Nonlinear Vibration

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Lecture - 34
Nonlinear dynamics of turning operation with delay and internal resonance

Welcome to today class of Non-linear Vibration. So, we are continuing in module 9. So, where we have already discussed about 2 applications. So, today we are going to study about the third applications that is on a manufacturing process.

So, we will take the turning operation as the application and we will study. So, if the vibration of the tool or the vibration of the workpiece to be non-linear and we will derive these equation on motion and see how to solve these equation motion.

So, in the previous 2 weeks. So, you have studied regarding these energy harvester and also vibration observer. So, where we have studied regarding the passive and active vibration observer. So, in case of energy harvester. So, we have seen how this piezoelectric excited cantilever beam can be used as a, energy harvester.

So, in that case. So, we have taken both external and internal resonance conditions and we have studied the equation governing equation of motion and then we have solved those governing equation of motion and we have seen how we can obtain this voltage from this vibration or we can harvest the energy from this ambient condition.

So, in case of vibration observer. So, which can find many applications in industry. So, we have seen the passive type of vibration observer and active type of vibration observer. So, today we will extend our analysis to a manufacturing process. So, where we are going to study regarding the vibration of the tool and work piece in case of the turning operation. So, the first work I will show you.

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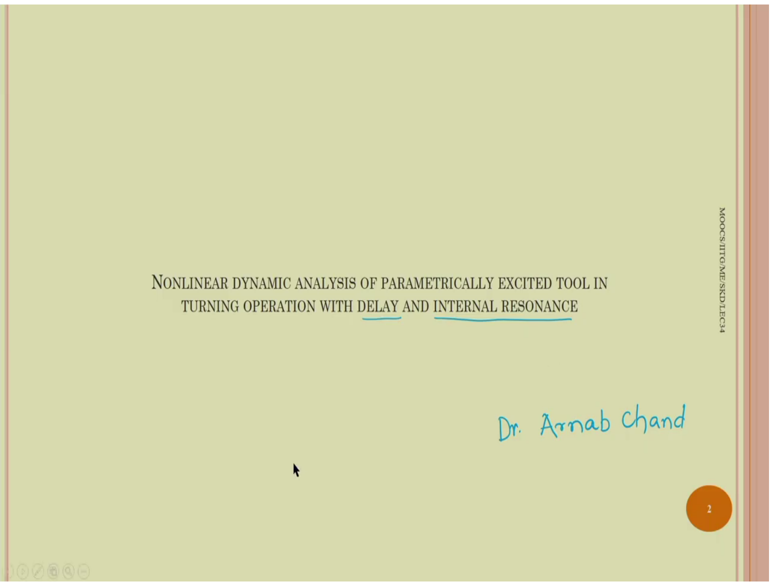

So, this is part of the PhD work of my PhD student Doctor Arnab Chand and some of these work have been already published in the journal of sound and vibration and so, these work.

So, you just see regarding these non-linear dynamic analysis of parametrically excited tool in turning operation with delay and internal resonance. So, here we are going to consider the delay and also the internal resonance condition and we will find the instability regions to study the conditions in which there will be no vibration.

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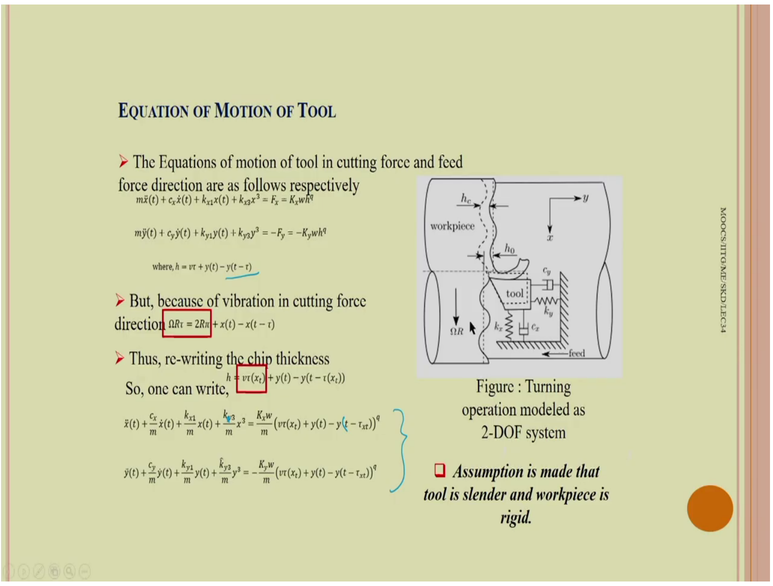

So, this is the turning operation. So, you can see the turning operation. So, in this turning operation. So, we have a work piece and this is the tool. So, the tool holder can be modeled or tool is supported the supporting condition of the tool can be model by using the spring and dumper system. So, it can be modeled. So, here we are assuming the workpiece to be rigid and the tool to be flexible.

So, as we are considering a flexible tool. So, here these flexibility is represented by using the spring constant or spring and damper in both x and y direction. So, in x direction. So, we have taken these vertical direction. So, this is x direction and the horizontal direction as y direction in this case. So, here. So, the spring in x direction is taken to be k x and the dumper is taken to be c x and in y direction it is k y and c y.

As the work piece is rotating. So, we assume the workpiece is rotating with omega capital omega. So, there is initial undulation in the cutting. So, you can see. So, initially. So, due to this undulation. So, this is the chip. So, you can see this is the chip and so, the chip thickness is represented by h.

So, we will see the thing. So, here initially it is h 0 and after some time. So, it will be h c. So, we have to find how much force is acting on the by the tool on the workpiece or the how much force is exhorted by the work piece on the tool in a reverse direction.

So, this is the feed direction. So, the equation of motion of the tool can be retained in both x and y direction. So, in the cutting force direction and feed direction. So, this is the feed direction y is the feed direction and this the cutting force direction. So, x is the cutting force direction and y is the feed direction. So, in cutting force direction the equation of motion can be retained equal to m x double dot.

So, in x. So, this is the, this is the cutting force direction, x is the cutting force direction. So, if the displacement is x in cutting force direction. So, it can be retained as m x double dot plus c x, x dot plus k x 1.

So, we have written this is k x 1 t, k x 1 into x t plus k. So, we are taking a cubic. So, you are modeling this spring as a non-linear spring. So, generally in most of the literature you can find they have taken this linear spring. So, here it is taken to be a non-linear spring. So, that is why this k x is represented by k x 1 is the linear part k x 1 x t and the non-linear part is written k x 3 x q.

So, we are taking a cubic non-linear spring. So, this way we can model the spring force as, k x 1 t into x t plus k x 3 into x cube. So, this will be equal to the cutting force that is x f x. So, cutting force f x, these cutting force can be retained by using this equation that is k x w into x q.

So, similarly in case of y direction. So, this is the y direction this oriental direction is the y direction. So, in this or the feed direction. So, in the feed direction. So, the equation motion can be written as m y double dot plus c y dot plus similarly here in y direction also we have taken we have taken these cubic nonlinearity.

So, that is why it can be written k y y t plus k y 3 y cube. So, this will be equal to minus f y. So, would be feed direction. So, the work piece will absorb a force on the. So, on the tool. So, that is why it is written as minus F y that is equal to minus K y w into h to the power q.

So, where h. So, that is the chip thickness can be written as. So, this is equal to b t plus y t minus y t minus tau. So, here you just see this is t minus tau. So, this is the tau is the time delay. So, delay term is used here. So, before previous cut let the previous cut in y direction, previous cut in y direction what is the displacement and this present cut what is the displacement and into. So, this is the v t, but because of the vibration in the cutting force direction. So, omega R tau will be equal to 2 pi R plus x t minus x t minus tau.

So, substituting these 2 in this equation and writing that h equal to v tau x t plus y t minus y t minus tau x t. So, we can write these equation in this form that is x double dot plus c by m x dot plus k x 1 by m x plus k y 3 by m into x cube. So, this is k x 3. So, this is not y. So, this is k x 3 by m x cube.

So, this will be equal to k x w by m in to v tau x t plus y t minus y t into y t minus x tau to the power q. So, similarly in the y direction equation can be written as y double dot t plus c y by m y dot t k y 1 by m, y t k y 3 by m y cube equal to minus k y w by m into similarly we can put these equation w and h q in this form.

So, h replacing h in this way. So, it will be equal to v tau x t plus y t minus y t minus tau x tau to the power q. So, here the tool is assumed to be rigid and the tool is assumed to be flexible and the work piece assume to be rigid. So, that is why we got these equations. So, here we have taken the delay also into account to find what is the chip thickness actually due to undulation. So, it will depend on the previous cut. So, the chip thickness will depend on the previous cut. So, that is why. So, taking that into account. So, it can be written in this form.

