Nonlinear Vibration
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Lecture - 05
Extended Hamilton's principle

So, welcome to today class of Non-linear Vibration. So, this is the 2nd class of 2nd module. So, in the last class we have started with the derivation of non-linear equation of motion.

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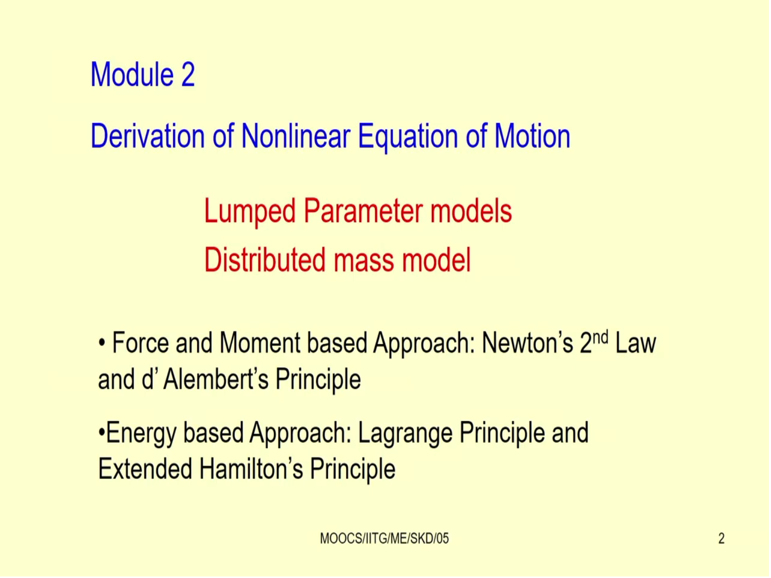

So where, we have discussed regarding the lumped parameter model and or the distributed mass model. So, two approaches just I have introduced and I have told you regarding the force and moment based approach; where we have used these Newton 2nd law and d' Alembert principle to only the lumped parameter models.

So, today class we will see these application of this force and moment based approach that is the Newton 2nd law or d' Alembert principle to this distributed mass system or continuous system. So, distributed mass is otherwise also known as continuous system.

So, today class first half we will study regarding the force and moment based approach that is application of Newton's 2nd law and d' Alembert principle to continuous system. Two examples we will see how to derive this equation of motion non-linear equation of motion, how they are different from this lumped parameter models. And then we will go for this energy based approach where briefly I will tell the application of this Lagrange principle and extended Hamilton's principle to derive the equation of motion.

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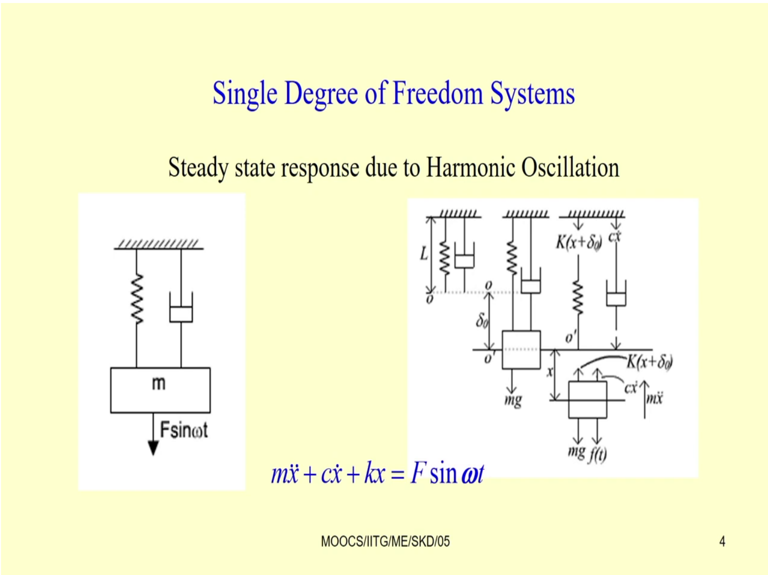

So, last class we have started with the derivation of equation motion of a simple spring mass system. So, initially taking these spring mass system as linear. So, we have derived this equation of motion as mx double dot plus cx dot plus kx equal to F sin omega t. So, here you have to draw the free body diagram and write down the equation of motion by using this Newton 2nd law or the d' Alembert principle.

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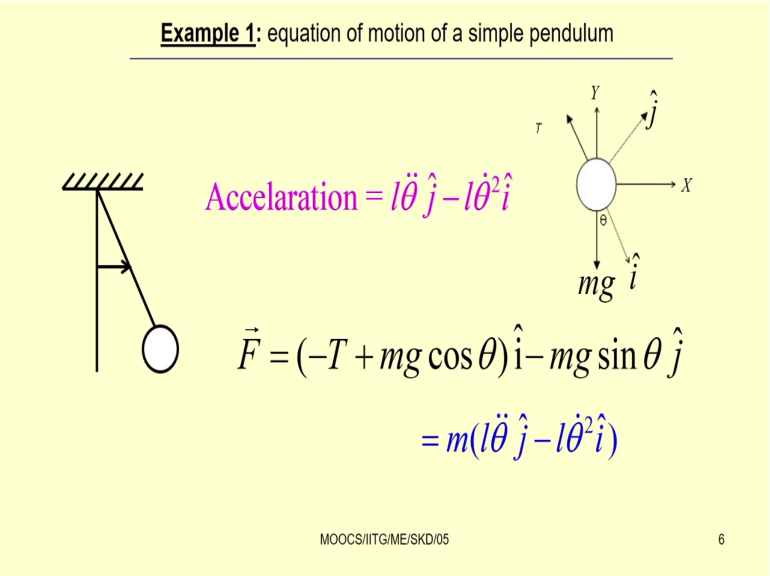

So, later the spring and the damper can be converted to non-linear spring or damper by considering a soft spring or hard spring. So, then this equation of motion of the spring mass systems can be written using the non-linear equation of motion.

We have taken a second example of these a pendulum simple pendulum and in the simple pendulum already by writing the forces and this acceleration. So, we can write in vector form we can write down the equation of motion.

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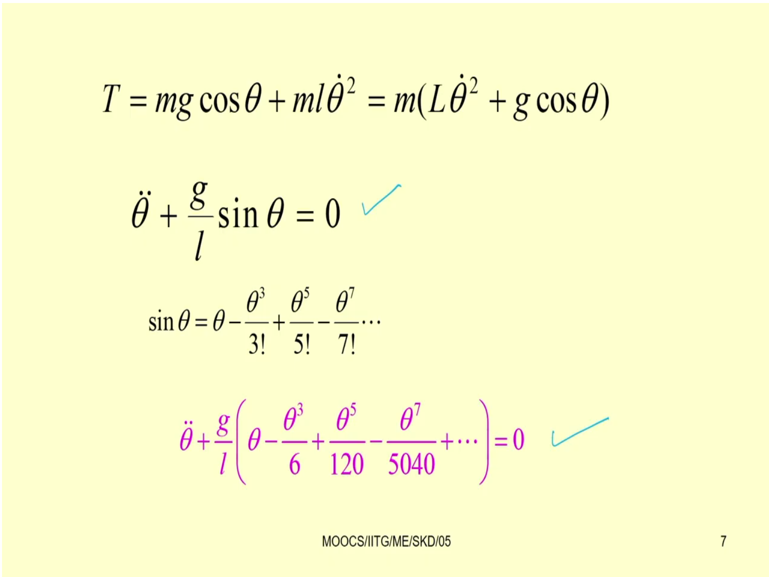

So, the equation of motion governing equation of motions we got in this form that is theta double dot plus g by l sin theta equal to theta. When this theta is small so, then we can consider the sin theta equal to 0 and the governing equation reduces to theta double dot plus g by l sin theta equal to 0.

