Nonlinear Vibration
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Lecture - 04
Force and moment based Approach, Lagrange Principle

Welcome to today class of Nonlinear Vibration. Today, we will start the 2nd module and in this module we are going to derive the equation of motion. So, particularly we were interested for deriving this non-linear equation of motion.

So, already you are familiar with derivation of linear equation of motion. So, it will be extensions of the equation of motion what you have derived in non-linear in linear case and we are going to get the non-linear equations of motion.

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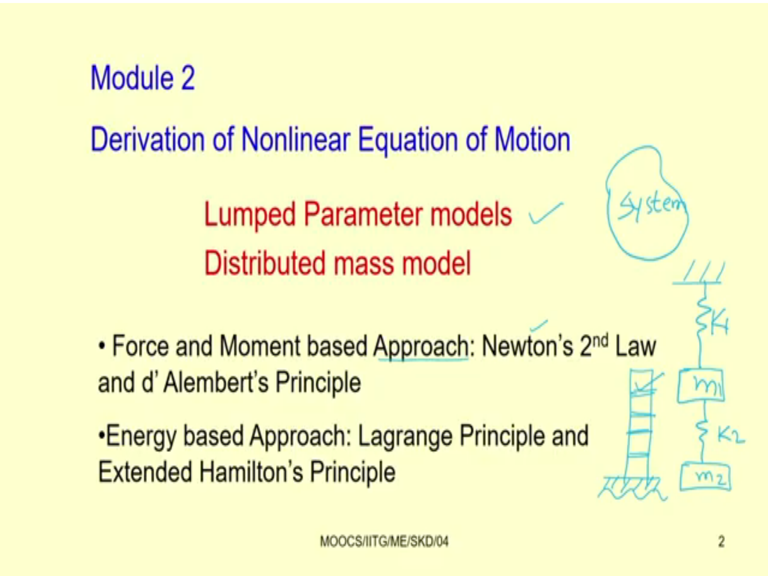

So, already you are familiar with two different type of models; one is the discrete or lumped parameter models and the second one is the distributed or continuous model. So, you have a system, the systems can be divided. So, a physical system can be modeled either by using this lumped parameter model or discrete model.

So, in discrete model you will have discrete number of degrees of freedom. For example, a single spring mass damper system, this is a single degree of freedom system single spring mass damper system so, this is a single degree of freedom system. So, if you put another mass so, it can be two degrees of freedom system.

So, this is m 1, this is m 2 so, now, this is K 1 and this is K 2. So, you have a two degrees of freedom system. Similarly, by adding number of mass and spring or a damper, so, your number of degrees of freedom system will increase and you can have a lumped parameter model.

Similarly, you can have a continuous system; the system may be so this is the system. The system may be continuous system for example; almost all the systems you are considering may be continuous system. For example, you just take the high rise building or a bridge or a cantilever beam or any other structure you just take the model of a let us take the model of a building.

So, in this building, so, there are several floors. So, these systems either you can model by multi degrees of freedom system or you can model this as a continuous systems also. So, almost all the systems or most of the systems you can model by continuous system, but in continuous system at the number of degrees of freedom system tends to infinite, it is very difficult to model and many times you have to truncate that continuous system to that of a multi degrees of freedom system.

Or the distributed mass system you can be model as a lumped parameter system by truncating the number of degrees of freedom. So, depending on the applications for example, if the system is subjected to earthquake excitation this building is subjected to lead earthquake excitation. So, as the excitation frequency is will be within few natural frequencies of the system, then by considering only few degrees of freedom systems, the modeling will be sufficient to have the response of the system.

So, depending on the applications these distributed mass models can be reduced to that of a lumped parameter models also. So, now, in lumped parameter model already I told you. So, we can have single degree of freedom model, two degrees of freedom model or multi degrees of freedom models.

And, in so these multi degrees of freedom system analysis can also be or it will depends on the single degree of freedom system model so, by knowing the single degree of freedom model. So, we can extend our analysis for multi degrees of freedom model. So, let us see how to derive this equation of motion.

So, equation of motion can be derived by force or moment based approach particularly we may use this Newton's 2nd Law or d Alembert Principle or energy based approach by using this Lagrange Principle or Extended Hamilton Principle. So, these equations or these principles are for fixed coordinate systems.

So, when the coordinate systems are moving coordinate system that time this Newton's 2nd Law or Newton's method can be converted to Newton Euler formulation, similarly this Lagrange formulation can be converted to Lagrange Euler formulation. Particularly these are useful in case of the robotic system when you have both fixed and moving coordinate systems.

So, let us see what is how we can apply the Newton's 2nd Law. So, today class we are particularly interested to study the Newton's 2nd Law and d' Alembert principle application to lumped parameter model. And, next class we are going to study this application of Lagrange principle and Hamilton principle or the energy principle.

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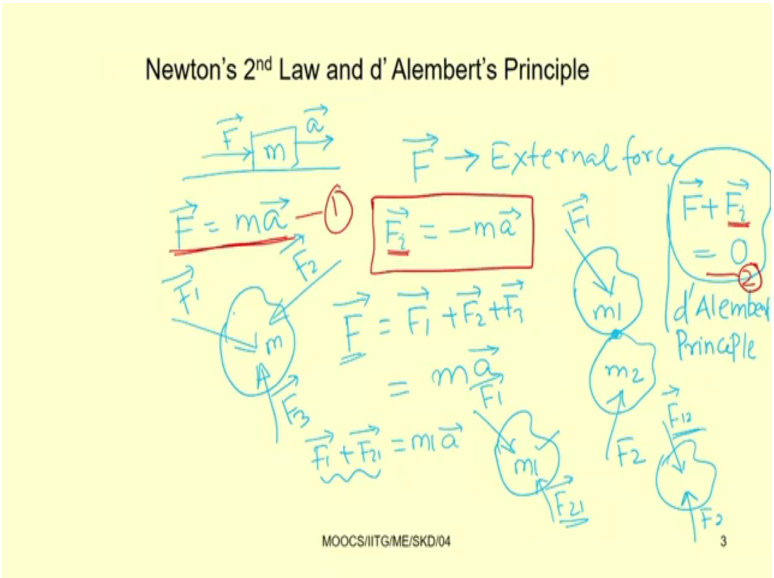

So, what is Newton's 2nd Law and how we can apply it to the system. Let us see that thing and then what is d Alembert principle, how is it related to Newton's 2nd Law, that thing also we can see. So, already you are familiar with Newton's 2nd Law where so, if you have a mass m, so, let this is mass m subjected to a force F, so subjected to a force F, so, it will have an acceleration a.