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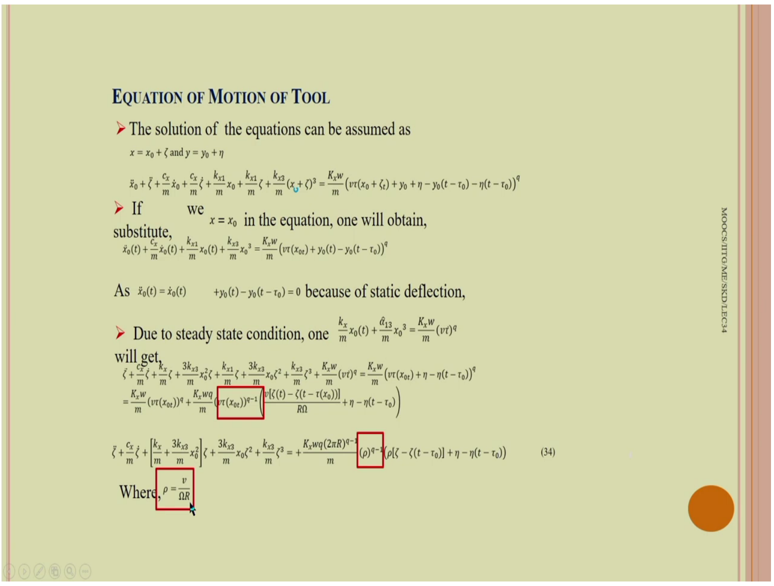

So, now assuming this x to be equal to x 0 plus zeta and y equal to y 0 plus eta. So, at time t for example, x equal to x 0 and time t y equal to y 0. So, at any time t. So, these x can be written as x 0 plus zeta and y equal to y 0 plus eta.

So, that is why this equation can be now for the written in this form, that is x double dot can be written as x 0 double dot plus zeta double dot and x 0 double dot plus zeta double dot plus c by m, c x by m x 0 dot plus c x by m zeta dot k x 1 by m x 0 plus k x 1 by m zeta plus k x 3 by m into x plus zeta cube equal to k x w by m.

So, v tau into x 0 plus zeta plus y 0 plus eta minus y 0 t minus tau 0 minus eta into t minus tau 0 to the power q. So, this q actually the number. So, depending on the conditions of the cutting operations. So, from experimentally this number is can be determined. So, for different materials and cutting conditions. So, q will be different.

So, in the numerical analysis we will see what is the value of q is taken in this case. So, if we will substitute x equal to x 0 in this equation we will obtain. So, for. So, this if x 0 is the steady state condition. So, if we substitute x equal to x 0 then this equation becomes. So, this x 0 double dot plus e by m x 0 dot plus k by m x 0 plus k 3 k x 3 by m x 0 cube equal to k x w by m into. So, these parameter to the power q.

So, as in case of the static deflection taking x 0 as the static deflection. So, this part will be equal to 0. So, then. So, now, neglecting this part. So, the remaining part can be written as zeta double dot. So, due to steady state condition. So, one can take this is equal to k by m x 0 t plus alpha 1 3 by m x 0 cube equal to k x w by m v tau to the power q. So, this is the condition one can take.

So, now. So, the equation will reduce to this form that is zeta double dot plus c by m zeta dot plus k x by m zeta plus 3. So, you just see when you expand these term that is x plus zeta cube. So, here you have put x equal to x 0 plus zeta cube.

So, you just see. So, if you expand these thing then this becomes x 0 cube plus zeta cube plus 3 x 0 square zeta plus 3 x 0 into zeta square. As zeta is very small the zeta cube and higher order terms can be neglected or if you want to keep also you can keep these up to cubic non-linear term and so, there will be a term of 3 x 0 square into zeta and another term with 3 zeta 3 x 0 into zeta square.

So, the perturbation will contain all these terms. So, that is why. So, this additional term you can see. So, zeta double dot plus c by m zeta dot k by m zeta. So, this is the additional term you are getting that is 3 k x 3 by m, x 0 square zeta plus k 1 by m zeta plus 3 k x 3 by m into x 0 into zeta square and plus k x 3 by m zeta cube plus k x w by m v tau to the power q.

So, it will be equal to k x w by m. So, v tau x 0 t plus eta into eta t minus tau 0 to the power q. So, this is due to the time delay, delay term. So, now, this can be written as, k x w by m. So, v tau, v tau x 0 t to the power q plus k x w q by m. So, this is equal to or v tau x 0 t to the power q minus 1 into this the term it can be written.

So, now we can give or rewrite these equation in this form that is zeta double dot plus c by m zeta dot plus. So, we can combine the term with zeta. So, this becomes k x by m plus 3 k x 3 by m into x 0 square zeta. So, you just see. So, now, the coefficient of zeta. So, equal to k x by m plus 3 k x 3 by m x 0 square. So, it depend on the initial condition that is initial x 0.

So, this will be the coefficient. So, this coefficient is generally the non-linear natural frequency of the system. So, that is omega n square. So, plus 3 k 3 by m x 0 zeta square plus k x 3 by m zeta cube. So, we have a quadratic term and a cubic term and other parts we have taken to right hand side. So, which can be acting as a forcing term. So, here we can take this rho equal to v by omega R.

So, these omega R into rho equal to v we have taken. So, that is why this rho equal to v by omega R. So, it is written in terms of now it is written in terms of rho. So, right hand side is written in terms of rho.

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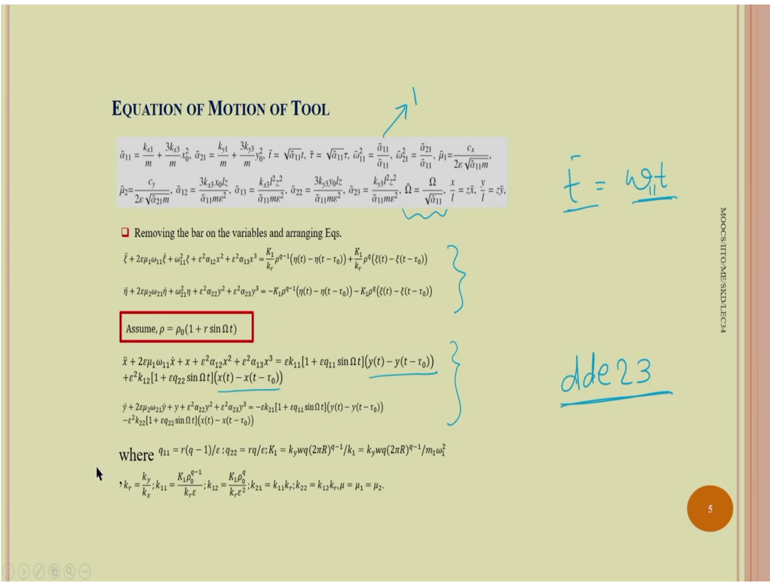

So, these are the parameters one can use. So, for example, one can to write these complicated equation in a simplified form. So, one can use these parameters for example, alpha 11 one can use is equal to k x 1 by m plus 3 k x 3 by m x 0 square. So, it will represent the natural frequency of the systems square of the natural frequency of the system similarly alpha 2 1 can be taken k y one by m plus 3 k y 3 by m into y 0 square.

So, these non-dimensional time can be written as these t bar equal to root over alpha 11. So, that is omega t. So, for example, generally we take these t bar equal to these is non dimensional time equal to omega into t. So, this for the x direction we have taken this is equal to omega 11 or alpha 11.

So, this is radian per second into radian per second into second. So, this becomes radiance. So, that is the angle. So, we are taking this non-dimensional time t bar. So, t bar equal to root over alpha 11 t then tau bar. So, this is the delayer time also we can non dimension h that thing. So, delay time also can be written root over alpha 11 into tau omega 11 square equal to. So, this is the non-dimensional omega 11 we are taking.

So, omega 11 square equal to alpha 11 by alpha 11. So, you just see we have taken it in such way that this term becomes 1. So, then omega 21 this is alpha 21 by alpha 11. So, mu 1. So, this is to represent the damping we are taking this mu term. So, mu 1 bar equal to c x by 2 epsilon.

So, here we are keeping or using the bookkeeping parameter epsilon that is why we have this epsilon here. So, mu 1 bar equal to c x by 2 epsilon root over alpha 11 bar by m. Similarly, mu 2 bar equal to c y divided by 2 epsilon root over alpha 21 m.

Similarly, alpha 12 can be written, alpha 13 and alpha 22 alpha 23 omega bar this is the frequency non dimensional frequency of rotation. So, rotational frequency we have taken equal to rotational frequency you have taken equal to capital omega. So, we can non-demisionalize this frequency that is omega bar equal to omega by root over alpha 11 bar.

So, then this link parameters x by l we have taken z x bar and y by l also we have taken z y bar. So, now, removing the bar from these. So, now, using these thing the equations can be written in using a bar form. So, that removing this bar we can have these two equation. So, one in zeta one in zeta and other in eta zeta is in x direction and eta in y direction.

