Nonlinear Vibration

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Lecture - 06
Use of Scaling and Book-Keeping Parameter for Ordering

Welcome to today class of Non-linear Vibration. So, this is the 3rd lecture, in the 2nd module of this non-linear vibration course.

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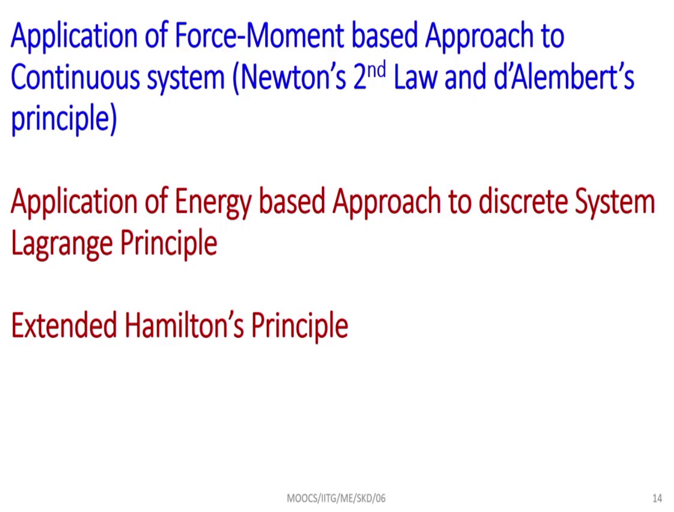

So, last two classes we have studied, how to derive this equation of motion using these Newton 2nd law or d'Alembert principle. For this discreet and distributed mass system; also last class, just I have started how to use this Lagrange principle to derive the equation of motion. So, today class we will solve some more examples to solve or to find the equation of motion by using this Lagrange principle. Also, we will use this extended Hamilton principle to derive some equation of motion.

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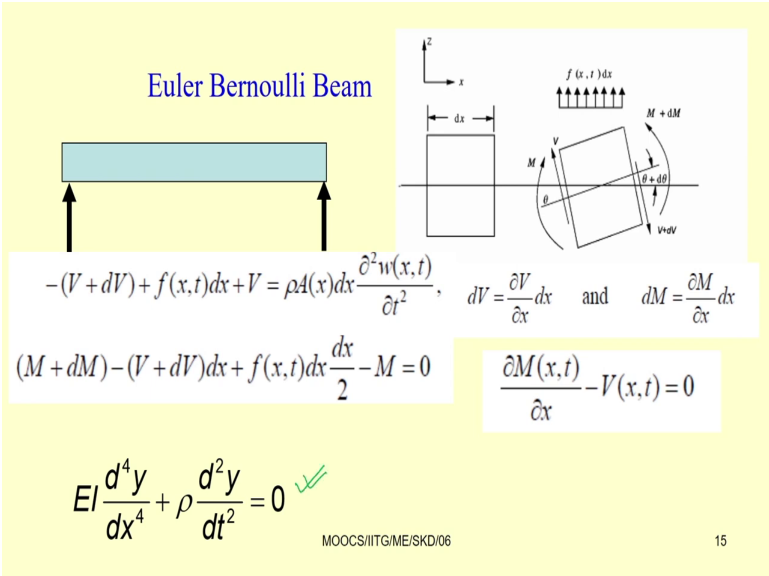

So, yesterday or last class, we have studied about this Euler Bernoulli beam equation.

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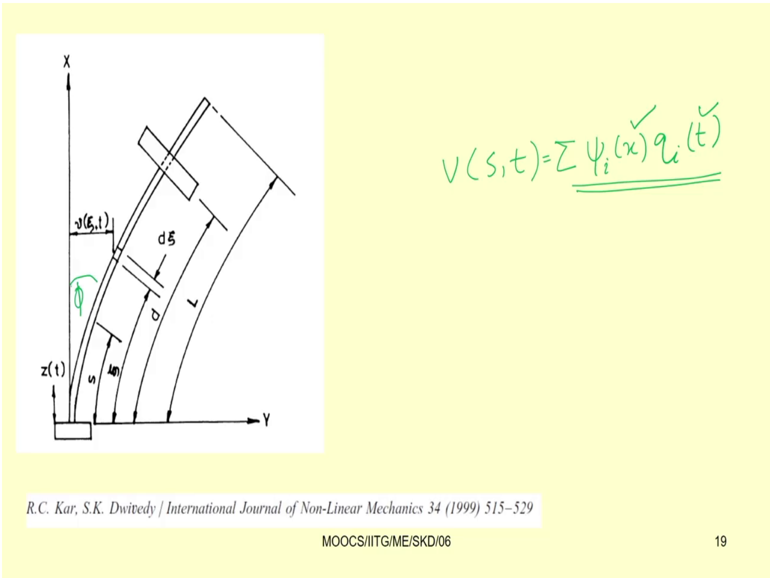

So, we have derived this beam equation using this Newton's 2nd law so, by drawing the free body diagram so, we have written the equation of motion or we have found the equation of motion. So, this is the equation of motion of a Euler Bernoulli beam equation.

So, similarly we have taken another example, where we have taken a cylinder beam subjected to base motion and we have derived this equation of motion so, which is a spatiotemporal equation of motion.

So, in case of spatiotemporal equation of motion, that is the v s t that is the transverse displacement of the beam can be written as by considering number of mode. So, you can write this equal to psi i x and q i t. So, it is a function of both displacement and time or like in case of long parameter system, where the displacement is a function of time only. So, here the displacement is a function of both space and time here, the displacement is a function of both space and time.

So, last class we have derived this equation of motion, first the spatiotemporal equation of motion by using this d'Alembert's principle. Where, we have taken this m s equal to e i into k s and that k s equal to phi dash and this this is phi this is the angle, and by writing the sign phi in terms of this v and or v dash. So, we have derived this non-linear equation of motion.

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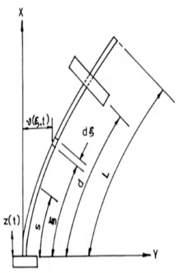
$$M(s) = EI\,\kappa(s)$$

$$\kappa = \frac{\partial \phi}{\partial s} = \phi' \quad \text{and} \quad \sin\phi = v', \qquad \phi' = v''/\sqrt{1 - v'^2}$$



$$M(s) = EI\, v''(1 - v'^2)^{-\frac{1}{2}} \approx EI\, v''(1 + \tfrac{1}{2} v'^2).$$

$$M_1 = -\int_s^L \{[\rho + m\delta(\xi - d)]\ddot{v} + c\dot{v}\} \left(\int_s^\xi \cos\phi \, d\eta\right) d\xi,$$

$$M_2 = -\int_s^L \{\rho[\ddot{u} - g] + m\delta(\xi - d)\,[\ddot{u} - g]\} \left(\int_s^\xi \sin\phi \, d\eta\right) d\xi,$$

$$M_3 - \int_s^L J\delta(\xi - d)\ddot{\phi}\, d\xi.$$

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$$EI\left(v^{iv}+\tfrac{1}{2}v^{iv}v'^2+3v'v''v'''+v''^3\right)+\left(1-\tfrac{1}{2}v'^2\right)\left\{\left[\rho+m\delta(s-d)\right]\ddot{v}+c\dot{v}\right\}$$
$$+v'v''\int_s^L\left\{\left[\rho+m\delta(\xi-d)\right]\ddot{v}+c\dot{v}\right\}\mathrm{d}\xi-\frac{\partial}{\partial s}\left[J\delta(s-d)\,(v')_{tt}\right]-\frac{\partial}{\partial s}(Nv')=0$$

$$N=\tfrac{1}{2}\rho\int_s^L\left[\int_0^\xi (v'^2)_{tt}\mathrm{d}\eta\right]\mathrm{d}\xi-\tfrac{1}{2}m\int_s^L\delta(\xi-d)\left[\int_0^\xi (v'^2)_{tt}d\eta\right]\mathrm{d}\xi$$
$$+m(\ddot{z}-g)\int_s^L\delta(\xi-d)\mathrm{d}\xi+\rho L\left(1-\frac{s}{L}\right)(\ddot{z}-g).$$

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So, the non-linear equation of motion after deriving, spatiotemporal equation of motion, then we have used generalized Galerkin principle.