So, if we know from our kinematic analysis how to derive this acceleration of the system, then we can apply this formula F equal to mass into acceleration. So, this is for a; this is for a simple systems we are telling. So, where F is the vector and a is the vector also F is the force vector and a is the acceleration vector.

So, F equal to ma, so this is the Newton 2nd Law. So, let thus this is a simple system I have shown where only single force is acting let in a force or let in a system there are multiple forces are acting on the system, then this F can be the summation of vector sum of these F 1, F 2, F 3. So, a number of forces are acting.

So, we can write this is mass m. So, you can find so, this let F will be the vector sum of this F 1. So, it will be F 1 plus F 2 plus F 3. So, then we can apply this force F equal to mass into acceleration and this acceleration will takes place in a direction in that of the resultant force.

So, here F is the resultant external force. So, F you should note that F is the external force acting on the system. So, this is the external force acting on the system. So, let us consider the system. So, let this is one system and this is another system, so they are in contact. So, this is a two degrees of let us take the system. So, here let this is the force acting and this is the force acting here.

So, this is m 1 force m 1 mass this is m 2 mass, so this is force F 1 this is force F 2. So, in this case so, you can observe that at this point contact point. So, there is a pair of constant force acting on the system; so pair of constant force. So, the second mass will exert a force on the first mass and the first mass also will exert a force on the second mass.

So, if we are drawing the free body diagram of this system. So, this is F 1; so this is F 1 and then so, we will have a constraint force here. So, that is F by 2 on 1 F 2 1. So, this is F 1 external force. So, this is constant force, this is mass m 1. Similarly, we can draw the free body diagram of the second mass.

So, in the second mass this is force F 2 external force, F 2 and this is the force F 1 2. So, you know this F 2 1 and F 1 2 are equal and opposite. So, they are pair of constant force. So, in this case now to write down this equation of motion so, we can write this F 1. So, let; so this vector sum actually it will be F 1 plus F 2 1.

So, F 2 1 will be equal to mass into acceleration, so mass into acceleration. So, we have to find the resultant of these two and we can find it. So, sometimes it is very difficult to find this constant force, when the number of degrees of freedom of the system increases. So, this is the and also we are dealing with the; dealing with the vector. So, that is this force and acceleration terms.

So, this is the disadvantage [laughter] of use of this Newton's 2nd Law. So, as the number of degrees of freedoms increases so, getting this constant force and or writing this equation motion using the constant forces are difficult. So, because it is difficult to find out all these constant force, we have to draw a number of degrees of number of free body diagrams to write down the equation of motion.

So, if a number of forces, or if a number of mass are acting or number of masses are there in the system. So, we can go for this multi degrees of freedom approach and then by drawing individual free body diagram, we can apply the Newton's 2nd Law to find the equation of motion. So, let us see what we mean by so, what we mean by d' Alembert Principle.

So, d' Alembert principle also it is nothing but so, you can write this d' Alembert principle like F plus F i. So, this is the external force is F and F i is the inertia force F plus F i equal to 0. So, this is the equation of motion or this is the d Alembert principle. So, in d' Alembert principle; so, in d' Alembert principle the external force plus the inertia force equal to 0.

So, what is inertia force? So, inertia force is nothing but so, F i is the inertia force. So, F i equal to minus m into a F i equal to minus m into a. So, this is the inertia force, inertia force is minus m into a. So, in this equation, so in this Newton's 2nd Law F equal to ma. So, if you take this ma to left hand, so this becomes F minus ma equal to 0.

So, this minus ma can be written as F i. So, this becomes F plus F i equal to 0. So, this equation that is your equation 1 and equation 2, they are not different, they are same, but by bringing this inertia force to left hand side and so, we can have converted this dynamic system to a equivalent static system.

So, we know by in static system we can apply this virtual work principle. So, by applying this d Alembert Principle, we are bringing the dynamic system to that of an equivalent static system and where we can apply this virtual work principle. So, applying this virtual principle is the advantage of using this d Alembert Principle.

So, later we will see the generalized d' Alembert principle also. So, if a number of degrees of freedom are there for each degrees of freedom, so we have to first find what is the inertia force acting. So, then after finding this inertia force, the external force plus the inertia force we have to make it equal to 0 to find the equation of motion.

So, now you know the Newton's 2nd Law and d Alembert Principle. So, let us apply this Newton's 2nd Law and d Alembert Principle to some systems and study the equation of motion of the system.