So, our equation now reduced to zeta double dot plus 2 epsilon mu 1 omega 11 zeta dot plus omega 11 square zeta plus epsilon square alpha 1 to x square plus epsilon square alpha 13 x cube equal to k 1 by k r rho to the power q minus 1 into eta t minus eta t minus tau plus k 1 by k r rho to the power cube zeta minus zeta t minus tau 0.

Similarly, zeta equation in y direction our equation is reduced to eta double dot plus 2 epsilon mu 2 omega 21 eta dot plus omega 21 square eta plus epsilon square alpha 22 y square plus epsilon square alpha 23 y cube equal to minus k 1 rho to the power q 1 into eta t minus eta t minus tau 0 minus k 1 rho to the power q zeta t minus zeta t minus tau 0.

So, here. So, this rho is a rotational is a function of the rotational speed. So, that thing can be written as a constant speed plus a variable speed. So, if we are assuming the variation to be sinusoidal, then we can ride these rho equal to rho 0 plus r sin omega t.

So, here either. So, if it is rotating with a constant speed then rho will be equal to rho 0, but when we are assuming the rotation to be varying then that variation we can write in a sinusoidal form. So, that is why it will reduce to. So, rho can be written as rho 0 into one plus r sin omega t.

So, here you just see. So, we can write now this equation in this form. So, it can be written x double dot. So, now, replacing we can write down these equation in this form that is x double dot plus 2 epsilon and mu 1 omega 11 x dot plus x plus. So, this zeta is replaced by x and this eta is replaced by y for better understanding. So, now, it is written in this form that is x double dot plus 2 epsilon. So, the these two equation you can check. So, here we got this omega 11 equal to as we are writing in this form.

So, x double dot plus 2 epsilon mu 1 omega 11 x dot and x plus epsilon square alpha 1 to square plus epsilon square alpha 13 x cube equal to epsilon k 11 1 plus epsilon q 11 sin omega t into y t minus y t minus tau plus epsilon square k 12, 1 plus epsilon q 22 sin omega t x t minus x t minus tau 0.

Similarly, in this y direction this is the equation can be written. So, here we have taken this q 11 equal to R q minus 1 by epsilon q 22 R q by epsilon k 1 equal to k y w q 2 pi r to the power q minus 1 divided by k 1 and that is equal to k y w q 2 pi R to the power q minus 1 divided by m 1 omega 1 square.

Similarly, k r is taken to be k y by k x k 11 equal to k 1 rho 0 to the power q minus 1 by k r epsilon k 1 2 equal to k 1 rho 0 to the power q by k r epsilon square k 21 equal to k 11 in to k r k 22 equal to k 1 2 into k r and mu equal to mu 1 equal to. So, here taken mu equal to mu 1 equal to mu 2. So, considering symmetric. So, we can write down these two equation in this form.

So now. So, as we got the governing equation of motion. So, to see these equation is not similar to the, what we have studied in case of the damping equation particularly in the right hand side the equation the forcing part in the damping equation is not similar to the what we have written it here.

Here the forcing is slightly different it is written in a slightly different way what. So, the left hand side contains these quadratic and cubic nonlinearity. So, if is equal to 0. So, we can have a damping equation with both quadratic and cubic non-linearity. Actually you know in a governing equation. So, you can have two-part complementary part and the particular integral.

So, the complimentary part also will contribute to the vibration of the system that is the free vibration of the system and these particular integral will be due to the forcing part of the system. So, now, these now we have to solve these equations you just say this equation is written in the form of a delay differential equation. So, here the delay part is considered in this equation. So, this is the delay differential equation.

So, you can solve this delay differential equation using these d d e 20. So, using d d e 23 command of the MATLAB or you can write your own equation solver to solve these things also we have discussed previously that by using this Runge Kutta method also you will be able do find the solution or solution of this non-linear governing equation of motion. So, let us take the example or how to solve it using this method of multiple scale.

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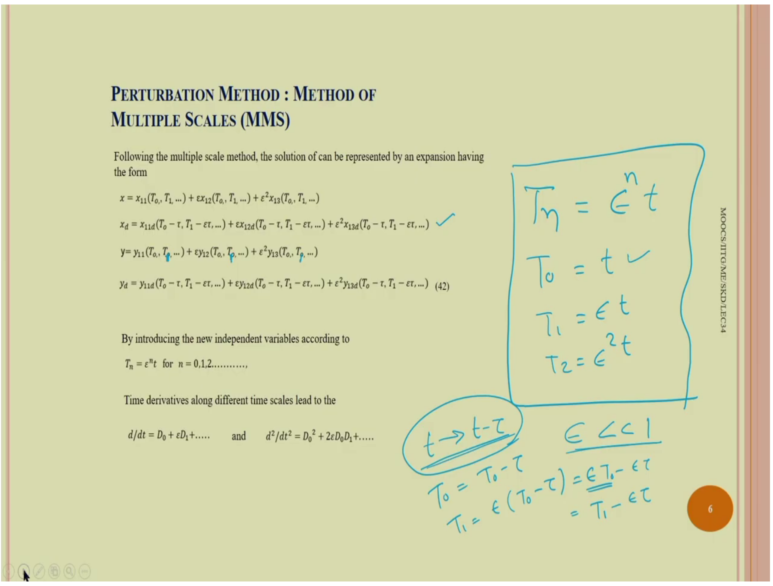

So, as you have seen due to the presence of the nonlinearity. So, it will not yield any close form solution. So, as it is not or we are not having any close form solution. So, we can go for some approximate solution. Approximate solution can be obtain by using many different methods such as methods of averaging, method of (Refer Time: 27:59) Poincare method, homotropy method and normal firm methods method of multiple scales also.

So, here method of multiple scales is used to solve these equation. So, here for the x and y we can take the conventional expansion. So, here of two second order expansion is taken. So, x is taken to be x 11 T 0 T 1 plus epsilon x 1 2 T 0 T 1 plus epsilon square x 1 3 T 0 T 1. Similarly, y is taken to be y 11 T 0 T 1. So, this is T 0 T 1 plus p epsilon y 1 2 T 0 T 1. So, that one T 1. So, this is T 0, T 1 can be written.

So, here T n. So, you can recall that a T n is nothing, but epsilon to the power n t. So, T n. So, that is why T 0 equal to t and T 1 equal to epsilon t and T 2 equal to epsilon square t. So, these are the different time scales similar to the time scale we are using in your watch that is for example, this hour hand, this minute hand and the second hand.

So, here we are taking three different time scale that is T 0 equal to t, T 1 equal to epsilon t and T 2 equal to epsilon square t where epsilon is a book keeping parameter which is very very less than 1. So, as it is very very less than 1e. So, these T 1 and T 2 r further less and less than these time scale t.

So, now, for the delay part you can note for the delay part we can use in a similar way. So, x d is written as x 1 1 d T 0 minus tau. So, this t is replaced by t minus tau. So, t is replaced by t minus t is replaced by t minus tau for the delay differential equation or a delay part. So, delay time can be written by using t minus tau. So, this T 0 can be written as T 0 minus tau.

Similarly, T 1 can be written multiplying epsilon we can write this is T 0 minus tau. So, this epsilon T 0 is nothing, but. So, this is equal to epsilon T 0 minus epsilon tau. So, this epsilon T 0 is nothing, but our T 1 time scale.

So, this is T 1 minus epsilon tau. So, that is why it is written T 0 minus tau, T 1 minus epsilon tau similarly we can have these epsilon square for T 2 it will be epsilon square T 0 minus tau. So, it will be epsilon square T 0 is nothing but T 2. So, it will be T 2 minus epsilon square tau. So, that way it can be written.

So, these x d that is the delay. So, this is the displacement in the previous cut that is x d can be written equal to x 11 d T 0 minus tau, T 1 minus epsilon tau and T 2 minus epsilon square tau. So, this is plus epsilon x 1 d 12 d T 0 minus tau T 1 minus epsilon tau T 2 minus epsilon square tau. Similarly, so, we can go up to epsilon square. So, epsilon square x 13 d into T 0 minus tau T 1 minus epsilon tau then it will be T 2 minus epsilon square tau similarly y d can be written using these y 11 d, y12 d and y 13 d.

So, writing these x y and the delay part of it in this form and using this d by d t equal to D 0 by D 0 plus epsilon D 1 and d square by d t square equal to D 0 square plus 2 epsilon D 0 D 1. So, we can reduced our equation and then separating of the order of epsilon.