But, if theta is not small then we can expand this sin theta equal to theta minus theta cube by factorial 3 plus theta phi by factorial 5, minus this theta to the power 7 by factorial 7 and we can write down the non-linear equation of motion in this form.

So, where this equation reduces to theta double dot plus g by l into theta minus theta cube by 6 plus theta 5th by 120 minus theta to the power 7 by 5040. You just see the coefficient of theta 5 and theta 7 are very very small. So, theta 5 coefficient is 1 by 120 and theta 7 coefficient is equal to 1 by 5040. So, these two terms can be neglected and one conveniently can keep the equation up to third order third.

So, the equation will reduce to theta double dot plus g by l into theta minus theta cube by 6. So, which is similar to that of a doping equation, what we have studied what we have seen in the introductory class.

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Moment of a force about a fixed point is equal to the time rate of change of angular momentum about the point

$$\vec{M}_o = \dot{\vec{H}}_o$$

$$M_0 = \vec{r} \times \vec{F}$$

$$\vec{M}_0 = (l\hat{i}) \times \left[(mg\cos\theta - T)\hat{i} - mg\sin\theta\hat{j} \right] = -mgl\sin\theta\hat{k}$$

$$\dot{\vec{H}}_0 = \frac{d}{dt}\left(ml^2\dot{\theta}\right)\hat{k} = ml^2\ddot{\theta}\hat{k}$$

$$-mgl\sin\theta\,\hat{k} = ml^2\ddot{\theta}\,\hat{k} \qquad \ddot{\theta} + \frac{g}{l}\sin\theta = 0$$

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So, also I told how by using this moment also you can find the equation of motion.

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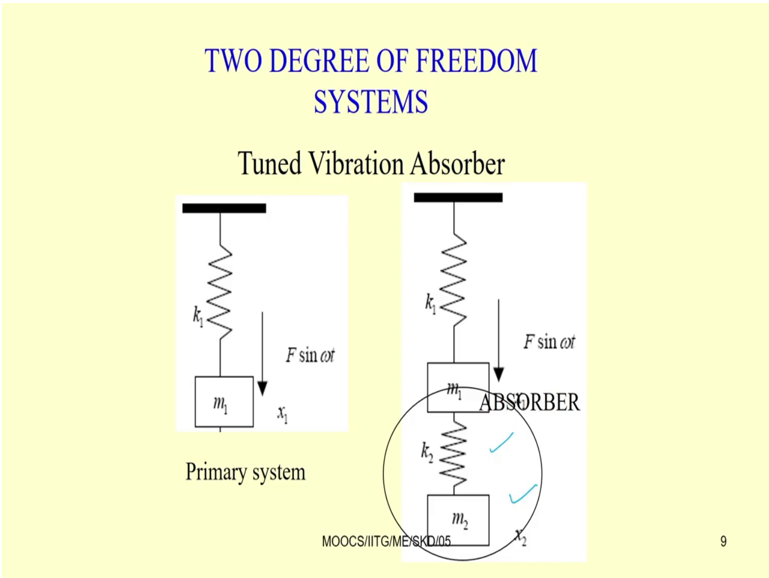

For the same system then we have considered the two degrees of freedom system. So, the primary system, in primary system we have added the observer. So, another spring and mass system and derive this equation of motion for the tuned vibration absorber. So, it can be made non-linear by using this non-linear springs also.

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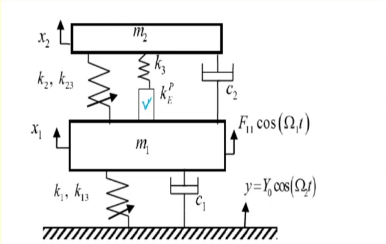

. 1 PZT stack actuator based active nonlinear vibration absorber.

$$m_1\ddot{x}_1 + c_1(\dot{x}_1 - \dot{y}) + c_2(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - y) + k_{13}(x_1 - y)^3 + k_2(x_1 - x_2) + k_{23}(x_1 - x_2)^3 = F_{11}\cos(\Omega_1 t) - F_c$$

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) + k_{23}(x_2 - x_1)^3 = F_c$$

$$F_c = k_r\left(x_1 - x_2 + n d_{33} k_c \ddot{x}_1(t - \tau)\right)$$

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So, we have seen the example of a active vibration absorber, whereby we have put these piezoelectric electric stack actuators. So, this element is the piezoelectric stack actuator by putting that piezoelectric stack actuator so and drawing the free body diagram. So, one can write the equation of motion of the system. So, this shows the equation of motion of the system. So, this F c is the actuator force. So, which can be written in this form; so, this way by using the free body diagram.

So, one can find the equation of motion for one degree of freedom system, two degree freedom system or multi degree of freedom system or one can find the equation of motion for lumped parameter system. So, today class actually we are going to study the equation of motion for the system with continuous system.

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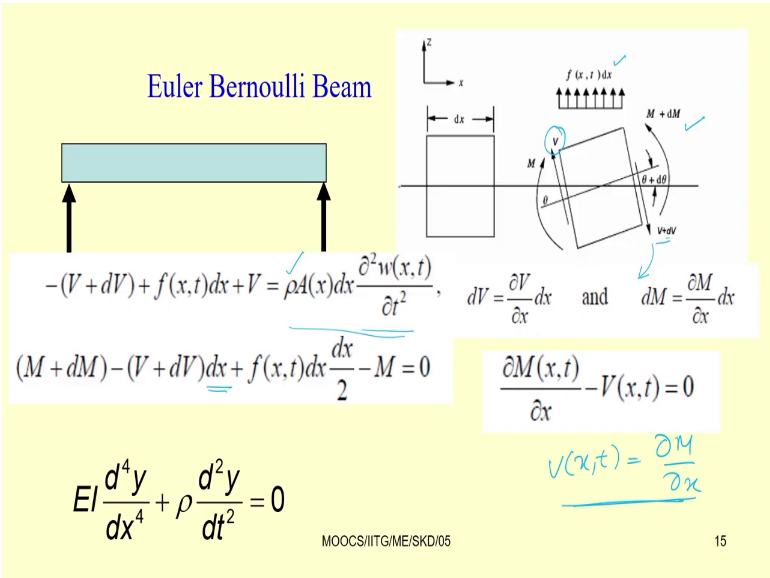

So, we will study the equation of motion for continuous system. So, let us start with the continuous system for example, this Euler Bernoulli beam.