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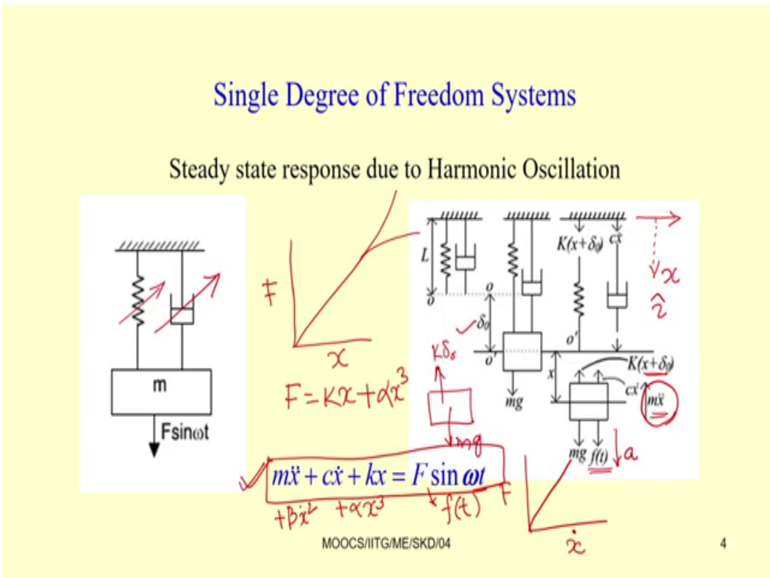

So, the familiar equation you know that is that of a spring mass damper system. So, in this case, so let us consider the initially first consider the spring to be linear and then we will consider the spring to be non-linear.

Initially we will consider the spring to be linear. So, the force displacement graph if you know. So, this is your x and this is force spring force the spring force it will be like this. So, initially it is spring force will be F equal to k x; F equal to k x. Similarly, if we are drawing, so if we are drawing this damping versus this x dot versus forcing. So, this is also c x dot.

So, this is x dot F versus x dot if you plot then also it is linear. So, you can have a linear system initially, then we will convert this system to that of a non-linear system. So, already I introduce to you two coordinate system; one is the generalized coordinate system and the second one is the physical coordinate system.

So, in the physical coordinate system so, you can introduce or you can take a physical coordinate. So, in this case let us take the physical coordinate for example, from the support condition. So, from the support let L is the length of the unstressed, spring and damper. So, now, by putting this mass; by putting this mass by attaching this mass there will be a displacement of delta 0.

So, it has come to this position. So, at this position so, if you draw the free body diagram. So, at this position so, if you draw the free body diagram. So, the weight is acting mg and the spring force. So, that is your K, so, delta 0. As there is no displacement, so, c x dot delta 0 dot equal to delta 0 dot will be equal to 0. So, this mg will be equal to K delta 0.

So, now from this position, so from this position if we slightly pull the system by an amount x, then it has come to this position. So now, at this position, so, the displacement equal to; so, we can write the total displacement of the system equal to x plus delta 0. So, if we are writing these equation of motions with respect to this fixed coordinate system.

So, let us take so, this is the position so, along these direction. So, we are taking this x direction and perpendicular to that we can take y direction, but this is a single degree of freedom system. So, we are using only one coordinate to express the motion of the system. So, now the displacement of this mass equal to so the displacement of this mass equal to x plus delta 0.

So, the spring force equal to k x plus delta 0 and damping force equal to c x dot at as delta 0 equal to constant. So, it is differentiation equal to 0. So, the damping force equal to c x dot and the spring force equal to k x plus delta 0. So, now, the inertia force equal to m x double dot. So, inertia force will takes place in a direction opposite to that of force. So, the force is acting in downward direction, so the body is moving in downward direction.

So, it will have a acceleration, so acceleration in this direction. So, body will accelerate in this direction. So, the inertia force will takes place in a direction opposite to that of the acceleration. So, it will take in the in this direction so, upward direction. So, the force is k x plus delta 0 is in upward direction.

Similarly, c x dot is in upward direction and mx double dot also is in upward direction and the downward forces are mg and that force what you are applying. So, let us apply some force F T which is a function of time, then this becomes downward force will be equal to m g plus F T and upward force becomes so, k x plus delta 0 plus c x dot plus mx double dot.

So, now by writing the equation of motion, so if we can write actually all of them in vector form. So, let us put a unit vector along this direction. So, this force so, we can write the total force acting on the system equal to mx double dot plus f t. So, it will be so, we are taking positive y in this direction; so, i minus k x plus delta 0 c x dot plus m x double dot.

So, all these things in opposite direction so, we will put minus I will be equal to 0. So, now, already you have seen this mg equal to K delta 0. So, by canceling that thing so, we can write the equation i reduces to this form m x double dot plus c x dot plus k x equal to F t. So, this F t we can write if we write this F t equal to F sin omega t, then this equation reduces to mx double dot plus c x dot plus k x equal to F sin omega t.

So, instead of taking this force F x equal to K x if you take it non-linear for example, let us take this is this way or this way. We can add plus alpha x cube, then so in the left hand side we can add another term alpha x cube and this will be the equation of motion. Similarly, if we are putting a damper non-linear damper, so we can put another term plus beta for example, beta x dot square or you can have the cubic nonlinearity also.

So, we can have a equation non-linear equation of motion. So, this way by using this Newton's 2nd Law or d Alembert Principle, so you can write down the equation of motion. Here directly I have told this d Alembert principle, that is mx double dot I have taken as the inertia force and I have written the equation of motion.

So, instead of taking this inertia force by applying Newton's 2nd Law first you can write the what are the forces acting for example, the forces acting will be equal to mg plus ft minus K x plus delta 0 minus c x dot. So, this will be equal to mx double dot that is Newton's 2nd Law.

So, by rearranging them, so, you can get the same equation of motion. So, this way either by using Newton's 2nd Law or by using d' Alembert Principle, so you can write the equation motion. So, for a non-linear spring, so you can draw an arrow. So, that will represent the non-linear, non-linearity in the spring.

