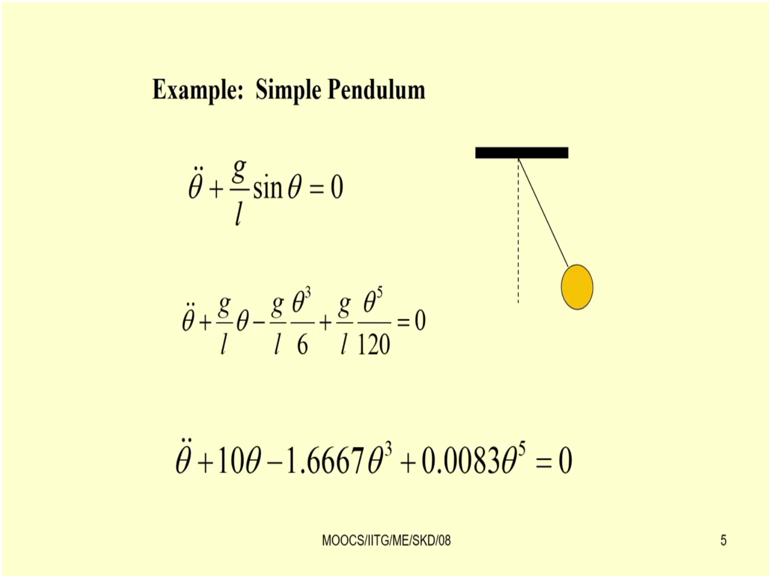
Nonlinear Vibration
Prof. Santosha Kumar Dwivedy
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Lecture – 08
Straight forward expansion

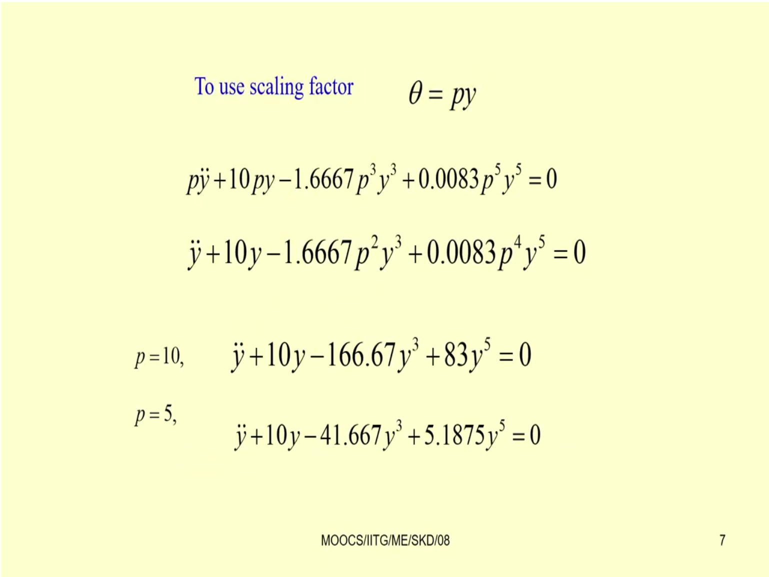
So, welcome to today's class of Non-linear Vibration. So, today class we are going to study regarding the solution of non-linear equation of motion by using Straight Forward Expansion and Lindstedt Poincare method.

So, last class I told you how to use these numerical methods to find the solution briefly we will review that part also. Last class also we have studied how to use this MATLAB to solve the problem or also briefly we will see the review of the MATLAB also today. So, let us first see how to solve the differential equation using numerical methods.

(Refer Slide Time: 01:12)



(Refer Slide Time: 01:13)



(Refer Slide Time: 01:15)

book-keeping parameter

$$\ddot{\theta}+10\theta-\varepsilon\left(\frac{1.6667}{\varepsilon}\right)\theta^{3}+\varepsilon^{3}\left(\frac{0.0083}{\varepsilon^{3}}\right)\theta^{5}=0$$

$\varepsilon=0.1$

$$\ddot{\theta}+10\theta-\varepsilon 16.667\theta^{3}+\varepsilon^{3}8.3\theta^{5}=0$$

MOOCS/IITG/ME/SKD/08 8

So, already we have know the ordering technique, ok, how to use the scaling factor bookkeeping parameter.

