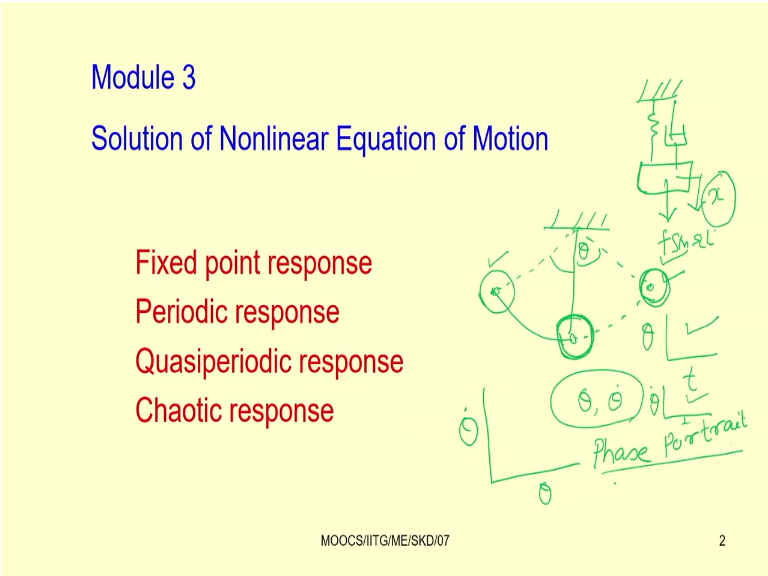
Nonlinear Vibration
Prof. Santosha Kumar Dwivedy
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Lecture – 07
Numerical solution, Analytical solutions: Harmonic Balance method

So, welcome to today class of Non-linear Vibration. So, today we are starting module 3, lecture 1. Last two classes we have studied regarding the developing this equation of motion by using this Lagrange principle and Hamilton principle.

(Refer Slide Time: 00:47)



So, in this module actually we are going to take 6 classes, 2 weeks.

(Refer Slide Time: 00:51)

3	Solution of Nonlinear Equation of Motion	3	1	Numerical solution method
			2	Harmonic Balance method
			3	Lindstd-Poincare′ method
		4	4	Method of Averaging
			5	Method of multiple scales
			6	Recent advanced method

MOOCS/IITG/ME/SKD/07 3

So, in the first for this, in the third week we are going to study regarding this numerical solution method, then harmonic balance method, Lindstd Poincare method. And in next week we are going to study method of averaging, method of multiple scales and recent advanced methods in solving this non-linear equations.

So, we should know what do you mean by the solution of a non-linear differential equation or for that matter what is the solution of a differential equation. So, by solving the differential equation so we can get actually different type of response in case of a vibrating system. So, for example, if you are taking the motion of a simple pendulum; so let this this is simple pendulum; so the simple pendulum when it is moving so if it is disturbed and brought to this position and release then so it will, so at this endpoint it has maximum potential energy, kinetic energy equal to 0. So, its velocity equal to 0 at this position.

So, at the velocity is 0. So, it is not moving upward. So, now, when it is coming back due to this restoring force so at this position, at this position so it has displacement equal to 0, but the velocity is maximum. So, the potential energy is 0. So, the potential energy is converted to kinetic energy. So, the velocity is maximum here at this position, but the displacement is 0.

So, this displacement that is theta and angular velocity theta dot. So, these two parameters are known as the state vector. So, these are the state vectors. So, this angle is the theta. So, by using these two state vector. So, we can represent the motion of the system.

So, here the velocity equal to 0 that is theta dot equal to 0, but theta is maximum. So, we have maximum theta at this position, but here theta equal to 0, but theta dot equal to maximum. Again, so if you can go to this position similarly, we can analyze and we can tell that here also angle is maximum this angle is maximum, but this theta dot is 0 at this positions. So, these position, these position and these position, these three positions. So, where we can have; so for example, in this position theta dot equal to 0. So, here also we have theta dot equal to 0. So, here we have theta. So, we have theta equal to 0.

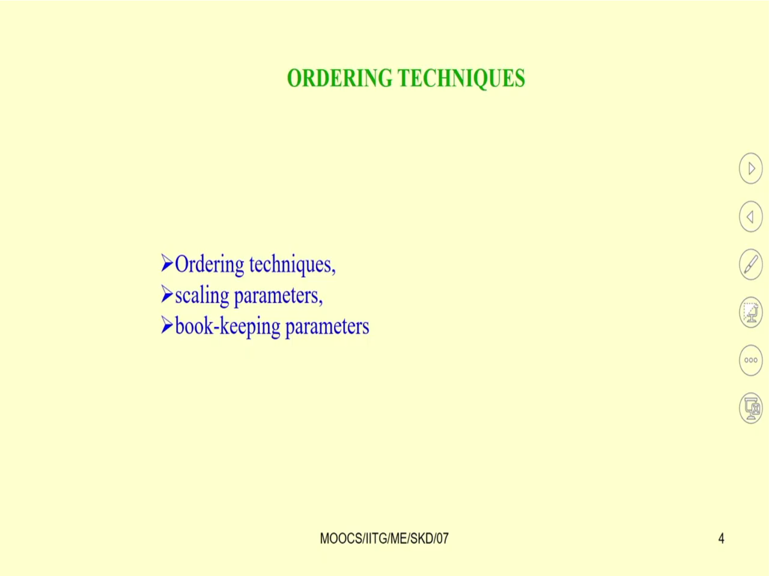
So, we can plot diagram with theta and theta dot and these type of diagrams are known as; these type of diagrams are known as phase portrait. So, we can plot the phase portrait phase portrait. So, this phase portraits can be plotted to find or to show the flow of the motion or the how the motion is taking place. With time so, one can plot this theta versus t how theta is changing with time. Similarly, one can plot this theta dot versus time also. So, combining these data and theta dot so one can plot this theta versus theta dot plot and that will be known as the phase portrait.

So, this variation of theta and theta dot with time when you are studying. So, these are known as the response of the system. So, how theta is varying with time or theta dot is varying with time when you are studying. So, those are known as the response of the solution, response of the system at a particular time t.

So, what is the response of the system, we can conveniently know by plotting this theta versus t or this theta dot versus t. Similarly, in case of a spring mass damper system also so we can plot; so we have a spring mass damper system. So, in this case of spring mass damper system; so this is the mass. So, we can study the motion of this mass x. So, let this is x.

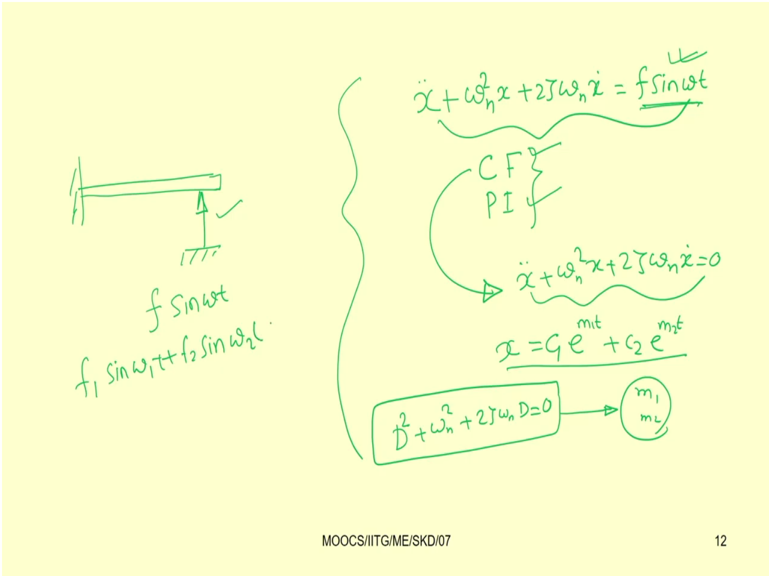
So, we can plot how x is varying with time and how x dot is also varying with time. So, in the introductory class we have discussed regarding this x. So, that response of the system; so the response of the system actually so let it is subjected to a force also response f sin omega t.

(Refer Slide Time: 05:59)



So, we have studied two different part; one is the complementary function and other one is the so for example, we have for this so for this prima systems we have we know the equation can be written in this form x double dot plus omega n square x plus 2 zeta omega n x dot equal to f sin omega t.

(Refer Slide Time: 06:03)



So, the solution of this equations contain two parts; one is the complementary function. So, complementary function or C F and another one is particular integral P I. So, this complementary function is the solution by putting this right hand side equal to 0. So, complementary function will give or will you can obtain this complementary function by solving this part x double dot plus omega n square x plus 2 zeta omega n x dot equal to 0.

So, already you know the solution of this part. So, you can find the auxiliary equation and by solving that auxiliary equation you can find the roots of these equation, auxiliary equation. And finding the roots of the auxiliary equations so always you can write this x equal to c 1 e to the power let m 1 and m 2 are the two roots of this auxiliary equation. So, your auxiliary equation is nothing but in this case this is D square plus omega n square plus 2 zeta omega n D equal to 0. So, this is the auxiliary equation.

So, the solution of auxiliary equation will give you two roots. Let this is m 1 and m 2. So, the solution x will be equal to c 1 m 1 t plus c 2 e to the power m 2 t. So, this way you can find the complementary function. So, complementary part then after getting the complementary part then you can solve for the particular integral also. Particular integral is the solution by taking the forcing term in the right hand side and that solution also already you know the solution.

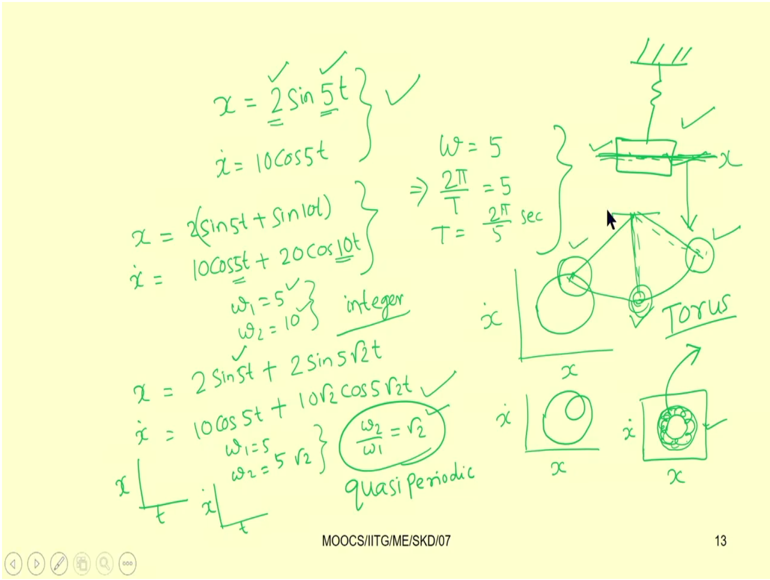
So, the total solution x will be this complementary part and the particular integral. So, in this case also this complimentary part when we have solved this c 1 and c 2 are unknown. So, the c 1 and c 2 will depend on the initial conditions. So, depending on the initial conditions so you have found the complementary part, but in the particular integral so you can find the solution, after getting the solutions we can add these two parts and you can get the complete solution.

So, but you can see as t tends to infinity particularly in case of the vibrating mechanically vibrating systems. So, if there is damping so this complementary part always tends to 0 or die down. So, as the complementary part die down then you will get the steady state solution or the particular integral part of the solution.

So, that thing we will see in a later time, but in today class first we should know what we are going to study or what we are interested to know about the response. For example, in this you just take a cantilever beam in cantilever beam also. So, let you give some vibration to this cantilever beam. So, in this case so let you just put a (Refer Time: 09:38) here. So, by putting the (Refer time: 09:40) and if you can Exide this using the (Refer Time: 09:42) so then you can get different type of response. Here, also you can give different type of forcing.

So, this forcing maybe so this forcing; so you can give a constant force or you can give a periodic force also. So, in case of periodic force the force may be f sin omega t or you can give a combination of force also for example, let you give a force of f 1 sin omega 1 t plus f 2 sin omega 2 t f 2 sin omega 2 t.

(Refer Slide Time: 10:23)



So, for example, in this case of spring mass system solid you know so, if there is no damping the solution x can be written as, so x equal to, let me write with some numerical number. So, let x equal to 2 sin, 2 sin 5 t.