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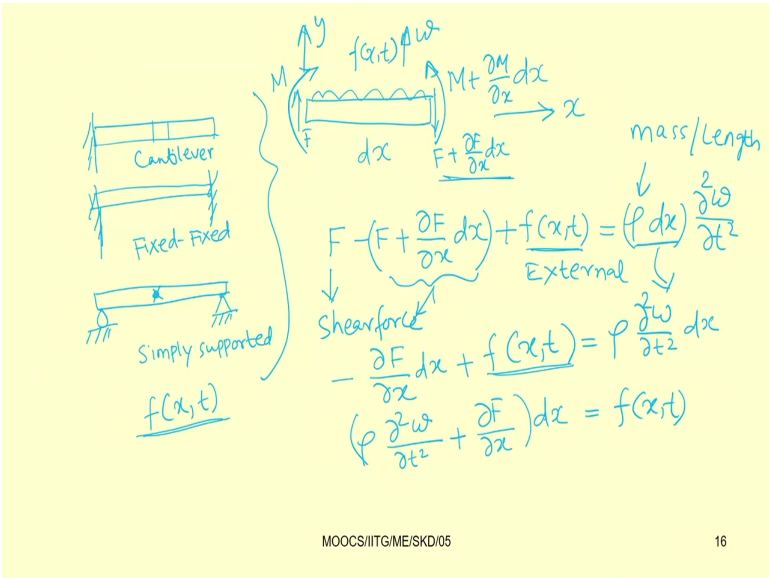

So, in case of Euler Bernoulli beam; so, this is a beam Euler Bernoulli beam is a beam slender beam slender beam subjected to bending pure bending. So, this is objected to pure bending. So, the so it can be so in this case the beam can be supported in many different ways. For example, it can be a cantilever beam so if the one end is fixed. So, it can be a simply supported beam also or fixed fixed beam, let us first draw the fixed fixed beam.

So, if both the sides we are fixing then this is a fixed fixed beam we can have a simply supported beam. So, if one end is so, let us put one end roller supported and other end hinged, if we are not putting a hinge here and both end roller support. So, in that case by applying an inclined load; so, the beam can slide over the roller. So, that is why one end must be hinged and other end must be roller supported. So, this is simply supported.

So, this case is simply supported beam. So, this is fixed fixed or ok. So, this is fixed fixed and these case is cantilever. So, on like the lumped parameter model; so, where the equation motions are retained by using the simple differential equations. So, here you can see the equations can be retained by using partial differential equation because the displacement of any points.

So, let us take this point the displacement of this point is a function of both space and time. So, it is a function of both space and time. So, we required a partial differential equation to write them the equation of motion.

So, let us first derive the equation motion of a simple Euler Bernoulli beam and then we will see how we can make this equation that of a non-linear equation. So, initially let us find the equation of motion of the Euler Bernoulli beam. So, let us take a small element for example, you take any of the beam and just take a small element let it is subjected to a loading condition that is f x t let f force f x t or g x, t let us write g x t already you have written f or something.

So, let this is the force acting keep with f x t no problem. So, let this is the force acting here it may be uniformly distributed loading or it may be any other loading acting on the system.

Then so, it will be subjected as it is subjected to pure bending so we can write M. So, this side it will be M plus del M by del x del M by del x into dx and also the shear force. So, let the shear force this side equal to F. So, this side it will be this direction. So, it will be equal to F plus del F by del x into the d x.

So, this is the force acting on the right side of these things. So, let us assume that there is small variation of the force along the length of the beam and also. So, you are taking a very small element. So, this elemental length let the elemental length is x which is at a distance x from any of the end, but let us take from the fix end of this thing if we are taking a cantilever beam.

So, now, by doing these force balance. So, let us do the force balance. So, by doing the force balance we can write the equation of motion. So, we have to do both force balance and moment balance to find the equation of motion of the system. So, it is derived this way so you can see so ok. So, let me write this equation or you can write F or you can write v also in some books this F is replaced by v.

So, the equation of motion will be F. So, f is upward let us take upward force positive and downward force negative. So, net force acting on the system will be equal to F then minus F minus del F by del x, del F by del x into d x. So, here F is this the shear force we are taking capital F is the shear force we are taken and plus the load acting here.

So, the load acting here we are writing. So, the load let us assume that the load acting. So, instead of taking the sign so, let us assume that load is f x t. So, it can be plus or minus so no problem. So, if your assuming this f x t force let the f x t force direction is upward small f x t. So, that is why we have taken this plus we have we are taking these upward force equal to positive then. So, this net force must be equal to. So, this net force will be equal to rho that is the mass this is mass per unit length rho and d x.

So, this is the mass so mass of the element into let w is the vibration the displacement in these direction transverse direction, then it will be del square w by del t square that is the acceleration. So, by applying this Newton’s 2nd law so, this is the net force acting on the element. So, it is equal to F so, this portion you can write that this way also F plus del F by del x into dx plus f x t. So, this is the external force acting this part is the external force acting.

So, this is the external force acting on the system. So, this F is the shear force so this part is the shear force. So, this the shear force in the left end. So, this is the shear force in the right end and this is the external force. So, this is the mass of that element mass of that element equal to. So, we can have the mass of the element rho dx rho is the mass per unit length. So, this the mass per unit length.

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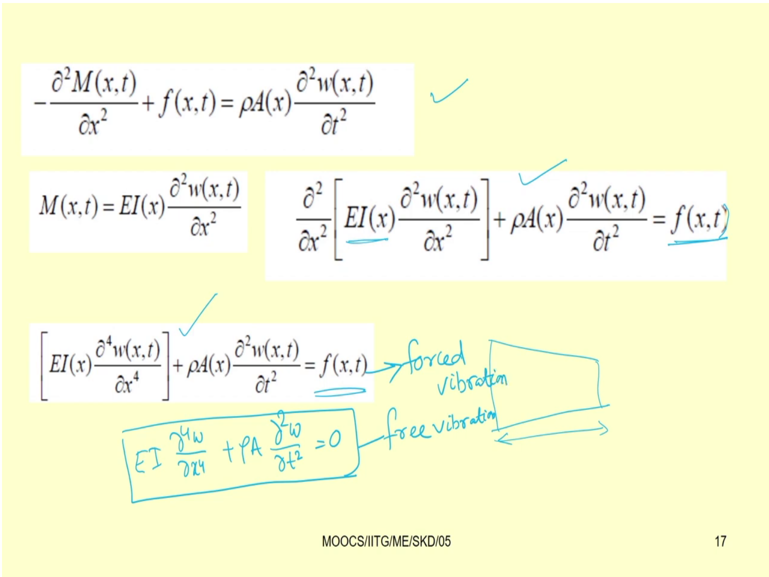

So, we got this and w is the transverse deflection transverse deflection so transverse deflection in this direction. So, we can write the equation motion in this form, so this is the mass into acceleration. So, now, by opening these thing this F and this F will cancel then we can write this equation of motion in this form that is, minus del F by del x. So, minus del F by del x into d x plus this external force f x t equal to rho del square w by del t square into dx.

So, in the absence of this external force in the absence of the external force; so, you can write; you can write rho or in presence of these things also you can write rho del square w by del t square; del t square plus del F by del x, del F by del x into dx. You can take this force also so equal to f x t, similarly you can take the moment also.

So, taking the moment, so; let us take the moment about this. So, let us take about this axis so let this is your y axis, this is the x axis this thing you have taken along x this is along y. So, you can write the moment equation.