(Refer Slide Time: 01:16)

Commonly used nonlinear equation of motion

Duffing Equation $\ddot{x}+\omega_n^2 x+2\zeta\omega_n\dot{x}+\alpha x^3=\varepsilon f\cos\Omega t$

Rayleigh's equation $\frac{d^2u}{dt^2}+\omega_0^2 u-\varepsilon(\dot{u}-\dot{u}^3)=0$

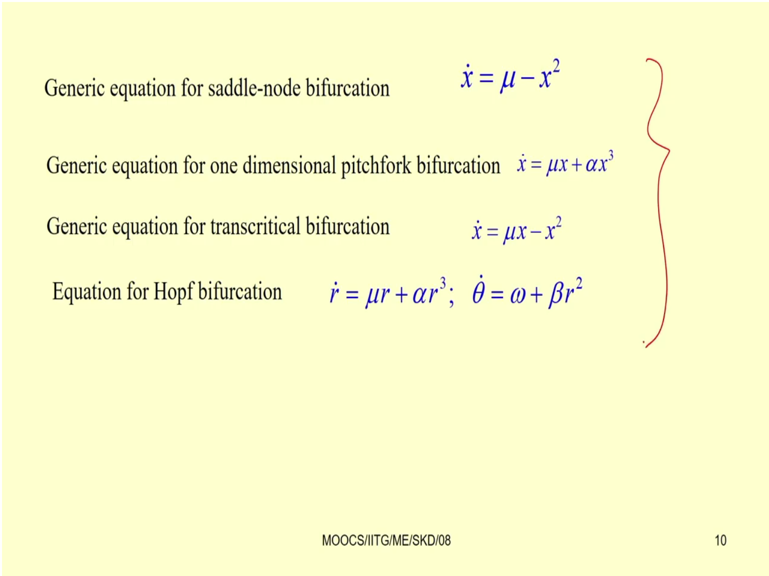
Van der Pol's Equation $\ddot{x}+x=\mu(1-x^2)\dot{x}$

Hill's Equation $\ddot{x}+p(t)x=0$

Mathieu's Equation $\ddot{x}+(\delta+2\varepsilon\cos 2t)x=0$

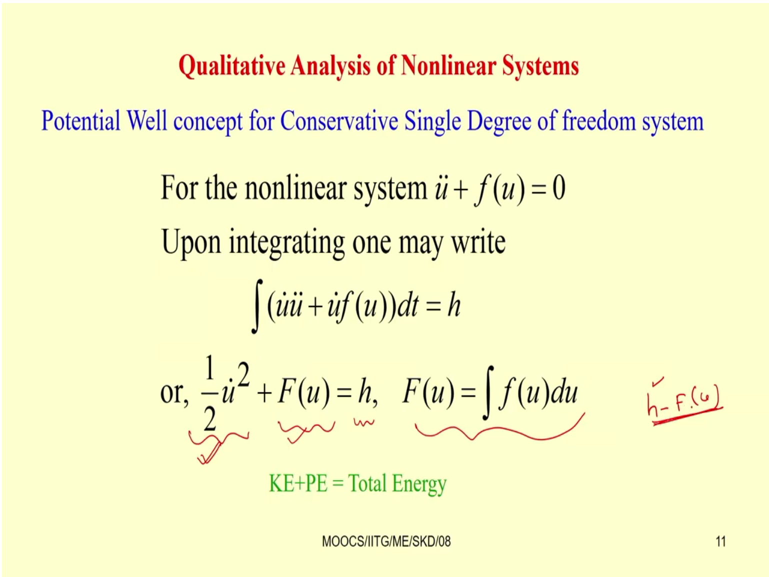
$$\ddot{x}+\left(\omega_n^2+2\varepsilon f_1\cos\Omega_1 t\right)x+\varepsilon\alpha x^3=\varepsilon f_2\cos\Omega_2 t$$

(Refer Slide Time: 01:20)



And commonly used non-linear equation of motions and then this generic form of bifurcations, those things we will study after few classes.