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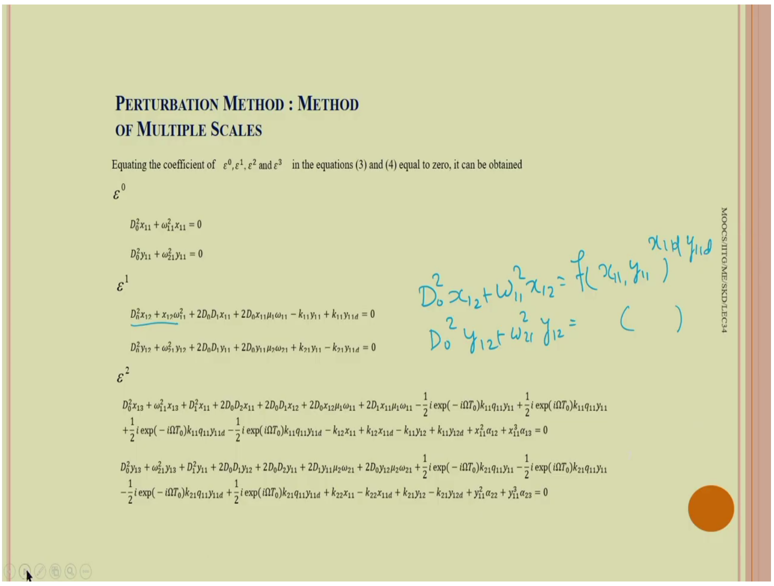

So, we can different order epsilon that is epsilon to power 0, epsilon to the power 1 epsilon to the power 2. So, we have these equations. So, our equation reduces to D 0 square x 11 plus omega 11 square x 11 equal to 0, D 0 square y 11 plus omega 21 square y 11 equal to 0. So, actually while writing this equation you must take care so, that the right hand side forcing term will be of the order epsilon so, that in the first case that is epsilon to the power 0 the right hand side you will get 0.

So, this will happen. So, in case the system is weekly the forcing is weekly non-linear. So, if the forcing is strong then in that case in the right hand side you we have a forcing term. So, which not will be equal to 0 and your equation of motion the solution of this equation will be different. It will be similar to that what we have studied in case of the hard excitation.

So, how this equation is similar to what we have studied in case of the weak excitation. So, that is why in the right side we have 0 0 term. So, our equation is reduced to this D 0 square x 11 plus omega 11 square x 11 equal to 0 D 0 square y 11 plus omega 21 square y 11 equal to 0.

So, already we know the solution of this thing where x 11 can be written by using. So, this is second order differential equation. So, by taking two constants it can be written similarly now for the after we get the solution for x 11 y 11.

So, we can substitute it in the second equation and then getting the solution for these x 12 and y 12 we can substitute those things in the third equation that is of epsilon square to get the required expression. So, here you can recall that we have to find the secular terms and eliminating those secular terms will get the reduced equation.

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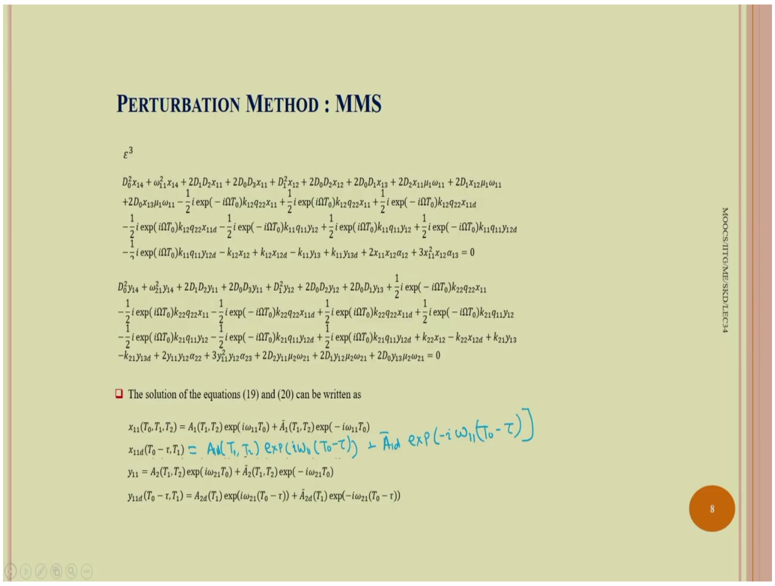

So, you just see as we are taking up to epsilon square. So, you can take up to epsilon q. So, these equation is written up to epsilon cube now the solution of this one solution of D 0 square x 11 plus omega 11 square x 1 equal to 0 and this D 0 square y 11 plus omega to 1 square y 11 equal to 0.

So, in this form. So, A it is equal to A 1. So, as this A 1 and A 1 bar will be constant. So, they will not be function of T 0. So, that is why they may be function of T 1 and T 2 that s why it is written A 1 T 1 T 2 it is not written T 0. So, you can note that it is not a function of T 0. So, x 11, T 0, T 1, T 2 can be written as A 1 T 1, T 2 sin e to power i omega 11 T 0 plus A 1 bar T 1 T 2 e to the power minus i omega 11 T 0.

So, here. So, you can note that. So, these i this A 1 bar is the complex conjugate of A1 A 1 bar A 1 and A1 bar are complex number and these A 1 bar is the complex conjugate of A 1 similarly. So, we can write down the second equation y 11, y 11 equal to A 2 T 1 T 2 e to the power i omega 21 T 0 plus A 2 bar T 1 T 2 e to the power minus i omega 2 1 T 0, Similarly, y 11 d can be written A 2 d.

So, A 2 will be replaced by A 2 d T 1 T 2 can be written A to the power i omega 21 t 0. So, T 0 will be replaced by T 0 minus tau similarly A 2 bar d equal to A 2 bar d T 1 e to the power minus i omega 21 T 0 minus tau.

Similarly, this x 11 d can be written equal to A 1. So, it will be A 1 d. So, here we are using this A 1 d T 1, T 2 e to the power i omega 11. So, these T 0 will be replaced by T 0 minus tau plus similarly A 1 bar d e to the power minus i omega 11 T 0 minus. So, it will be T 0 minus tau. So, this way it can be written.

So, now, writing these x 11, x 1 d, y 11, y 1 d. So, we can substitute these equation in these equation. So, you just see only we will take these 2 terms in the left hand side. So, these equation can be written D 0 square. So, if you recall. So, D 0 square x 12 plus omega 11 square x 12 omega 11 square x 12. So, other part will swift it to the right hand side.

Similarly, the second equation also we will write this is equal to D 0 square y 12 plus omega 21 square y 12. So, other part will shift to the right hand side. So, you can note that this other part is a function of both. So, it is a function of both x 11, y 11 and x 1 d 11 d and y 11 d. So, these are the function of both x and the delay part the system. So, that way we can write down these equations. So, now, substitute these solution they are.

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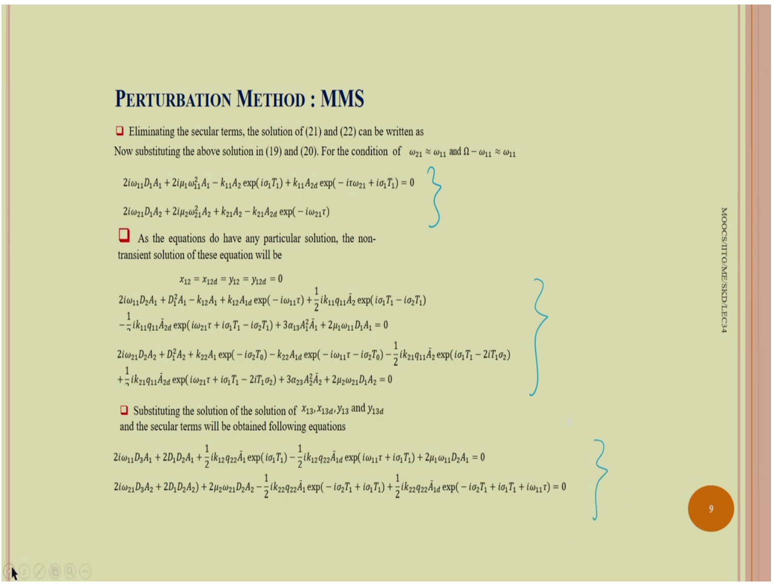

So, we can get. So, now, we can eliminate the secular terms. So, by eliminating secular term. So, our equation will reduce to these form and so, as you know that as the equation do not have any particular solution the non-transient solution of these equations will be x 1 2 equal to x 1 2 d equal to y 12 equal to y 1 to d equal to 0.

So, by putting these for steady state actually x 12 x 12 d equal to y 12 and y 2 d equal to 0. So, we can have these equation. So, now, we have these equation. So, substituting the solution of the x 13 x 13 d, y 13 and y 13 d in the secular term. So, we can obtain a set of equations like this and here also now, what we can write.