So, here the amplitude is 2 and this omega equal to 5 omega equal to 5 means, omega equal to 5 the meaning of this thing this is equal to 2 pi by T, T is the time period equal to 5. So, the time period of oscillation in this case of the spring mass system. So, can be 2 pi by 5 2 pi by 5 second.

So, this is the time period. So, in this way you can get the time period and this is the amplitude of oscillation. So, in this case if you write this x dot. So, this x dot will becomes 10 cos 5 t and if you plot the phase portrait so you can plot the phase portrait that is x versus x dot already, I have told you how to derive the equation ok.

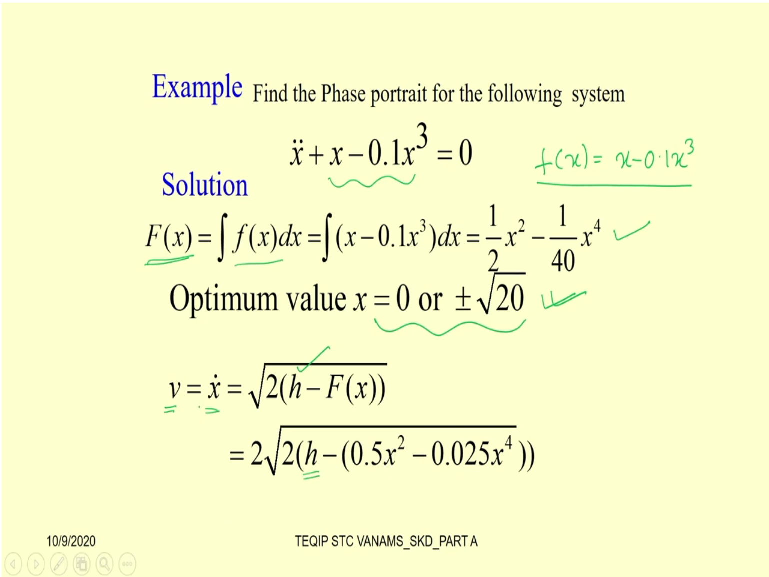
So, already I told you how you can use this excel file or MATLAB to plot this x versus x dot and let us take one more example. So, that is you take x equal to 2 sin 5 t plus sin 10 t. So, here x dot will be equal to so, it can be so for the first part so that is 2 sin 5 t. So, these becomes 10 cos 5 t plus here 20 cos 10 t.

So, in the previous case when you are plotting this x versus x dot so you can get a curve like this. One closed loop circular curve you can get, but in the second case if you plot these thing. So, you just see so, you have two frequencies here. So, one frequency is 5 omega 1 equal to 5. So, in this case omega 1 equal to 5 and omega 2 equal to 10. So, this is 10 cos 10 t. So, 10 so omega 1 equal to 5 omega 2 equal to 10. So, you have two frequency, but you just see the ratio of the frequency is an integer.

So, if the ratio of the frequency is integer, then you will get a periodic response also, but shape will be like this. So, you can plot this thing and verify the shape, but if you take two frequency response, but the response let you take in this way 2 sin 5 t plus 2 sin 5 t plus 2 sin 5 root 2 t then this x dot will be equal to 10 cos 5 t plus 10 root 2 cos 5 root 2 t.

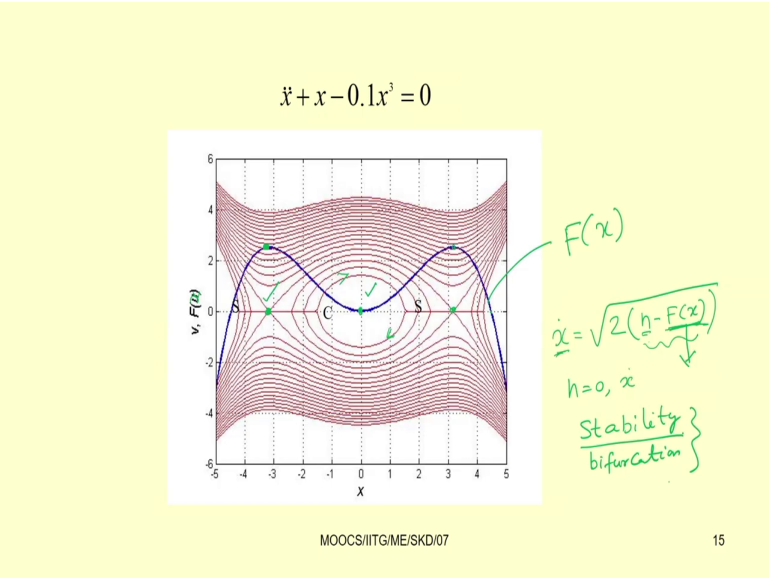
So, here the frequency are you are you are getting two frequency. So, one frequency is 5. So, one frequency omega 1 equal to 5 and omega 2 equal to 10 root 2. So, here the ratio so; if you see the ratio. So, the ratio is omega 2 by omega 1. So, omega 1 equal to 5 and omega 2 equal to 5 root 2 so the ratio is; the ratio equal to root 2. So, root 2 is an irrational number.

(Refer Slide Time: 14:23)



So, here you are getting the ratio to be integer, ratio is integer, but here the ratio is irrational number. So, if you have the ratio irrational number. So, you will get a response so that will look like this. So, that is x versus x dot.

(Refer Slide Time: 14:53)



So, if you plot it. So, this will look like a Torus. So, Torus is; that means, you can have two loops and they are interconnected interwoven. So, you can physically verify these response. So, this is Torus or these type of response is known as quasi periodic response, the response what you will get so, if you plot this x versus t you can plot x versus t and x dot versus t and verify and verify the response. So, these responses are known as quasi periodic response.

So, now, you know two different type of response; one is the periodic response, periodic you know. So, if it is can be written in terms of harmonic then it is harmonic, but in periodic also. So, why are the response will repeat with time that is periodic.

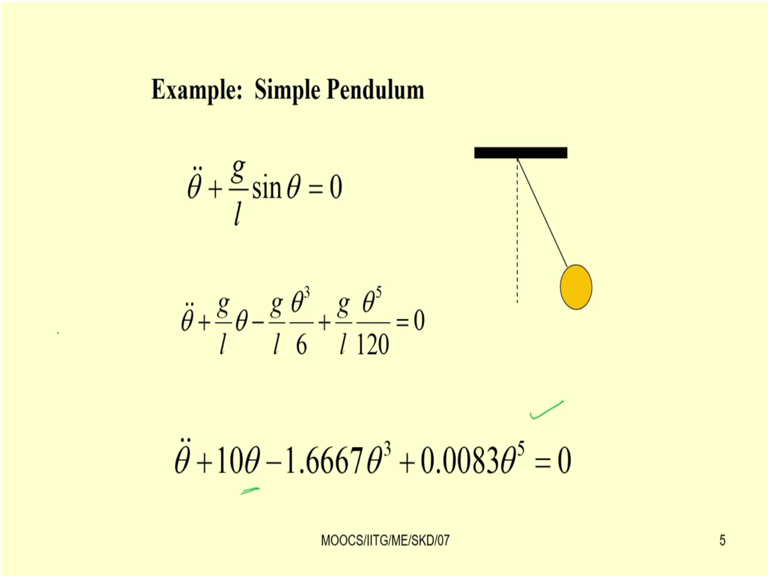
So, the harmonic responses are also periodic responses. You can note all harmonics are periodic, but all periodic are not harmonic response. So, you know the periodic response and also you know the quasi periodic response and in this case another type of response you can see.

So, when you bring these thing down and leave it. So, if you bring it down and leave it so, with time you can see. So, it is oscillating about a fixed point. It is always oscillating about a point. So, where you are displays where you are; where you can find this displacement x equal to 0. So, about this point this displacement x you can take equal to 0. So, this is the equilibrium point. So, we can find the equilibrium point about which the oscillation is taking.

So, those equilibrium points so, either it can be a fixed point, it can be periodic, quasi periodic or chaotic response. So, different type of response we will study later. So, first let us see so, in this particular next 6 classes we are going to discuss regarding this fixed point response, periodic response, quasi periodic response, and chaotic response. Also their bifurcations, so how in non-linear system the response frequency response behaves, how why there is branching, why you what do you mean by multiple solutions. So, all those type of things we are going to study in this module.

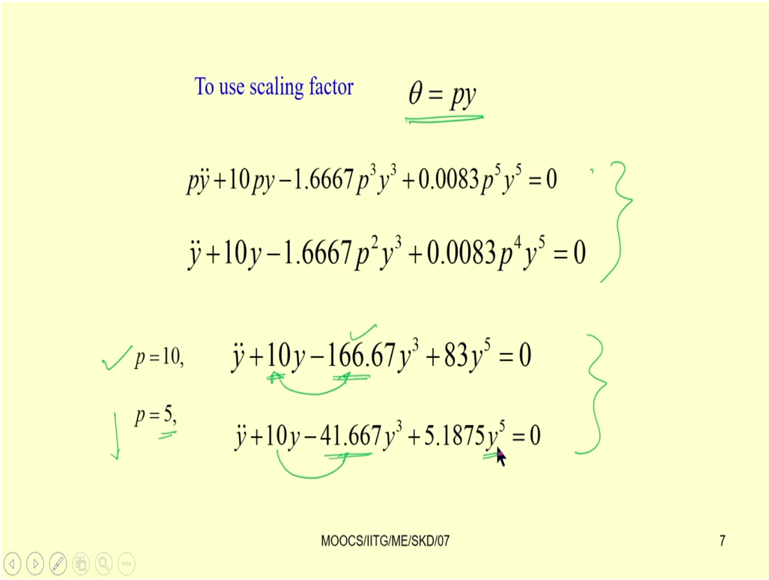
So, already we have discussed regarding the ordering technique ordering technique in case of a non-linear system. So, in ordering technique particularly we are interested or we have studied the scaling parameter and bookkeeping parameter.

(Refer Slide Time: 17:59)



For example in scaling parameter, this is the equation of motion of a simple pendulum. So, here so, this theta, by writing this theta equal to p y so ok.

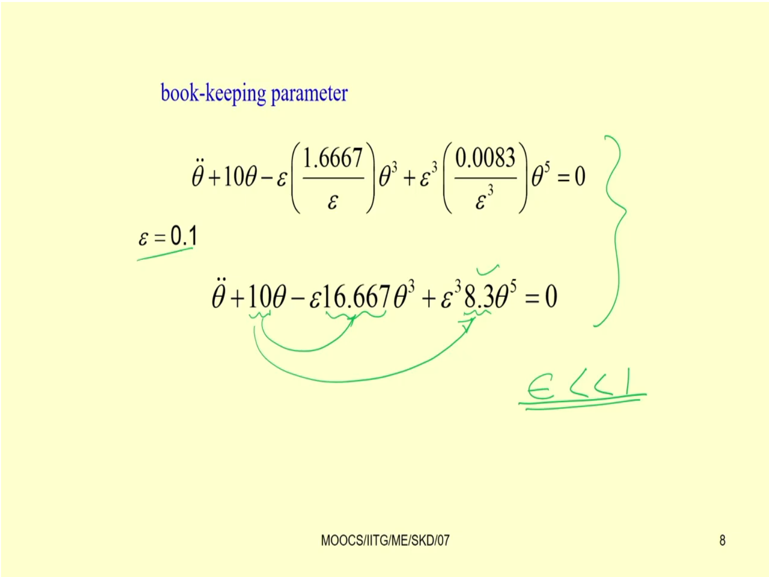
(Refer Slide Time: 18:11)



So, writing theta equal to p y. So, we have found this equation. So, by putting different value of p for example, p equal to 10 we have substitute, then you can check the coefficient and by putting p equal to 5 you can check the coefficient. So, in this way we can order. So, you can see this coefficient is 166 in comparison to 10 this is very large.

So, we have to take a parameter p in such way that these and these parameters would be comparable. So, now, it is very high, but now if I putting p equal to 5 you can check this is coming closer to 10. So, we can take or do some more iterative to find what should be the scaling parameter. So, that this coefficient of the linear term that that is coefficient of y should be similar or same order as that of the coefficient of y cube or y to the power 5th.

