Nonlinear Vibration
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Lecture - 28
Passive and Active vibration absorber with displacement and acceleration feedback

Welcome to today class of Non-linear Vibration. So, previous classes we are starting with the applications of non-linear vibrating systems. And, we have already covered the flexible non-linear systems; particularly we have taken a base excited cantilever beam. And in the cantilever beam, we have attached the we have attached one mass at arbitrary position which gives rise to internal resonance conditions.

Also we have considered three different type of beams, where we have taken this elastic beam, viscoelastic beam and elastomagnetic beam. And further we have applied Piezo electric pass to this cantilever type of beam and converted that thing to an energy harvester. So, we have studied all these cases, where we have seen these non-linear systems gives rise to two different stability and bifurcations.

So, particularly we are considered the fixed point response stability and bifurcation of the fixed point response, where we observed this pitch-fork type of bifurcation, saddle-node type of bifurcation and also Hopf type of bifurcation. In addition to that so, we have seen this periodic quasi periodic and chaotic responses.

So, this periodic response also we have seen. So, it may be single periodic, two periodic or it may give rise to period doubling route to chaos also quasi periodic route to chaos also. We have observed and this torus break down route to chaos and several type of crisis are also we have observed in the previous classes.

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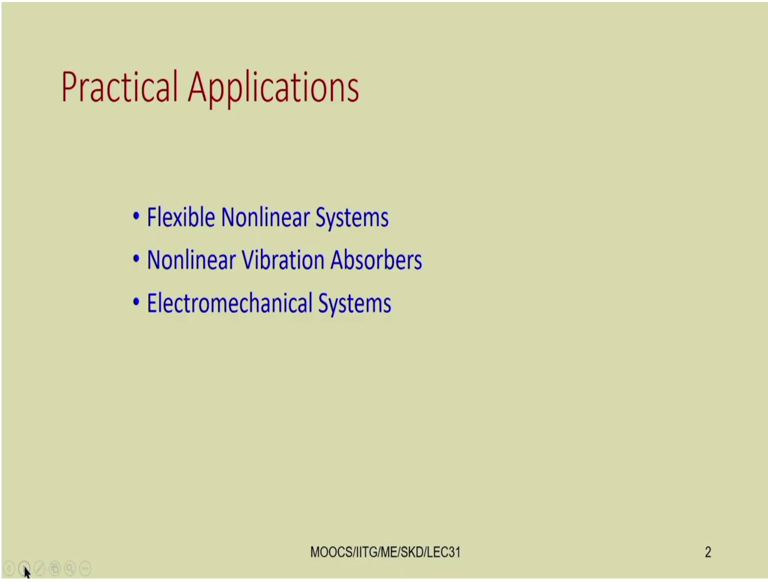

Now, next two classes we are going to study regarding these non-linear vibration absorber, because as there is vibration in the system. So, we must know how we have to absorb the vibration, or how you have to contain the vibration, or how you have to control this vibration to absorb the vibration.

So, we must know what is the principle of vibration absorber? So, how it is different from vibration isolation? So, already we know what is vibration isolation; that means, so for a linear system we can isolate the vibration of the system so, by operating it at a frequency vary away from the natural frequency of the system.

Similarly, in case of non-linear systems so, as the resonance is not occurring at a fixed point, but at a range of frequencies. So, in that case so we must have to develop some mechanism to control this vibration or to isolate this vibration causing in a system. Similarly this principle of vibration absorber in linear case we know. So, we know that by adding a secondary system to the primary system primary vibrating system. So, we can control or we can absorb the vibration of the primary system.

So, it will happen if the excitation frequency of the primary system is equal to root over k 2 by m 2, where k 2 is the stiffness of the spring system, we have attached to the secondary mass and the mass in the secondary system is m 2. We will see in case of non-linear systems how we can absorb this vibration, or whether this tuned mass damper vibration tuned mass absorber is applicable to a non-linear systems or not.

So, next two classes we are going to study on this vibration absorber. And later we will see some examples of electro mechanical systems and some other systems in the application of these non-linear vibrating systems.

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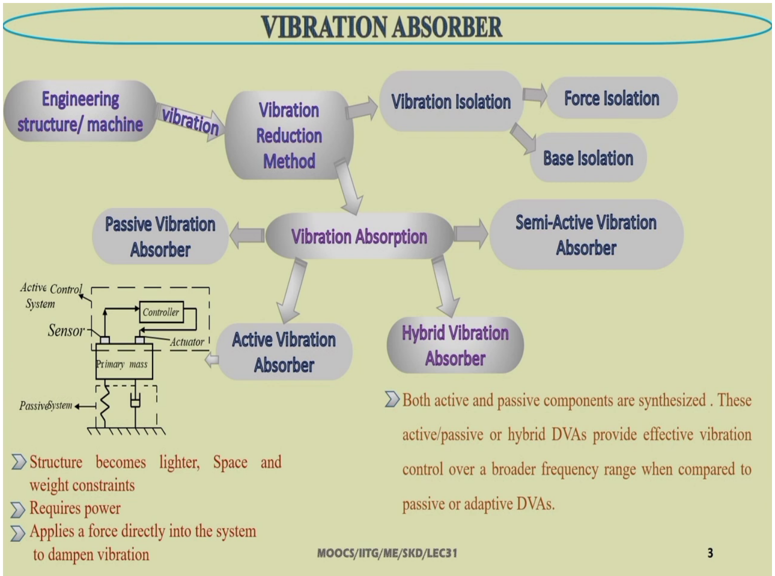

Vibration absorbers already we know that there are several engineering structure and machines. So, there will be vibration due to unbalanced force. So, this we must develop some vibration reduction methods. So, one is the vibration isolation and other one is the vibration absorber. So, in case of vibration isolation, we can do this force isolation or base isolation.

Similarly, in case of this vibration absorber so, we can have the passive vibration absorber or semi active vibration absorber or we can go for this active vibration absorber. This shows a schematic diagram of a active vibration absorber, where we have this primary system. So, this primary system this is the passive system passive spring and damper are attached to the system.

So, now we can put a controller we can put the actuator sensors and controller. So, sensors will sense the vibration and then we have to put a actuator which will actively. So, which will be actively controlled the motion of the actuator can be controlled in such a way that, it will achieve the absorption of the primary system absorption of the vibration of the primary system.

We can have the hybrid vibration absorber also so, in case of hybrid vibration absorber. So, both active and passive components are synthesized. These active passive or hybrid dynamic vibration absorbers provide effective vibration control over, a broadband frequency range when compared to passive and adaptive dynamic vibration absorber.

So, in case of the active vibration absorber structure become lighter space and weight constraints will be there it requires powers. So, when you are putting this active part though we have to put the actuator. So, for that actuator we require power applies a force directly into the system to dampen the vibration.

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There are several applications you can see so, several applications are there. For example, so this Boeing vertol ch 47, where five active vibration absorbers are used so, this is in aero space applications in also F 18 fighter plane wing with piezoelectric actuator for buffeting. Then, in car suspension system also, we can use this active vibration absorber.

So, active vibration absorber reduction gears can be used also, these are used already technical cooperation program USA Canada and Australia. So, then this RIVD J Damper then this sports car in sports car also these things have been put. So, in bioengineering applications also you can see this vibration.

So, this to control the vibration of the hand movements so, this vibration bracket can be used bracelet can be used. So, you can see this small video you can see you just see when he is taking the food so there is so, due to this shaking of the hand. So, it is difficult to difficult for him to take the food properly.