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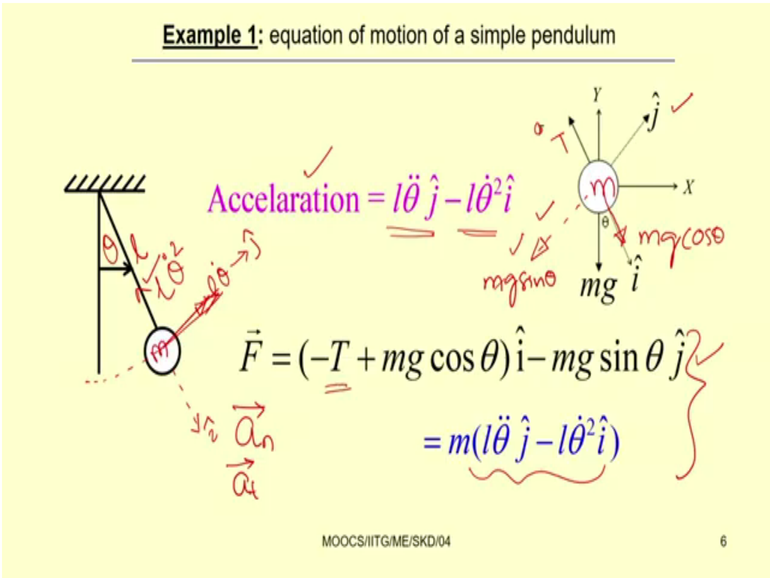

So, let us take another example. So, in this example let us take the simple pendulum. So, already in the introduction lecture also I told you about the simple pendulum. In simple pendulum, so you have a mass less string with a mass. So, let the mass of the let this mass equal to m. So, we have to find the equation of motion. So, let us use both Newton 2nd Law and d Alembert principle to find this equation of motion.

So, here as it is a rotating system, so either we can use the translation. So, in terms of this, so, let this angle is theta. So, either we can write in terms of the linear acceleration or the linear force will be equal to mass into acceleration in that form F equal to ma equation we can use or we can use this angular acceleration also. So, both a way we can solve this problem.

So, let the length of the pendulum equal to l. So, as this is theta, so it will be subjected to or it will have a velocity. So, velocity will be l theta dot. Similarly, so, let us take this direction.

So, let us take this direction as i unit vector along this direction as i, and perpendicular to that as j. So, as this is rotating or making a circular motion so, we know so, it will have acceleration two component of acceleration.

So, if this theta is uniform. So, if theta dot is uniform then we can write, so it will have; so it will have two components. So, we know that as this is rotating we have two component; one is the normal component and another is the tangential component.

So, normal component of acceleration and tangential component of acceleration. So, tangential component of acceleration will be tangent to the system so, along this direction. So, this is the tangential component and normal component will be towards this. So, normal component equal to l theta dot square so, it will be equal to l theta dot square towards the center and tangential component will be along the tangent to this thing and it will be l theta double dot.

So, we have l theta double dot and l theta dot square. So, these two terms will be there. So, this acceleration then we can write in vector form this acceleration as l theta double dot that is tangential. So, this is l theta double dot j minus so, as this i direction we have taken in downward direction. So, minus l theta dot square i l theta dot square will be opposite to the i direction.

So, the acceleration can be written as l theta double dot j minus l theta dot square i. So, this is the acceleration we know so, after knowing this acceleration so, our if we apply this Newton’s 2nd Law, then what is the external force acting on the systems we have to find. So, after getting the external force we can write this external force will be equal to mass into acceleration.

So, let us see what is the force acting on the system only two forces are acting on the system; one is the weight component that is mg and another one is the tension T. So, this is tension T; tension T in this string and this is mass m weight mg. So, this weight we can divide into two component.

So, one will be; so, one will be along this direction, which will balance, so which will balance the tension and the other component this component, so if this angle is theta. So, this is mg cos theta; mg cos theta and this term is mg sin theta. So, mg sin theta. So, T will balance this mg cos theta and mg sin theta is the restoring force acting on the system.

So, this is the external force acting on the system, but we can write down. So, write down the total force acting on the system in the vector way. So, as this i directions we have taken downward, so this mg cos theta minus T i so, the net force acting on the systems can be written in this form.

So, minus T plus mg cos theta i, then mg sin theta, so this is the mg sin theta so, this direction. So, mg sin theta. So, as we are taking this direction as positive j, so this direction equal to negative j, so minus mg sin theta j. So, this is the net force acting on the mass. So, this thing will be equal to according to the Newton's 2nd Law, this thing will be equal to mass into acceleration.

So, acceleration equal to already we have written this is equal to l theta double dot j minus l theta dot square i. So, if we are applying d Alembert principle, then we could have written. So, minus m into l theta double dot j, minus l theta dot square i equal to 0. So, this part can be brought to left hand side with a negative sign and putting that thing equal to 0. So, that is d Alembert Principle.

So, this is Newton's 2nd Law. So, this is Newton's 2nd Law we have applied. So, this is net force equal to mass into acceleration. Now, equating the i component and j component. So, we can get what is the tension in the string and also the resulting equation of motion.

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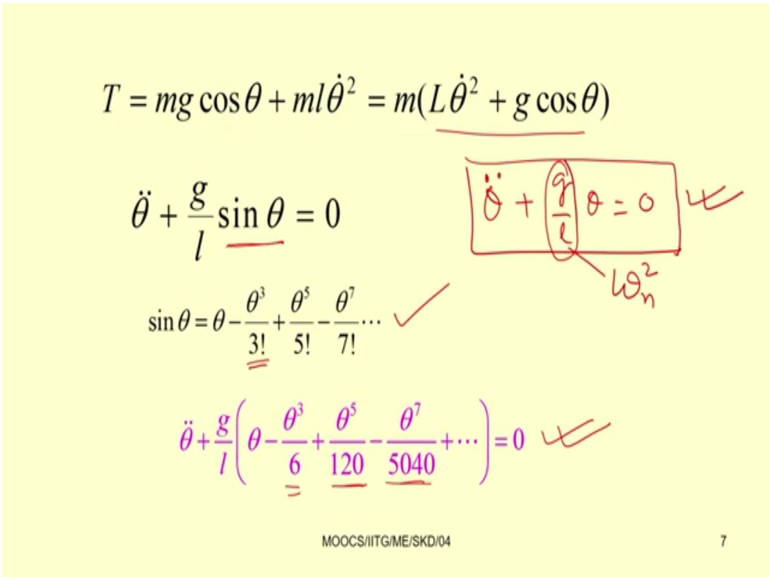

So, we will get this T equal to mg cos theta plus m l theta dot square or taking this m common so, we can have L theta dot square plus g cos theta. And, the second part, so taking the other component that is this j component, so, by taking j component so, it becomes minus mg sin theta will be equal to m l theta double dot. So, or we can write this theta double dot plus g by l sin theta equal to 0. So, already in the introductory class I have introduced or I have expanded this sin theta.