(Refer Slide Time: 19:15)



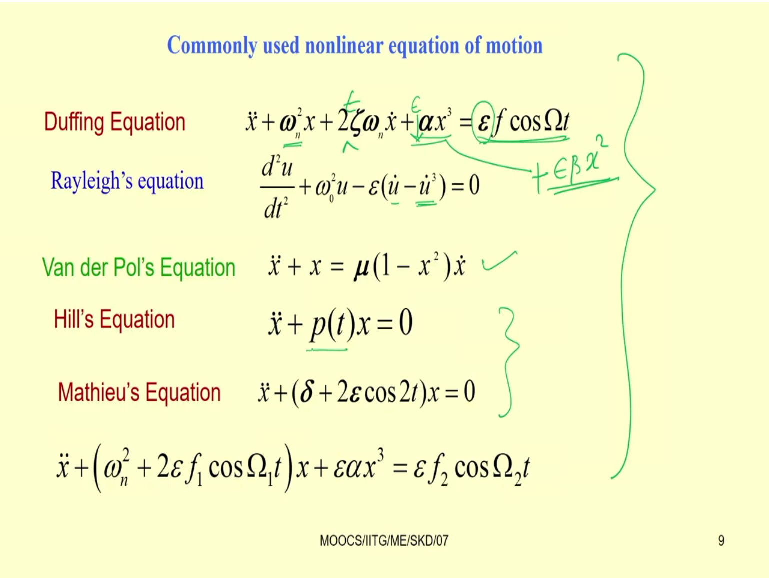
So, that way we can study. Similarly, by keeping this book keeping parameter book keeping parameter is nothing but a parameter which is this epsilon which is very-very less than 1. So, this is very-very less than 1. So, this parameter is very-very less than 1.

So, you can use the scaling parameter for example, if you are using epsilon equal to 1 so, we can write the same equation in this form that is theta double dot plus 10 theta minus epsilon into 16.667 theta cube here; you can note these term that is 16.667 is comparable to this term 10; that means, they are of the same order.

So, you have to make it in such way that they should be in same order. Here, also you can see these non-linear term is also of the same order. So, it may not be exactly equal to 10, but it should be near to 10. So, here it is 16.0 something and this cubic order here equal to 8.3, the coefficient is 8.3.

So, by choosing properly this value of this parameter that is scaling parameter and this book keeping parameter so, you can write down this non-linear equation. Non-linear equation of motion in many different ways so, same non-linear equations can be written in many different ways.

(Refer Slide Time: 20:37)



So, these are the commonly used non-linear equations already we know and a particular non-linear equation when we will see, this may be the combination of all these equation. For example; so, we have seen the duffing equation. So, duffing equation with a cubic non-linear term so, here we have a forcing term is also added.

So, the equation is x double dot plus omega n square x plus 2 zeta omega n x dot plus alpha x cube equal to epsilon f cos omega t. So, here one can put a value of epsilon here to so that the damping is 1 or less than that of the linear term. Similarly, if this non-linear term is 1 or less then here, also we can put epsilon.

So, epsilon can be put here epsilon alpha x cube and here is the forcing. So, if you we have weak forcing then we can use this term epsilon.

So, if the forcing it strong then this epsilon term may not be used in this equation. So, this duffing equation can be written in many different way; many different variations can be done. So, already I told you, you can introduce this epsilon, in the damping you can introduce the epsilon order or you may introduce this higher order epsilon also epsilon square of the damping is very-very small, then instead of epsilon it can be written 2 epsilons square zeta omega and t.

Similarly, this non-linear cubic non-linear term with epsilon or we can write epsilon square also order of order. So, instead of this cubic non-linear we may have. So, we can add some term like this plus epsilon beta x square. So, this is quadratic non-linearity also we can add to the system. So many different way we can write down the duffing equation. So, this alpha may or may not be positive always it may be negative also.

So, depending on the hardening or softening we can have the plus or minus sign of the alpha. Similarly, for the forcing already we know the forcing may be weak or the forcing may be ok, the forcing may be weak or the forcing may be strong forcing and we have studied this Rayleigh equation also.

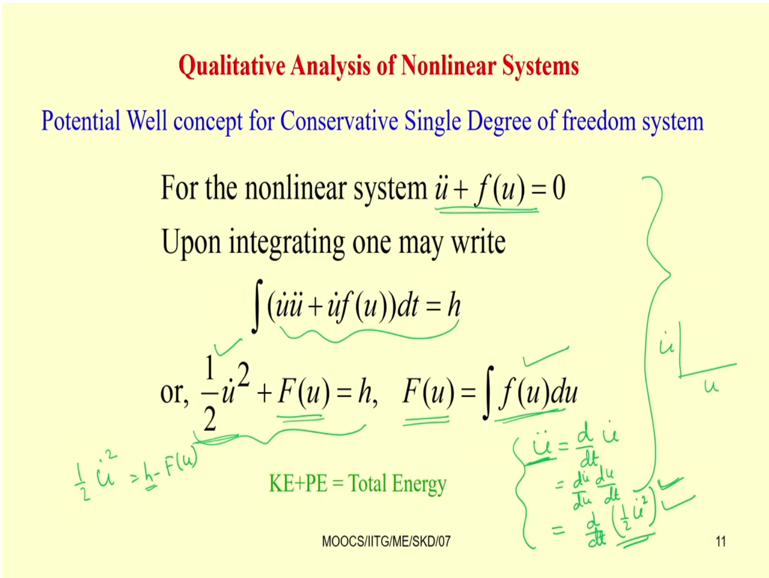
So, this Rayleigh equation is d square u by d t square plus omega square u minus epsilon u dot minus u dot cube. So, here you can see so, we have the non-linearity in damping. So, in duffing equation also we can add, we can modify the duffing equation by adding cubic non-linear also in that equation. So, here we can show this wonderful equation is a special case of Rayleigh equation. So, that thing can be written in this form that is x double dot plus x equal to mu into 1 minus x square into x dot.

Similarly, we can write another equation; Hill's equation for particularly these are for parametrically excited system Hill's equation and Mathieu equation. So, Hill's equation so, we have a periodic coefficient here x double dot plus p t x equal to x double dot plus p t x equal to 0 then we can have Mathieu equation. So, in case of Mathieu equation our equation becomes x double dot plus delta.

So, this p t is replaced by delta it is replaced by delta plus 2 epsilon cos 2 t. So, this is a periodic term. So, in terms of harmonic it is written, p t can be any periodic about this Mathieu equation is written by using this harmonic term that is 2 epsilon cos 2 t. Also, we can instead of putting a single forcing so we can have multiple forcings also. So, this way we can have different non-linear equations also, we have studied some more non-linear equations.

So, we have derived many non-linear equations in module 2. So, those equations also can be used. So, to solve in this model so particularly in this module, we will be interested to solve different non-linear equations.

(Refer Slide Time: 25:11)



Let us see the qualitative. So, when you are doing or we are going to find the solution so, we should know the qualitative and other different methods for solving this non-linear equation. So, already in the introductory class we have studied these things. So, let us have a equations, any general equation, any general non-linear equation can be written in this form that is u double dot plus f u equal to 0.

So, this u double dot actually can be written u double dot equal to so d by d t of u dot. So, d by d t of u dot is u double dot, but it can be written in this way also that is d by d u d by d u into d u by d t or this is d u dot. So, this is d u dot by d t. So, this is d u dot by d u into d u by d t. So, this is nothing but so, this is equal to d by d t. So, this is d by d t of half u dot square.

So, d by d t of half u dot square equal to u double dot. So, this acceleration term u double dot can be written by using this energy term. So, this energy term that is half u dot square kinetic energy already you know half m b square or half m into mass into velocity square. So, this u dot that is the acceleration term can be written by using a energy term this way.

So, here by multiplying this u dot using the same principle, but what is written here. So, you can write down this u double dot plus f u in this form that is u dot into u double dot plus u dot into f u d t equal to h. So, integration 0 that is a constant or this integration equal to half u dot square what we have seen here integration of d by d t of half u dot square equal to half u dot square plus this F u equal to h. So, this F u equal to integration of f u d u.

So, here so, this f u is nothing but this potential energy and this represent the kinetic energy of the system so; that means, this equation the governing equation which is written in the terms of the force can be also written using this energy and this energy equation becomes half u dot square plus F u equal to h or this F u equal to so, here F u equal to this. So, you can write this f u equal to h minus half u dot square or in other words also this u dot you can write this u dot square.

So, half u dot square equal to h minus F u, then this u dot square equal to 2 into h minus F u and u dot will be equal to root over. So, it will be root over 2 into h minus F u. So, for a particular value of h so, we can find the relation between this displacement and velocity so, if you plot this displacement and velocity so let us if we plot this u versus u dot so, that will give us the flow in case of the non-linear system.

So, let us take this example. So, for example, this is x double dot plus x minus 0.1 x cube equal to 0. So, here F x will be equal to small f x into d x. So, here F x equal to small f x equal to so, this equation can be written as x double dot equal to minus x plus 0.1 x cube. So, f x so, this equation is written x double dot plus f x equal to 0. So, this part is f x so, f x equal to x minus point 1 x cube. So, knowing f x equal to x minus point 1 x cube so we can write this capital F x.

So, capital F x equal to integration of that thing so, integration of x equal to half x square; so minus 0.1 x cube so that will give 1 by 40 x 4th. So, this is the potential energy term similar to potential energy so, this is half x square minus 1 by 40 x 4th.

So, the optimum value so optimum value of x, so if you recall this spring mass system or the case either spring mass system or the double pendulum. So, already you know so, in case of double pendulum so, the potential energy is maximum at this position. Similarly, the potential energy is maximum at this position. Kinetic energy is 0 at this position. So, or it has a minimum value 0 means minimum value. So, it has maximum potential energy is maximum here, potential energy is minimum here, and potential energy is again maximum here.

So, here optimum value of x we can get from this thing optimum value of optimum value we can get at x equal to 0 or x equal to plus minus root 20. So, this velocity also you can find. So, by putting this thing so, you can get the optimum value of potential energy so, which is occur at 0 or plus minus root 2.

So, you can already you known to write this x dot, x dot is nothingbut the velocity. So, velocity equal to 2 into h minus F x. So, by taking different value of this h already F x equal to so point 5 x square minus point 0.025 x 4th so, by taking different value of h, so one can plot this x versus x dot.

So, by plotting this x versus x dot so first so, this is the plot f x. So, this is F x versus. So, this is curve of F x this is not x this is x this is v this is x. So, you can see the potential energy of a maximum value at this position it has a minimum value at this position, and it has a maximum value again, at this position and again it will have a minimum value at this position. So, this is the variation so, if you plot this F x versus x. So, you can get this curve.

Now, if you plot this x versus x dot for example, so, let us see the equation what we got that is x dot that is v equal to root over 2 into h minus F x 2 into h minus F x. So, if this F x is greater than h F x is greater than h. So, in that case this part will be negative. So, negative root over negative part imaginary.

So, there will be no velocity the velocity so, there will be so, the roots will be imaginary. So, there will be no flow actually, as this is not a real number so, there will be no flow taking place in this case. So, you just see so, let us take this line, we can take any line. So, let this is constant h, if you take this one so, you take any value of x and then you start floating the x versus x dot. So, by taking a value of h less than this thing there will be no flow.

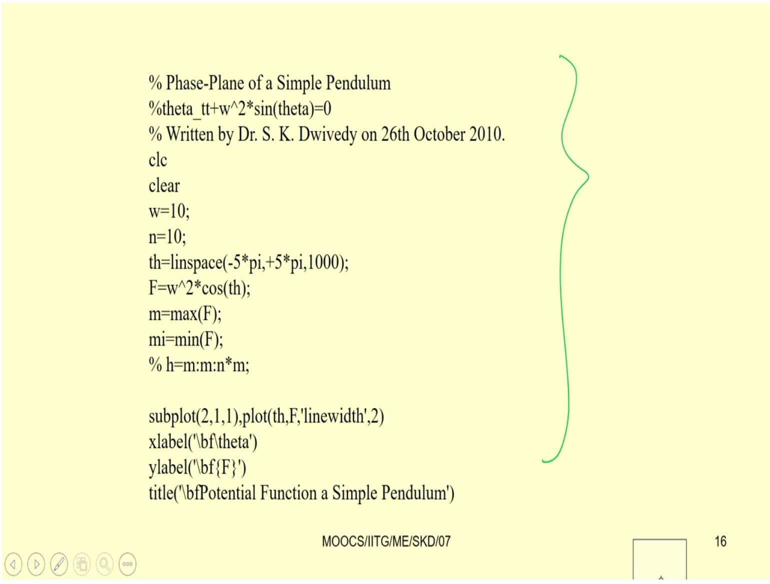
So, if you take a value of F x greater than this h then only there will be some motion in the system so; that means, so, there will be no flow. So, let us take these 0 line. So, if you take this total energy equal to 0 total energy h equal to 0. So, if h equal to 0 then x dot becomes so if h is equal to 0. So, then x dot is so, it is imaginary ok. So, this will have an imaginary value. So, there will be no motion.