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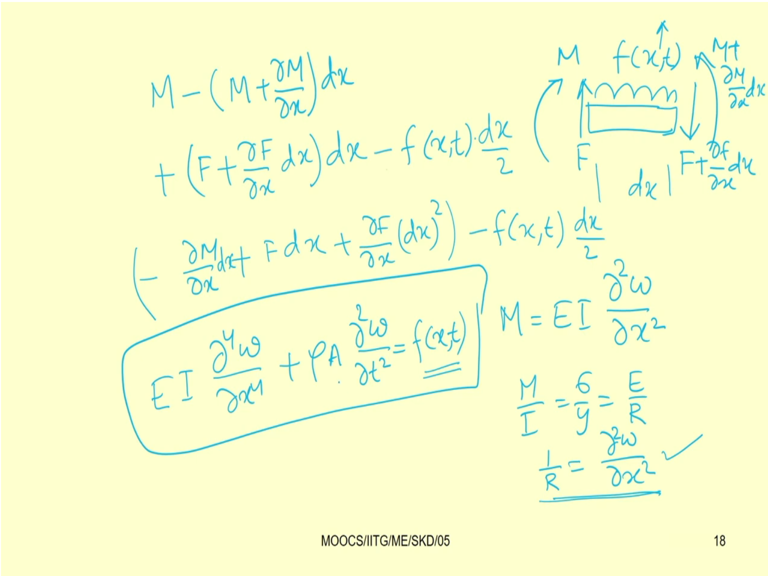

So, moment equation will be equal to. So, let your moment equation will be equal to. So, if you are taking M so M minus M plus del M by del x into dx. So, we are taking the moment so you just see, so we have this thing. So, this is the loading; loading we have taken f x f small f x t. So, this is shear force, so this is shear force. So, this is F capital F we have taken. So, this is F plus delta F by del x into d x.

So, we have taken this is M and this is on this is M plus del M by del x into dx. So, now, taking moment about this line, so it will be your y axis. So, M minus M plus del M by del x into d x that is the moment. So, this moment you have taken positive. So, clockwise moment you have taken positive.

So, anticlockwise moment is negative. So, due to this force F, so, F is passing through that point. So, if this is equal to 0 then clockwise moment equal to as it is positive we were taking. So, then it will be F plus del F by d x into dx.

So, multiplied by as this distance equal to dx, so this distance equal to d x. So, this is the thing. So, then we are taking this f x direction in upward direction. So, it can be retained f x t. So, this let us assume that this total force is f x t into. So, it will act in a distance d x by 2. So, this way we can write now this M and this M will cancel. So, the equation can be written in this form.

So, that is minus del M by del x minus del M by del x plus. So, you can write this equation F dx plus del F by del x into d x whole square.

So, this part del M by d x into d x; so, these f x t into d x by 2. So, this way you can write, so this moment equation and force equation now you can simplify these equations and by simplifying these equations. So, you can get these Euler Bernoulli beam equation. So, which is nothing, but your del EI. So, you can substitute M equal to you can substitute M equal to EI del square w by del x square.

So, because you know in pure bending M by I equal to M by I equal to sigma by y equal to E by R. So, this 1 by R is nothing but, so for a linear system so these 1 by R for linear curvature.

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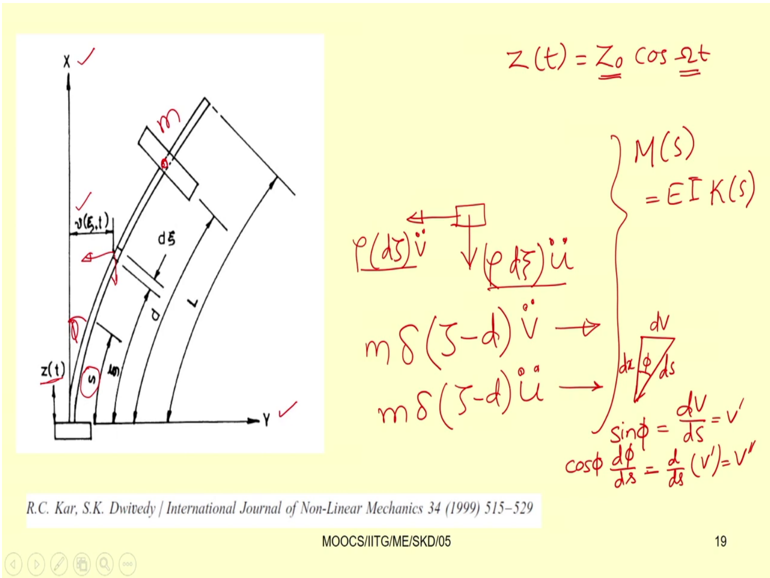

So, this 1 by R you can take equal to del square w by del x square del square w by del x square. But so, if you will take large curvature; so, this 1 by R you need not use cannot write in this form and it can be expanded. So, in that case in case of this is not small the curvature is not small then you can use some other relation. So, that relation I will tell you. So, when we will derive the equation of motion for a large deflection beam just after this thing.

So, by reducing so, you can reduce this equation by using these two fundamental equations and knowing the relation between these shear force and bending moment. So, you can derive this equation of motion of the Euler Bernoulli beam equation which is nothing but your EI del 4th w by del x 4th plus rho A del square w by del t square.

So, this will be equal to your f x t or in case of in case of no external load or free vibration. So, this equation, so this right hand side will be equal to 0.

So, it will be equal to EI del 4th w by del x 4th plus rho A del square w by del t square equal to 0. So, this part correspond to the inertia force of the system and this part correspond to; and this part correspond to the force due to the elasticity or the force due to the stiffness part of the system. So, this way you can derive this equation of motion. So, let us see this derivation again.

So, in some books so instead of writing F; so, they are writing these shear force as V. So, this is shear force as V. So, for the small element f x t; so, the total force can be written as f x t into dx or. So, that means, this f x t is the force per unit length and this is M is acting in left side this is the bending moment M plus dM is the moment in the right hand side. So, if you neglect the shear deformation. So, that is this theta is very small then you can write down this equation motion.

So, first you just write down the force equation motion. So, that is V minus V plus dV plus f x t dx plus V equal to rho A x dx del square, so this is the inertia force inertia rho A. So, rho so there I have taken rho as the mass per unit length. So, if you are taking rho as the density.

So, then this way also you can write rho into A that is mass per unit length into dx that will give you the mass of that small element, into del square w by del t square. So, from this equation so this V V and cancels. So, you got this dV you can write dV always by del by del x into dx and dM equal to del M by del x into dx.

So, similarly by taking the moment about the left side, so you can write down this equation. So, here they have taken this anticlockwise moment as positive. So, if you are taking this anticlockwise moment positive then you can write M plus dM minus V plus dV. So, this is V plus dV into so this distance equal to dx. So, you just see.

So, this V plus dV will produce a clockwise moment. So, that is why it is negative and then this f x t f x t dx which will produce as it is taken in upward direction f x t dx.

So, it will be into d x by 2. So, you can assume that it is acting the total force is acting at the middle of the section. So, that is why you can multiply this dx by 2 and minus M. So, here the assumption is that this anticlockwise moment is positive. The previous case when I described so I have taken the clockwise moment as positive. So, from this equation so you can get this del M by del x minus V x t equal to 0.