So, if theta is small then this motion will be; then this motion the sin theta can be written as theta and the motion can be written as a as theta double dot plus g by l theta equal to 0. So, this is the motion of a simple pendulum in linear form. So, already you are familiar with this thing, where this g by l is nothing but this omega n square g by l equal to omega n square.

But, if theta is not small then you can expand that theta and you can write sin theta equal to theta, minus 3 factorial plus theta to the power 5 by 5 factorial, minus theta to the power 7 by 7 factorial or expanding these 3 factorial, 5 factorial and 7 factorial. So, the equation of motion can be written as theta double dot plus g by l into theta minus theta cube by 6 plus theta to the power 5 by 1 20 minus theta to the power 7 by 5040 equal to 0.

So, you can check or you can see that so, these terms this is 1 by coefficient is 1 by 6, here the coefficient is 1 by 20 and here the coefficient is 1 by 5040. So, these numbers can be neglected. So, if theta is very small, then these numbers can be neglected, but if theta is large then this number cannot be neglected and you can have this type of equation of motion so, this non-linear equation of motion.

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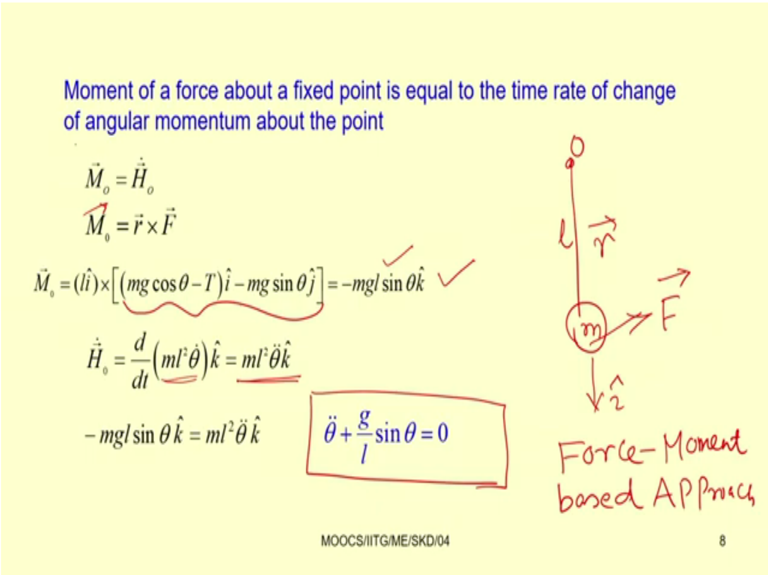

So, this way you can apply this Newton’s 2nd Law or d Alembert Principle to derive the equation of motion. So, let us apply this moment method also to find this equation of motion. So, here we have applied F equal to mass into acceleration, but we can apply this moment also moment we know moment of a force about a fixed point. So, this is the fixed point about which it is rotating so, this mass. So, this is mass m, so this is o. So, we know moment of a force about a fixed point is equal to the time rate of change of angular momentum about the point.

So, that is m 0 moment of the force that is m 0 will be equal to the rate of change of angular momentum that is H 0 dot or d by dt of H 0. So, where mg equal to so, if I am writing this as r vector that is position vector. So, the net force acting here we have seen. So, we can write the net force acting here equal to F, already we have written the expression for F. So, m 0 equal to r cross F.

So, we can write this m 0 equal to r cross F, so m 0 is a vector. So, m 0 equal to r cross F. So, this r here so, this length is l already we have taken this direction i. So, we can write this li. So, that is r equal to li and F already we have written got this expression for so up to this. So, we got this expression for F. So, this is the expression for F that is mg cos theta minus T i minus mg sin theta j.

So, now this i cross i equal to 0 and i cross j will give k. So, this becomes minus mgl sin theta k. So, m 0 so, we got the moment of the force equal to minus mgl sin theta k. Now, this H 0 dot equal to d by dt of H 0. So, H 0 equal to the angular momentum; so, angular momentum equal to ml square theta dot.

So, angular momentum so, already we know this angular momentum and theta dot equal to this angular velocity. So, angular momentum equal to i omega, so that is equal to m l square into theta dot. So, this H 0 dot equal to d by dt of ml square theta dot so, this is taking place about k so, this equal to ml square theta double dot k.

So, now equating this m 0 equal to H 0 dot. So, we can write minus mg l sin theta k equal to m l square theta double dot k. So, from this thing also we can write this equation theta double dot plus g by l sin theta equal to 0, which we can expand up to the desired order depending on our application.

So, this way also by using this angular momentum, so moment and this angular momentum principle, so we can derive this equation of motion. So, now, we know both force and moment based approach. So, this that is why this approach is known as force and moment based approach.

So, if you have a rotating system. So, either you can apply this force based approach or moment based approach or if you have only translating system. So, you can go for the force based approach. So, this is this way you can derive this equation motion of a single degree of freedom system. So, you can similarly derive this equation of motion of multi degree of freedom system.