So, to absorb the shaking of the hand so, we can develop one bracelet. So, that or we can develop some vibration absorber so, which will absorb the vibration of the subject. So, similarly there is several civil structure, where this 634 meter tall Tokyo skytree so, Japan.

So, here this tuned vibration absorbers are been used. So, then we have this tallest statue in the World Statue, Statue of Unity India. So, here also in the lake part you can see so, two tuned mass damper systems are used here. So, it can prevent the vibration of the system.

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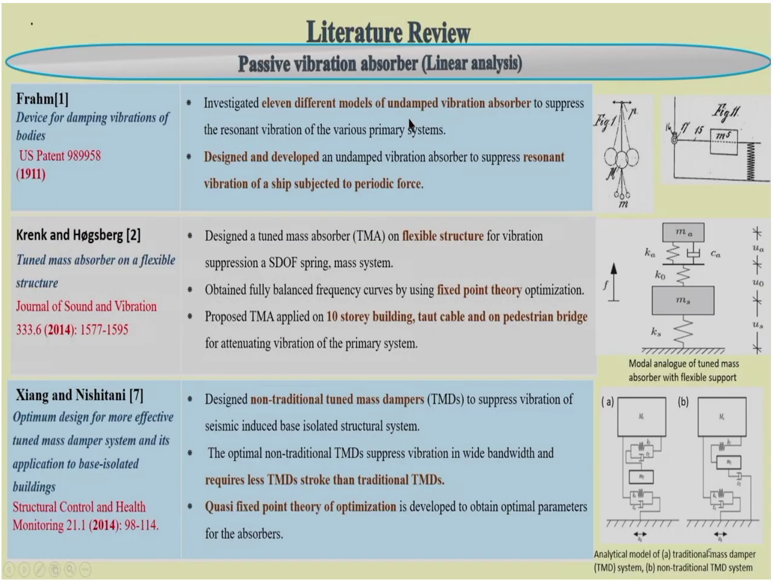

That several literatures are available and some of these literature for example, you can see this Frahm in 1911 so, this is US patent. So, here investigated eleven different models of undamped vibration absorber, to suppress the resonant vibration of the various primary system.

So, these vibration absorbers are the passive vibration absorber the principle is simple. So, here so in this case the secondary system. So, we are going to use a secondary system in the primary system. So, the secondary spring and mass are chosen in such a way that. So, the external excitation frequency will be equal to root over k 2 by m 2.

So, similarly this Krenk and Hogsberg so, they developed this tuned mass absorber in a flexible structure. So, it is published in 14 JSV 2014 so, designed a tuned mass absorber on flexible structure for vibration separation of a single degree of freedom spring mass systems. So, obtained fully balanced frequency curves by using fixed point theory optimization. So, proposed tuned mass absorber applied to 10 storey building taut cable and on pedestrian bridge for attenuating vibration of the primary system.

Similarly, this Xiang and Nishitani they in 14 in they were the optimum design for more effective tuned mass damper system. And, its application to base isolated buildings they have designed non traditional tuned mass damper. So, it is non traditional tuned mass damper to suppress the vibration of seismic induced base isolated structural system.

The optimal non-traditional tuned mass dampers suppress vibration in wide broadband and required less tuned mass dampers stroke than the traditional TMD. So, quasi fixed point's theory of optimization is developed to obtain the optimal parameter for the absorber. So, here you can see the structure so, traditional mass damper. So, so this is the traditional one and this is the nontraditional 1. So, in traditional 1 you can see how the spring and dampers are used.

And in case of the non traditional 1 so, this is the primary m 1 is the primary system. So, here the secondary system is directly connected to the ground. So, here the secondary system in case of the traditional the secondary system is connected to the primary system, but in case of the non traditional the secondary system is connected to the ground itself. So, secondary spring and damper part can be directly connected to the ground.

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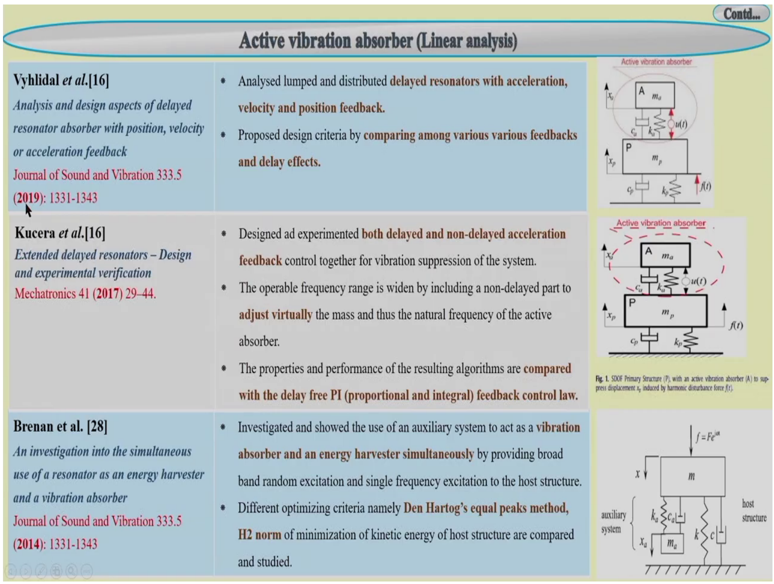

Let us see some active vibration type of absorber for example, so you can take the work of Vyhlidal so, Vyhlidal in 19 so this is recent paper 2019. So, analysis and design aspect of delayed resonator absorber with position velocity and acceleration feedback. So, we can give either the position velocity or acceleration feedback or a combination of all these 3 or a combination of 2 of any two either position velocity position acceleration or velocity acceleration position velocity.

So, that way any combinations of these three can be taken and these absorber can be designed here this is the primary mass. So, this is the primary system. So, in the primary system it is subjected to a force f t and it has a displacement of x p that is x primary. So, to that thing this spring and damper this is the passive part in addition to passive. So, a active part where u t is written. So, this is the active force is acting to the system. So, this active part is attached to or is connected these primary system and the secondary system.

This analysed lumped and distributed delayed resonator with acceleration velocity and position feedback. So, when we are using some feedback system. So, there may be some time lag and for that purpose. So, one can consider a delay differential equation to write down this equation of motion.

In that case so, one can use a delayed resonator to study the system. So, the proposed design criteria by comparing among various feedback and delay systems. So, then you can see this work of Kucera et al, Kucera et al in 2017. So, they extended the delayed resonator and design and experimental verification also carried out.

So, they carried out design and experiment both delayed and non delayed acceleration feedback control, together with vibration suppression of the system. The operable frequency range is widen by including a non delayed part to adjust, virtually the mass and thus the natural frequency of the active absorber.

The properties and performance of the resulting algorithms are compared with the delay free proportional integral feedback control law. So, then this Brenan et al also in 2014. So, they investigated the simultaneous use of a resonator as an energy harvester and vibration absorber.

Previous class we studied regarding this energy harvester. So, we have seen so, in the parametric instability region when the system response becomes parametrically unstable. So, that time we can use them as a energy harvester. And, here the opposite of energy harvesting will be the energy absorbing property.

So, one can use the same system as an energy harvester, or as an vibration absorber depending on the range of frequency in which it is working, or depending on some other system parameters also changing the system parameter can control the vibration. And sometimes it can be used as a vibration absorber, or sometimes it can be used as the energy harvester.

So, in this work they investigated and showed the use of auxiliary system to act as an vibration absorber, and an energy harvester simultaneously by providing broadband random excitation and single frequency excitation to the host structure. So, different optimizing criteria namely Den Hartog's equal peaks method H 2 norm of minimization of kinetic energy of the host structure are compared and studied, here this is the host structure.