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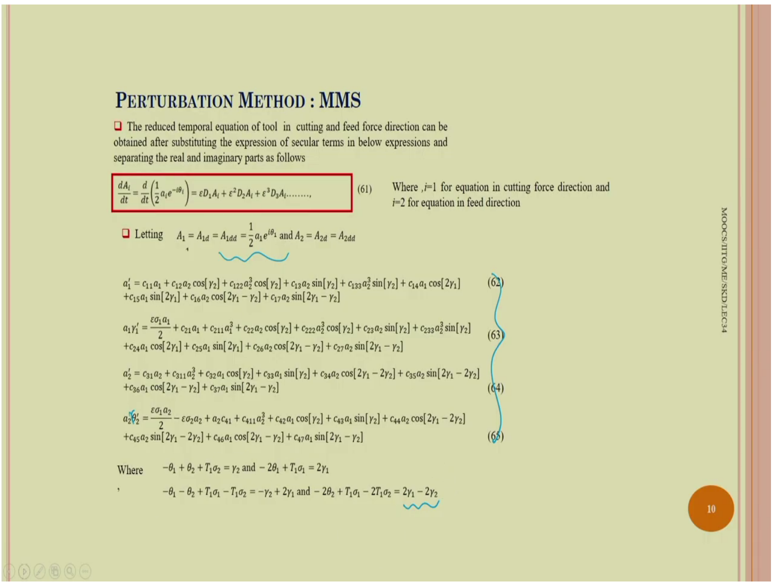

So, we can write these taking these A equal to A equal to half a e to the power i theta form. So, we can write A equal to as is a complex number. So, we can write A equal to half a e to the power i theta and this d A i by d t that is d A 1 by d t for example, will be equal to d by d t half A 1 e to the power i theta 1.

So, which will be equal to epsilon D 1 A1 plus epsilon square D 2 A 1 plus epsilon cube D 2 A 1. So, in that way if we can write. So, we can have these four equations. So, these four equations that is A 1 dash A 1 gamma 1 dash A 2 dash and A 2 theta 2 dash. So, this can be written in terms of gamma 2 dash because we are reducing this theta to gamma form to make the equation in non-autonomous form.

So, theta 1 can be written theta 1 minus theta 1 plus theta 2 plus T 1 sigma 2. So, here we are using the detuning parameter sigma 1 and sigma 2. So, minus theta 1 plus theta 2 plus T 1 sigma 2 equal to gamma 2 and minus 2 the theta 1 plus T 1 sigma one equal to 2 gamma 1.

And it can be written this minus theta 1 minus theta 2 plus T 1 sigma one minus T 1 sigma 2 equal to minus gamma 2 plus 2 gamma 1 and minus 2 theta 1 plus T 1 sigma 1 minus 2 T 1 sigma 2 equal to 2 gamma 1 minus gamma 2. So, by writing in this form.

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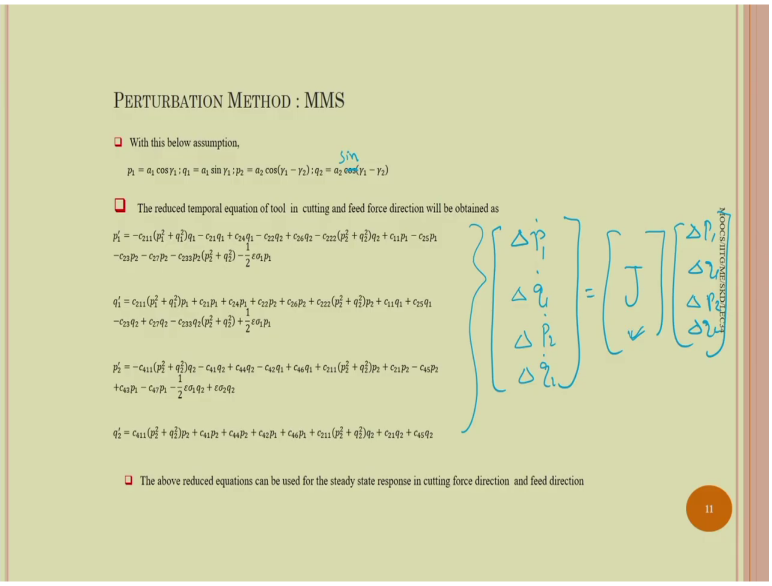

So, our equation is reduced and it can be observed that due to the present of the term a 1 gamma 1 dash, the trivial state conditions will not contain the perturbation. So, it can be. So, we cannot obtain the stability from the earlier equation. So, now, we have to make the transformation in such way that. So, we will get distinct equations which can be perturb to get these turbulent equation.

So, here we can assume this p 1 equal to a 1 cos gamma 1 q 1 equal to a 1 sin gamma 1, p 2 equal to a 2 cos gamma 1 minus gamma 2 and q 2 equal to a 2 cos gamma 1 minus gamma 2 q 2 equal to sin a. So, it will be a sin. So, it will be a sin gamma 1 minus gamma 2.

So, then substituting these equation. So, we can obtain this p 1 dash q 1 dash p 2 dash and q 2 dash equations. So, which we will perturb to get these delta p 1, delta p 1 dot delta q 1 dot, delta p 2 dot and delta q 2 dot. So, this thing can be written equal to this Jacobean matrix into delta p 1, delta q 1, delta p 2, delta q 2.

So, in this form can be written and so, this is the Jacobean matrix. So, now, finding these Eigen value of the Jacobean matrix. So, we can find whether the system is stable or not. So, as we are interested for the trivial state stability region or to find the instability region. So, we can find for the system parameter for which the system becomes stable or unstable.

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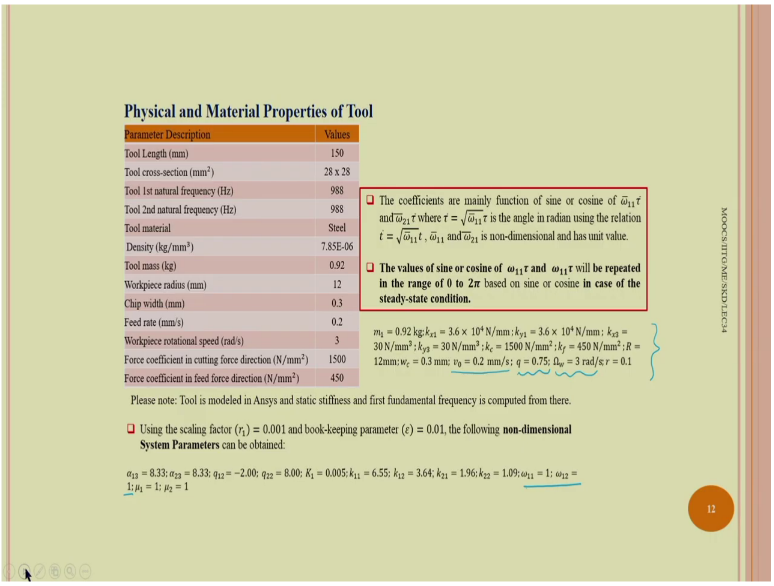

So, if the real part of the Eigen values of the Jacobean matrix is positive then the system is unstable and if the real part of the Eigen values of the Eigen values are negative, then the system is stable. So, taking these conditions. So, we can study the stability of the system.

So, here these parameters have been taken for example, you have taken the tool length to be 150 mm. So, tool cross section 20. So, it is square cross section we have taken 28 into 28 mm square. So, tool first natural frequency it can be observe that it is around 988 hertz, tool second natural frequency. So, as we are taking a square cross section. So, it is same that is 988.

So, tool material is taken steel. So, density is taken to be 7.85 2 E minus 6 kg per mm cube. So, this tool mass we have taken only 0.92 kg and work piece radius to be 12 mm. So, chip width that is 0.3, feed rate is taken to be 0.2, work piece rotational speed is 3 radian per second. So, force coefficient in cutting forces direction Newton per mm square this taken to be 1500 and the force coefficient in speed force direction. So, it is taken to be 450.

So, taking these parameters. So, the other parameters are obtained the coefficient are mainly function of sin and cosine of omega 11 tau and omega 21 bar tau where tau equal to omega 11 bar tau is the angle in radian using the relation tau bar equal to root over omega 11 bar t. So, omega 11 bar and omega 21 bar are non-dimensional and has unity value. So, we have taken it is in such way that. So, they have the unity value. The values of sin and cosine of omega 11 tau and omega 11 omega 21 tau will be repeated in the range of 0 to 2 pi.

So, based on the sin and cosine in case of the steady state condition. So, this value taking these value. So, the tau term have been determined and they have been taken in the analysis. So, here you just see the other parameter what we have taken in this analysis m 1 equal to 0.92. So, here it is written the mass is taken to be 0.92.

So, that is why m 1 is taken to be 0.92, k x 1 equal to 3.6 into 10 to the power 4 newton per m m, k y 13.6 into 10 to the power 4 newton per mm, k x 3 equal to 30 newton per mm cube, k y 3 30 newton per mm cube as we are taking these cross section square cross section. So, the stiffness are taken similar in both x and y direction. So, k c is taken to be 1500 Newton per mm square, k f equal to 450 newton per mm square, R equal to 12 mm, w c equal to 0.3 mm and v 0 you just see this is the feed taken to be.

So, feed v 0 equal to 0.2, v 0 equal to 0.2 millimeter per second. So, q that coefficient that power term you have seen q. So, q is taken to be 0.75 and this is taken this rotational speed of the spindle is taken to be 3 radian per second and r is taken to be. So, this the scaling parameter r. So, that is taken to be 0.1. So, taking all these parameters. So, you can absorb that the coefficients are coming to be in the. So, coefficients are coming in the proper order.