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$$EI\{v_{ssss} + \tfrac{1}{2}v_s^2 v_{ssss} + 3v_s v_{ss} v_{sss} + v_{ss}^3\}$$
$$+ (1 - \tfrac{1}{2}v_s^2)\{[\rho + m\delta(s-d)]v_{tt} + cv_t\}$$
$$+ v_s v_{ss}\int_s^L \{[\rho + m\delta(\xi - d)]v_{tt} + cv_t\}\, d\xi$$
$$- [J_0\delta(s-d)(v_s)_{tt}]_s - (Nv_s)_s = 0$$

$$N = \frac{1}{2}\rho\int_s^L\left\{\int_s^\xi (v_s^2)_{tt}\,\mathrm{d}\eta\right\}\mathrm{d}\xi + \frac{1}{2}m\int_s^L \delta(\xi - d)$$
$$\times\left\{\int_0^\xi (v_s^2)_{tt}\,\mathrm{d}\eta\right\}\mathrm{d}\xi + m(z_{tt} - g)$$
$$\times\int_s^L \delta(\xi - d)\,\mathrm{d}\xi + \rho L\left(1 - \frac{s}{L}\right)(z_{tt} - g)$$
$$- J_0\delta(s-d)\{\tfrac{1}{2}v_{stt}v_s^2 + v_s v_{st}^2\}$$

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So, either 1 can use this multimode generalized Galerkin principle or single mode Galerkin principle, were this n equal to only 1.

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$$v(s,t) = \sum_{n=1}^{\infty} r\psi_n(s)\,u_n(t),$$

$$\ddot{u}_n + 2\varepsilon\zeta_n\dot{u}_n + \omega_n^2 u_n - \varepsilon \sum_{m=1}^{\infty} f_{nm} u_m \cos\phi\tau$$
$$+ \varepsilon \sum_{k=1}^{\infty}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty} \{\alpha_{klm}^n u_k u_l u_m + \beta_{klm}^n u_k \dot{u}_l \dot{u}_m$$
$$+ \gamma_{klm}^n u_k u_l \ddot{u}_m\} = 0, \quad n = 1, 2, \ldots, \infty$$

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So, if you take n equal to only 1, then it will reduce to r psi n s into u n t. So, or n will be equal to 1 so, in that case it will be r psi into u. So, in that case by substituting that equation so, you will get a equation, in its temporal form.

By substituting this equation in this spatiotemporal equation and so, as this is; this shape function may not be the exact Eigen function. So, there will be some residue, now by minimizing this residue one can get these temporal equation of motion.

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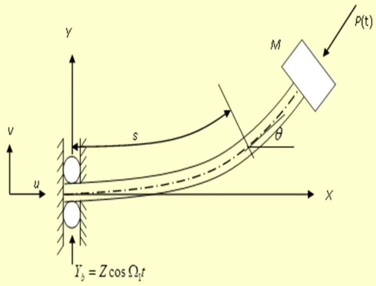

$$M(s) - M_\xi(s) - M_L(s) = 0$$

$$M(s) \approx EI\left(v_{ss} + \frac{1}{2}v_s^2 v_{ss}\right)$$

$$v_s^2 + (1+u_s)^2 = 1$$

$$u(\xi,t) = \xi - \int_0^\xi \left(1 - v_\eta^2\right)^{\frac{1}{2}} d\eta$$

$$M_\xi(s) = -\int_s^L \rho A \ddot{u} \int_s^\xi \sin\theta \, d\eta \, d\xi - \int_s^L \rho A\left(\ddot{v} + \ddot{Y}_b\right) \int_s^\xi \cos\theta \, d\eta \, d\xi$$

$$M_L(s) = -M\ddot{u}\int_s^L \sin\theta \, d\xi - M\left(\ddot{v} + \ddot{Y}_b\right)\int_s^L \cos\theta \, d\xi - P(t)\int_s^L \sin\theta \, d\xi$$

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$$EI\left(v_{ssss}+\frac{1}{2}v_s^2 v_{ssss}+3v_s v_{ss} v_{sss}+v_{ss}^3\right)+\rho A v_s\left(\int_0^s\left(\dot{v}_\xi^2+v_\xi \ddot{v}_\xi\right)d\xi\right)+M\left(\ddot{v}+\ddot{Y}_b\right)v_s v_{ss}+v_s v_{ss}$$

$$\left(\rho A\ddot{Y}_b(L-s)+\int_s^L\left(\rho A\ddot{v}+c_d\dot{v}\right)d\eta\right)-v_{ss}\left(\int_s^L\rho A\int_0^\xi\left(\dot{v}_\xi^2+v_\xi\ddot{v}_\xi\right)d\xi d\eta+M\int_0^s\left(\dot{v}_\xi^2+v_\xi\ddot{v}_\xi\right)d\xi\right)+$$

$$\left(1-\frac{1}{2}v_s^2\right)\left(\rho A\left(\ddot{v}+\ddot{Y}_b\right)+c_d\dot{v}\right)+\left(P(t)v_S\right)_s=0$$

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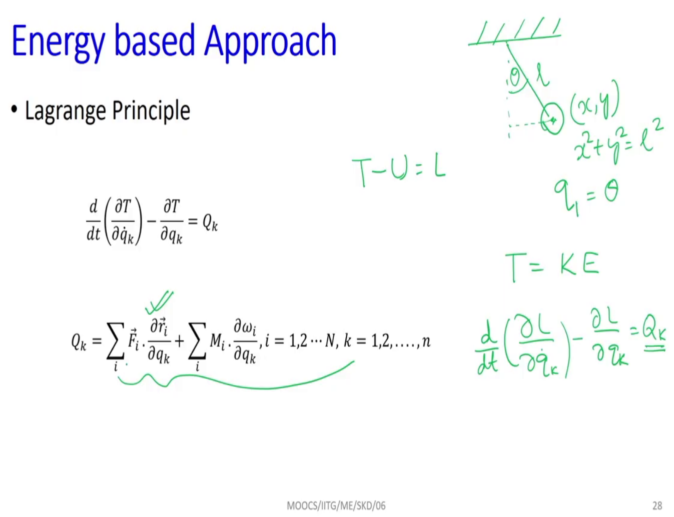

So, in this way one can get the equation of motion. So, then in Lagrange based approach so, last class we know what is the Lagrange’s equation. So, this is equal to d by dt of del T by del q k dot minus del t by del q k equal to capital Q k. So, here q k is the generalized coordinates.

So, we have discussed regarding the generalized coordinate for example, in case of this simple pendulum the example of simple pendulum as given. So, here the position of the bob can be written by its coordinate x and y. So, this is the physical coordinate.

So, but these physical coordinate x and y are interrelated by the constraint equation that is x square plus y square equal to l square x square plus y square equal to l square. But, instead of using these two coordinate x y one can use this coordinate that is theta to express the position of the mass. So, here x will be equal to so, you can write what is x so, x equal to l sin theta and y equal to l cos theta.

So, that way one can write these coordinate or the position of the mass by using these generalized coordinate theta. So, generalized coordinate is nothing, but the minimum number of coordinates required to express, the motion of a body here by using only theta. So, we can express the motion of the mass.

So, now you know now the generalized coordinate. So, here q k at q or k equal to 1 here k equal to 1 only single degree of freedom system. So, q equal to 1 so, this q 1 equal to theta by using q 1 equal to theta we can derive this equation of motion. So, here T is the kinetic energy of the system and q 1 dot or q k dot is the velocity term so, differences on of q k is q k dot.

So, d by dt of del T by del q k dot minus del T by del q k equal to capital Q k. So, here by taking this T minus U equal to L that is the Lagrangian of the system. So, this equation can also be written by d by dt of d by dt of del L by del q k dot minus del L by del q k equal to capital Q k. So, this is the generalized force. So, the generalized force can be written by using this expression.