So, you can see this del M by del x equal to so you cancel these thing. So, del M by del x minus V x t equal to 0, that is rate of change of your; rate of change of angular this; rate of change of bending moment equal to the shear force or this V x t equal to. So, you know this V x t equal to del M by del x, V x t equal to del M by del x.

So, from these thing so you can find the equation motion which we will see. So, from the second equation so you can write this minus del square M x t by del x square plus f x t equal to rho A del square w by del t square.

So, M after canceling this M, so you got this thing and by putting this M x t equal to EI del square w by del x square. So, we can write down this equation in this form that is del square by del x square into EI del square w by del x square plus rho a del square w by del t square equal to f x t.

So, here so if you are taking some cross section. So, for example, if the cross section is not uniform for cross section is not uniform. So, that time you can use this equation that is del square w by del x square outside these thing. So, del square by del x square of EI x. So, if your sometimes this E also may be a function of x when you are using some functional degraded material.

So, I x I may be a function I may be a function of x also I for example, in this case I is. So, in this case this cross section is not uniform. So, if the cross section is not uniform. So, you can have different I, so the I will vary along the length of the; length of the beam. So, if I is varying along the length of the beam.

So, you can take this del square by del x square outside and you can write down this equation this del square by del x square into EI x into del square w by del x square plus rho x del square w by del t square equal to f x t.

So, if these external force is not there then this f x t equal to 0 and this equation can be written as del square by del x square EI x del square w by del x square plus rho a into del square w by del t square equal to 0. Now if we are taking this both e and I uniform that is constant, then this EI can be taken out and it will reduce to the simpler form of this Euler Bernoulli beam equation that is equal to EI del 4th w by del x 4th plus rho A del square w by del t square equal to f x t.

So, here in case of the free vibration. So, this equation can be written so already we have written that equation that is del 4th the w by del x 4th plus rho A del square w by del t square so equal to 0. So, this is for free vibration so this is for free vibration case and this is for the forced vibration case.

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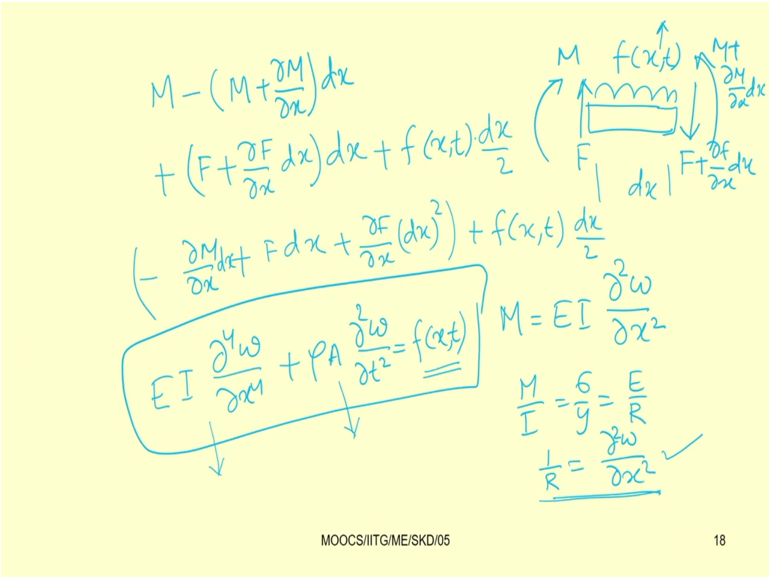

So, you can see on like the long parameter model. So, this w is not a function of only time, but is a function of both time and. So, it is a function of both time and this x that is the position. So, it is a function of both time and position. So, let us see another example.

So, in this case a simple slender beam is also taken. So, which just moving in vertical direction that is z t. It is moving in vertical direction with z t. So, that is for example, let me write this z equal to z 0 cos omega t. So, here z 0 is the amplitude of the motion and omega is the frequency of that motion. So, that is we have taken a base excited beam.

So, let us use this Newton's 2nd law or this moment base force and moment based approach to derive the non-linear equation of motion of the system. So, in this case so let us take a small element at a distance zeta. So, along the length of the beam; so, we have taken the parameter as S, S is the parameter along the length of the beam. So, this is the x axis and this is the y axis. So, here in this beam so let us so a mass M is attached at an arbitrary position and. So, our objective is to derive the equation of motion for this beam.

So, in this case as it vibrate. So, let us take this v t v zeta t is the displacement along the transverse direction and u zeta t is the displacement along the axial direction. So, here considering inextensibility condition; so, we can see there is relation between this u and v and we can reduce the two degrees of freedom motion that is u and v to that of a single degree of a freedom motion that is along u direction along v direction.

So, what are the forces acting? So, as we are considering this as an slender beam. So, let us take this rho mass per unit length, let rho is the mass per unit length of the beam. So, as we have taken the so we have taken at a distance zeta a small element d zeta. So, the length of this or the mass of this element will be rho into d zeta and acceleration of this element. So, in 2 directions we can take the acceleration. So, in u direction that is along these axial directions.

So, let it is acceleration will be u double dot and in trans in this transverse direction this acceleration will be v double dot. So, the inertia force so if you are writing in terms of d' Alembert principle. So, then we can find these inertia force. So, the inertia force will be so for the small element the inertia force will act along this direction and along this direction.

So, in this direction it will be equal to. So, if we are taking this small element. So, in this direction it will be rho.

So, d zeta so this is the mass of the element. So, into acceleration; so, acceleration equal to. So, acceleration will be equal to u double dot and along these direction. So, it will be equal to similarly it will be rho d zeta into v double dot. So, these are the acceleration or this inertia force acting in this element similarly for the attached mass.

So, if the mass is m. So, we can write this inertia force in the similar way. So, it will be equal to mv. So, at that position for example, so let it is at a distance d. So, we can use this Dirac delta function and to write these position of these things.

So, we can write using this Dirac delta function to write this position of this thing. So, we can write using this Dirac delta function zeta minus d zeta d. So, it will be the force can be written m u m delta zeta minus d v double dot and another force it will be equal to or this will be inertia force zeta minus d into u double dot. So, this is in this longitudinal direction and this is along the transverse direction the inertia force is acting.

So, like this Euler Bernoulli beam equation. So, we can write this moment M. So, moment M can be written equal to EI into these EI into phi dash or EI into or you can write this EI into K, K S that is radius that is curvature K S will be the curvature. So, generally we can write this is equal to K S and this K S can be written as phi dash.

So, you can see so if we are taking this angle equal to phi. So, this phi so this is v so, at a distance so, we can draw the small. So, for example, so this is the x direction. So, displacement equal to square along the beam we have taken. So, long the beam we have taken a distance dS.

So, this is d x along the x direction and this is our dV small displacement along the transverse direction. So, this angle equal to phi. So, one can easily write this sin phi. So, sin phi will be equal to dV by dS.

So, as sin phi equal to dV by dS or v dash; so, we can differentiate these and we can write. So, this is cos phi into d phi by dS cos phi by d phi by dS equal to. So, this is equal to v double dash or d by d by dS of v dash. So, that is nothing, but v double dash. So, we got this cos phi into d phi by dS equal to v double dash or phi dash equal to. So, phi dash you can write this now you can write this phi dash.