So, you just see corresponding to maximum value of F x. So, when F x is maximum. So, if your potential. So, this part is having a maximum value the corresponding. So, you can see the corresponding displacement. So, the corresponding displacement and velocity you can find. So, this point correspond to maximum potential energy, so this is maximum potential energy and this is the minimum potential energy.

So, the when the potential energy is minimum so later, we will study about the stability. So, when we will study about the stability of the system that time I will tell stability and bifurcation. So, that time I will explain more regarding this curve and we will know what you mean by stable response. What do you mean by unstable response. So, that time we will discuss that this is this point is known as saddle point. So, this is a center and this is also a saddle point and we can have the motion.

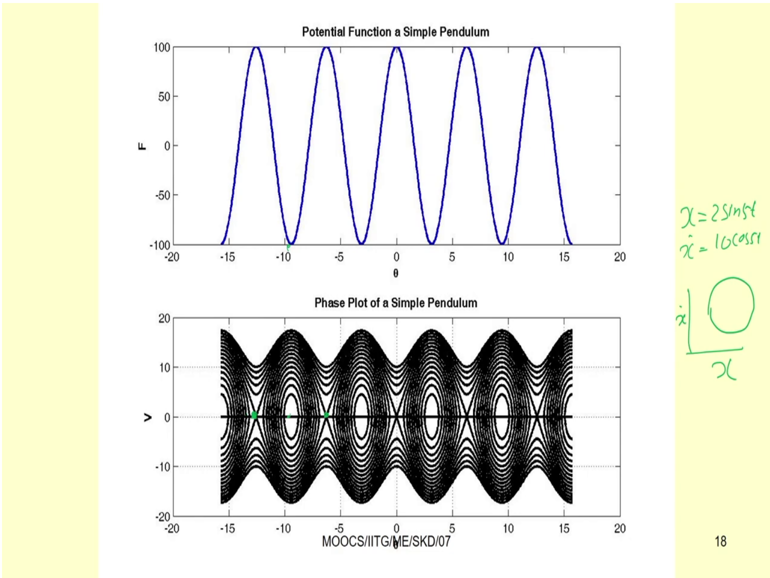
So, in the forward direction what is the motion and in the reverse direction what is the motion. So, those type of things in details we will study there so, but for the time being you should know how to plot this x versus this velocity. Its interpretation we will see in other class when we will know exactly. What we mean by bifurcation and stability then it will be easier for you to understand this problem.

(Refer Slide Time: 35:31)



So, you can write one MATLAB code, this is simple MATLAB code written to plot that curve. So, you can write this thing.

(Refer Slide Time: 35:41)

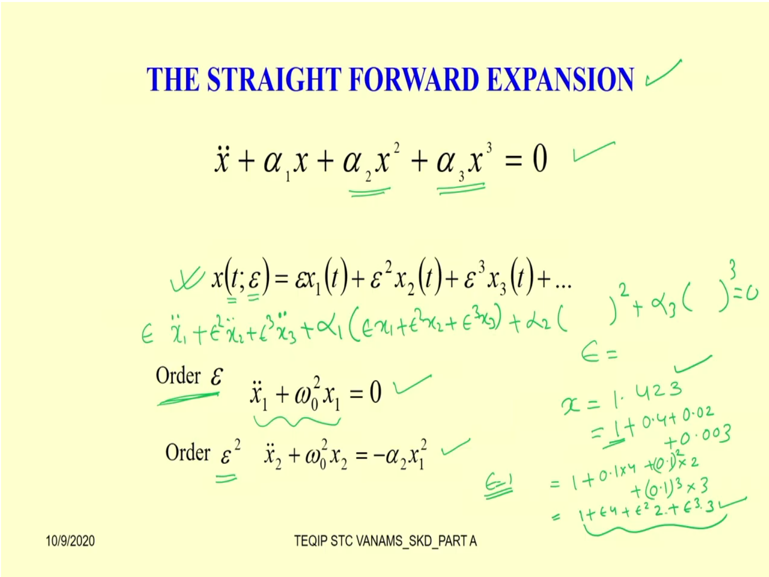


So, here also same thing is plotted. So, for many cycles it is plotted. So in that curve we have seen that is for a single cycle, but you can plot this thing for many cycles. So, what you can see between these two point, between these two saddle point. So, you can have a response clearly you can have periodic response.

So, already you have seen. So, if x equal to I have given you this example x equal to sin 5 t and so in that case your x dot equal to 10 cos 5 t and when you have plotted you just see, if you compare this curve with this curve, you can see similar curves here. So, these are periodic response. So, you can have periodic response, a number of periodic response here. So, this corresponds to minimum potential energy.

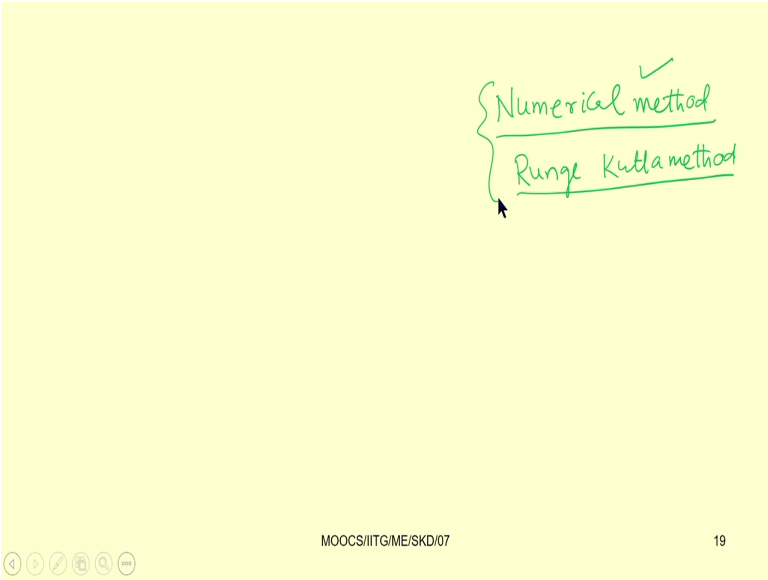
So, corresponding to maximum potential energy we can see later we can discuss and see that this will give rise to unstable fixed point response. So, this is a stable fixed point response and this will be unstable fixed point response. So, those things we will study in next classes.

(Refer Slide Time: 36:53)



So, there are several solution procedures are there for studying this non-linear system. So, one such method is so we will study. So, there are several methods are there one.

(Refer Slide Time: 37:09)



Let us first study the simplest one that is the numerical method, numerical methods so, directly by solving these numerical method, you can find the response of the system. So, generally this Runge Kutta method is used, Runge Kutta method so, you can use the Runge Kutta method to solve this non-linear equation. We will see how can use this Runge Kutta method ok. So, you can use this Runge Kutta method to solve this non-linear equation.

Similarly, there are other method approximate methods are there also. So, some of the approximate methods we are going to study in this module. So, let us now study the straight forward expansion method and then we will briefly see how you can use this MATLAB to solve this governing differential equation motion. So, straightforward expansion like previously, so in linear system also so you can do the straight forward expansion, in non-linear case also you can do the straight forward expansion to find the solution of the equation.

So, let us take a generalized case that is x double dot plus alpha 1 x plus alpha 2 x square plus alpha 3 x cube equal to 0. So, this is a duffing equation with quadratic and cubic non-linearity quadratic and cubic non-linearity. So, the solution we can write the solution x, we can write equal to epsilon x 1 plus epsilon square x 2 plus epsilon cube x 3.

So, here epsilon is a bookkeeping parameter epsilon is a bookkeeping parameter. For example, let us assume that the solution can be written, you are getting a number let the number equal to 1.423. So, this x equal to 1.423 you can divide it in this way 1 plus 0.4 plus 0.02 plus 0.003 or you can write this equal to 1 plus 0.1 into 4 plus 0.1 into 2.1 square into 2 plus 0.1 cube into 3.

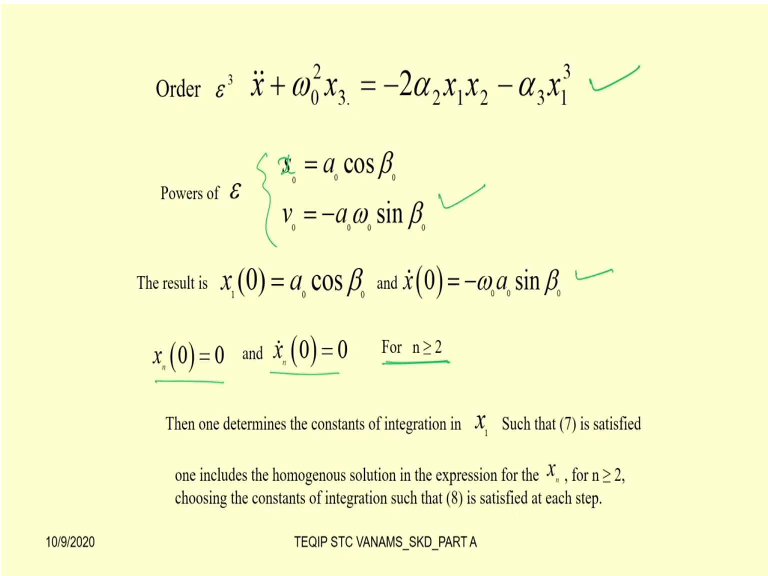
So, if you are taking this epsilon equal to 0.1 so, if you are taking epsilon equal to 0.1 so, this can be written equal to 1 plus epsilon into 4 plus epsilon square into 2 plus epsilon cube into 3. So, this example of these number 1.423 which can be written as 1 plus 4 epsilon into 4 plus epsilon square into 2 plus epsilon cube into 3 is similar to here this number x t epsilon. So, t is the time and epsilon is the book keeping parameter.

So, by using this book keeping parameter epsilon so you can write down this number. For example, the solution solution is nothing but a number we will get. So, that number can be written in this form that is epsilon into x 1 plus epsilon square into x 2 plus epsilon cube into x 3. So, that we can put so here you can add a x 0 also. So, if you want sometimes you can add x 0 plus epsilon x 1. So, here x 0 equal to so x 0 equal to 1, this is epsilon into x 1 plus. So, this way also you can write.

So, now substituting this equation in this original equation so, you can write for example, this will be written as epsilon. So, if I am substituting this way this equation can be written as epsilon into x 1 double dot plus epsilon. So, epsilon into x 1 double dot plus epsilon square into x 2 double dot plus epsilon cube into x 3 double dot x 3 double dot plus alpha into alpha 1 into epsilon x 1 plus epsilon square x 2 plus epsilon cube x 3.

Similarly, plus alpha 2 into so same thing into square plus alpha 3 into same thing into cube equal to 0. So, then we can take out the terms with order of 0 yeah order of epsilon. So, if we are putting this x 0 here also then you can take the terms with x to the power 0. So, order of epsilon that is x 1 double dot plus omega 0 square x 1 equal to 0. Similarly, epsilon square will be equal to x 2 double dot plus omega 0 square x 2 equal to minus alpha 2 x 1 square. So, the solution, the solution of this equation already is known to you.