If only force is there, then the first term only can be written if both force and moment are acting on the system, then this moment term also can be written. So, capital Q k that Q k equal to summation F i dot del r i by del qk what is r i r i is the position vector. So, in this case the position vector of this mass that thing can be written by using the r i.

So, r i in this case it will be equal to so, x i plus y j so, by using the coordinate system i and or the unit vector along i along x direction as i and y direction as j. So, in a vector form r can be written as x i plus y j and in that way also force can be if some force is acting on the system, then that force also can be written in a vector form.

Then, by doing the dot product we can find the; find this these generalized force. Similarly from the moment side also M i dot del omega i by del q k will give you the q k.

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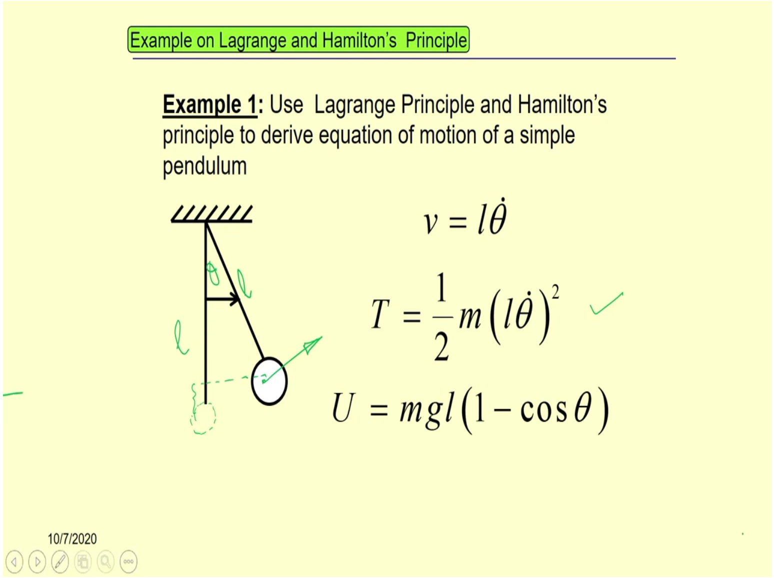

So, let us take the example one simple example of the simple pendulum. So, here the velocity so, for finding the kinetic energy, we know we must have to find the velocity. So, one has to first write down the position vector r so, or let this length equal to l so, if the length equal to l.

Then so, one can write the velocity equal to l theta dot or l omega. So, then kinetic energy equal to half m into l theta dot square, similarly potential energy so, potential energy is due to change in this position. So, initially the mass was here so, a tail length l.

So, now, as it has move to this position so, this is the change in position. So, this U equal to so, U will be equal to m g l into 1 minus cos theta so, this angle is theta. So, this angle is theta. So, this is l cos theta so, total length equal to l.

So, this is l cos theta after moving an angle theta so, the change in height equal to l into 1 minus cos theta. So, change in potential energy that is your U equal to m g l into 1 minus cos theta. So, now, taking T equal to half m l theta dot square and U equal to m g l into 1 minus cos theta. So, we can derive this equation of motion.

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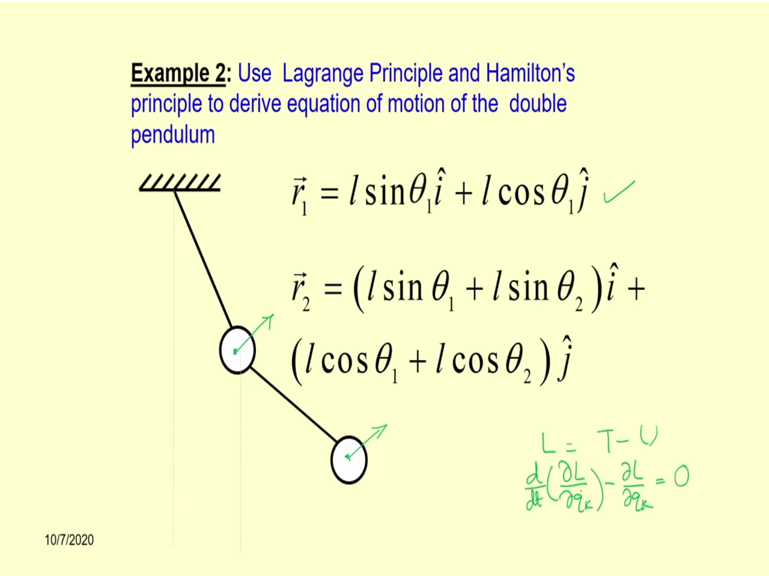

So, the equation of motion can be derived, similarly for a double pendulum. So, you can write this first you can write this position vector r 1, r 1 of these and then so this is position vector of the first mass. Similarly, the position vector of the second mass you can write. So, after writing the position vector of first mass and second mass by differentiating that thing you can write down, the velocity term.

Then, after getting the velocity term, then the kinetic energy term can be written and potential energy can be written so, as this energy term are scalar term.

So, you need not have to worry much regarding the direction. So, after finding this velocity term, and the kinetic energy and potential energy, then by writing L equal to T minus U, L equal to T minus U and then using the Lagrange principle d by dt of del L by del q k dot minus del L by del q k. So, equal to as no force is acting, then you can put it equal to 0 and you can derive this equation of motion.

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$$\begin{bmatrix} (m_1+m_2)l^2 & m_2 l^2 \\ m_2 l^2 & m_2 l^2 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{Bmatrix} + \begin{bmatrix} (m_1+m_2)gl & 0 \\ 0 & m_2 lg \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

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So, the equation of motion can be written in this matrix form like this. So, here it has been taken that the sin theta 1 so this is theta 1. So, sin theta 1 and theta 2 are small. So, if theta 1 and theta 2 are not small, then it will be replaced by sin theta 1 and sin theta 2 term. So, this way one can derive this equation of motion by using Lagrange principle.

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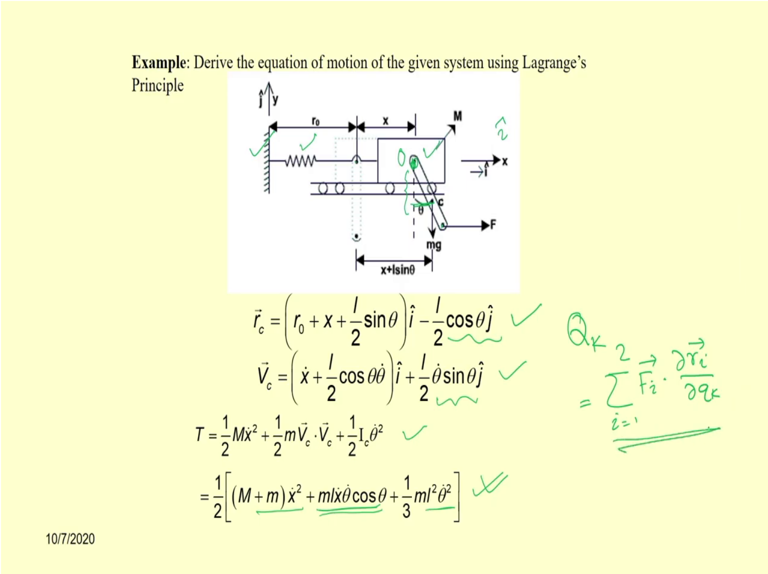

So, for example, let us take another example so, here this is a 2 degrees of freedom system. So, initially the mass this cart is at this position this dotted position, now it has shifted to this position. So, in this cart so, there is a pendulum compound pendulum or a rod hanging from this cart. So, when this is moving when the cart is moving towards right, the pendulum this compound pendulum or the bar is rotating by an angle theta.

So, initially you must ride the to solve this type of problem. So, first you write the position by using this position vector. So, here we have fixed the coordinate system physical coordinate system about this axis. So, about the y axis we have fixed the coordinate system. So, we can write the position vector. So, after writing the position vector, then we can write down the velocity term.