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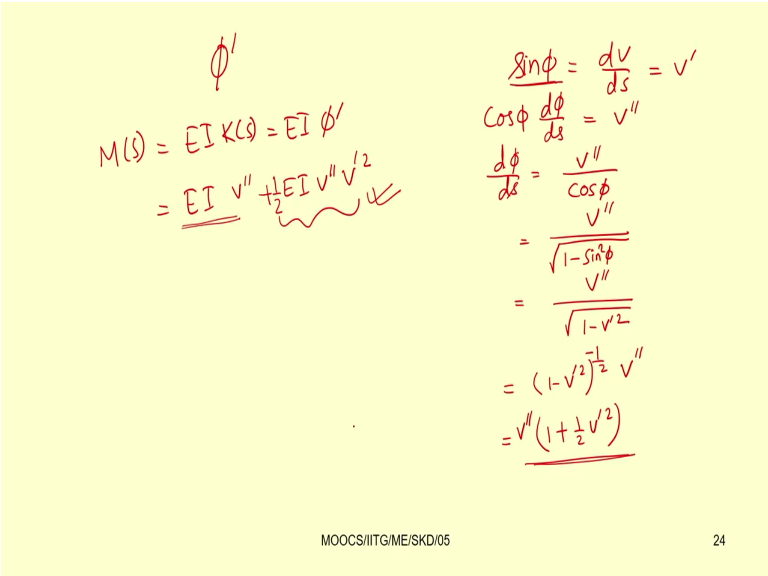

So, we can write so now, these phi dash can. So, already you have written the sin phi sin phi equal dV by dS or cos phi d phi by dS, if you differentiate this thing equal to v double dash. So, now, this d phi by dS can be written as so d phi by dS equal to v double dash by cos phi, but this cos phi can be written as. So, this is v double dash by cos phi can be written as 1 minus sin square phi root over.

So, this is nothing, but v double dash by. So, already we have taken these sin phi equal to V dash. So, it is equal to root over 1 minus v dash square. So, we can write this equal to 1 minus v dash square to the power minus half into v double dash. So, this is equal to so by expanding this thing by normally we can write this equation equal to half v dash square into v double dash.

So, you can see this d phi by dS can be written as V double dash into 1 plus half v dash square. So, previously you have seen this M that is moment equal to EI K S. So, that is nothing, but your EI phi dash. So, this is further can be written EI v double dash plus EI v double dash into. So, this is half into v dash square. So, when you have not consider this phi to be small in that case.

So, we have this additional term that is half EI v double dash into v dash square. So, this introduces the nonlinearity in the system, otherwise in case of Euler Bernoulli beam equation. So, you have seen only this term due to this bending that is M equal to EI del square v by del s square, but now we are considering an additional term that is half EI v double dash into v dash square.

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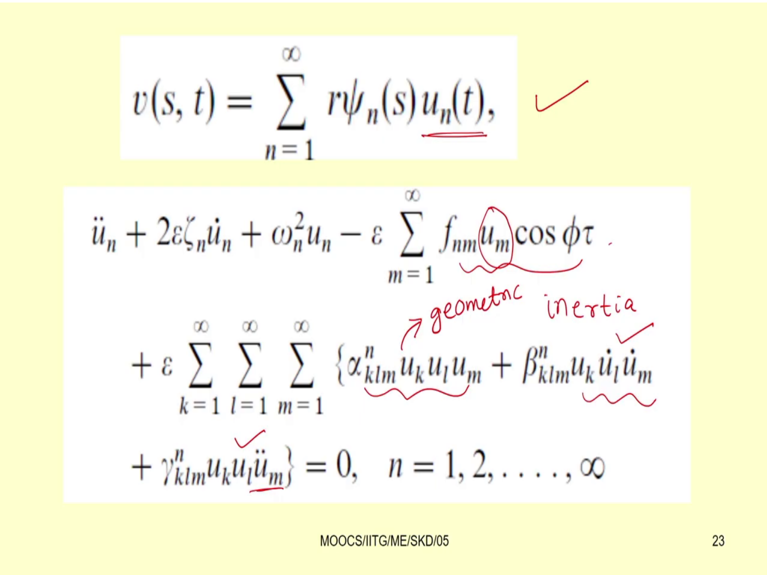

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$$EI\left\{v_{ssss} + \tfrac{1}{2}v_s^2 v_{ssss} + 3v_s v_{ss} v_{sss} + v_{ss}^3\right\}$$

$$+ \left(1 - \tfrac{1}{2}v_s^2\right)\left\{\left[\rho + m\delta(s-d)\right]v_{tt} + cv_t\right\}$$

$$+ v_s v_{ss}\int_s^L \left\{\left[\rho + m\delta(\xi - d)\right]v_{tt} + cv_t\right\} d\xi$$

$$- \left[J_0\delta(s-d)(v_s)_{tt}\right]_s - (Nv_s)_s = 0$$

$$N = \frac{1}{2}\rho\int_s^L\left\{\int_s^\xi (v_s^2)_{tt}\,\mathrm{d}\eta\right\}\mathrm{d}\xi + \frac{1}{2}m\int_s^L \delta(\xi - d)$$

$$\times\left\{\int_0^\xi (v_s^2)_{tt}\,\mathrm{d}\eta\right\}\mathrm{d}\xi + m(z_{tt} - g)$$

$$\times\int_s^L \delta(\xi - d)\,\mathrm{d}\xi + \rho L\left(1 - \frac{s}{L}\right)(z_{tt} - g)$$

$$- J_0\delta(s-d)\left\{\tfrac{1}{2}v_{stt}v_s^2 + v_s v_{st}^2\right\}$$

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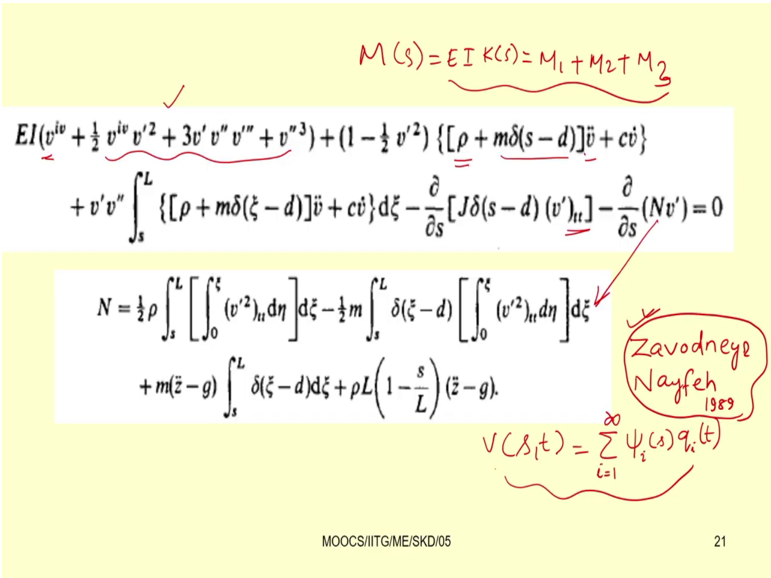

So, we if we can you can see now this derivation. So, quickly we can visit this derivation.