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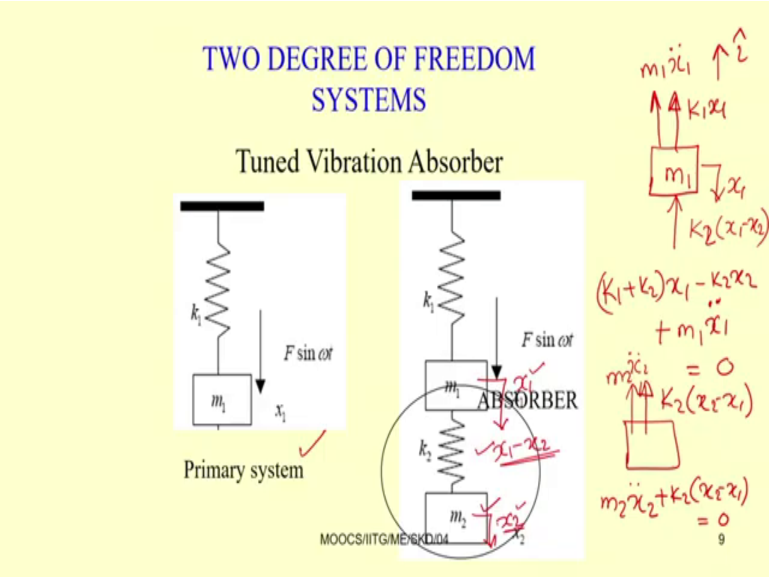

So, let us take the case of a tuned vibration absorber. So, already in the introductory class I told so, how we can observe the vibration of a primary system. So, this is the primary system by adding the secondary system. So, the secondary system contained the spring and mass. So, here now we can write the equation of motion of the system.

So, for example, for the primary system let the displacement equal to x 1 and for the secondary system the displacement equal to x 2. So, the equation of motion now can be written. So, this is 2 degrees of freedom system, so it can be written. So, by drawing the free body diagram and applying either the Newton’s 2nd Law or the d Alembert principle.

So, first we will derive this equation of motion of a linear system, then we will convert the system to that of a non-linear system and we will see some of the applications also. So, for example, so in this case this mass m 1. So, let us give a displacement x 1 downward. So, if we have given a displacement x 1 downward the spring will be stressed and it will exert a force in upward direction.

So, that force equal to k 1, x 1, but the spring k 2; the spring k 2 as it is compressed. So, as it is moving downward by x 1 it is compressed by x 1 and also due to the displacement of this mass m 2. So, it has a displacement of so, if the mass x 2 is moving downward. So, the spring will exert a force in upward direction.

So, the relative motion of the spring will be x 1 minus x 2. So, as the relative motion of the spring will be x 1 minus x 2. So, we can write this k 2 x 1 minus x 2 force also will act in upward direction. So, that is k 1 x 1 minus x 2. So, our equation of motion now can be written. So, our equation of motion we can write. So, this is the two forces acting on this thing.

And, either if we are applying this Newton's 2nd Law, then this will be equal to so, net force will be equal to k 1 x 1 plus this is k 2; this is k 2 k 2 x 1 minus x 2. So, this is the net force acting, so it will be k 1, then this is k 1 plus k 2 x 1 minus k 2 x 2. So, this is the net force acting will be equal to m 1 you just see this is this will be equal to m 1 x 1.

So, we can write down this thing plus we can take it that side inertia force let the inertia force let us take. So, inertia force will be equal to m 1 x 1 double dot plus m 1 x 1 double dot equal to 0. If, we are writing using Newton's 2nd Law so this direction, so let we are taking this is positive i direction. So, then it will be negative direction.

So, it could have been written k 1 plus k 2 x 1 minus k 2 x 2 will be equal to minus x 1 double dot or we can write this equation by taking this inertia force that side equal to so, this for. So, k 1 plus k 2 or m 1 x 1 double dot plus k 1 plus k 2 x 1 minus k 2 x 2 equal to 0.

Similarly, now for the second mass only the spring force is acting. So, the spring force so, if we are putting in this direction and so, the spring force can be written. So, I can write this way also so, k 2 into x 2 minus x 1. So, you just see it could have been downward direction k 2 into x 1 minus x 2 or upward direction in k 2 x 2 minus x 1, and similarly inertia force by putting this inertia force m 2 x 2 double dot.

So, we can write this equation m 2, x 2 double dot plus k 2, x 2 minus x 1 equal to 0. So, this way we can write by drawing the free body diagram. So, we can write down the equation of motion.

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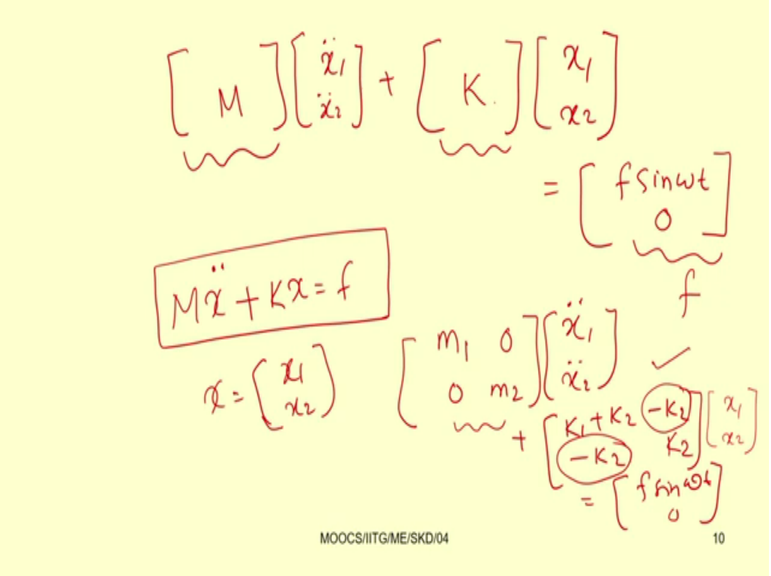

So, here one can write this equation actually in matrix form also. So, this into x 1 double dot, x 2 double dot plus x 1, x 2 ok. So, here one thing we have forgotten.