So, after writing velocity we can find the kinetic energy and potential energy and then we can find the equation of motion. So, for example, in this case let us assume that the mass this cart has moved by an amount x, initially it was at this position the center of this this point was here.

So, at a distance r 0 and it has a displacement of x. So, when it is moving by an amount x let the bar the bar hanging from this position. So, it has rotated by an amount theta. So, the position so, we can write down write down the position vector of this point c let this point is c.

Then, the position vector r c will be equal to r 0 plus x plus this distance. So, let us take that of the center of mass center of mass is c. So, we have taken the center of mass as c. So, let the center of mass is at l by 2 distance from this thing so, this is l by 2 into so, this distance equal to sin theta. So, l by 2 sin theta. So, these angle we have taken theta so, this is l by 2 sin theta.

So, r c equal to r 0 plus x plus l by 2 sin theta. So, we have taken these direction i and along the vertical direction as j so, minus so, this the horizontal component and vertical component will be equal to minus l by 2 cos theta vertical component, this is the vertical component r it is in downward direction. So, that is why this negative sign so, minus so we can have these minus l by 2 cos theta 2.

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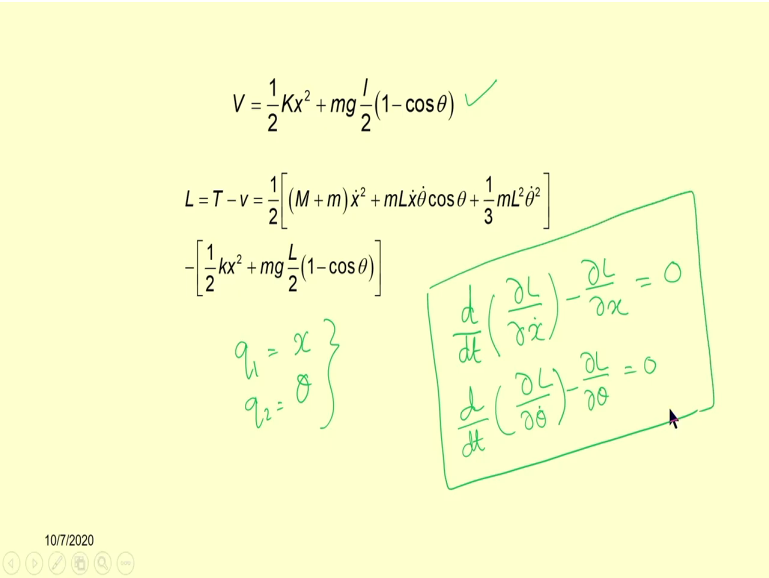

So, the kinetic energy now, this velocity differentiating these term. So, the velocity equal to r 0 so, differentiation of r 0 equal to 0. So, differentiation of x equal to x dot, then this l by 2 sin theta will give you half l by 2 cos theta into theta dot i plus, then this cos term cos theta term. So, this part differentiation gives l by 2 theta dot sin theta j. Then, this kinetic energy kinetic energy is due to this mass translation of the mass.

So, that is equal to half M x dot square and for this rod also so, kinetic energy of this rod. So, the center of mass so, two way you can find the kinetic energy of this rod either pure rotation about this point; about this point let me write this point as O pure rotation about O you can write or both rotation and translation about point d point c, that is the center of mass.

So, by writing about c so, you can write this is equal to half m V c dot V c that is velocity dot velocity. So, you know that is equal to half m V square plus then half I c into theta dot square.

So, you can write this by expanding this thing, then you can write this is equal to half M plus m x dot square plus m l x dot theta dot cos theta plus one third ml square theta dot square.

So, here actually by without solving this equation or expanding this equation by hand manually, you can use symbolic software like this Mathematica or maple or you can write the equation or write this thing in MATLAB also. In symbolic form to differentiate first you write down your r c that is position vector.

Then differentiate to get velocity then write the expression this way in symbolic form so, as to get this equation of the kinetic energy. So, when the expressions are very big. So, it will be difficult to write down or find the equations by manually doing this problem.

So, in that case you must have to use this mathematical software or symbolic software to derive this equation of motion. So, now, you know how to find this kinetic energy here, you just see either. So, if you will take actually rotation about O so, that will be equal to one third ml square theta dot square. So, you can see a couple term here so, which contain both x dot and theta dot.

So, this term contain both x dot and theta dot. So, here you have a term x dot square. So, here you have a term theta dot square. So, completely this is a two degrees of freedom system. So, with x as one degree of freedom and then these theta is another degree of freedom.

So, now potential energy so, we have a spring here. So, as the spring is having a displacement so initial so spring is having a displacement of x. So, let us write or take the take the stiffness of the spring as K. So, in that case it will be half K x square and as the mass position of mass is changing by l by 2 1 minus cos theta. Then, the potential energy of the system equal to V equal to half K x square plus m g l by 2 into 1 minus cos theta.

So, now, by writing this L equal to T minus v so, T minus v so, half M plus m x dot square plus m L x dot theta dot cos theta plus one third ml square theta dot square minus half k x square plus m g L by 2 1 minus cos theta. You can use this Lagrange principle d by dt you just see you have to coordinate.

So, this is two degree of freedom system. So, here you have q 1 you can take q 1 equal to x and q 2 equal to theta. So, as q 1 equal to x and q 2 equal to theta so, two equations you can find so, d by dt of del L by del u k dot that is del x. So, first you can do with respect to this, then del L minus del L by del x.

So, this will be equal to so you just see. So, if there is no force then it will be equal to 0. So, if some forces are acting, then it will be generalized coordinate capital Q 1. Similarly your second equation will be d by dt of del L by del theta dot minus del L by del theta. So, this is equal to 0. So, by using these two formula so, you can derive this equation of motion.

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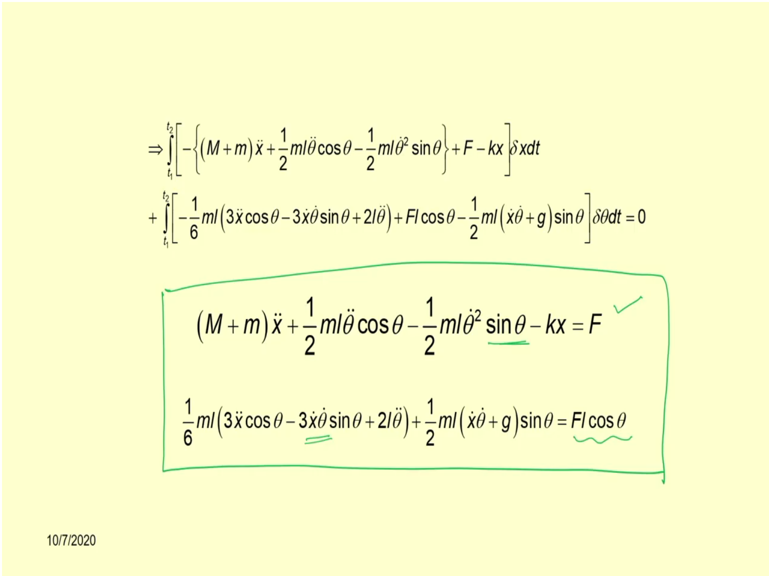

So, this way you can use this Lagrange principle to derive this equation of motion so, here there is a force acting. So, if there is a force acting then that forcing will be there. So, let us see if there is a force. So, here a force F is acting so, as F is acting. So, you can find so, how to find this Q? So Q is nothing but, so here in this case you can have Q 1 and Q 2.

So, you know the formula Q k equal to summation i equal to 1 to n. So, in this case you can have 2. So, this will be equal to F i dot del r i by del F i so, this is del r i by del q k. So, this capital Q k equal to F i dot del r i by del q k. So, here assuming the force acting in this horizontal direction and also you have a mass also acting m g downward is also acting so, taking both the forces. So, you can derive you can find the equation of motion.