So, in the host structure they have put the auxiliary system. So, you just see a the auxiliary system is connected to the primary system, with a dumper and a spring. So, spring and dumper is used and they have studied this system as both harvester and actuator.

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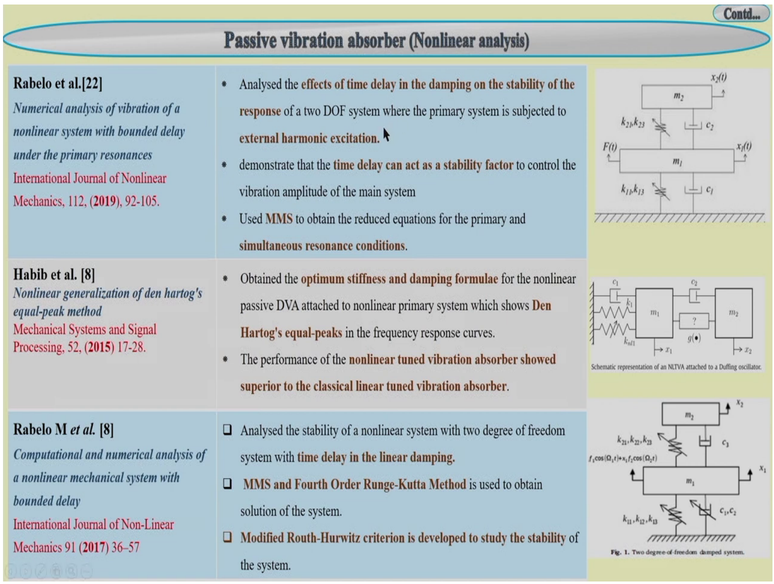

Harvester and absorber so, there are several passive vibration absorber where this non-linear analysis also carried out. So, you can see some of them for example, this work by Rabelo et al so, in 2019 they carried out this they analysed the effects of time delay in the damping of the stability of the response of two degrees of freedom system, where the primary system is subjected to external harmonic excitation.

Demonstrate that the time delay can act as a stability factor to control the vibration amplitude of the main system. They used this method of multiple scales to obtain the reduced equation for primary and simultaneous resonance condition. Similarly this Habib et al, they studied this non-linear generalization non-linear generalization of Den Hartog’s equal peak method.

So, they obtained the optimum stiffness and damping formulae for the non-linear passive, dynamic vibration absorber attached to the non-linear primary system which shows Den Hartog’s equal peaks so, in the frequency response curve. So, already we know so when we are adding a secondary system to the primary system.

So, now, we have a two degrees of freedom instead of a single degrees of freedom system, when we are considering only the primary system. As we have a two degrees of freedom system so, we have two natural frequency and when we are adding the spring mass system, secondary spring mass system in such a way that so, the natural the external frequency equal to root over k 2 by m 2.

So, in that case so, at resonance conditions that is when the external frequency equal to the first or the natural frequency of the primary system at that position the response amplitude of the primary systems becomes 0. So; that means, it is completely the secondary system completely absorb the vibration.

But, though it is absorbing the vibration at omega equal to omega n of the primary system, but what one can obtained two peaks near, the two natural frequencies, two modal frequencies obtained in this case as we have a two degrees of freedom system. So, we have two natural frequency or two modal frequencies.

So, when excitation frequency equal to these two modal frequencies. So, that time now we are getting two peaks, but we are getting 0 response so, equal to the 0 response when it is equal to the previous the external frequency equals to the previous natural frequency of the system or the natural frequency of the primary system ok, that ways exactly. What we are doing?

So, by adding the extra spring mass system so, we are shifting the natural frequency or shifting the resonance frequency to the left and right of the original frequency natural frequency of the primary system. That is why one can get two peaks in place of a single peak in case of a two degrees or freedom or this vibration absorber case.

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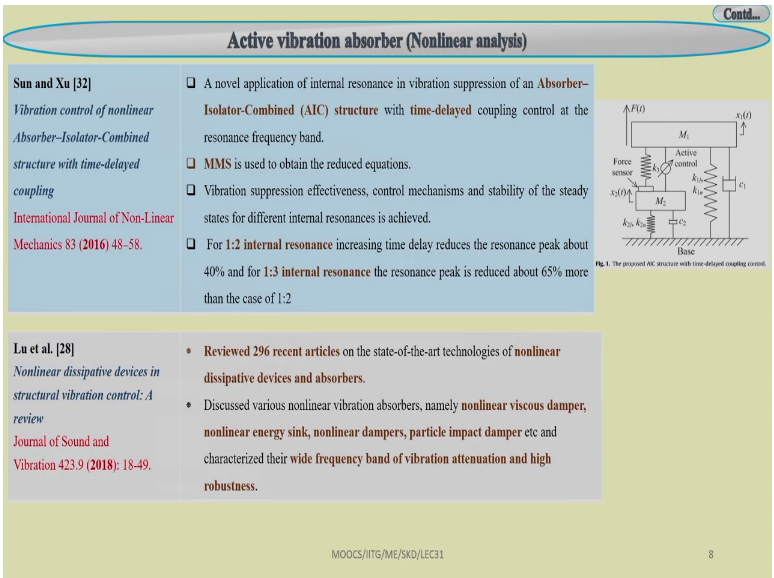

So, there are several studies are also on this active vibration absorber of non-linear system, many researchers they have also considered the non-linear spring and damper in case of the secondary system also. For example, this Sun and Xu in 2016, they have developed a novel application of internal resonance in vibration separation of an absorber isolator combined. Structure with time delayed coupling control at the resonance frequency band, they have used method of multiple scales to obtain the reduced equations.

So, vibration separation effectiveness control mechanism and stability of the steady state for different internal resonance conditions have been studied, they have used or they have considered 1 is to 2 internal resonance, for 1 is to 2 internal resonance increasing time delay reduces the resonance peak about 40 percent and with 1 is to 3 internal resonance condition. So, they have shown the resonance peak is reduced about 65 percent more than that of the 1 is to 2 internal resonance.

What is 1 is to 2 internal resonance? If the second natural frequency or second modal frequency is two times the first modal frequency, you have this 1 is to 2 internal resonance. Similarly the second natural frequency or second modal frequency, if it is three times the first or primary frequency, of the primary system then it is 1 is to 3 internal resonance condition.

By adjusting or by taking different mass and spring in the secondary systems, we can generate either 1 is to 2 or 1 is to 3 internal resonance conditions. And by using these different internal resonance conditions we can reduce the amplitude of vibration of the system. So, Lu et al in 2018 so, they have reviewed 296 recent articles on the state of the art technology for non-linear dissipative devices and absorber.

So, discussed various non-linear vibration absorber namely non-linear viscous damper, non-linear energy sink, non-linear dampers, particle impact damper and characterized their wide frequency band of vibration attenuation and how to obtain high robustness in this type of analysis also has been discussed in the work of Lu et al.