(Refer Slide Time: 01:27)

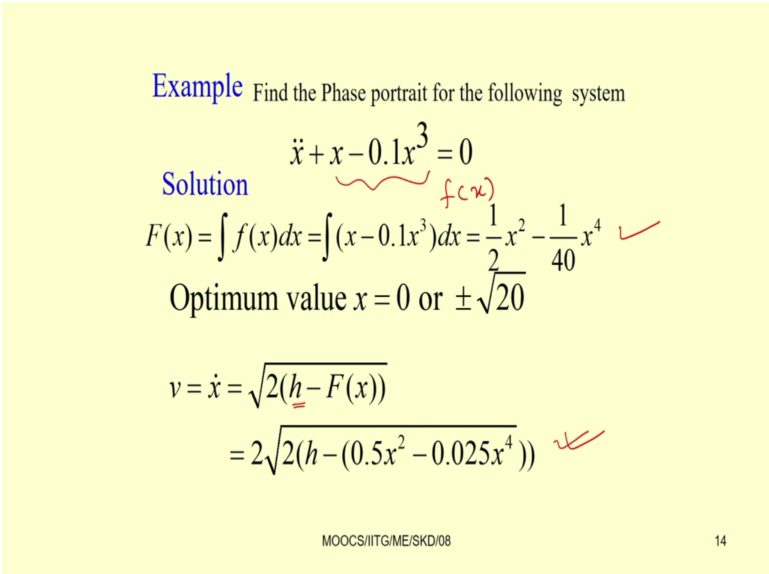


So, then this qualitative analysis, so here any differential equations; so, we can write it in this form that is u double dot plus f u equal to 0, then multiplying u dot and integrating these things. So, you can reach one equation which can be written in this form that is half u dot square plus f u will be equal to h. So, this is nothing but the energy equation. So, where have u dot square is the kinetic energy correspond to kinetic energy, this capital F u correspond to the potential energy and h is the total energy.

So, given the total energy of a system, so we can find the relation between this u dot that is the velocity and u that is the displacement of the system; so, by plotting this displacement and velocity, so you can plot or we can know the phase portrait of the system or how it flows, how the response flows that idea we can get. So, here this f u is capital F u is nothing but f u that is the function from this u double dot plus f u equal to 0 into d u. So, integration of f u by doing this integration of f u we get this capital F u, so which is potential energy.

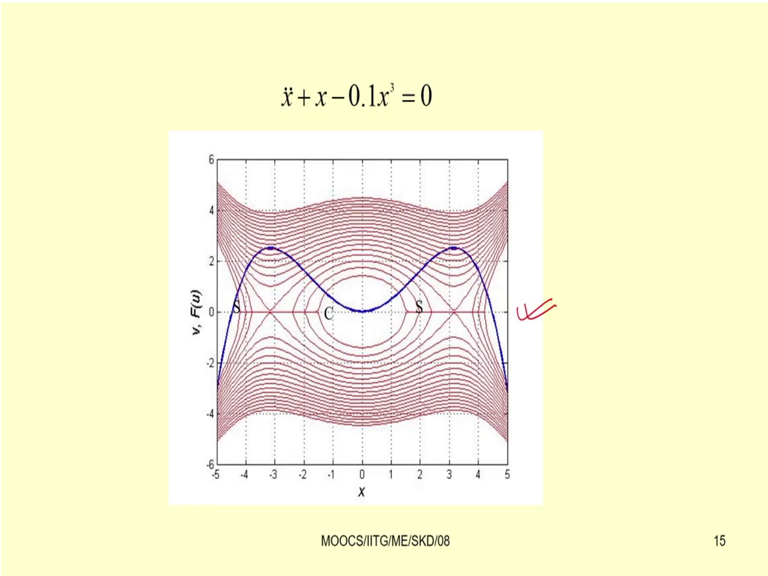
So, total energy minus potential energy equal to kinetic energy. So, from this thing, so you can get the relation between u dot and u. So, you can note it here. So, when h minus F u, so h minus F u is negative h minus F u cannot be negative for a real system. So, because this kinetic energy plus potential energy equal to total energy, so maximum this h can be equal to f u. So, in that case that u dot will be equal to 0. So, velocity will be 0 when h equal to f u and if h greater than f u, then only we can have some velocity. So, the system will have a motion, so only if the total energy is greater than the potential energy.