So, already you have found this r i and then by using that thing so, you can find this equation. So, in the first case your Q 1 becomes F and in the second so, Q 2 becomes F l cos theta. So, you have this is the equation of motion so, you got so you just see. So, this is not a equation of motion of a linear system.

So, in this case the equation of motion so, you can further expand. So, the sin theta and cos theta term here also you have a combination of x dot and theta dot term so, by expanding the sin theta cos theta and taking note or making them of higher order. So, you can get this non-linear equation of motion. So, this way you can find the equation of motion by using Lagrange principle.

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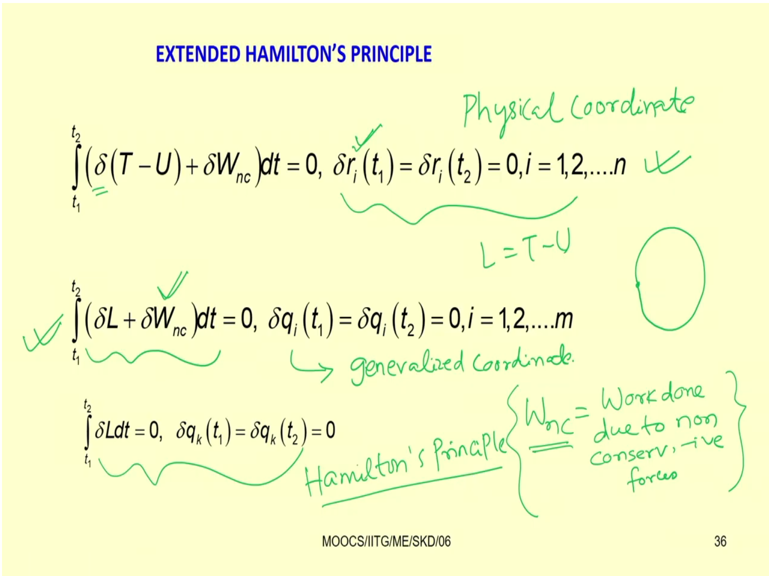

So, let us see if we want to use these extended Hamilton principle, then how we can derive this equation of motion? So, to derive this extended Hamilton principle actually you must know this principle and how this principle works.

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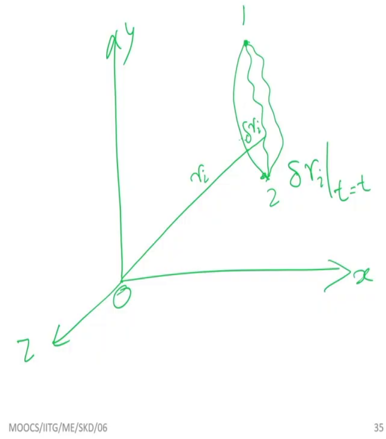

So, in case of the extended Hamilton principle the assumption is that so, let this is state 1 this is state 2. So, let the body is moving from state 1 to state 2. So, let this is your state 1 this is state 2. So, you can have a so, let this is the reference coordinate system. So, from the reference coordinate system so, this is you can take the position vector at any time. So, you can take that position vector you can find and so, there are several paths from 1 to 2.

So, one can follow several path let for this is one, let this is another path also. So, out of many path so, this is another path also so, out of many path 1 path will be the true path and other paths will be the varied path. So, if this is the true path let it is equal to you can write this position r i and the position vector of any of the varied path can be written by using so, this is delta r i. So, it will be r i plus delta r i.

The position vector so, this is O so, you can take this x y z; x y z. So, coordinate systems you can take and you can find the position vector. So, let the so, it is moving from position 1 to 2. So, the assumption is that so, in this extended Hamilton principle the assumption is that the valid path and the true path intersect at this point 1 and 2 so, it meets at point 1 and 2. That means, this delta r i delta r i at t 1 t equal to t 1 or at t 1 and t 2 you can tell that way also.

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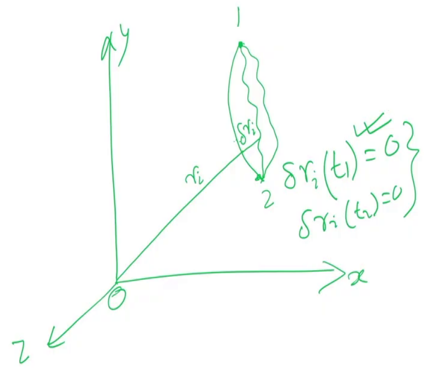

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So, at t 1 t 2 so, that is equal to 0; delta r i at t 1 equal to so, at this position 1 so, that is position 1 equal to 0 and delta r i at t 2 also equal to 0. So, this is the basic assumption in this extended Hamilton principle.

So, taking or knowing this varied path and the actual path. So, this extended Hamilton principle is derived. So, that principle is del T minus U plus delta W nc dt equal to 0 and already I told you the condition. So, this is delta r 1 at t 1 equal to delta r delta r i at t 2 equal to 0. So, here t is again the kinetic energy and U is the potential energy and W n c is the non-conservative work done.

So, W nc is the work done due to non conservative forces; work done due to non conservative forces ok. So, already you known so, there are two type of force; one is conservative force and second one is the non conservative force, in case of the conservative force the work done in a cycle. So, let it start from here and come back to this position.

So, the work done equal to 0 and in non conservative force as there will be some loss of energy. So, this will not be equal to 0.

So, for example, work done due to friction and where the losses occur. So, there the work done is known to be that due to non conservative force. So, writing this L equal to T minus U, one can write down this equation in this form also delta L plus so, delta is the del delta this is the delta operator differential operator. So, integration t 1 to t 2 delta L plus delta W nc dt equal to 0. So, this is the extended Hamilton principle.

So, here so r i is written in terms of the physical coordinate system. So, you know already the physical coordinate system, r i is the physical coordinate and q i is the generalized coordinate so, q so these are the generalized coordinate ok. So, by using this generalized coordinate q i. So, you can derive the or you can write down this equation of motion or you can find the equation of motion.

So, if there is no conservative force, there is no conservative force that is delta W no non conservative force that is delta W nc equal to 0. Then, this equation reduces to this that is integration t 1 to t 2 delta l dt equal to 0 qk at t 1 equal to q k at t 2 equal to 0. So, this equation is known as Hamilton's so this is from the so, this is your Hamilton's principle. [vocalized-voice].

And this is extended Hamilton principle. So, the extended Hamilton principle is for by extending or including these non-conservative work done, due to non conservative force. So, when this non-conservative force equal to 0. So, this extended Hamilton principle reduces to

Hamilton's principle. So, so according to your Hamilton's principle so, integral t 1 to t 2 del L dt equal to 0.

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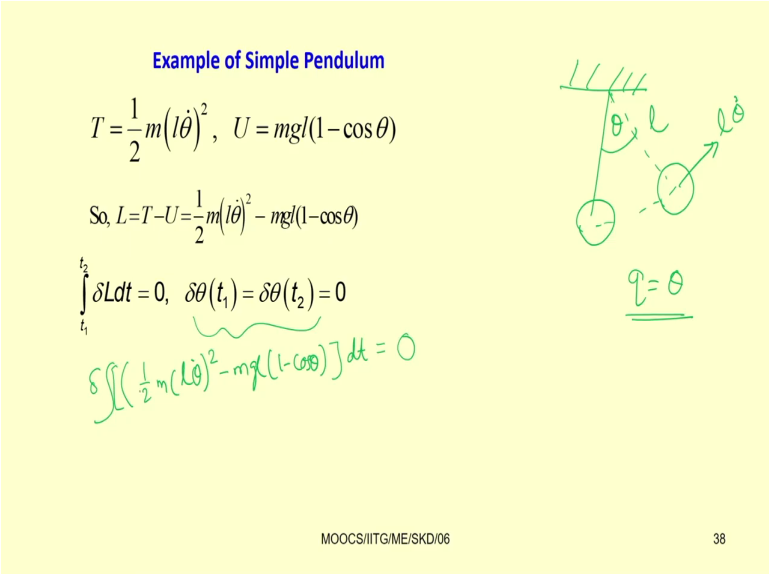

So, let us see with some examples. So, we can see this thing we can take the example of the simple pendulum, same example of simple pendulum and derive this equation of motion by using this Hamilton principle. So, already we have seen that velocity, velocity if this angle is theta if this length is l. So, this velocity equal to l theta dot.