(Refer Slide Time: 42:53)

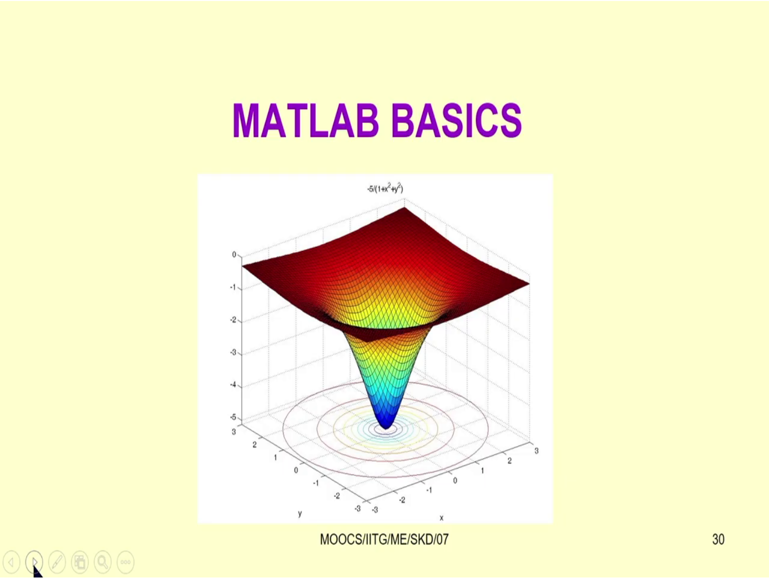


So, the solution is known then it can be written x 1 equal to so, x 1 equal to similarly order of epsilon cube equal to this so, solution power of x 1 so solution s 0 can be written as a 0 cos beta 0. So, this is solution x can be written x 0 equal to a 0 beta 0 and its velocity will be this way.

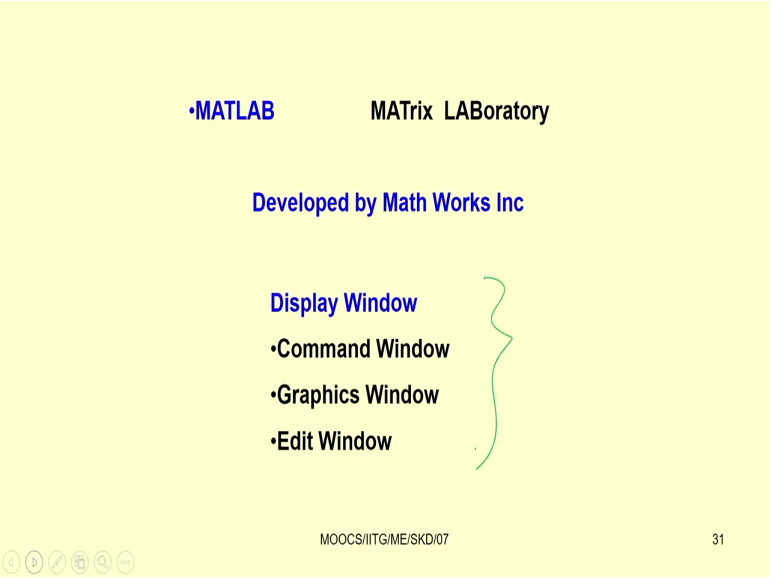
So, you can write the solution of x 1 also can be written solution x n equal to 0 x n dot equal to 0 for n greater than equal to 2. So, this way by getting this solution of x 1 x 2 x 3 so, you can write down this equation, but this way writing this equation has some disadvantage.

So, it leads to actually some erroneous result. So, those things we will study in the next class and before that thing, I just want to give a demo or want to tell you how you can the basics of the MATLAB so which you are going to use in this course.

(Refer Slide Time: 44:01)

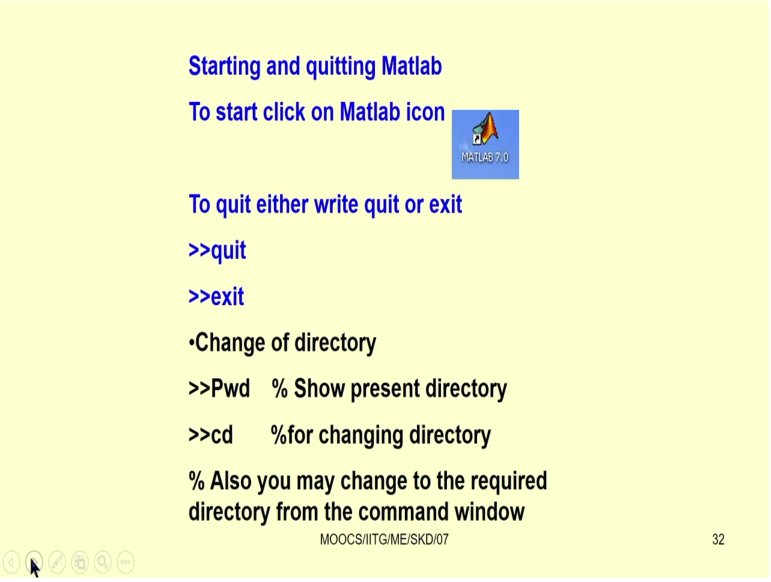


(Refer Slide Time: 44:07)



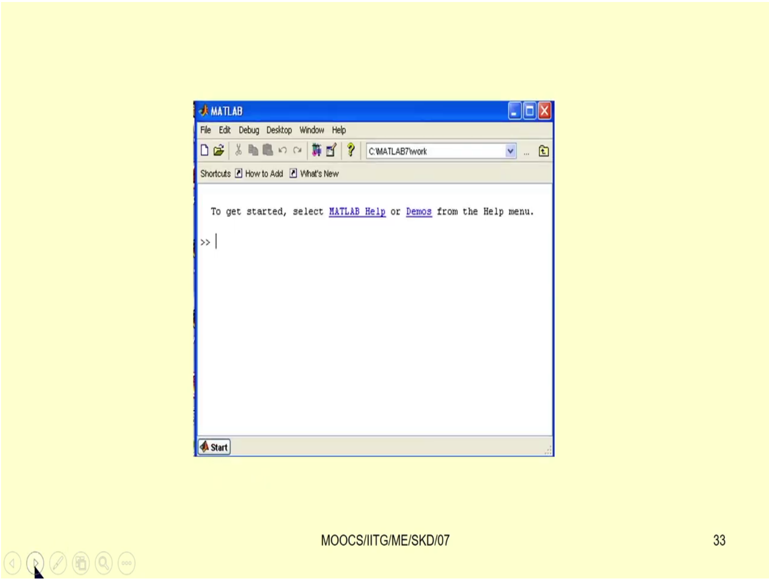
So, you can so actually this MATLAB contains these three windows. So, you can have display window, command window, graphics window, and edit window.

(Refer Slide Time: 44:17)



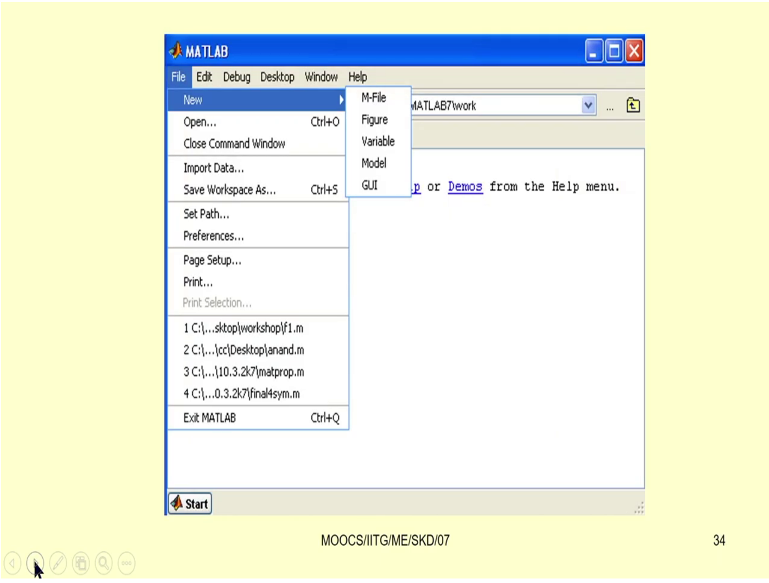
And then, so you can start by double clicking this you can get this icon and you just double click to start this MATLAB. So, to quit this thing you can use this word exit and this so, this is the command window.

(Refer Slide Time: 44:29)



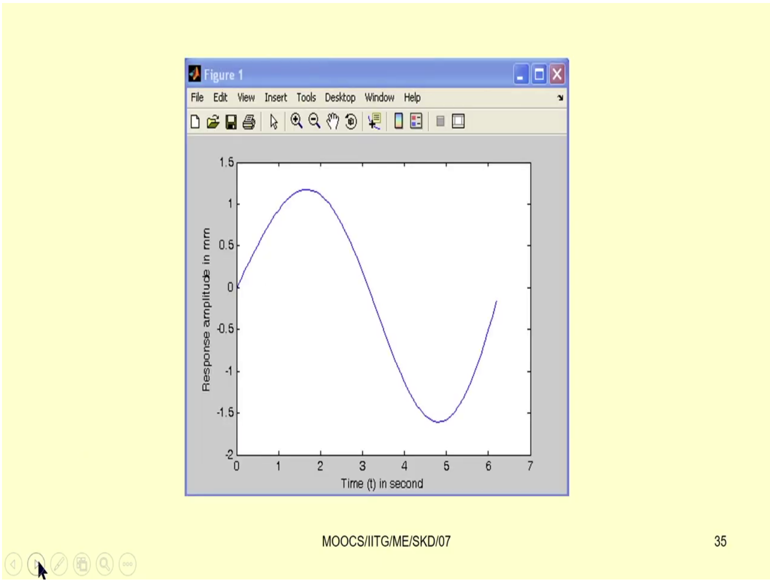
So, in this command window you can write any command to execute the thing.

(Refer Slide Time: 44:39)



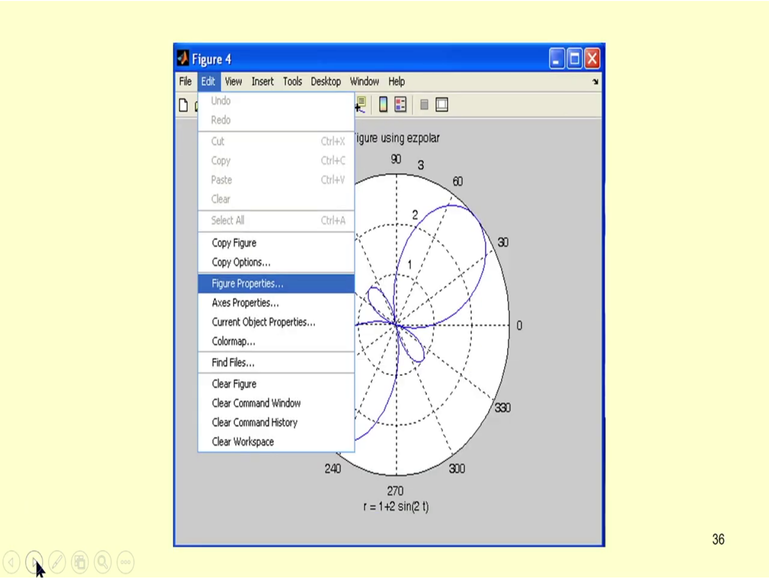
Similarly, you can have a edit window. So, in this case you can create a M-File or you can open. So, you can have a new file, you can you can have a new file, it can be M-File, figure file also, variable file, model or you can go for this graphics user interface also.

(Refer Slide Time: 44:57)



Then you can have a figure file, figure window also.

(Refer Slide Time: 45:01)



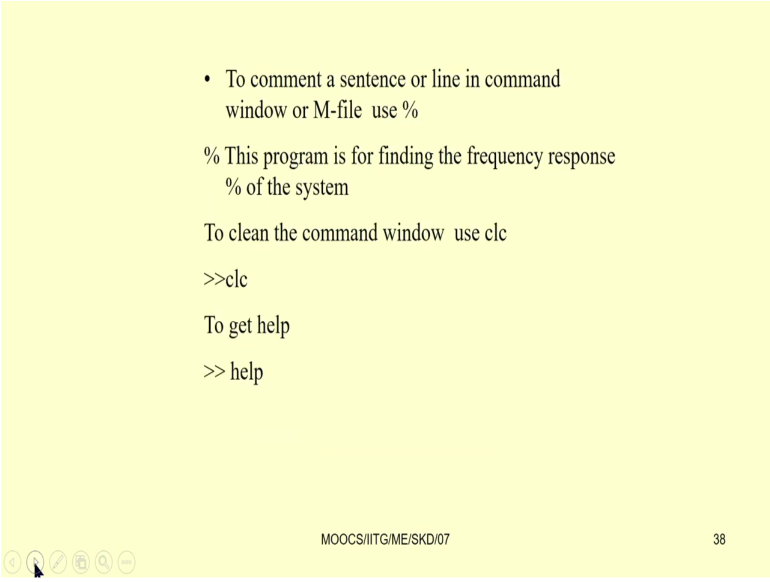
This way figure window several figure window so in this figure window you can go for this change in axes property.