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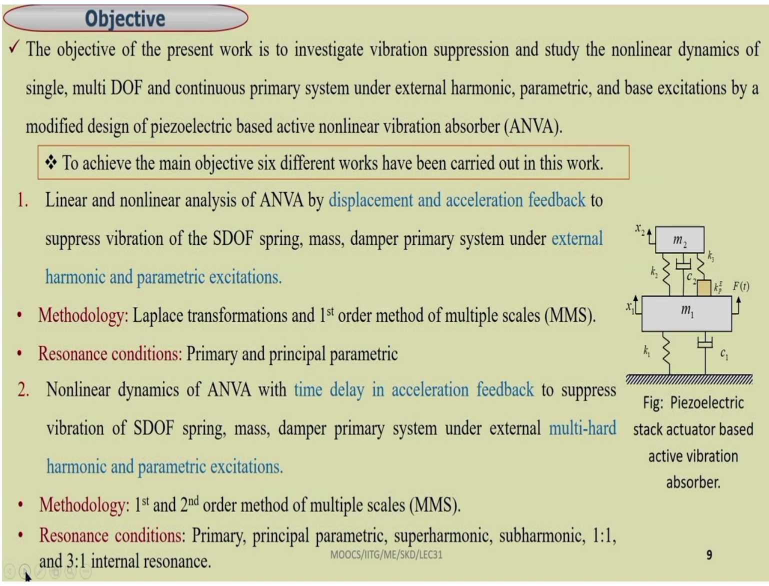

Here, we can see now we can study the following system. So, now, you have seen the literature review and from the literature review, we can see that we have several type of either we can have a passive vibration absorber, we can have active vibration absorber or we can have this hybrid vibration absorber.

In this absorber also we can have these internal resonance of 1 is to 1; that means, when the actually 1 is to 1 internal resonance is similar to this tuned vibration absorber, where the natural frequency of the primary system equal to the excitation frequency. And, it is also equal to the natural frequency of the secondary system.

Natural frequency of the secondary system means root over k 2 by m 2. So, in that case we have seen this tuned vibration absorber also, we can have this internal resonance of 1 is to 2 internal resonance of 1 is to 3 by arranging different spring mass damper system. Let us now consider so, let us now see how we can model this type of system, how we can analyze this type of systems and what are the results and observations in this type of systems.

So, when we are taking let us take a piezoelectric stack actuator based active vibration absorber. We can take several systems so, in one case we can take both the primary and secondary systems to be linear. So, then we can take the primary system to be non-linear, secondary system to be linear also we can take the case when both primary and secondary systems are non-linear.

Similarly the forcing conditions also we can change. So, we can have a single harmonic or we can have multiple harmonic excitation present in this system when we have this linear systems. So, particularly one can find the response by applying this Laplace transform or one may use this first order method of multiple scale also to study the system.

So, one may study both primary and principle parametric resonance conditions, one may use this harmonic balance method also so, in modified harmonic balance method will be shown here in this work where by using that things. So, we can find the response of the system. So, we will study two or three different type of different systems in this work. Actually this work is the PhD work of my student mister Shivanand Mahanthi.

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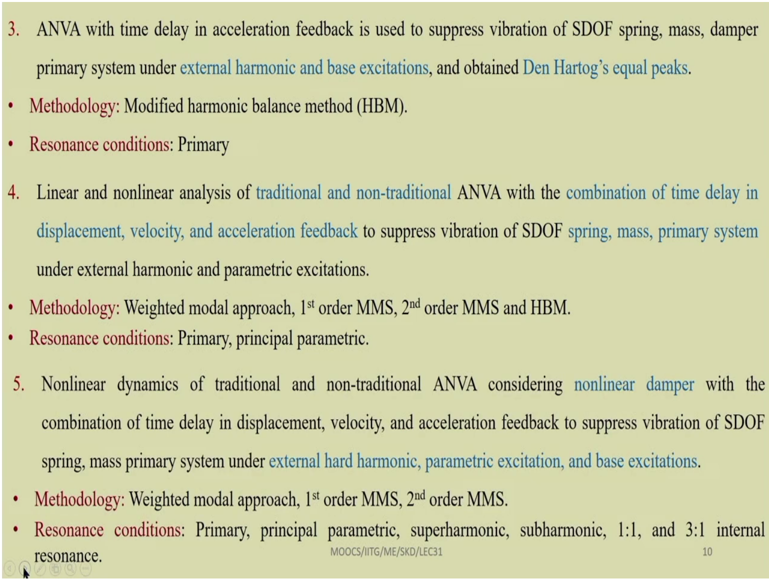

So, he is working on these passive and active vibration absorber. We can see this active non-linear vibration absorber with time delay. So, previous cases so you have not studied the time delayed system here we are going to study the time delay system. So, active non-linear vibration absorber with time delay in acceleration feedback, we may use this velocity feedback and displacement feedback also. So, generally we will take a single degree of freedom spring mass damper system.

So, will see by applying this external harmonic and base excitation, how we can obtain the Den Hartog equal peaks 3 4 systems we are going to study one by one let us see.

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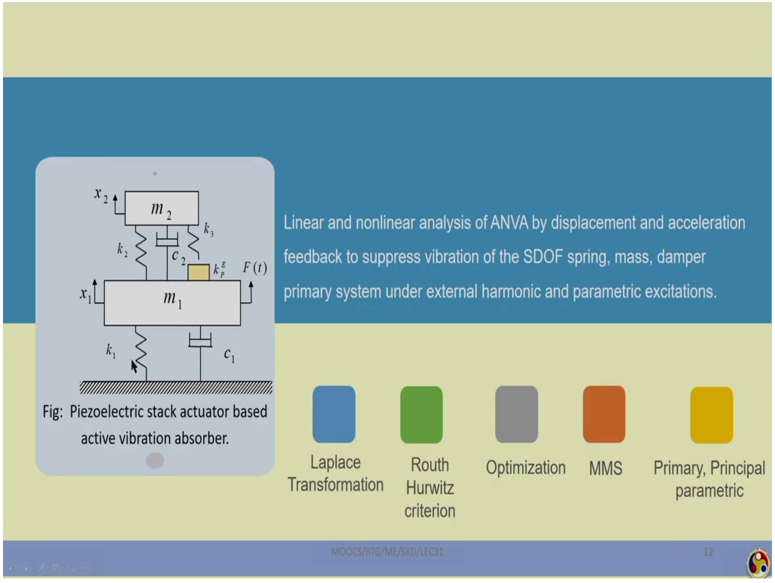

So, let us see the first case so, when we are going to study a piezoelectric stack actuator based active vibration absorber. So, this is the primary system so, in the primary system. So, this m 1 is the mass of the primary system k 1 is the spring then c 1 is the damping coefficient.

So, it is subjected to the primary system is subjected to a force F t and having this displacement x 1 about the equilibrium position, we have added a spring k 2 we have added a damper c 2 and we have added a piezoelectric stack actuator having this property this k p E. So that means, when we are applying this voltage to the piezoelectric stack actuator. So, it will behave as a actuator and it can be so, if it is proportional.

So, we can take this force to be k p into this displacement of this thing. So, in addition to this stack actuator so, we have added a spring you just see a spring is added to the in addition to the stack actuator. So, this is the stack actuator. So, this part is the stack actuator. So, in the stack actuator we have attached a spring which can actually be useful. So, in case this piezoelectric stack actuator fails to work.

So, here we will see by using this Laplace transformation we will find the response, then the stability criteria in case of the linear system. So, you just see the system is linear system we have taken. So, by using this Routh Hurwitz criteria we can study the stability, then we can see how we can use the optimization method to find the equal peak. Further we will use method of multiple scales and study the primary and principle parametric resonance conditions.