So, this kinetic energy T equal to half m l theta dot square and U potential energy equal to m g l into 1 minus cos theta. So, L equal to T minus U equal to half m l theta dot square minus m g l into 1 minus cos theta. So, now, let us find let us derive this thing so, this delta L dt equal to 0 so, according to this principle delta theta so, here q k equal to q k equal to 1. So, and q equal to theta that it the generalized coordinate q equal to theta so, theta at t 1 equal to theta at t 2 equal to 0.

So, let us integrate this things so, by now you have to apply this del operator. So, del of half m l theta dot square minus m g l into 1 minus cos theta. So, you just see this will be integrated dt delta l dt so, here non conservative work done delta W nc equal to 0. So, we are not there is no force acting on the system.

So, you can take it is equal to 0 so, and you can make it equal to 0. So, now, if this delta operator can enter inside this thing so, you can write or by slightly expanding this thing.

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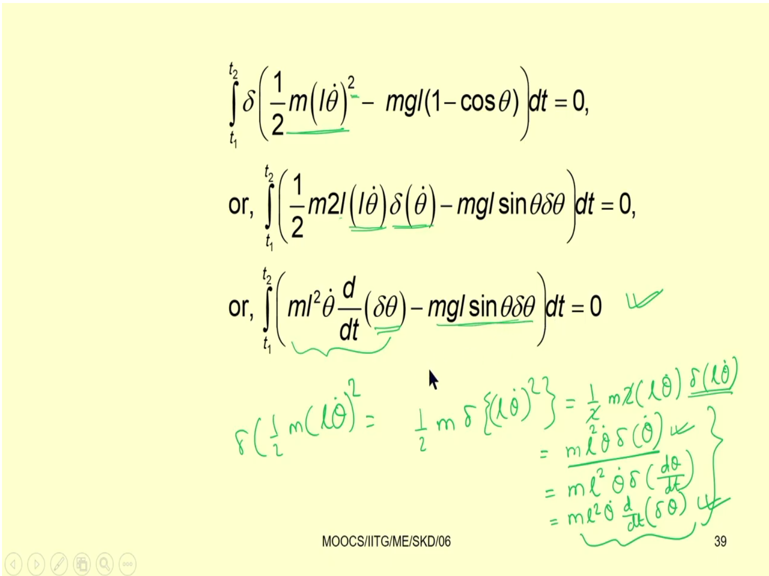

So, this integral t 1 to t 2 so, this becomes integral t 1 to t 2, then delta of half m l theta dot square minus m g l into 1 minus cos theta dt equal to 0 or this is equal to t 1 to t 2 half. So, this thing now this operate this del operator here so, by putting this del operator here this constant half m is also constant.

So, l theta dot square so, this becomes 2 into you just see this is so, this is 2 into you can see this part. So, 2 into l theta dot so, this is l theta dot then del of l theta dot you have to write. So, from del of l theta dot this l has come constant taken out, then this becomes theta delta theta dot.

So, now this part by putting this del operator. So, this becomes half m into 2 l so, this 2 has come here so, this 2 has come 2 into l theta dot then del so, actually you can write this way. So, del of half m l theta dot square equal to half m del of l theta dot square. So, this is equal to half m 2 into l theta dot into del of l theta dot. So, this becomes half so, this 2 and half cancel so this has gone. So, this becomes m l this is theta dot then from this thing.

So, we can write this is equal to another l will this l is constant so, ml square theta dot into del theta dot so, here is the trick you have to play. So, this part by expanding so, you can write this thing in this form ml square theta dot. So, this del theta dot you have to write this way del of so, theta dot is nothing, but d theta by dt.

So, you can exchange these things and you can write this equal to ml square theta dot into d by dt of del theta. So, this is the trick actually you have to play so, that you can derive this equation easily.

So, this part you just see it is written this way. So, ml square theta dot d by dt of del theta d by dt of del theta. So, here delta theta dot is written in this form. So, you have to remember this part only while deriving the this equation motion. So, delta theta dot equal to del of d theta by dt so, this d theta by dt is taken out. So, that this becomes ml square theta dot d by dt of delta theta.

So, this is only the tricky part of this derivation, otherwise you can follow the standard procedure of differentiation and integration and you can have it. So, then this part is minus m g l so, from the potential energy differentiating these thing.

So, m g l m g l del of m g l equal to 0 so, then minus minus plus so, this becomes m g l cos theta ;so, del of m g l cos theta. So, that gives rise to minus m g l sin theta and delta theta so dt. So, now, you can easily integrate this thing so, this ml square theta dot d by dt of delta theta into dt so, dt dt will cancel. So, this integration becomes ml square theta dot d of d theta.

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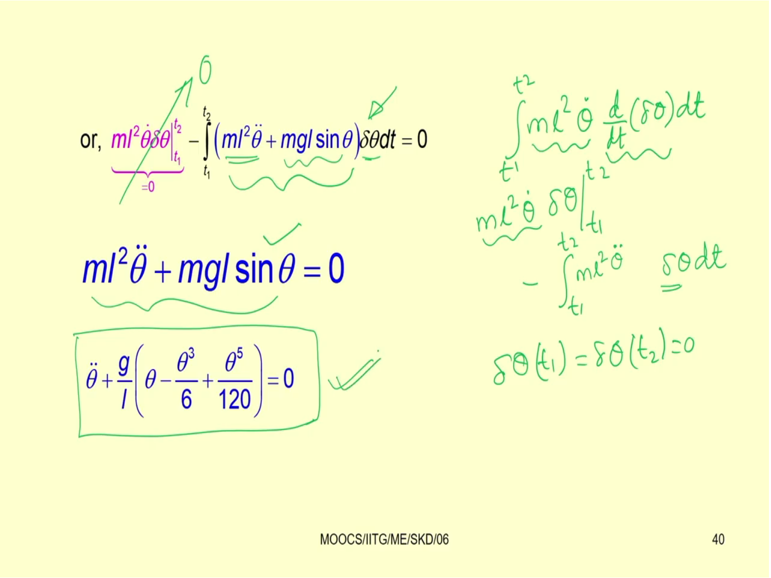

So, now, you can integrate easily by applying this integration by parts. So, this becomes so, you have the integration so, you just see the integration was ml square theta dot into del theta. So, you have to integrate so, this was let us see this again. So, this is ml square theta dot d by dt of delta theta into dt. So, this is this is written d by so, you just see this part. So, this is d by dt of delta theta into dt. So, these integration you have to do.

So, let us take this as the first part and this as the second part and do this integration by parts. So, this integration is from t 1 to t 2. So, first part remaining remain as it is so, you can put it ml square theta dot and integration of d by dt of del theta dt. So, this becomes integration of this differentiation part this becomes delta theta so, this from t 1 to t 2 then minus integration t 1 to t 2. So, this is integration by parts. So, this delta theta remain as it is then differentiation of this part.

So, differentiation of this ml square theta dot. So, differentiation of ml square theta dot is nothing, but ml square theta double dot. So, this becomes ml square theta double dot d square theta by dt square. So, the first term you got in this integration this is ml square theta double dot into this delta theta is taken outside into delta theta into dt.

Then from the potential energy so, the from potential energy terms so, you have another term. So, this term equal to minus mgl sin theta delta theta dt. So, integration of this thing so, this is sin theta delta theta dt so, that integration becomes so, you just keep it as it is so, you need not have to expand that thing. So, just you have to keep it is as it is so, this becomes mgl sin theta delta theta dt.