So, using scaling factor r 1 equal to 0.001 and book keeping parameter epsilon equal to 0.01, the following non dimensional system parameters are obtained. So, that alpha 13 equal to 8.33 alpha 23 8.23 q 12 minus 2 q 22 equal to 8 and k 1 equal to 0.005, k 11 equal to 6.55, k 12 equal to 3.64, k 21 1.96, k 22 1.09 omega 11 you just note. So, it is non-dimensionalize in such way that this omega 11 equal to 1 and omega 12 also equal to 1 and mu 1 equal to 1 and mu 2 also it is coming to be 1.

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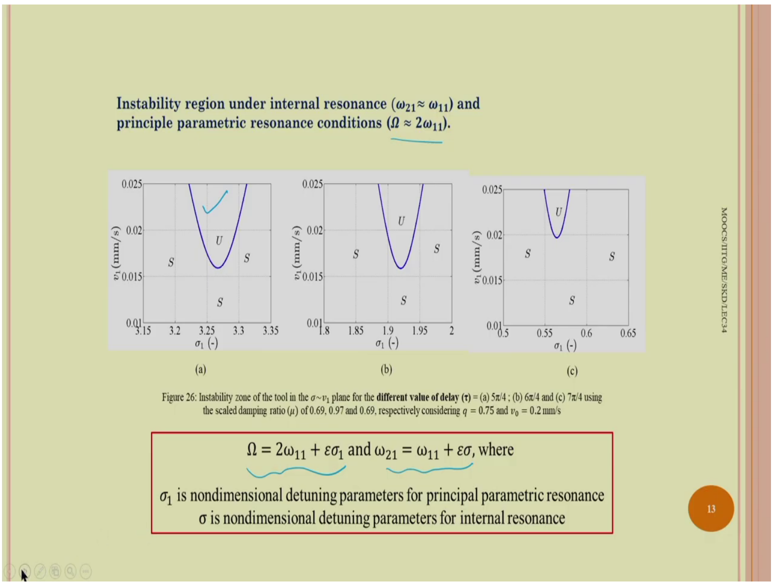

So, taking these parameters. So, and from the Jacobean matrix. So, one can find the stability just by finding the Eigen values and checking the Eigen values. So, for different value of b 1 and sigma. So, that is the feed rate. So, and sigma one is the detuning parameter. So, you can get this instability region.

So, the region outside these curve its stable inside this is unstable and outside it is stable. So, the operator has to take the feed rate and also the rotational speed in such way that it should in these stable region to have a vibration free or there will be no vibration of the tool.

So, if it is within these range in this instability region then the tool will vibrate. So, the obtained region. So, will be very much useful to find the operating conditions for which there will be no chatter in the turning operation. So, here the instability region under internal resonance condition here we have taken. So, you just see we have taken this omega 21 equal to omega 11.

So, that is why. So, we have this as this is 1 is to 1. So, we are considering this 1 is to 1 internal resonance condition in this case and along with these thing. So, we are considering the principle parametric resonance conditions also, that is capital omega equal to 2 omega 11 plus epsilon sigma 1 and omega 21 equal to omega 11 plus epsilon sigma.

So, this is scattering the internal resonance condition and this is scattering the this is scattering the condition for though we can tell this is external resonance condition that is principle parameter resonance condition, but this is due to the rotational speed the variation in the rotational speed in a periodic wave. So, due to that assumptions we are getting this thing.

So, sigma 1 is non dimensional detuning parameter for principle parameter resonance and sigma is non dimensional detuning parameter for internal resonance condition. So, you can observe that. So, in stability zone for tool in sigma v 1 plane for the different value of delay. So, three different values of delay has been taken.

So, in the first case the delay is taken to be 5 pi by 4, in the second case it is 6 pi by 4 and in this third case it is 7 pi by 4 for mu equal to 0.69, 0.97 and 0.69 respectively considering q equal to 0.75 and v 0 equal to 0.2 millimeter per second.

So, you can observe how the instability region varies. So, in this case in the last case the instability region when the delay is taken to be 7 pi by 4. So, it is to be. So, you can have a very small instability region and for the most to cases most of the cases the system remain in stable region. So, taking different delay conditions.

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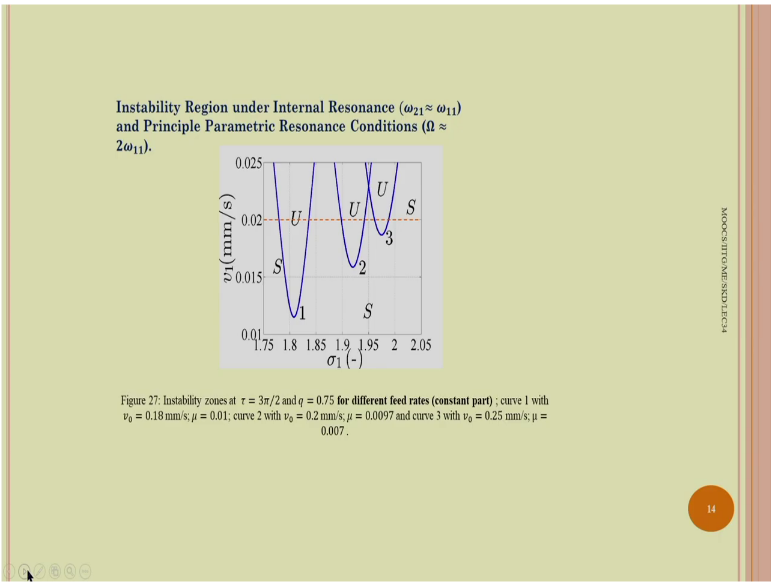

So, one can actually obtain this instability region and the operator will get the sense to operate it at a system parameter where the system will not have any vibration or there will be no chatter in the system. So, the instability region and under these internal resonance condition for principle parameter resonance it is studied here.

So, here tau is taken to be 3 pi by 2 q equal to 0.75 for different feed rates. So, 3 different speed rates have been taken. So, first curve is for feed rate v 0 equal to 0.18, second mu or first case v 0 equal to 0.18 mu equal to 0.01 for the second curve. So, you can see with v 0 equal to point 0 0.2 millimeter per second mu equal to 0.0097 and for the third curve you can see v 0 equal to point 0 0.25 millimeter per second and mu equal to 0.007.

So, here also you can see. So, in this third case we have the we have more stable zone and the operator can operate it freely in this or for a wide range of system parameter.

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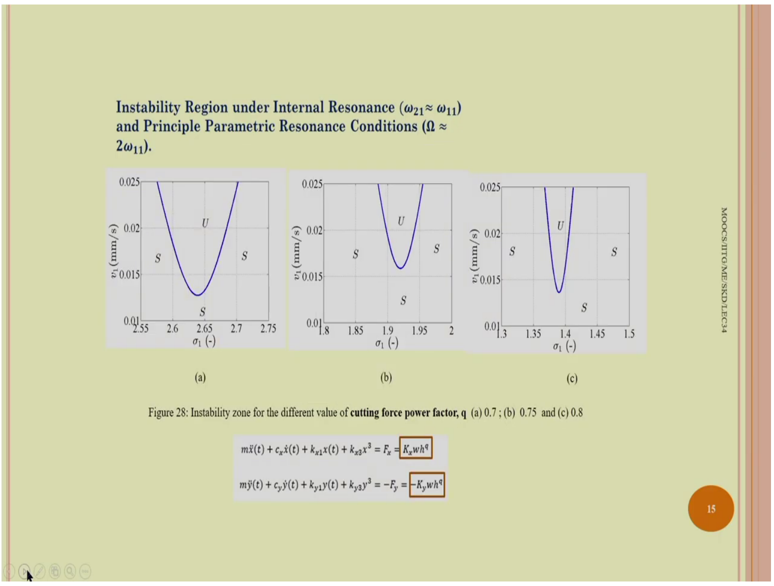

Similarly, so, we can vary different parameters and study the instability region.

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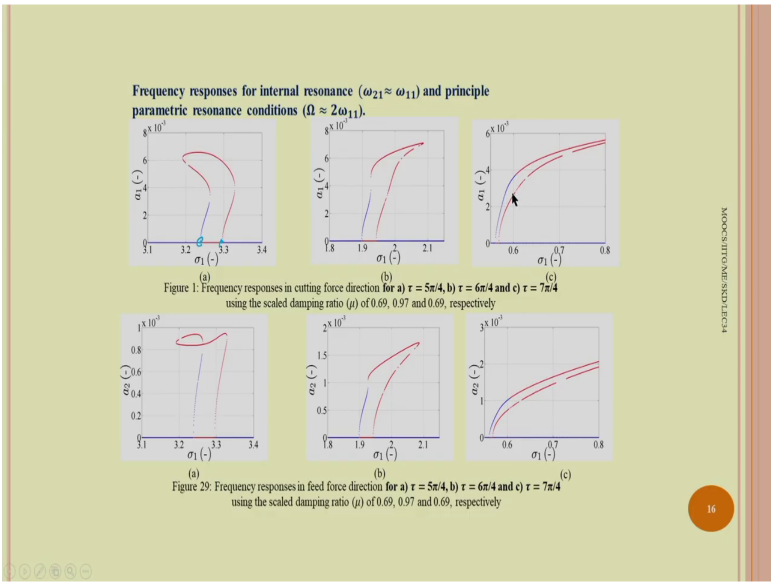

So, in addition to the parametric instability region. So, one can plot the frequency response plot also. So, in these unstable range. So, if someone operate this thing in unstable range for example, 1 to have the texture in the turning operation. So, in that case. So, he can operate it in the unstable region. So, when it is operated in this unstable region. So, it is one should know the frequency response plot and one can find so, how much response or how much vibration will be there in the system or in the tool.