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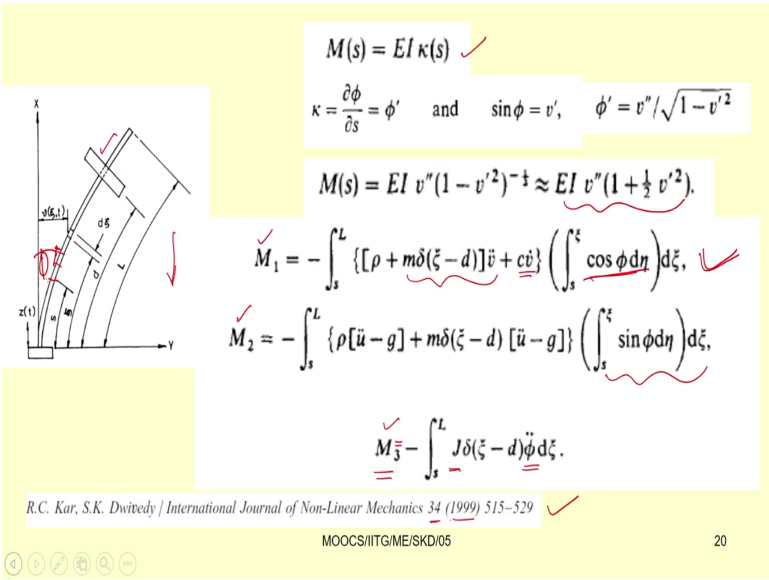

So, already I have written this derivation. So, you can write this M s equal to EI K S and k equal to del phi by del s equal to del phi dash and taking this sin phi. So, this angle we have taken as phi. So, sin phi already I have explained that sin phi equal to v dash. So, phi dash equal to v double dash by root over 1 minus v square and by taking that thing to numerator and expanding using binomial theorem.

So, we can write so this is equal to EI v double dash into 1 plus half v dash square; so, this taking the moment, so taking the moment about this s. So, at a distance s let us find what is the moment. So, I am taking dummy variable d eta. So, now, we know what are the forces acting already I explained the forces acting in this system.

So, let us take the forces inertia forces along the transverse direction as M 1 moment due to inertia force along transverse direction as M 1, moment of the forces along longitudinal direction is equal to M 2 and the moment due to the; due to the rotary inertia equal to M 3.

So, we have these 3 terms so we can write. So, this way we can write. So, M 1 will be this, so already we know the force equal to rho v double dot minus rho v double dot also for the mass this for this attach mass M. So, it is M using this Dirac delta function it can retain. So, M delta zeta minus d v double dot; that means, when zeta equal to d. So, the term will equal to 1 otherwise this is equal to 0.

So, this integration M delta zeta minus d v double dot equal to 0, when zeta not equal to d. When zeta equal to d only it will have a value otherwise it is equal to 0. So, if we include damping also, so by putting this dumping c v dot. So, we can write this M 1. So, dumping for example, c let us take coefficient of dumping in these transverse direction as c. So, then this moment will be equal to say if we are taking moment so this is the moment. So, this moment will takes place along v direction.

So, then the perpendicular distance so we have taken a small let this is the dummy variable zeta. So, the perpendicular distance will be along this. So, it will be; so it will be the cos component of that thing. So, that is why this cos component is multiplied. So, it is cos phi into the d eta. So, this integration is taking place from s to zeta.

So, this is the, so this is the moment equation for taking the force in the transverse direction, similarly taking the force along the longitudinal direction that is u direction. So, you can write this equation of motion. So, rho u double dot then minus rho g rho g is the weight component then M delta into zeta d into u double dot minus g as g is acting vertically downward. So, this is u double dot into g and the distance multiplication of the distance, distance is nothing, but sin phi d zeta; sin phi d eta into d zeta.

Similarly, taking the rotor inertia let J is the rotor inertia of the system. So, due to this mass the rotor inertia of the system then this will be multiplied by rotor inertia into phi double dot that will give you; that will give you the M 3 integration of s 2 L J delta zeta minus d phi double dot d zeta. So, this is the rotor inertia due to this attached mass at an arbitrary position.

So, this part of the derivation actually published in this journal of International Journal of Non-Linear Mechanics volume 34 in 1999; so, by Professor R. C. Kar and myself. So, this is part of my PhD work actually. So, you can see this derivation. So, now, you can see so by writing this M equal to. So, you can write M s is equal to EI, now already we got this is equal to K S equal to this M 1 plus M 2 plus M 3 and now M 3 and differentiating it twice; and differentiating it twice.

So, one can get this equation. So, this equation becomes EI del 4th V by del x 4th plus half del 4th w by del s 4th into del v by del square plus 3 into, so let us write in terms of v dash. So, 3 into v dash into v double dash into v triple dash plus v double dash cube plus 1 minus half v dash square into rho plus M delta s minus d into v double dot plus v dot plus v dash into v double dash, integration s to l rho plus M delta zeta minus d v double dot plus c v dot d zeta minus d del by del s into J delta s minus d v dash.

tt means differentiation with respect to twice differentiation with respect to time minus del by del s into N into v dash equal to 0 where this N term is written here. So, you just see you can note here that only for the Euler Bernoulli beam. So, we have the first term that is EI del 4th w by del s 4th and one more term that is there that is rho. So, one more time we have so this is the term rho v double dot equal to 0.

So, other terms so other terms are coming so for example, this term is coming due to the additional mass in the system and also. So, there are several non-linear terms present in the system. So, you can see all these terms are non-linear terms.

So, by taking this large amplitude oscillation. So, we have seen so the equation of motion contain many non-linear term. So, here you can note that this v s t is a function of position function of position s and time t. So, you can convert this thing to its temporal form by using this Galerkin procedure.

So, where you can use this v s t equal to summation. So, you can take several modes into account; so, psi i s into qi t. So, this is variable separation methods you can use. So, where you can take let us take single mode approximation. So, if you take single mode approximation, so this work reduces to that of Zavodneye and Nayfeh; Zavodneye and Nayfeh paper of 1989. So, published in the journal of non-linear mechanics related to this vibration of a similar system.

So, but if you take 2 mode into account; so, it will reduce to my own paper in this same journal and you can take 3 modes also. So, I have several works related to 3 modes also. So, you can take i equal to 1 to infinity or you can take I only single mode, 2 modes or 3 modes depending on different type applications. So, then you can reduce these thing by applying this Galerkin method.

So, by substituting this in this equation and using some weight function you can find the residue and equate the residual equal to 0 to find the equation of motion. So, you can find the equation of motion in this way for example, by substituting these so you can write down these equation of motion in this form u n double dot plus. So, here you just see u n is function of time only.

So, u n double dot plus 2 epsilon zeta n u n dot plus omega n square u n minus epsilon m equal to 1 to infinity f n m u m cos phi tau plus epsilon k equal to 1 to infinity l equal to 1 infinity m equal to 1 to infinity alpha n klm uk ul plus beta n klm uk ul dot dot plus gamma n klm uk ul double dot.

So, you can see these term that is uk ul u m. So, if klm are equal to 1 then it reduces to u cube only similarly. So, here it will reduces to u 1 into u 1 dot square. So, product of 2 velocity term is a acceleration term, similarly here it is double dot if it is single mode then it is u 1 double dot. So, u 1 double dot is nothing but this acceleration. So, this is also an acceleration term and this also an acceleration term.