So, now, you just see already we know our delta theta or delta r i at t 1 equal to delta theta at t 2 equal to 0 so, already we know this thing. So, this part tends to 0 so, this part as delta theta at t 1 equal to delta theta at t 2 equal to 0. So, this part becomes 0 and this is the part as delta theta is arbitrary delta theta is arbitrary so, it cannot be 0 so, the integral to be 0. So, we must have this part equal to 0. So, this part is nothing but the equation of motion of the system.

So, from this thing so you can find so, this ml square. So, this way we can find the equation of motion of the system. So, now, expanding the sin theta so, we can write this sin theta equal to theta minus theta cube by 6 plus theta 5th by 120. So, the equation of motion becomes theta double dot plus g by l, theta minus theta q by 6 plus theta 5th by 120 equal to 0. So, this way you can apply this extended Hamilton principle to derive this equation of motion.

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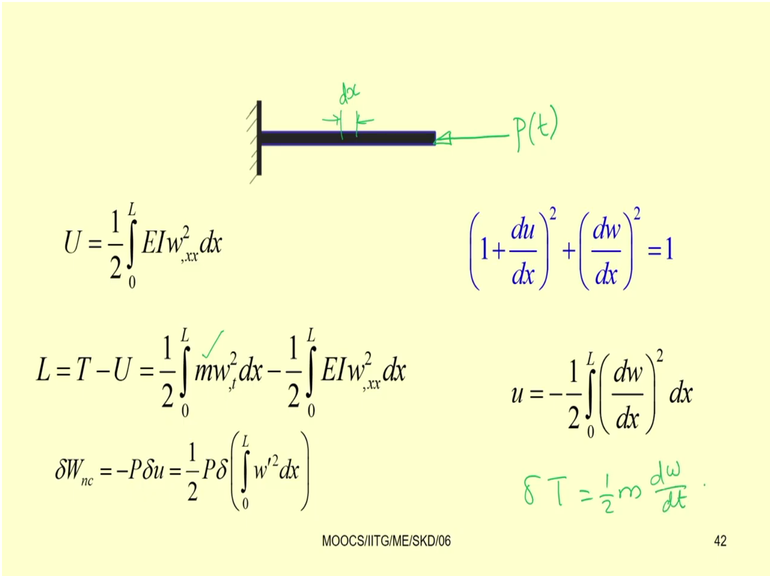

So, let us take one more example. So, previous thing we have taken a discrete system. So, let us take one more example where we have a continuous system and so, let a force P t is acting axial in axial direction. So, we have a cantilever beam subjected to a axial force P t and for the system so, we want to derive this equation of motion.

So, here let us take this m equal to mass per unit length then m so, by taking a small element dx at a distance x we can take a small element dx. So, mass of the small element becomes m into dx. And the kinetic energy of the systems will be equal to for the small mass first you find that is delta T.

So, delta T will be equal to half m. So, let the transverse displacement can be written as w, then this differentiation dw by dt can be written but here instead of write d.

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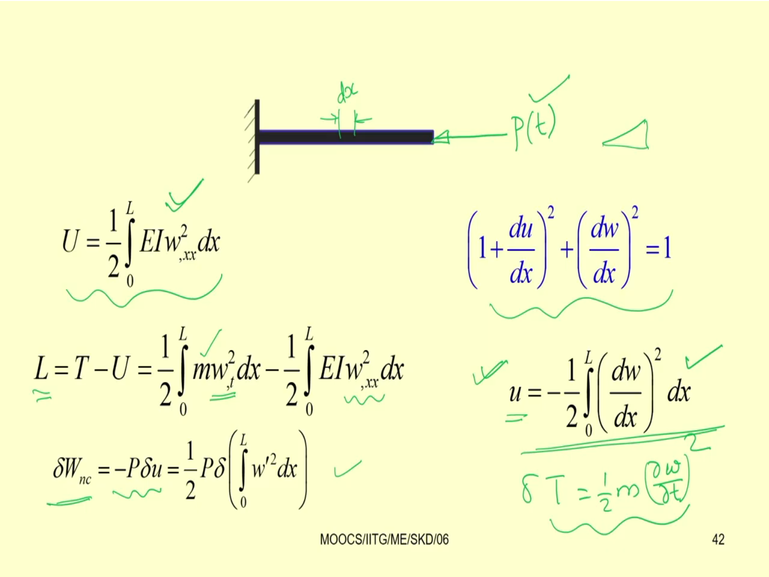

So, you can put the del w by del t, because w is a function of both space and time ok. So, w by del w by del t. So, that is velocity or it is written as w comma t. So, this is velocity square so, half mv square so, this is velocity del w by del t is velocity. So, this delta T that is delta T that is kinetic energy of the small element can be written as half m del w by del t square. So, total kinetic energy will be integral of that thing integration 0 to L mw t square dx.

And then potential energy so, strain energy already you know so, strain energy can be written in this form so, half integration 0 to l EI del square w by del x square whole square. So, this is the potential energy or strain energy of the system and T is the kinetic energy. So, you can write L equal to T minus U equal to half 0 to L mw t square dx minus half 0 to L EI w square. So, double EI double differentiation of w whole square dx.

So, now, by taking this in extensibility condition so, we can take so for example, this is a small element if you take. So, in the small element let the displacement along x direction equal to U and in the y direction or in this x transverse direction it is w. So, you can find so, this inextensibility conditions by using this expression. So, that is 1 plus del u by del x whole square plus del w by del x whole square equal to 1.

Or from this thing so, you can get u that is the displacement in this form. So, u equal to minus half integration 0 to l dw by dx square dx. So, this way you can write or you can use this partial derivative also. So, if you are considering w is a function of x and t, otherwise if you are considering for the time being w is a function of t. So, this way also you can write.

So, you got u that is the deflection or displacement in x direction. So, you have to find the work done due to non conservative force that is due to this force P t the work done. So, delta W nc will be equal to minus P delta u. So, that is equal to as we are putting this compressive force and assuming that U is taking place towards right, then you can put this minus delta u work done equal to minus P into delta u.

So, that is equal to half P into for u you can substitute this expression so, that you can get half P delta 0 to L w dash square. So, this is w dash square dx. So, now by knowing this l and W nc delta W nc so, we can easily derive the equation of motion.

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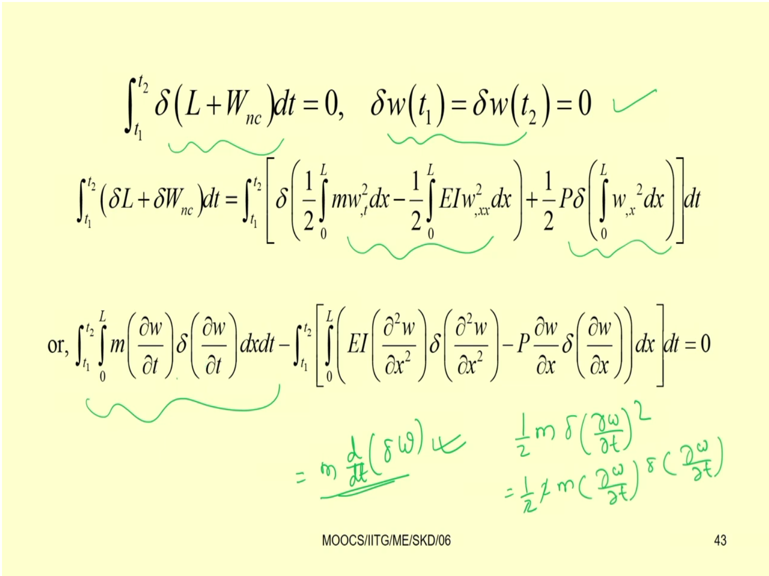

So, the equation of motion can be easily derived by using that expression. So, now, by applying this extended Hamilton principle so, that is integral t 1 to t 2 delta L plus W nc or delta L plus delta W nc dt equal to 0. And condition is that delta w at t 1 equal to delta w at t 2 equal to 0. So, we can find the equation of motion.