So, if one can plot the frequency response plot so, one can see that these point correspond to these point correspond to the super critical pitch for bifurcation point and these correspond to the sub critical pitch for bifurcation point. So, here. So, in the super critical pitch for bifurcation point. So, after the supercritical pitch for bifurcation point with increase in the sigma.

So, that is with increase in the speed of the speed rotational speed. So, it will vibrate and these response can be obtained. So, from this a 1 one can see. So, what will be the response similarly for a 2 a 1. So, for a different value of tau it has been plotted. So, in the first case tau equal to 5 phi by 4, second case it is 6 phi by 4 and third case it is 7 phi by 4.

Similarly, for frequency response can be plotted for different feed rate. So, this is the first case it is plotted for a different tau value that is delayed and in the second case different feed rate for tau equal to 5 phi by 4, 6 phi by 4 and tau equal to 7 pi by 4. So, this is in the cutting force direction and the second set of figures are in the feed force direction that is a 2. So, this is a 1 versus the sigma 1, this is a 2 verses versus sigma 2.

So, you can see. So, in the first case it is 8 into 10 to the power minus 3, 8 into and in the second case in y direction that is in the feed direction. So, it is 1 into 10 to the power minus 3. So, in feed direction. So, it is less and in the cutting force direction the response is found to be more. So, this way one can study different frequency and force response plot for a different system parameters.

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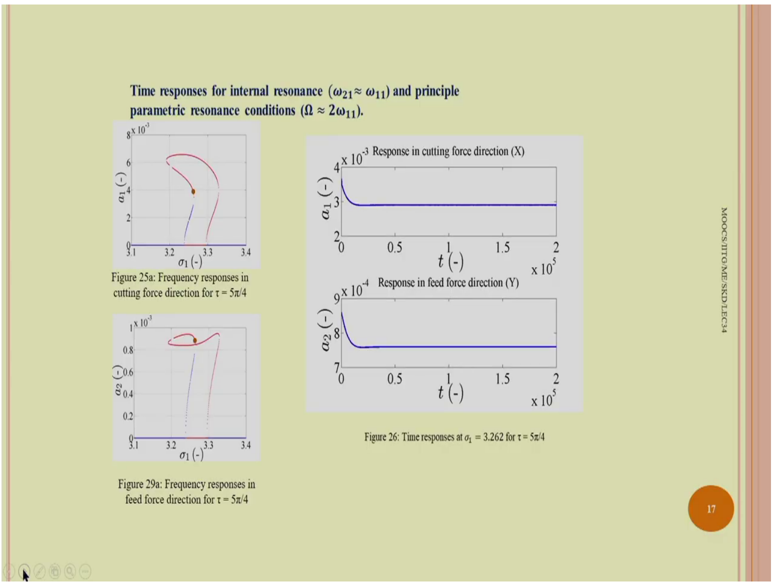

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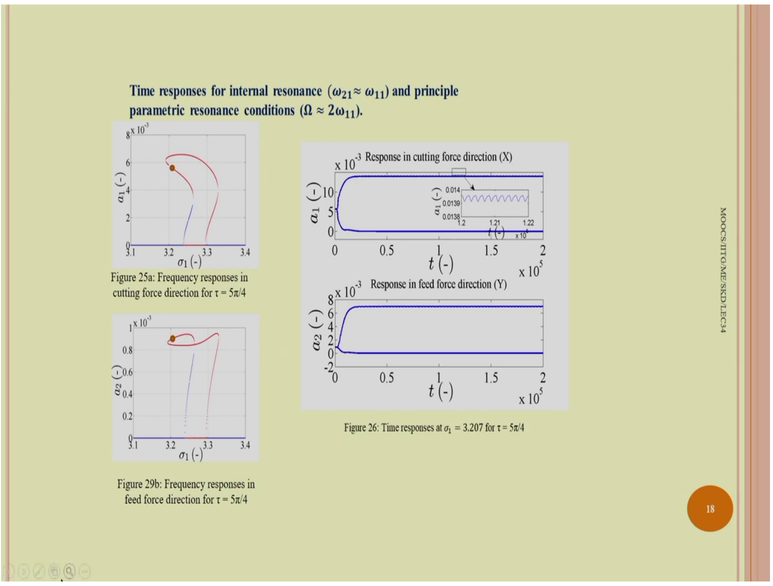

So, here the time response is also plotted to show the response in the cutting force direction and in the feed direction. So, two points have been taken. So, in this unstable zone. So, while taking in this unstable zone two points. So, you can see there is a there is a vibration or so, due to these vibration there will be chatter mark on the workpiece.

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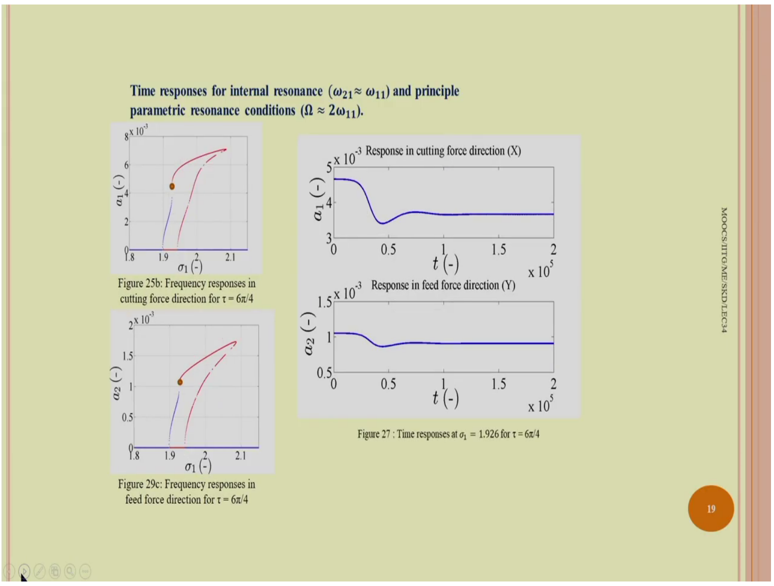

So, for different point similarly it has been obtained.

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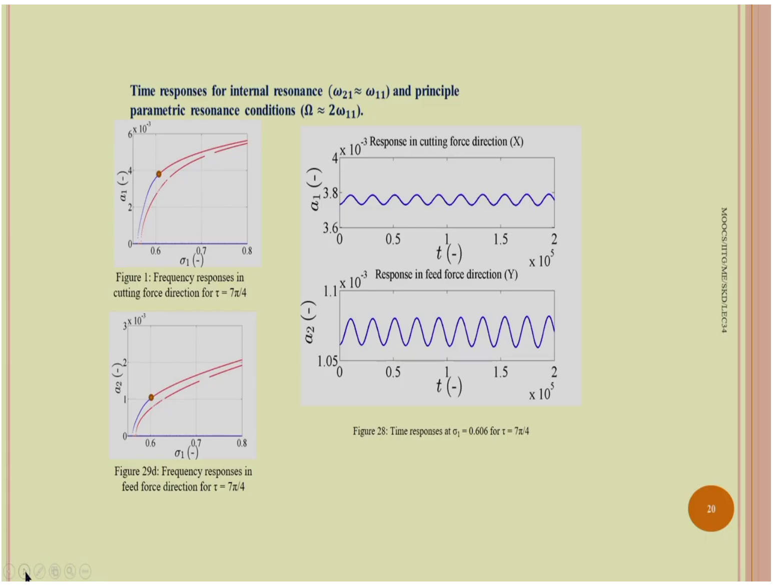

So, you can see this undulation is more in this case. So, time response for sigma 1 equal to 0.606 and tau equal to 7 phi by 4. So, one can study different marking or one can study different response of the turning operation in this way. So, in today class. So, we have discussed regarding the vibration of the tool. So, one can similarly study the vibration of the workpiece also.