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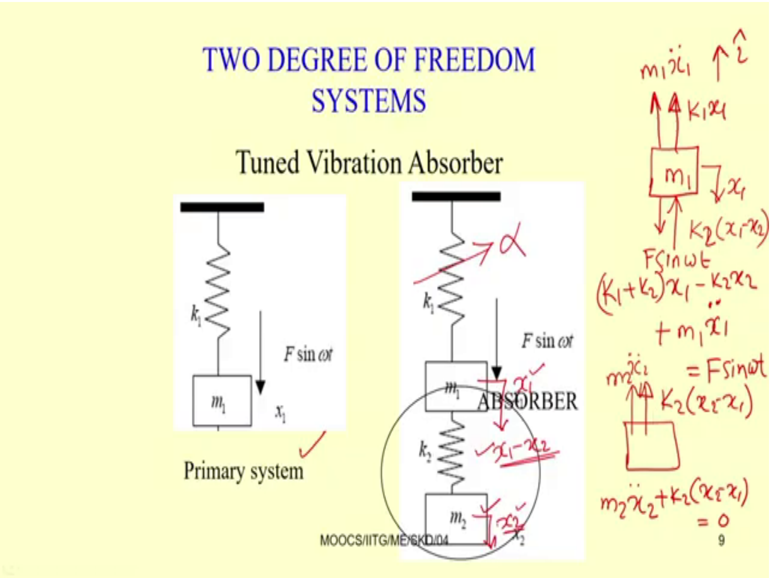

So, in this right hand side, so this is not equal to 0. So, we have a; so, here we have a force acting. So, this force can be written as F sin omega t. So, if this force is not there then it will be 0 at the force is acting so, we can write this force equal to F sin omega t. So, here we have to write the force external force acting that is F sin omega t.

So, here this will be equal to f sin omega T and second mass no force is acting no external force is acting. So, this will be equal to 0. So, this matrix so, we can write it in matrix form. So, this matrix is known as the mass matrix and this matrix is known as the stiffness matrix. So, we can write and this vector can be written as the force vector.

So, the equation of motion can be written in this form M x double dot where x is a vector. So, this is x 1 and x 1 and x 2. So, this x can be written as x 1 x 2 vector, so, M x double dot plus

K x equal to f. So, this is the so, you just see this looks like similar to that of a single degree of freedom system, but here this mass and stiffness are in matrix form.

So, in this particular case you can note that this mass matrix. So, if we have we are writing for this present system. So, we have this m 1 0, 0 m 2 the equation is m 1 0 0 m 2, x 1 double dot, x 2 double dot plus. So, we can have this k 1 plus k 2 minus k 2 minus k 2.

So, let us see and k 2 equal to F sin omega t and 0. So, you can see this mass matrix is uncoupled. So, this is uncoupled, but the stiffness matrix is coupled. So, here the off diagonal terms are there. So, as the off diagonal terms are there. So, it is coupled due to the presence of this term and the this term. So, it is coupled.

So, in case of non-linear system so, now, we can introduce the non-linearity in the system for example, let us put this to be non-linear. So, in that case the equation so, you can write this k 1, let us put alpha non-linearity alpha. So, in this case we can have another force that is alpha x 1 cube.

So, this equation becomes non-linear that way, similarly we can introduce the non-linear force between this and this. So, in that case it will be k 2 or in addition to k 2 we can have another non-linear term and we can write down this equation of motion. So, later or in linear case you know the modal analysis method is used to uncoupled this equation of motion. So, if it is in linear, then we may apply the modal analysis method to make the system non-linear.

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So, what is modal analysis method? As our aim so we can write this equation for example, this way K x equal to let us take the case of a pre vibration. So, let it is equal to 0. So, to or if it is force vibration then it will be f.

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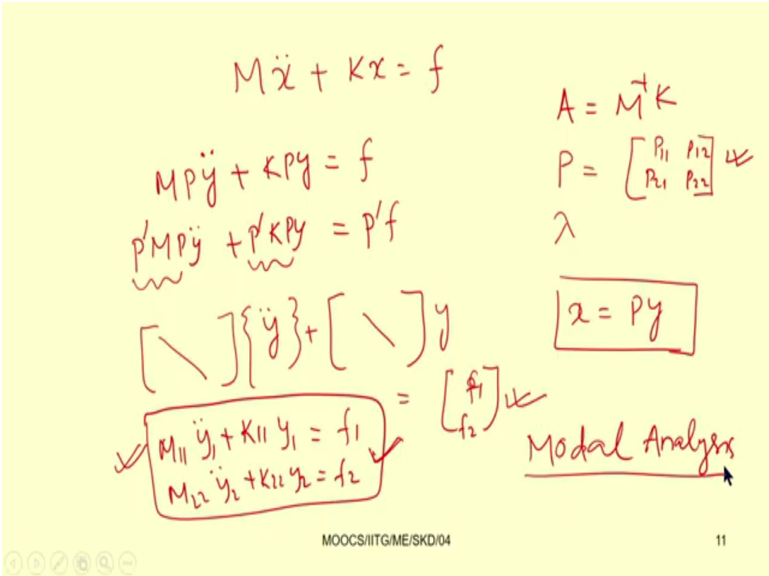

Now, we can write let A equal to M inverse K; let A equal to M inverse K and P is the eigen value P is the eigen vector not eigen value lambda is the eigen value. So, if this is a two degrees of freedom system, so we will have two eigen values corresponding these two eigen values. So, we can have you have two eigen values, and corresponding these two eigen values, so we have two eigen vectors.