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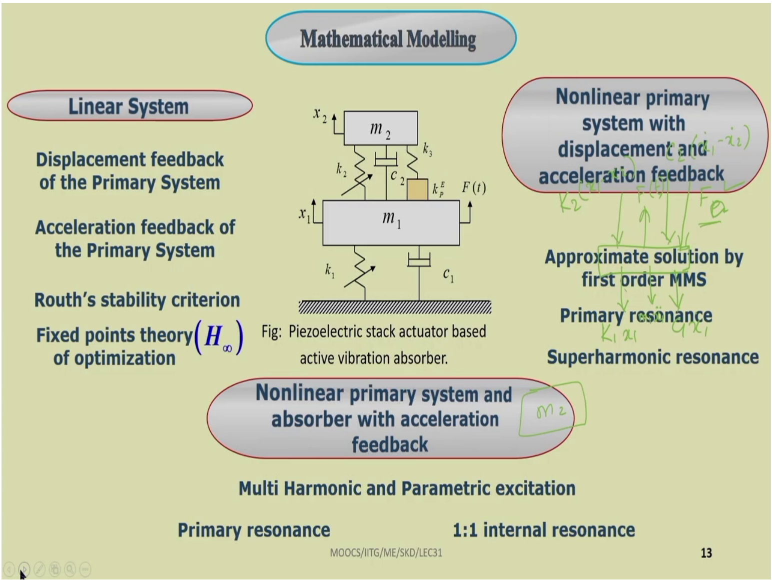

So, let us see the mathematical modeling so, mathematical modeling is simple. This is a two degrees of freedom system. So, either you can use the Newton's second law or this D Alembert principle or this energy based method like the Lagrange principle and Hamilton principle to study or to find the equation of motion. In this particular case we can one can conveniently used this Newton’s law to derive the equation of motion.

So, for this purpose so one can draw the free body diagram for example, drawing the free body diagram of mass m 1 so, if this spring is pulled upward, then it will exert a force k 1 force downward and c 1 x 1 dot force in downward direction. So, as the mass is moving upward direction, then it will be subjected to a inertia force in the downward directions.

So, inertia force equal to m 1 x 1 double dot if one draw the to find the equation motion. So, you can draw the free body diagram. So, now, mass is moving upward so, in that case this is k 1, x 1, k 1, x 1 then c 1, x 1 dot k 1 x 1, c 1 x 1 dot then this is m 1 x 1 double dot.

So, this is the inertia force. So, in addition to these things it is connected to mass m 2 through the spring k 2 and damper c 2 and this piezoelectric actuator. For the spring k 2 so, we can have a force. So, the force will be in this direction. So, it will be k 2 x 1 minus x 2 similarly we can have another force that is c 2 x 2 x 1 dot minus x 2 dot. So, in addition to that we have the force due to this piezoelectric actuators force, due to this piezoelectric actuator.

So, the equation motion can be written so, in this form. So, it will be m 1 x 1 double dot plus k 1 x 1 plus c 1 x 1 dot plus k 2 into x 1 minus x 2 plus c 2 into x 1 dot minus x 2 dot minus or plus F p equal to 0 ok. So, another force is also acting instead of 0. So, this will be equal to F t so, F t is the force acting. So, F p is the force given by the actuator or F a you can write also is the force by the actuator. So, total it will be equal to this.

Similarly, so we can draw the free body diagram for the mass m 2. And we can find the equation of motion. So, let us see what are the equation of motion we are getting here. So, we can give a displacement feedback of the primary system. So, acceleration feedback of the primary system and use this Routh’s stability criteria and use this fixed point theory of optimization, we have this non-linear primary systems with displacement and acceleration feedback.

Approximate solution we have to do by using first order method of multiple scale so, primary resonance and super harmonic resonance conditions. We have to study the non-linear primary systems and absorber with acceleration feedback further we will study. So, initially we will study this as a linear system, then we can make the spring to be non-linear.

So, when he was making this non-linear. So, one additional term will be added to the equation of motion. Further also we can take this k 2 spring to be non-linear also. So, a non-linear term can also be added. So, further it can be more complicated by using multi harmonic and parametric excitation to the system.

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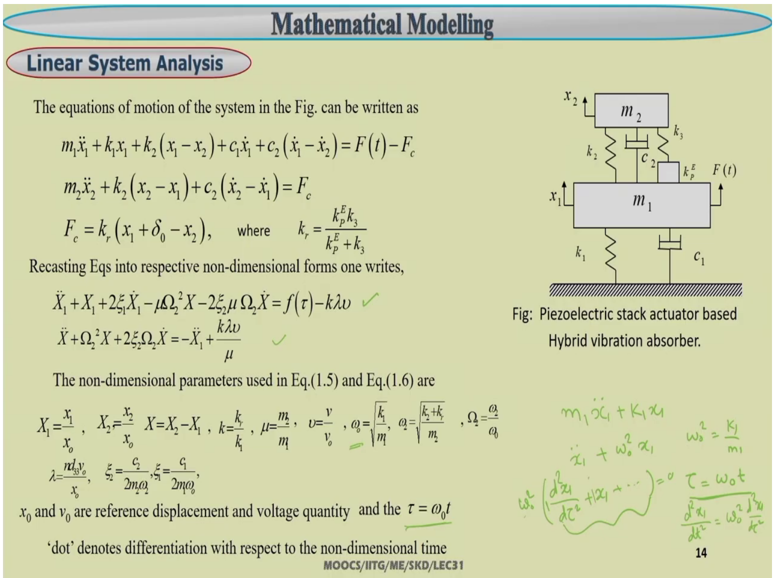

So, primary resonance conditions and 1 is to 1 internal resonance conditions may also be studied in this type of system. Slowly let us see so, let us see the linear system analysis. So, in this linear system already I have shown how to derive this equation of motion. And you have seen it is written in the same way. So, that is m 1 x 1 double dot plus k 1 x 1 plus k 2 into x 1 minus x 2 plus c 1 x 1 dot plus c 2 into x 1 dot minus x 2 dot equal to F t minus F c.

So, this control force is written as F c. This F t force is acting downward and this F c force is acting on the mass m 1 by mass m 2 this F c. And, this F c ok the for the second system also we can write this equation m 2 x 2 double dot plus k 2 so, it is connected to the first mass by k 2 and c 2 and then this k 3 by the stack actuator. So, m 2 is connected by the spring. So, this mass m 1 and m 2 are connected by the spring k 3 and stack actuator k P E.

So, that is why this for the free body diagram of m 2, we can write this equation in this form that if m 2 x 2 double dot plus k 2 into x 2 minus x 1 plus c 2 into x 2 dot minus x 1 dot equal to F c. So, now, this F c so if we are taking only proportional type of controller, then we can write this equal to k r into x 1 plus delta 0 minus x 2, where delta 0 is the initial displacement of the stack actuator so, minus x 2.

Here k r so as we are using k r can be written. So, if we are writing this stiffness of the actuator equal to k P and k 3 is also a spring. So, the stiffness of the actuator we have taken k P E and so, this k 3. So, both of them are in series so, the equivalent stiffness can be written as k r equal to k P E into k 3 by k P E plus k 3.

So, that is why this F c can be written as k r into x 1 plus delta 0 minus x 2. So, this way we obtain two equation motion and then so, from this equation motion. So, we can write by non dimensional form also we can write, to write it in non dimensional form. So, first we can we can divide this m 1 everywhere, m 1 in first equation or m 1 by dividing m 1 this equation becomes x 1 double dot plus k 1 by omega 1 k 1 by omega 1.

Now, it becomes k 1 by omega 1 equal to omega 1 square or k 1 by omega 1 root over is taken to be omega 0 in this case, then k 2 by m 1 c 1 by m 1 c 2 by m 1 so, m 1 is initially divided. Now, by taking a time non dimensional time non dimensional time one can take tau equal to omega 0 t. For example, you just see only this two terminate mean derive and show you so, m 1 x 1 double dot plus k 1 x 1.