So, that is why these 2 terms are known as inertia non-linear term; inertia non-linear term and this is geometric non-linear term. So, if you neglect this acceleration then these 2 inertia term should be 0, if the acceleration is very small then these 2 terms can be neglected and the only term will be the geometric non-linear term. And this equation can be reduced to that of a; that of a equation which you have seen before. So, this inertia so let me tell these forcing term the I will tell the equation.

So, in this case you can see in the forcing term this coefficient of u m coefficient of u m is a time bearing term. So, that is why this equation is known as the equation of a parametrically excited system. So, this is the equation of a parametrically excited system with non-linear term. So, you have seen the equation of a parametrically excited term as Mathieu equation and hails equation in the introductory class.

So, in this case so, you have these additional non-linear additional geometric and inertia non-linear term, in addition to that Mathieu type of equation so we can see. So, in this way you can derive this equation of motion of any single any continuous system. So, by taking a small element you can write down this moment equation and by writing this moment equation.

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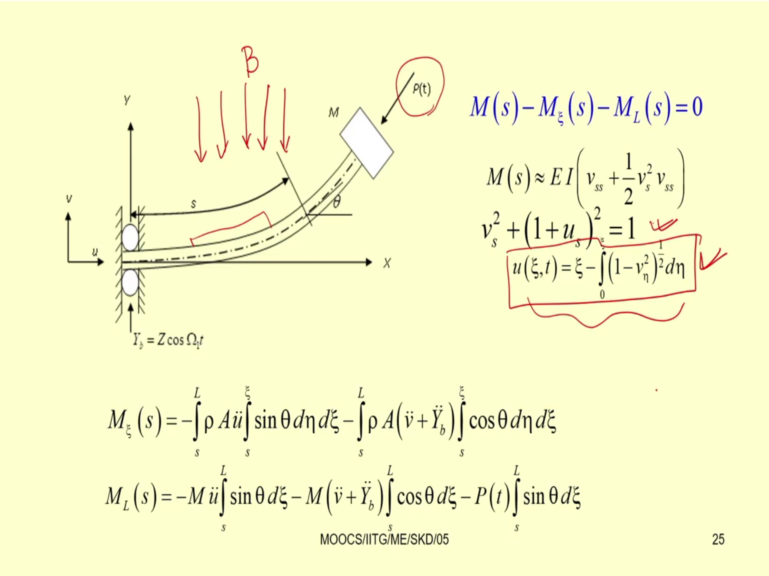

So, you can derive this equation of motion similarly this work you can, this as an assignment. So, you can derive this equation of motion of a base exited cantilever beam with a tip mass attached to these things and subjected to a follower force. So, let a force pt is also acting in this beam. So, in this case; so, you can derive this equation of motion. So, previously in the extensibility conditions was taken.

So, this displacement in x direction is written as a function of the displacement in y direction by using this relation, that is v s square plus 1 plus us square equal to 1, where v s equal to del v by del s, u s equal to del u by del s. So, using this relation, so this is the relation for the inextensibility condition. So, the previous equation was derived here also you can derive this equation of motion by using similar approach. So, you can have many different applications or many different these type of continuum systems for example, you may put one piezoelectric patch also here.

So, for sensing and actuator actuation purpose also you can take this as magnetic material and apply the magnetic field. So, you can apply the magnetic field v also in this system. So, there can be several applications of these type of system for example, microm or micromechanical electromechanical systems or smart systems. So, these derivations will be very much useful in different applications. So, you should know how to derive this non-linear equation of motion.

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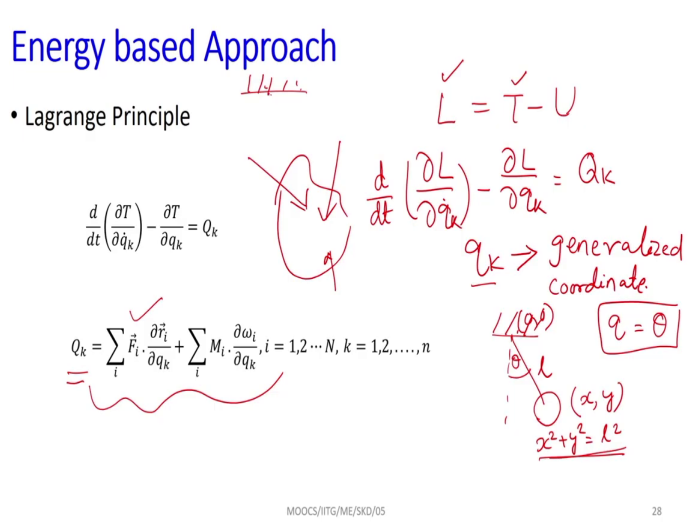

So, briefly just I will tell you how to use now this Lagrange principle. So, initially we have done these force and moment based approach where we have this Newton's 2nd law or d' Alembert principle. So, then in this energy based approach; so, we are going to use this

Lagrange principle where we can write this like Lagrange principle as d by dt of del T by del q k dot minus del T by del q k equal to capital Q k.

Or using this Lagrange of the system that is L equal to T minus U, where U is the potential energy T is the kinetic energy. The same equation can be written as d by d t of del L by del q k dot del l by del q k dot minus del L by del q k equal to capital Q k. So, here this qk is known as the generalized coordinate. So, this is generalized coordinate. So, this generalized coordinate what do you mean by generalized coordinate?

For example, so already I told for a given systems. So, we have both physical coordinates and generalized coordinate. So, for example, in this case of these simple pendulum. So, you can write down this equation using the coordinate x and y that is the physical coordinate ok. So, physical coordinate, but you can see that this x and y they are depends on each other for example, they are related by this constraint equation that is x square plus y square equal to l square.

So, if this point we have taken 0 0 coordinate this is x y, then the length of the simple pendulum is constraint. So, that is x square plus y square equal to l square. So, by using this you can write either x in terms of y or y in terms of x, but conveniently you can use another parameter that is theta for write down; for writing the equation of motion.

So, here theta is the generalized coordinate and using this generalized coordinate theta. So, you can write this kinetic energy and the potential energy of the system and you can find the Lagrangian of the system then you can write this d by dt of del l by del q k dot. So, here q k k equal 1. So, this q equal to theta in this case. So, q equal to theta you can find this equation of motion.

Similarly, this capital Q k is known as the generalized force. So, these generalized force equal to let so in the system a number of forces are acting f 1 f 2 f 3. So, from some position so this is the position physical coordinates you have taken. So, let this force is at a distance position vector of this force is at r 1 r 2 r 3 r 4. So, then this q k can be written as i equal to summation i equal to F i dot del r i by del q k plus i summation i M i dot del omega i by del q k.

So, this is equal to generalized force. So, if moment is not there moment it is not working then it will be only this force q k will be equal to summation F i dot del r i by del q k. So, tomorrow class will solve or next class we are going to solve some problems on this using this Lagrange principle also we will study the extended Hamilton principle to derive this equation of motion.

So, we will see both linear and non-linear equations and we will complete the derivation of equation of motion and this will complete the module 2nd module in the next class.

So, thank you, thank you very much.