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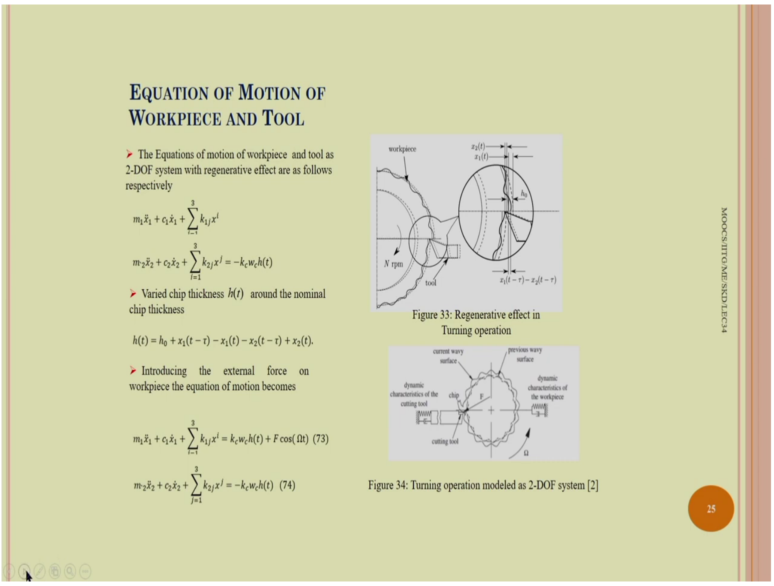

So, here. So, this part is given as an assignment to you to study the equation of motion of both work piece and tool when they are flexible. So, in that case this equation can be written in these form and following in the previous way. So, you can write down this governing equation.

So, after write down writing the governing equation of motion so, you can reduce it to this delay differential equations and then in by solving these solving using method of multiple scale. So, you can find the response of the system.

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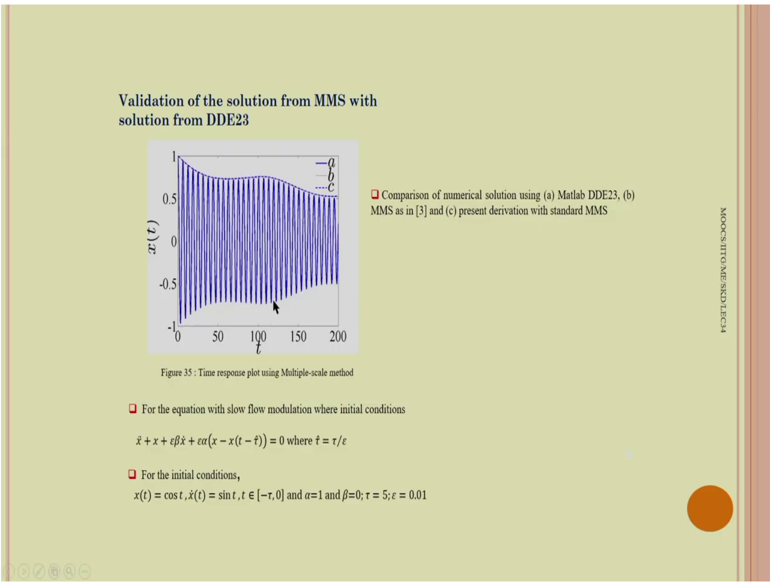

So, this part I am not going to discuss in detail but you can see that using this method of multiple scale on this DDE 23. So, you can verify that the results are coming to be similar or same.

So, the advantage of using this method of multiple scale is that in addition to getting the time response accurate time response you can get the frequency and force response plot of the system and the analysis can be extended to know the effects of different system parameter on the turning operation.

So, initially we have plotted this instability region. So, from the instability region we know the zone at which the system will not vibrate or the tool will not vibrate. So, in this case there will be no vibration of the tool and the workpiece.

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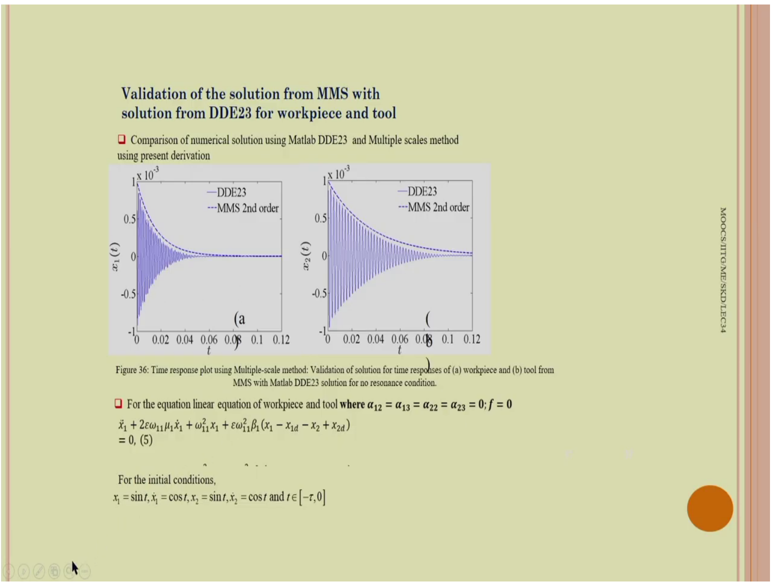

So, knowing those regions actually we can operate the system in that range. So, as to get the response of the system here more comparisons have been given between DDE to 23 and method of multiple scale second order method of multiple scale and these are the instability regions plotted.

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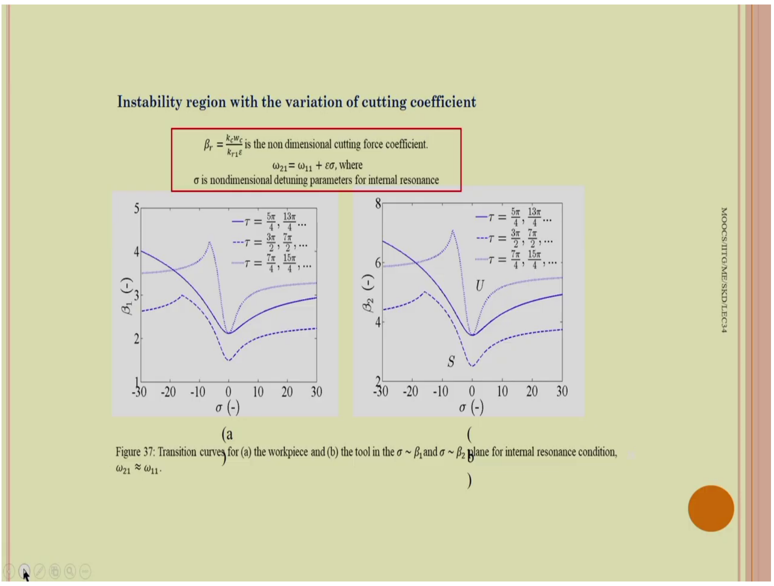

So, when both the tool and workpiece are flexible. So, this is the instability region. So, previous case when only the tool was flexible. So, you have seen we had a wide range of stability region, but in this case. So, the stability region is reduced because both the tools and the work piece are vibrating.

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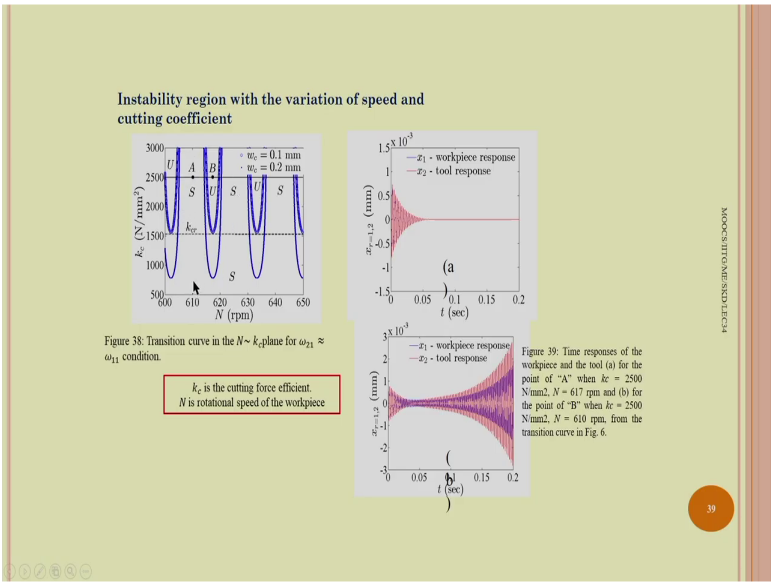

So, these are the instability region you can plot and you can see between these the response will in the unstable range the response will grow are the response is growing. So, there will be mark on the tool, a mark will be there will be mark in the workpiece.

So, there will be chatter mark in the workpiece. So, we have to avoid those to avoid those chatter mark. So, the system has to be operated in this stable range. So, this way you can study different manufacturing system. So, this is the turning operations I have told similarly you can extend this work for the milling operation also you can derive this equation for the forming operation like this rolling operation.

So, in many manufacturing system as well the process is non-linear. So, you can extend your analysis your linear analysis to the field of non-linear analysis and you can find the operating system parameter for which the systems will be stable and you can operate the system in the stable zone two half chatter free surfaces in the workpiece. So, thank you very much.