So, first you divide this by m 1 so, this becomes x 1 double dot plus omega 0 square x 1 as omega 0 square equal to k 1 by m 1. So, now, taking tau this non dimensional time equal to omega 0 into t omega 0 is radian per second time is in second, second second cancel. So, this becomes radian which is non dimensional. So, by taking tau equal to omega 0 t so, this x double dot which is differentiation with respect to time t.

So, we can write this equal to so, d square x 1 by d tau square. So, it will be omega 0 square will be multiplied with this thing, because twice if you are differentiating with respect to so, this is d square. So, d square x 1 by d t square will be nothing, but it will be omega 0 square d square x 1 by d tau square. So, this omega square you can take this omega 0 square can be taken common so, this becomes x 1. So, this into omega 0 square yes or no. So, this term will be there and it will be equal to 0.

So, now, you can divide as omega 0 is not 0. So, you can divide this term and you can get this as the equation motion. So, here you just note the coefficient of d square x x 1 by d tau square is 1 and also the coefficient of x 1 is also 1 which is shown in this equation and in the second equation also.

Second equation you just see. So, first equation you got in this way so, by taking this tau equal to omega 0 t. And, you have used this non dimensional parameter and you have written down this equation. Similarly we know that we can write this c by m equal to 2 zeta omega 0 as omega 0 is so, that way you can write, then we can use this non dimensional parameter.

For example, let us use x 1 equal to capital X 1 equal to x 1 by x 0 capital X 2 equal to x 2 by x 0 x equal to capital X equal to capital X 2 minus capital X 1 k equal to k r by k 1 mu equal to mass ratio mu 2 by mu 1 and v equal to v by v 0 omega 0 already you have taken equal to root over k 1 by m 1. And omega 2 equal to k 2 plus k r by m 2, and this external forcing frequency omega 2 equal to omega 2 by omega 0.

Also we can consider some other terms like lambda equal to n d 3 3 v 0 by x 0. So, this is due to the piezoelectric stack actuator and zeta 2 equal to c 2 by 2 m 2 omega 2 and zeta 1 equal to c 1 by 2 omega 1 omega 0 so, by writing these ways we can have this two equation using these two equations.

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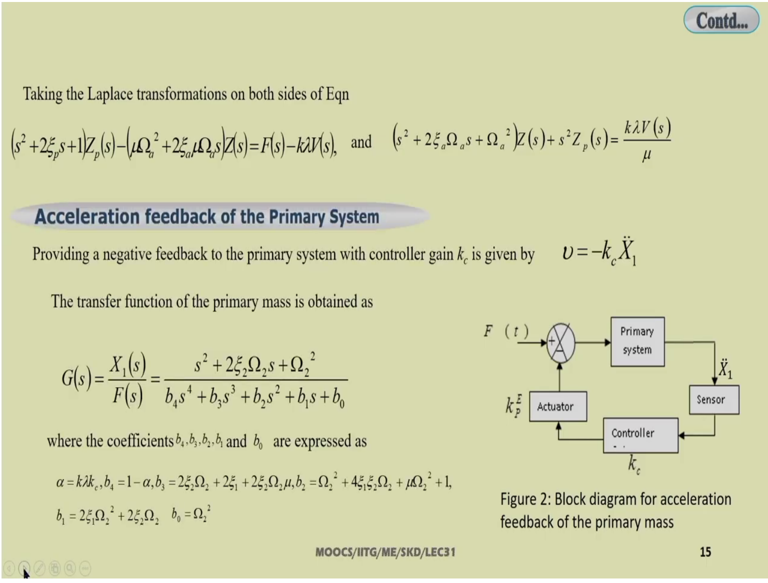

So, these are you just see initially we are finding for the linear case, then we will consider the non-linear case. So, if you take the Laplace transform. So, taking the Laplace transform the equation becomes s square plus 2 zeta p s plus 1 into Z p s minus mu Z s square plus 2 zeta a mu Z a s into Z s equal to F s minus k lambda V s and second equation becomes s square plus 2 zeta a omega a s plus omega a square into Z s plus s square Z p s equal to k lambda V s by mu.

This way we can derive this equation of motion. So, let us see the previous slides here, you just check after writing this taking the Laplace, we can take different type of feedback loop also and we can study the system. So, if were taking this acceleration feedback of the primary system. We can take acceleration as acceleration equal to X double dot, if the displacement is X acceleration is X double dot.

So, we can write so, in this feedback case that will be equal to the forcing term will be equal to minus k c X 1 double dot. So, the transfer function of the primary mass is obtained as G s equal to X 1 s by F s. So, in that case it will be equal to s square plus 2 zeta 2 omega 2 s plus omega 2 square plus b 4 s 4 plus b 3 s cube plus b 2 s square plus b 1 s plus b 0, where the coefficient b 4 b 3 b 2 b 1 and b 0 are expressed as you can express in this way.

So, these are given how you can express these coefficients. So, this is the control block diagram for the acceleration feedback of the primary system, this F t is applied to the system. So, we have a primary system sensor feedback so, sensors so we are getting the acceleration.

Actually, we you know that it is easier to get the acceleration by putting a accelerometer on the vibrating system. So, by putting the accelerometer, we can sense the acceleration and that thing can be used effectively as the feedback and getting that feedback we can control the vibration of the system.

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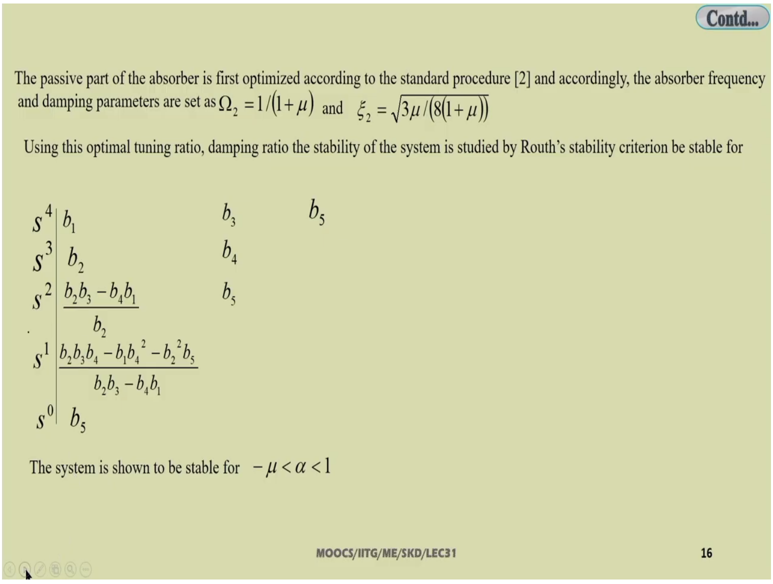

Now, the passive part of the vibration absorber is first optimized according to the standard procedure. And accordingly the absorber frequency and damping parameters are set as omega 2 equal to 1 by 1 plus mu and zeta 2 equal to root over 3 mu by 8 into 1 plus mu.

So, using this optimal tuning ratio so, damping ratio the stability of the system is studied by Routh's stability criteria. As we have this S 0, S 1, S 2, S 3 and S to the power 4. So, we can write down this coefficient after writing this coefficient, then we can check the first column of the we can construct the this Routh array.

So, from this Routh array so, you just see we have this fourth order equation. So, this fourth order equation can be written conveniently so, after writing this fourth order equation. So, we can write the coefficient then of S 4, then S 2, S 3, S 2, S 1 and S 0. So, the first column will tell. So, if there is a change in the sign then you can tell the stability will change.

