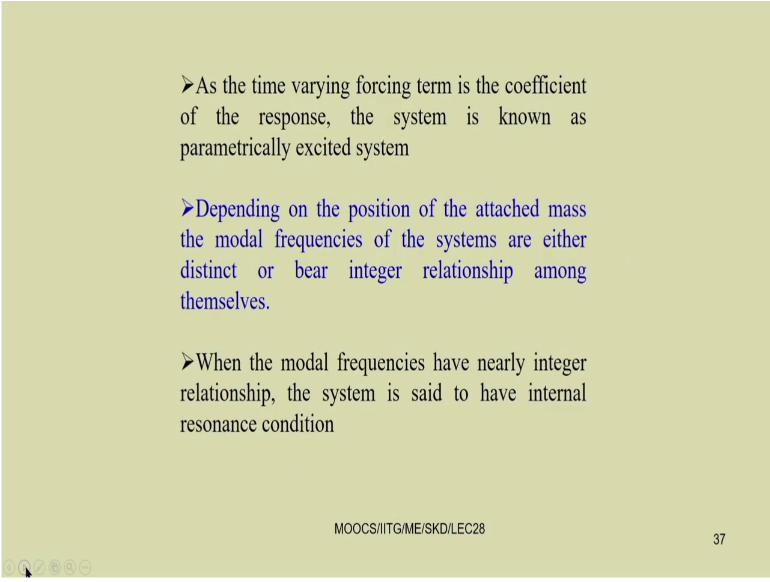

In these ways, we know how to derive this equation of motion. So, particularly in the second case, so when we have derived this equation using this energy method and subsequent first order using these only one mode approximation. So, you have seen this equation. And we have here used this control system also. So, here this parameter E and I we had we have used, E is for the Young's modulus for elastic material it is constant.

So, later we will take an viscoelastic material where the property will be different. For viscoelastic material this E cannot be a real number, so it will be a complex number. Next class we are going to see, so if we have a viscoelastic or magneto elastic beam, then how we can find the equation of motion and how the beam will respond to non-linear vibration.

Thank you very much.