So, here so this now by expanding this term so, L equal to T minus U already we have written this is L and this is from P. So, now, just you have to expand this thing by integration by parts. So, here m delta so, this part actually we have to expand and other part just you have to take as it is. So, this is delta of del w by del t square. So, this part you have to just expand. So, this will be equal to half into 2 into m into delta of del w by del t.

So, this thing already I explained. So, this will be equal to m this is del w so, you can exchange this thing so, by exchanging this so it will be m d by dt of delta w. So, if you can write this half m delta del w by del t square in this form and then by using this integration by parts. So, easily you can derive this equation of motion. So, you just see this term, now it can be written in this form.

So, integration t 1 to t 2 integration 0 to L m into del w by del t del of del w by del t dx dt minus integration t 1 to t 2 integration 0 to L EI del square w by del x square del of del square w by del x square minus P into del w by del x del of del del x by del w by del x dx dt. So, you just see now this part you have to integrate by parts. So, if you integrate by parts, then this equation of motion can be obtained easily.

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or, $$\int_0^L\left(\int_{t_1}^{t_2} m\left(\frac{\partial w}{\partial t}\right)\frac{\partial}{\partial t}(\delta w)\,dt\right)dx - \int_{t_1}^{t_2}\left[\int_0^L\left(EI\left(\frac{\partial^2 w}{\partial x^2}\right)\frac{\partial}{\partial x}\left(\delta\left(\frac{\partial w}{\partial x}\right)\right) - P\frac{\partial w}{\partial x}\frac{\partial}{\partial x}(\delta w)\right)dx\right]dt$$

$$\int_0^L m\left(\frac{\partial w}{\partial t}\right)\underbrace{(\delta w)_{t_1}^{t_2}}_{=0\text{ as per defination}}\,dx - \int_0^L\left(\int_{t_1}^{t_2} m\left(\frac{\partial^2 w}{\partial t^2}\right)\delta w\,dt\right)dx$$

$$-\int_{t_1}^{t_2}\underbrace{\left[EI\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\delta\left(\frac{\partial w}{\partial x}\right)\right)_0^L + EI\left(\frac{\partial^3 w}{\partial x^3}\right)(\delta w)_0^L - P\left(\frac{\partial w}{\partial x}\right)(\delta w)_0^L\right]}_{\text{Boundary conditions}}dt$$

$$-\int_{t_1}^{t_2}\left(\int_0^L\left(EI\frac{\partial^4 w}{\partial x^4} + P\frac{\partial^2 w}{\partial x^2}\right)\delta w\,dx\right)dt = 0$$

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So, you can obtain the equation of motion easily and you just see. So, this is the equation of motion now you just see this part. So, when you are integrating it by parts. So, this becomes m del w by del t into del w so, this is t 1 to t 2. So, you know del w t 1 equal to del w t 2 equal to 0. So, this part actually will be 0 as for the definition and so, this part is the boundary conditions. So, this is integral t 1 to t 2 EI del square w by del x square del of del w by del l del x 0 to L.

So, either this part will be equal to 0 that is del square w by del x square that is bending moment or this del w by del x. So, del w by del x is nothing, but the slope or this part you can see. So, this is the shear force this component is the shear force and or this del w. So, here del w is nothing but, that is the displacement.

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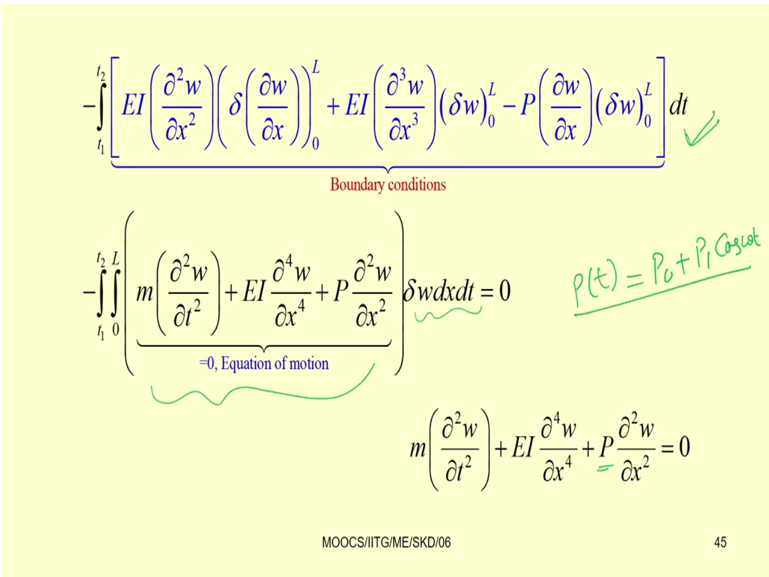

So, either the slope will be 0 or so, four different boundary conditions are generally there in case of the beam. So, those four different boundary conditions are displacement either displacement will be 0, slope will be 0, bending moment will be 0 or the shear force will be 0.

So, in this case so this is the boundary conditions you have got and this part you just see this part which is coefficient of delta w dx dt.

So, as delta w is arbitrary. So, this is the equation of motion of the system. So, the equation of motion of the system becomes m del square w by del t square plus EI del 4th w by del x 4th plus P del square w by del x square so, if 1 will take this P.

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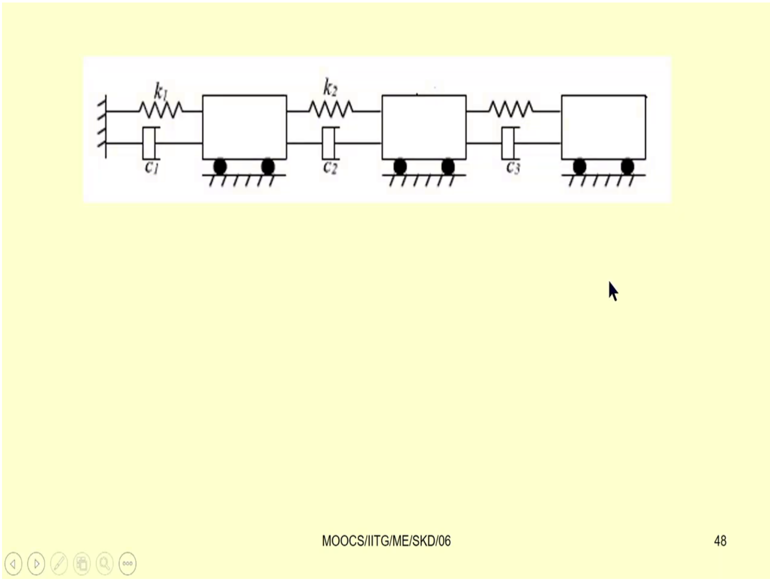

So, one can take this P equal to so, as I told P t I have written there. So, you can take this t P, P t equal to by taking P t equal to P 0 plus P 1 cos omega t you can take also. So, this is equal to P 0 plus P 1 cos omega t so, by taking this so, you can have a non-linear equation of motion. So, which will be that of a parametrically excited system so with this. So, today class we will conclude here.

So, this way you can derive this equation of motion of any system any physical system by using either this Newton second law, or d'Alembert's principle or by using this Lagrange principle or this Hamilton principle. So, these equations actually we have derived when you have a fixed coordinate system. So, if the coordinate systems are not fixed. So, that time so, you have to use either this Lagrange Euler formulation or this Newton Euler formulation.

So, we will study or we will take some examples of Lagrange Euler formulation or Newton Euler formulation during project phase of this course. And also so, you have to derive this equation motion by using the symbolic software like Mathematica, or by Maple or some software like this MATLAB also in MATLAB also you can derive this equation of motion.

First you write down the position vector, then differentiate that thing to find this velocity then, write down the kinetic energy then write down this potential energy. Then, either apply Lagrange principle or Hamilton principle and derive this equation of motion. So, these things will be given you as assignment in this course, to derive this equation of motion for a number of systems using symbolic software. So, thank you. So, this is the end of this module second module. So, next class we will study the third module.

Thank you very much.