So, this is P 1 1, P 1 2 and P 2 1, P 2 2, so, we have two eigen vectors. So, now, after getting this P, so what we can do? So, let us write this x equal to P y. So, we can put x equal to P y. So, then this equation reduce stress to M P y double dot plus K P y this is equal to K P y equal to f.

Now, pre multiplying by P transpose; P transpose M P y double dot plus P transpose K P y equal to 0. So, from the property of this eigen values and eigen vectors. So, we can see that this matrix is diagonal, also this matrix is diagonal. So, we have a diagonal matrix here. So, this matrix is diagonal. So, this is y double dot plus this matrix is also diagonal y this is not 0.

So, this is we have multiplied P transpose. So, this is P transpose into f. So, this is equal to P transpose into f. So, you have two terms here, so that is f 1 f 2. So, now, you just see independently you can write this two degrees of freedom system in a in the form of a single degree of freedom. So, then it will be for example, it will be M 11 y 1 double dot plus K 11 y 1 equal to f 1.

The second equation will be M 22 y 2 double dot plus K 22 y 2. So, this is equal to f 2. So, this way you can write 2 equation of motion. So, in case of non-linear so, in the linear part you can uncoupled this way, but you will have some coupled term in this non-linear part. So, which you can take into right hand side and you can use as a forcing form.

And, you can still write this equation in this form M 11 y 1 double dot plus K 11 y 1 equal to f 1 minus some other term, so which is a function of your y 1 and y 2. Similarly, you can write the second equation it will be f 2 minus g y 1 by 2 and some other terms also will be there. So, in that way by using this modal analysis method so, you can reduced. So, you can reduce this coupled equation to that of a uncoupled equation.

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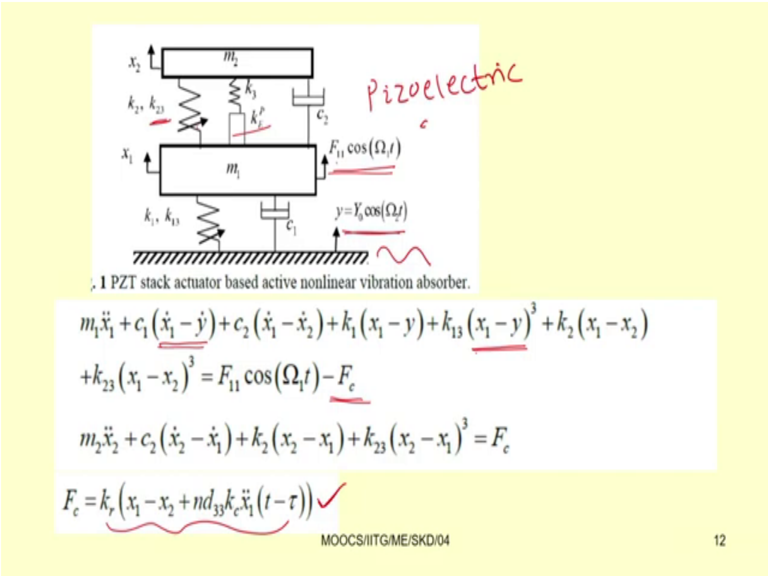

So, let us see one more example of vibration observer. So, in this case you can make the system non-linear. So, this is the primary; this is the primary bottom one is the primary system.

So, it is subjected to base excitation, the base is also moving and also it is subjected to a force F 11 cos omega 1 t. So, here we are using a piezoelectric stack actuator this is a piezoelectric. So, this is piezoelectric stack actuator. So, in this piezoelectric stack actuator so, if you apply this voltage then it can behave as a actuator and it can give a force.

So, the equation of motion of the system you can write by drawing the free body diagram. So, here in this case you just see this is active system. So, you can control the vibration on this active system by applying an external energy that is in this case you can apply this voltage and you can control the vibration of the system.

So, you can write the draw the free body diagram. By drawing the free body diagram, so you can write down the equation of motion for example, as this is moving x 1. So, directly so, this is m x 1 double dot plus c x 1 dot minus. So, this is x 1 this is x 2, so this is not y dot. So, this is c 1 you just see the bottom is moving with y equal to y cos omega t.

So, as this is moving y cos omega t. So, this will be this is moving x and this is y. So, that will be relative displacement between these two. So, that is why it is x 1 dot minus y dot plus c 2. So, here the damper is between so, damper is between mass 1 and mass 2. So, this become c 2 into x 1 dot minus x 2 dot plus k 1.

So, this k 1 is having x 1 minus y plus we have a non-linear term here. So, that is x 1 minus y cube plus k 2. So, you just see from k 2 here k 2 is between this mass 2 and mass 1. So, then the displacement relative displacement equal to x 1 minus x 2 so, k 2 into x 1 minus x 2.

Similarly, for k 23 so, the non-linear term is written in terms of k 23 so, k 2 3 into x 1 minus x 2 cube. So, will be equal to the forcing acting here, that is F 11 cos omega T minus the forcing due to this stack actuator, so that is equal to F c. So, the forcing of the stack actuator can be written in this form.

So, later you will know more regarding this stack actuator when we will study the application of this thing, then the second equation can be written this way m 2 x 2 double dot plus c 2 into x 2 dot minus x 1 dot plus k 2 into x 2 minus x 1 plus k 2 3 into x 2 minus x 1 cube equal to F c. So, this way by drawing the free body diagram. So, you can either apply Newton's 2nd Law or d Alembert principle to find the equation of motion for discrete system.

So, tomorrow class we will take some example of continuous system for example, Euler Bernoulli Beam equation we will take with large curvature also with small curvature that will be Euler Bernoulli simple Euler Bernoulli Beam equation. And we will take with large curvature, so that this Euler Bernoulli Beam can be converted to some non-linear equation of motion.

So, we will see some application oriented examples also and in addition to that we will apply the Lagrange principle to discrete and distributed mass system.

Thank you.