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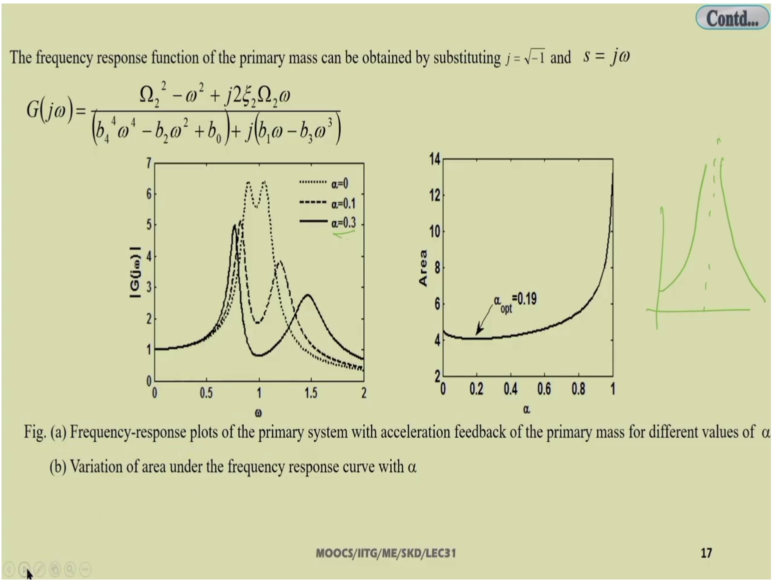

From this thing so, we can see that G j omega will be equal to omega 2 square minus omega square plus j 2 zeta 2 omega capital omega 2 omega divided by b 4th omega 4th minus b square omega square plus b 0 plus j b 1 omega minus b 3 omega cube. So, now one can plot this omega versus this G j omega.

So, this is similar to the magnification factor versus this omega or the response amplitude versus omega. So, you can easily observe that by using that thing when there is no feedback. So, you can see the system will be so, system will the primary system will have a resonance condition.

So, now for the secondary system so now, by taking these parameters so one can plot it and for three different value of alpha so, alpha equal to 0 alpha equal to 0.1 and alpha equal to 0.3, one can plot this omega versus G j omega and clearly one can observe for alpha equal to 0. So, we have two peak and the response amplitude is very high.

For alpha equal to 0.1 by taking alpha equal to 0.1. So, now, it is reduced and, but at omega equal to omega 1 still it is not equal to 0. So, here also still it is not equal to 0 or it is not reduced by taking this alpha equal to 0.3. So, you can see the response amplitude at omega equal to 1 so, is reduced very much. So, omega equal to one is nothing, but. So, this omega is the frequency ratio so; that means, when the external frequency equal to the natural frequency of the system.

So, previously you have seen. So, from the primary system it tends to infinite, but by adding the spring mass system in a proper way you can see that it can be reduced considerably. So, one can plot this area verse area to find this optimal value of alpha. So, here the optimal value of alpha is obtained to be 0.19. So, variation of area under frequency response curve with different value of alpha.

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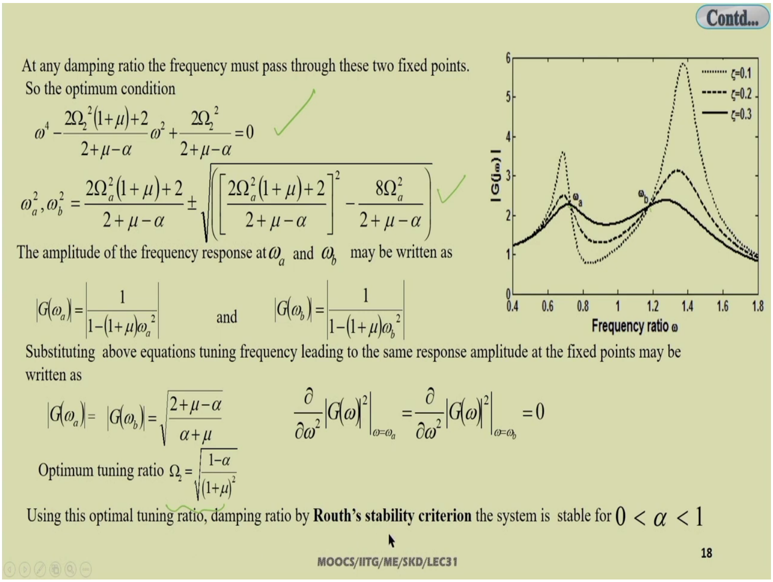

So, at any damping instead of if we are taking or when we are plotting this frequency ratio versus G j omega. So, we can see you can see irrespective of damping so, for this curve is plotted for three different value of zeta that is the damping ratio. So, in that case you just see irrespective of damping. So, there are two points that is omega a and omega b about which all the curves pass one can note these two points so, at any damping ratio the frequency must pass through these two fixed points for the optimal condition.

So, this omega 4th minus 2 omega 2 square into 1 plus mu plus 2 by 2 plus mu minus alpha omega square plus 2 omega 2 square by 2 plus mu minus alpha equal to 0. And we can have this omega a square and omega b square by solving this equation. So, we can get two value of omega.

So, this is the characteristic equation from the characteristic equation, we can get this two value of omega for which this will satisfy this equation. So, this is omega a square and omega b square will be equal to omega a square into 1 plus mu plus 2 by 2 plus mu minus alpha. So, this is the quadratic equation in omega square. If you take omega square equal to a, then this equation becomes s square minus b or if you take omega square equal to x.

So, this becomes x square minus a b x plus c equal to 0. So, this is a quadratic equation in omega square you can find the root of this quadratic equation by using this formula. So, that is minus b plus minus root over b square minus 4 a c by 2 a. So, using that things so, you can get this equation. So, you can observe the value of this omega a and omega b and from this thing so, you can get these two points where this these two fixed point.

The amplitude of the frequency corresponding to omega a and omega b can be written as G omega equal to 1 by 1 minus 1 plus mu into omega a square and 1 by 1 minus 1 plus mu into omega b square. So, substituting the above equation tuning frequency leading to the same response amplitude at the fixed point may be written as G omega a equal to G omega b that will be equal to root over 2 plus mu minus alpha by alpha plus mu and as these two points are fixed points.

So, if you differentiate this thing with respect to omega square. So, del by del omega square G omega square omega equal to omega, a will be equal to del by del omega square G omega square omega equal to omega b must be equal to 0. So, the optimum tuning ratio omega 2 can be obtained in this way. So, this is the optimum tuning ratio omega 2 equal to root over 1 minus alpha by 1 plus mu whole square so, this way by applying this so, using this optimal tuning ratio damping ratio by Routh stability criteria, the system can be said to be stable for 0 less than alpha less than 1.