(Refer Slide Time: 45:13)

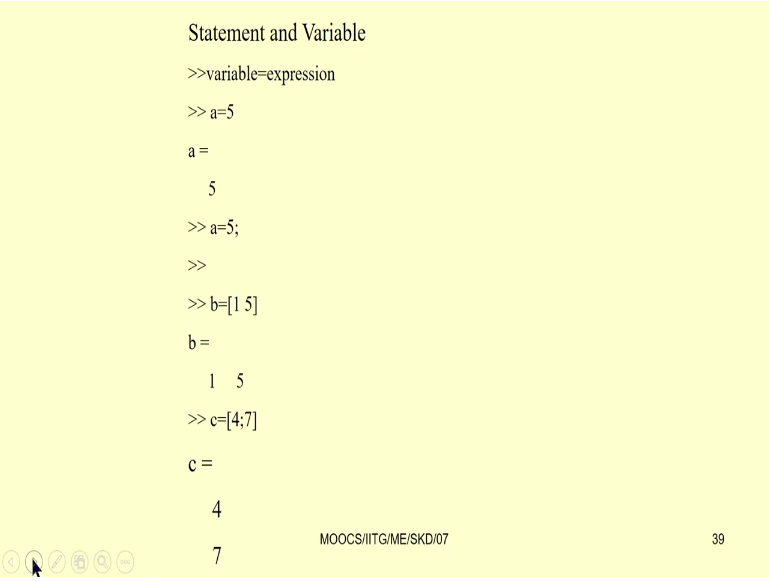
So, you can change the axes property by clicking this edit and then so this use of semicolon. So, if you put you have written for example, you have written t equal to 0 to 0 to 2 pi with an increment of 0.1. So, if you are putting the semicolon then it will not display. So, if you are not putting then all the number of t that is t equal to 0, 0.1, 0.2, 0.3 of 2 to pi it will write down all the things.

So, when you do not want it to be displayed. So, you put a semicolon otherwise if you wanted to be displayed, then that semicolon command you may not put.

(Refer Slide Time: 45:47)

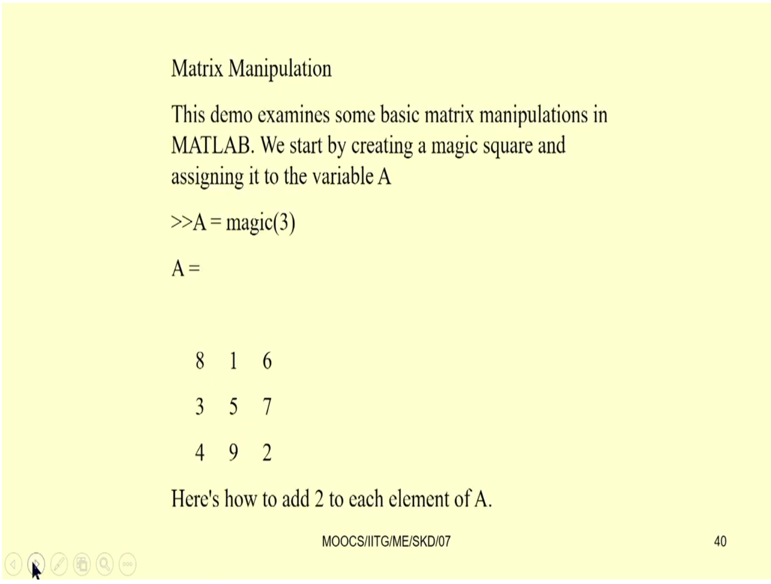


(Refer Slide Time: 45:49)



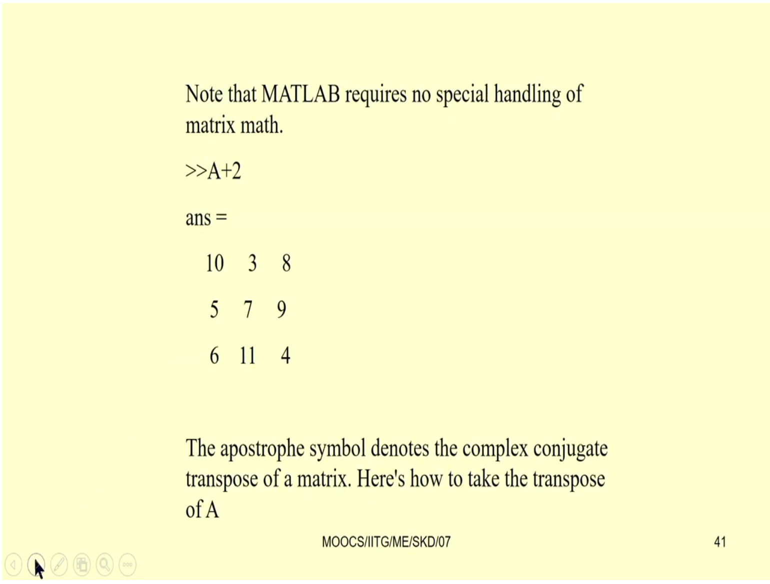
You can write if you if your program is very small you can write in command window, but if you want to keep the program and execute it again and again so that you can write it in a script file and here some examples are given for example, a equal to 5. If you are not putting the semicolon so, it will display like this a equal to 5. So, if you are putting semicolon so, it will not display. So, it is written b equal to 1 5. So, if it is semicolon is not there so, it is displaying 1 5, if I writing c 4 semicolon 7 so, this will display like this 4 7. So, here you are writing a row vector.

(Refer Slide Time: 46:29)



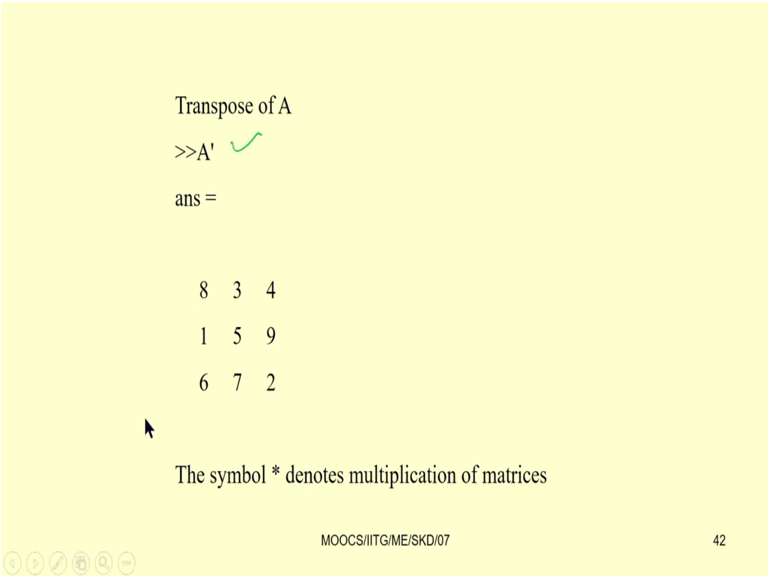
So, here this way you can write one column vector ok. So, this way you can write some matrix also.

(Refer Slide Time: 46:33)



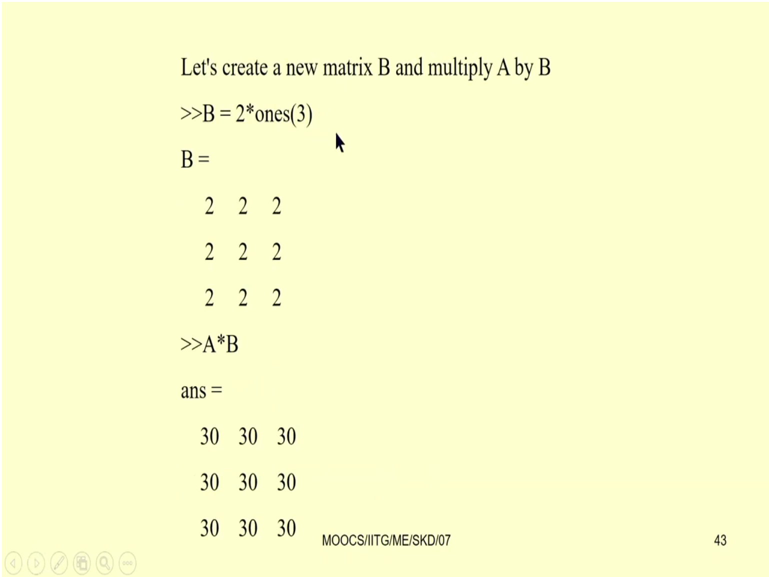
Matrix operation simple matrix operation also you can learn so for example, A plus 2. So, it will add 2 all the elements.

(Refer Slide Time: 46:43)



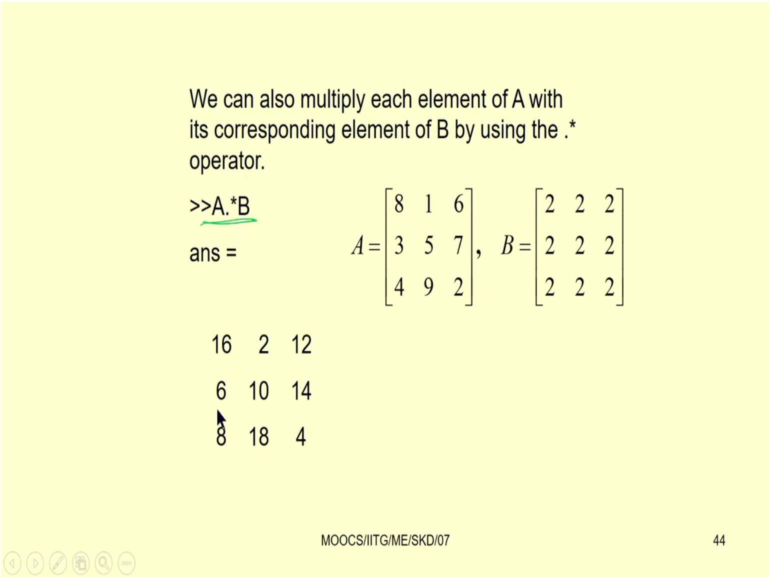
Then transpose just by putting a dash so, you can have the transpose of these thing, transpose of that matrix, then you can create any other matrix.

(Refer Slide Time: 46:53)



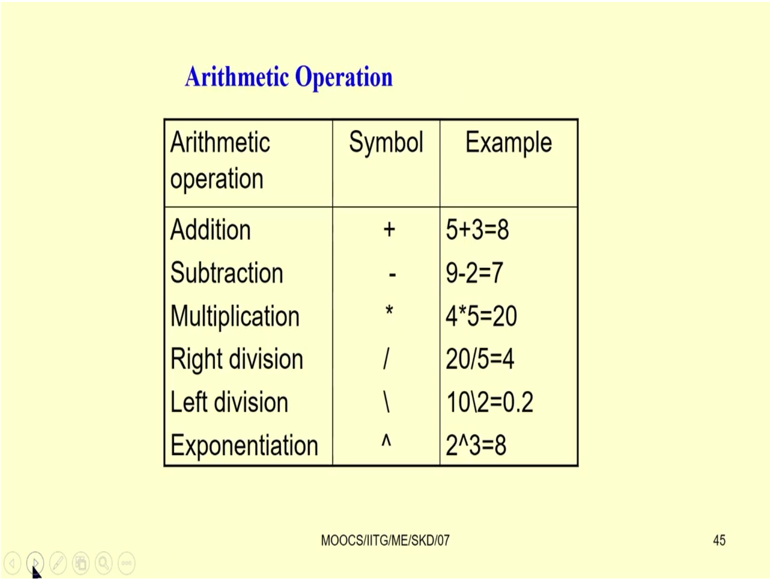
For example, 2 into these thing A cross B.

(Refer Slide Time: 47:01)



So, A matrix B matrix, what element wise if you want to multiply then you just put a command like this A dot star element wise operation can be done by putting this dot ok.

(Refer Slide Time: 47:13)

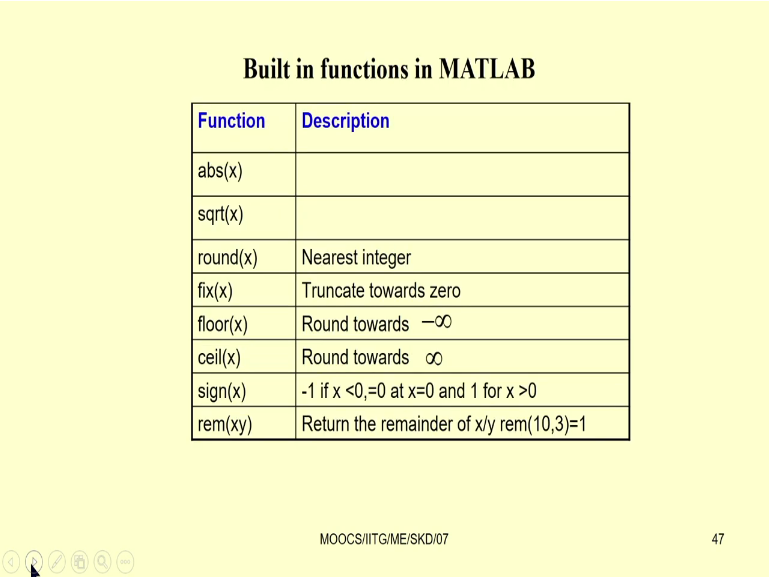


So, these are some of the addition subtraction the symbols you can use ok.

(Refer Slide Time: 47:16)

Command	Description	Example (250/3)
format short	Fixed point with 4 decimal digits	83.3333
format long	Fixed point with14 decimal digits	83.33333333333333
format short e	Scientific 4 decimal digits	8.3333e+001
format long e	Scientific 15 decimal digits	8.333333333333333e+001
format short g	Best of 5 digits fixed or floating	83.333
format long g	Best of 15 digits fixed or floating	83.3333333333333
format bank	Two decimal digits	83.33
format compact	Eliminate empty lines	
format loose	Add empty lines	

(Refer Slide Time: 47:17)



Function	Description
abs(x)	
sqrt(x)	
round(x)	Nearest integer
fix(x)	Truncate towards zero
floor(x)	Round towards $-\infty$
ceil(x)	Round towards ∞
sign(x)	-1 if x <0,=0 at x=0 and 1 for x >0
rem(xy)	Return the remainder of x/y rem(10,3)=1

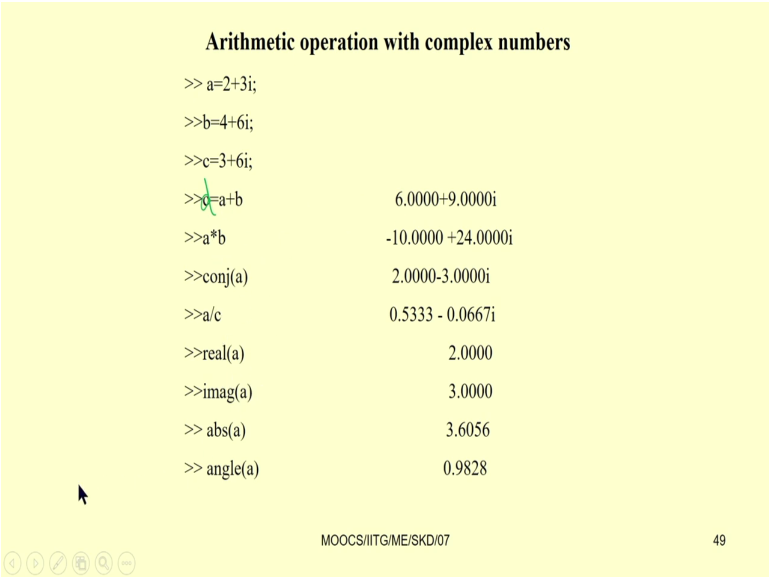
So, all these things have been briefly given in this.

(Refer Slide Time: 47:23)

exp(x)	e^x
log(x)	ln(x)
log10(x)	Base 10
sin(x)	
cos(x)	
tan(x)	
asin(x)	$-\pi/2$ to $\pi/2$
acos(x)	0 to π
atan(x)	$-\pi/2$ to $\pi/2$
sinh(x)	
cosh(x)	
tanh(x)	
asinh(h)	
acosh(x)	
atanh(x)	
atan2(y,x)	atan2(-0.5,0.5)=0.79 and atan2(0.5,-0.5)=2.36

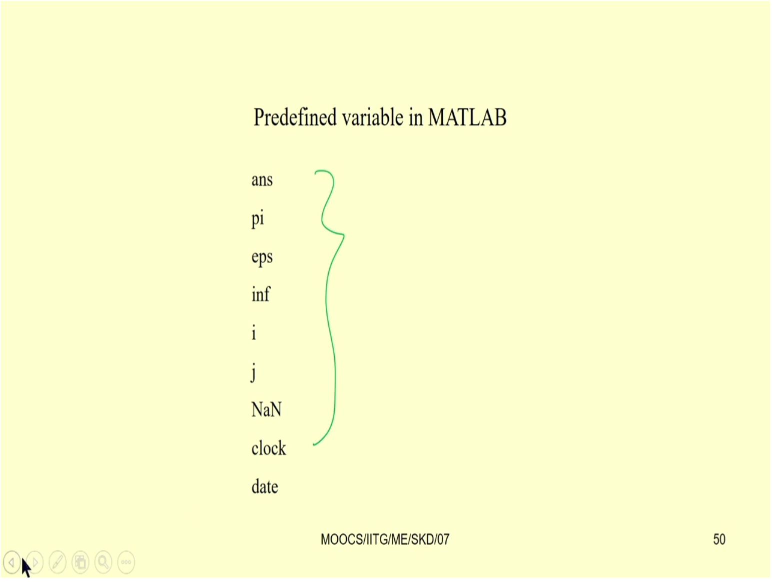
So, the slides you can see. So, these are the some of the usually used commands in the MATLAB code.

(Refer Slide Time: 47:29)



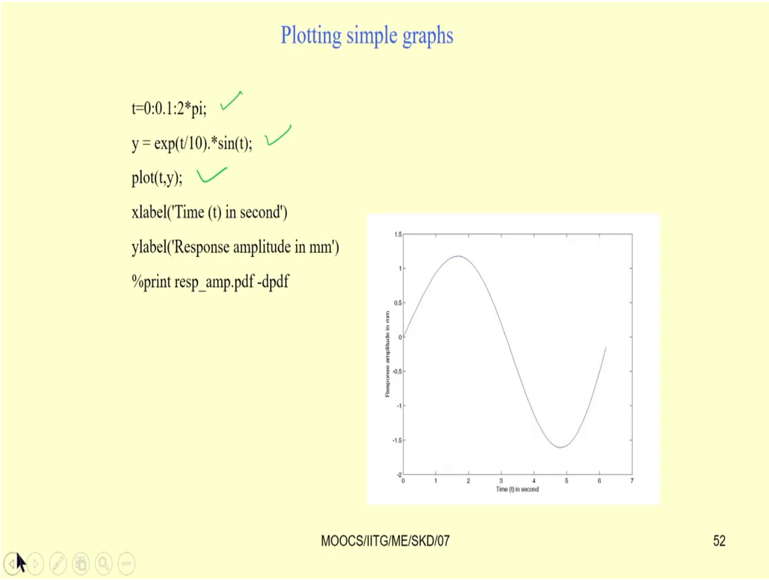
So, these are the commonly arithmetic operation a equal to, complex number you can write 2 plus 3 i b equal to 4 plus 6 i c equal to 3 plus 6 i so, you can do ok. So, you can write d equal to a plus b ok. So, you can do a plus b a cross b conjugate a by c real number. So, all these simple commands which are available in MATLAB you can use for this purpose.

(Refer Slide Time: 47:57)



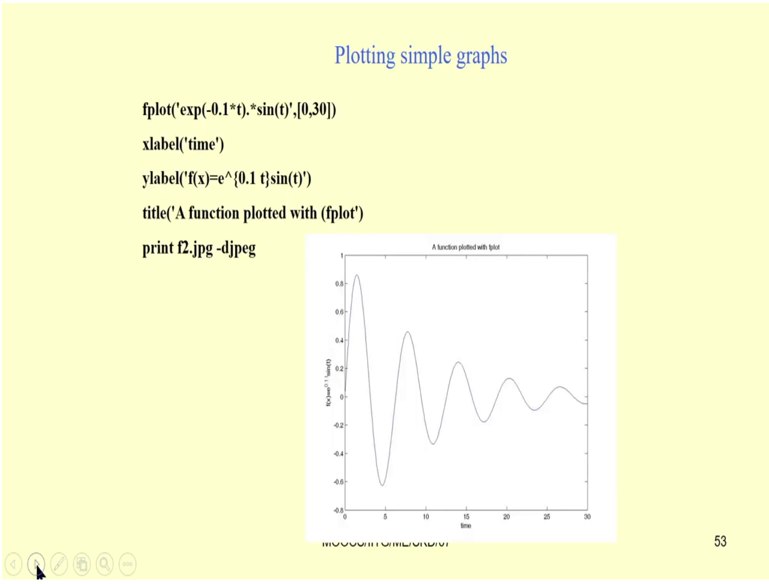
So, these are the predefined variables in MATLAB. You should not use this terms for your variable ok.

(Refer Slide Time: 48:07)



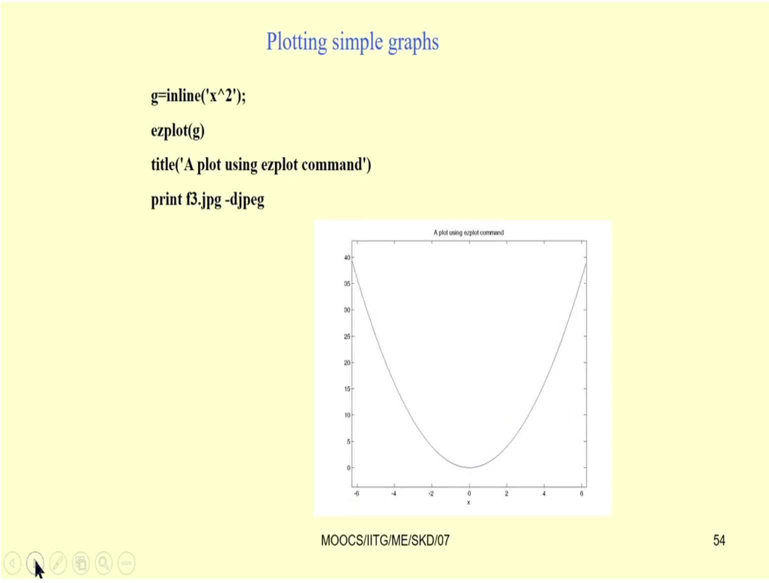
So, to plot you can use a simple plot command. So, for example, t equal to this, so y equal to this. So, simply you can use plot t comma y. So, for x level so, you can put x level equal to whatever you want to put, this x level y level. So, that way you can plot it.