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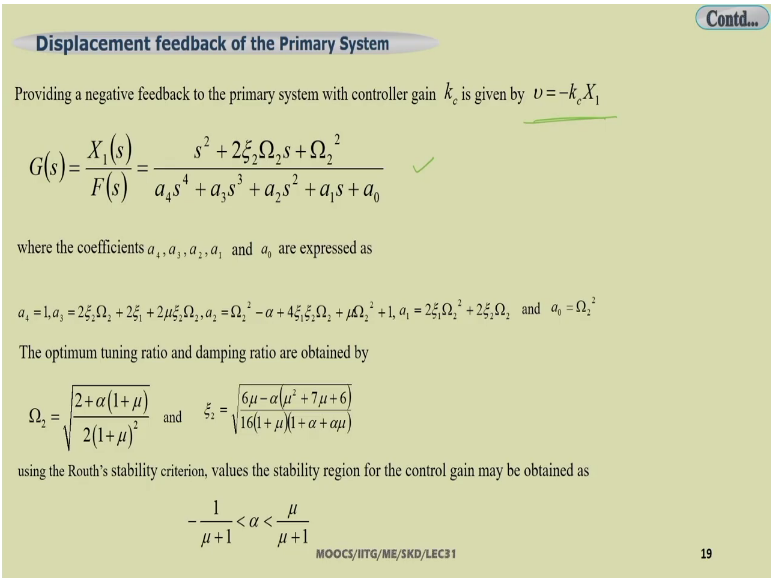

Similarly, one can study the displacement feedback of the primary system. So, in case of displacement feedback so, it can be taken as v equal to minus k c into X 1 so, in that case this G s term can be written in this form that is X 1 by s by F s equal to s square plus 2 zeta 2 omega 2 s plus omega 2 square by a 4 s 4 plus a 3 s cube plus a 2 s square plus a 1 s plus a 0.

Where the coefficient a 4, a 3, a 2, a 1 and a 0 are expressed as so, this is the expressions for all this parameter. So, the optimum tuning ratio and damping ratio can be obtained in the previous way that is omega 2 will be equal to 2 plus alpha into 1 plus mu by 2 into 1 plus mu square and zeta 2 equal to root over 6 mu minus alpha into mu square plus 7 mu plus 6 divided by 16 into 1 plus mu into 1 plus alpha plus alpha mu.

So, using Routh stability criteria the value of stability region for the control gain can be obtained as minus 1 by mu plus 1 less than alpha less than mu by mu plus 1.

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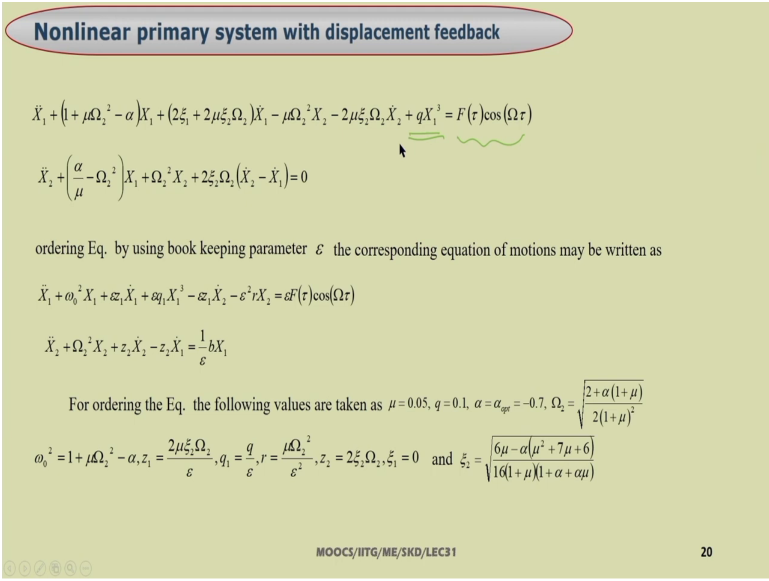

This way one can study the systems previously, we have taken these systems to be linear. So, now, one can take the non-linear systems also so, in this case the equation. So, now, we have added a non-linear term that is q X 1 cube here we have added. So, you have taken the nonlinearity in the primary system and the system is subjected to a forcing F tau cos omega tau. So, as we have taken the nonlinearity in the primary system. So, this equation can be written in this form.

So, now in that case so what we can do. So, here we can order the equation, we can put order them by using the order by epsilon. So, actually while taking this order of epsilon you must know the relative magnitude of each term by assuming that the cubic non-linear is of the order of epsilon. And the forcing is also of the order of epsilon and the damping is also of the order of epsilon.

The equation can be written in this form. So, the equation can be written X 1 double dot plus omega 0 square X 1 plus epsilon z 0 X 1 dot plus epsilon q 1 x 1 q minus epsilon z 1 X 2 dot minus epsilon square r X 2 equal to epsilon F tau cos omega tau and second equation equal to X 2 double dot plus omega 2 square X 2 plus z 2 X 2 dot minus z 2 X 1 dot equal to 1 by epsilon b X 1.

So, for order ordering the equation the following values are taken so, these are the values taken so, from here. So, one can obtain this omega 2 and this zeta 2 and z 2 also. So, this way the equation you can write by taking the nonlinearity in the primary system.

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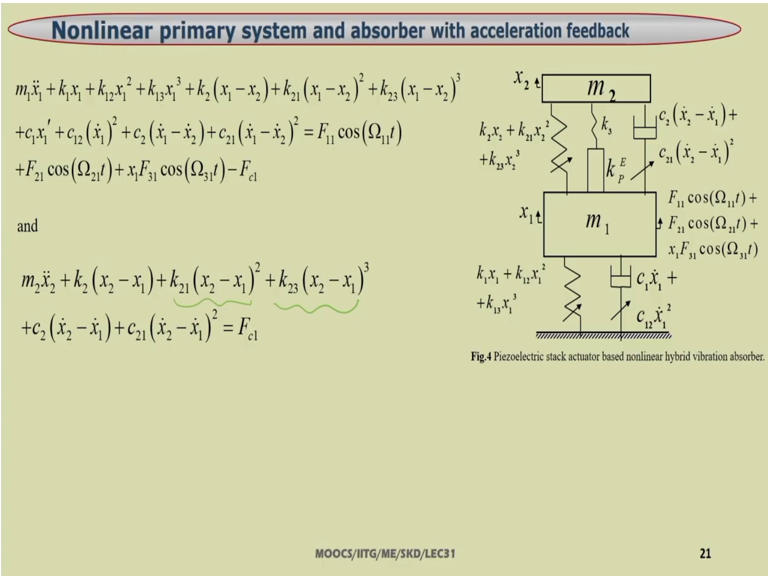

So, if you want to take the nonlinearity in both primary and secondary system. So, the equation can be modified. So, you just see the this is the term non-linear term added to the secondary system. So, if you are taking this one so, one can add the cubic non-linear term and quadratic non-linear term the equation will be m 2 x 2 double dot plus k 2 into x 2 minus x 1 plus k 2 1 x 2 minus x 1 square plus k 2 3 into x 2 minus x 1 cube plus c 2 into x 2 dot minus x 1 dot plus c 2 1 into x 2 dot minus x 1 dot square equal to F c 1.

Similarly, for the primary systems so, it will be m 1 x 1 double dot plus k 1 x 1 plus k 1 2 x 1 square plus k 1 3 x 1 cube plus k 2 into x 1 minus x 2 plus k 2 1 into x 1 minus x 2 whole square plus k 2 3 into x 1 minus x 2 whole cube plus c 1 into x 1 dash plus c 1 2 into c 1 2 into x 1 c 1 into x 1 dot. So, it will be c 1 into x 1 dot dash dot c 1 2 into x 1 dot square plus c 2 into x 1 dot minus x 2 dot c 2 1 into x 1 dot minus c 2 x 2 dot square. So, you are taking the non-linear damping also. So, it depends how you are defining your nonlinearity in damping and in the spring.

So, depending on that thing you can write down this equation of motion so, by taking all the forces acting in the system. Today class you have learnt how we have derived this equation motion for different vibration absorber system, and also you have seen how we can obtain these equal peaks, or we can obtain two fixed point at which irrespective of damping all the observers will have the same value.

Next class will see how we can solve these equations and we can obtain effectively, or we can have the effective vibration absorber in case of the non-linear systems.

Thank you.