(Refer Slide Time: 48:27)



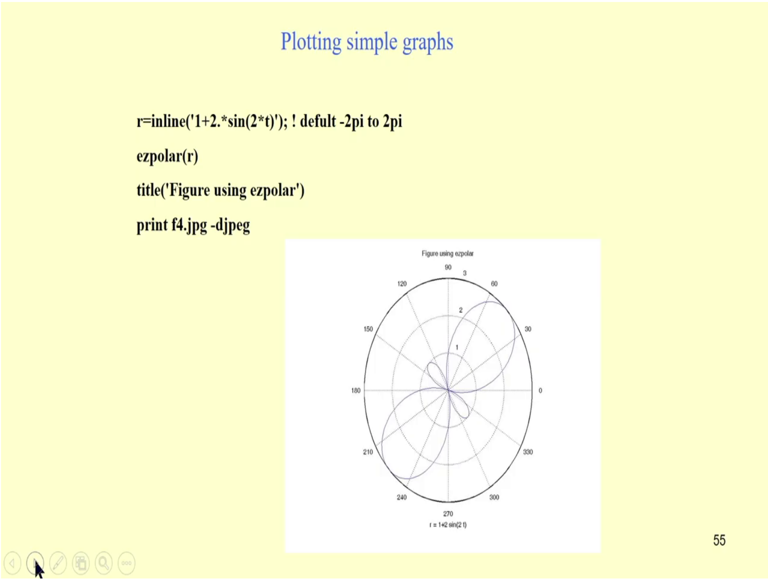
So, you can go through these commands. So, fplot; so for plotting a function so you can use a plot ok.

(Refer Slide Time: 48:35)

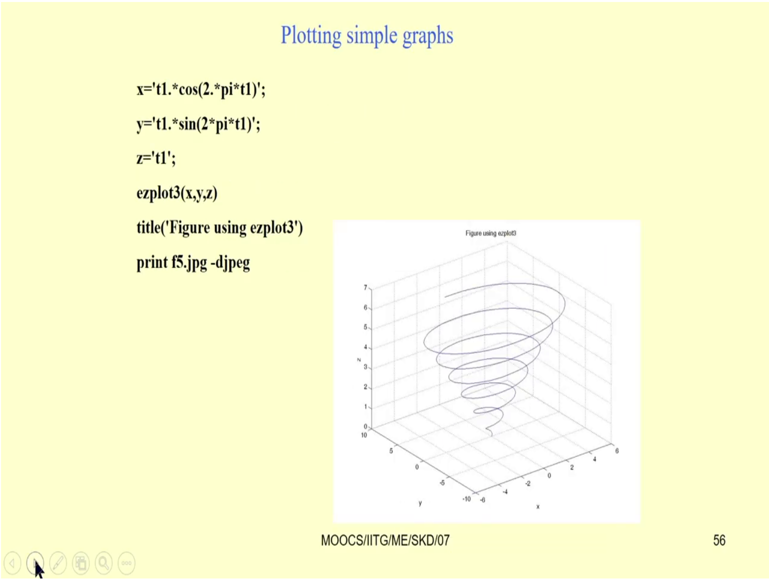


So, e z plot command also you can use ezpolar command you can use.

(Refer Slide Time: 48:39)

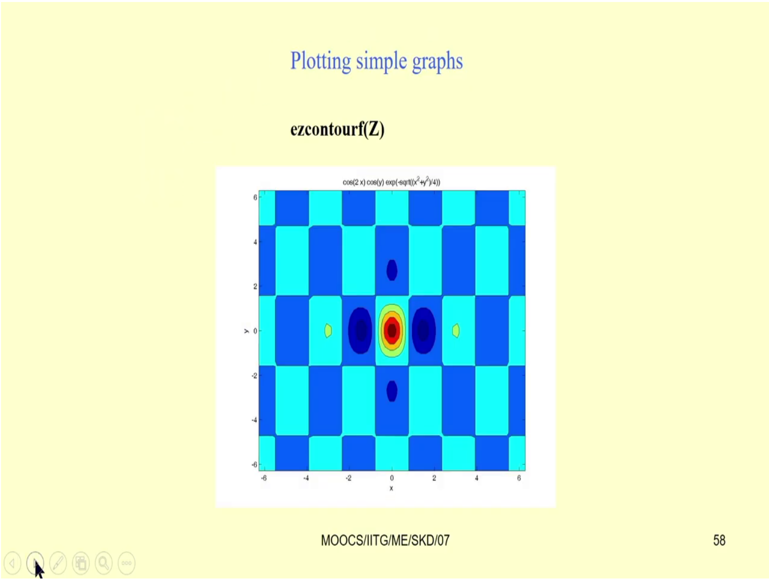


(Refer Slide Time: 48:41)



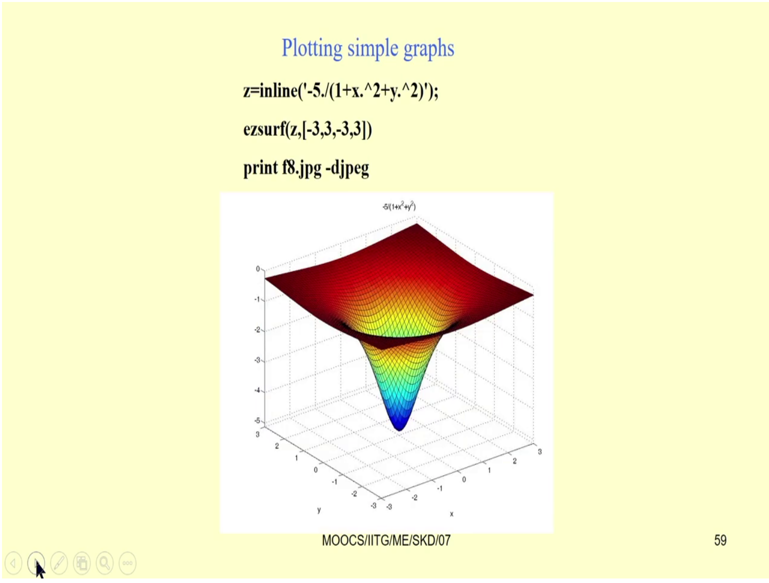
Then you can use this ezplot3 for 3D plot.

(Refer Slide Time: 48:47)



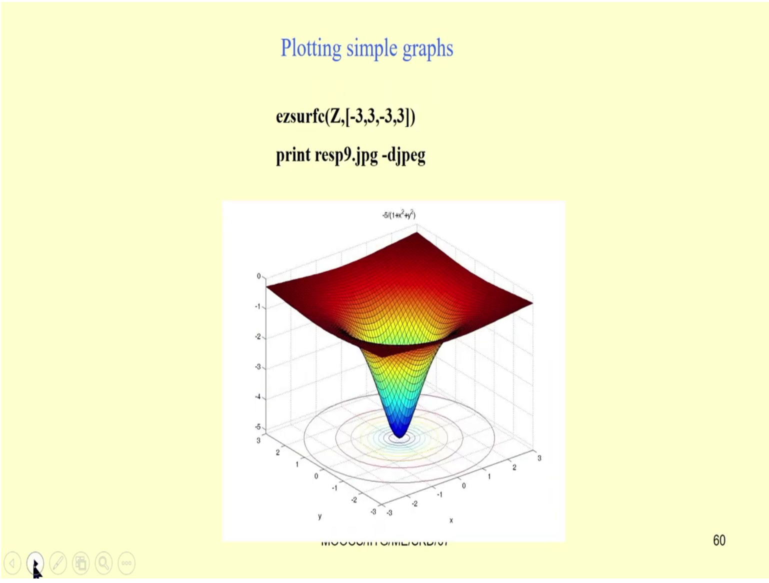
So, simple graphs.

(Refer Slide Time: 48:49)



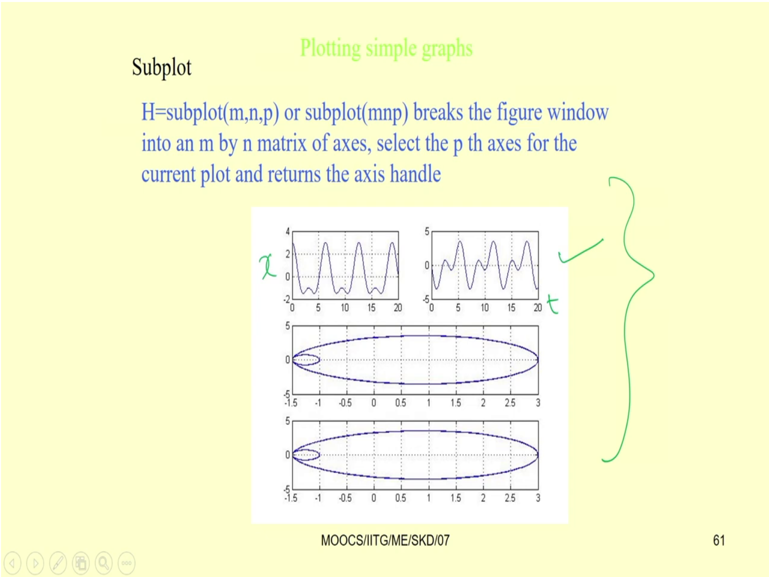
These way contour plot you can do.

(Refer Slide Time: 48:53)



And then surface plot you can do.

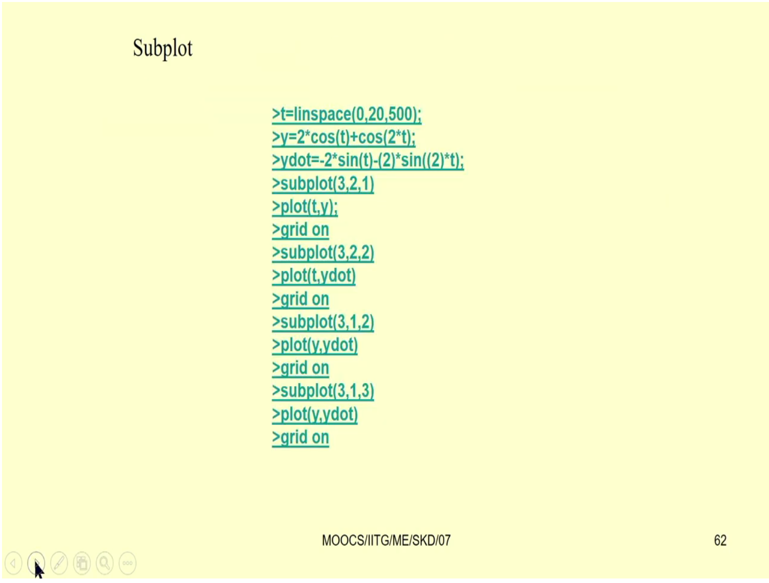
(Refer Slide Time: 48:53)



So, many other features are there. So, we can utilize those things. Here also you can use this subplot command by for plotting multiple for plotting multiple plots. So, you can use the subplot command.

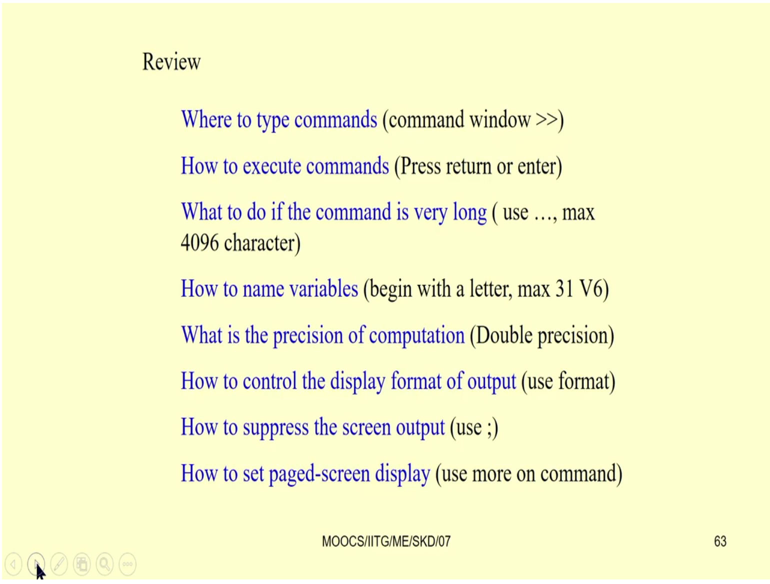
So, here we have plotted. So, you can see. So, this is this is a for example, this is x versus t you can plot. So, this way you can plot different response. So, this program is given also here.

(Refer Slide Time: 49:21)



So, we can see this program how to plot using this subplot command. So, this is the demo for today class.

(Refer Slide Time: 49:29)



So, next class we will see we will give some more demo regarding this MATLAB and also we will study this numerical methods that is Runge Kutta method in detail and also the straightforward expansion and Lindstd Poincare method to find the response of the non-linear equation of motion.

So, thank you very much.