(Refer Slide Time: 03:45)



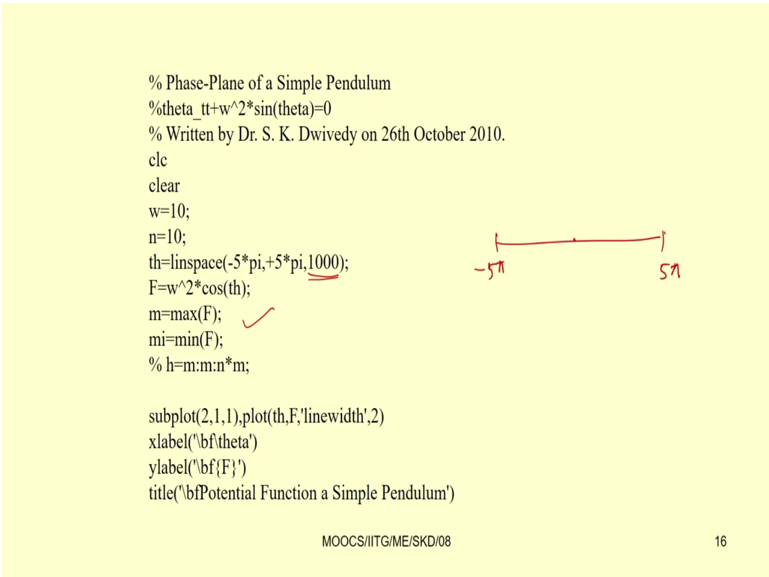
So, we have taken this example. Also we can take this example for example x double dot plus x minus 0.1 x cube equal to 0. So, here f x is written as, so this part is equal to f x. So, this is small f x. So, now integrating these things, so you can get half x square minus 1 by 40 x forth equal to capital F x. So, by taking different parameter of h and substituting these value of f x, so we can get that is the velocity x dot equal to this and from these expression, so we can plot this x versus x dot.

(Refer Slide Time: 04:23)



The physical interpretation of these things we will learn after most probably in next week, so when we are going to study regarding the stability and bifurcation of the response.

(Refer Slide Time: 04:37)



So, you can write conveniently write a simple MATLAB code to plot these things. So, you can divide for example, this theta you can divide from minus 5 by 5 pi by 5 minus 5 pi to plus 5 pi by 1000 points. So, this minus 5 pi to plus 5 pi, it is 10 pi. So, this range minus 5 pi to plus 5 pi you just divide into 10,000 points.

So, with this improvement starting with, that means starting with minus 5 pi. So, we can go up to this 5 pi and with this thing. So, by knowing this theta, so we can find. So, maximum value of f minimum value of f, we can find, then we can plot this to we can plot. First we can plot the potential energy variation of potential energy with theta, then we can plot f dot also. So, we can plot f dot.

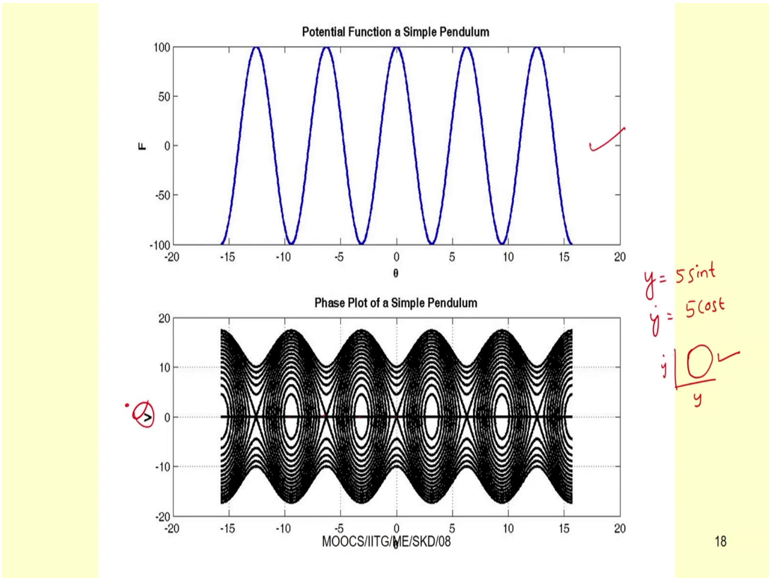
(Refer Slide Time: 05:37)

```
for i=0:15
h=((m-mi)/n)*i+mi;
thdot=sqrt(h-F);
r=real(thdot);
subplot(2,1,2),plot(th,real(thdot),'k','Linewidth',2);

hold on
subplot(2,1,2),plot(th,-real(thdot),'k','Linewidth',2)
end

grid on
hold off
xlabel('\bf\theta')
ylabel('\bf{V}')
title('\bfPhase Plot of a Simple Pendulum')
```

MOOCS/IITG/ME/SKD/08 17

So, this is the code. So, theta dot equal to square root of h minus F. So, by knowing this thing, then we can plot both the things that is this is the plot you can, so potential function.

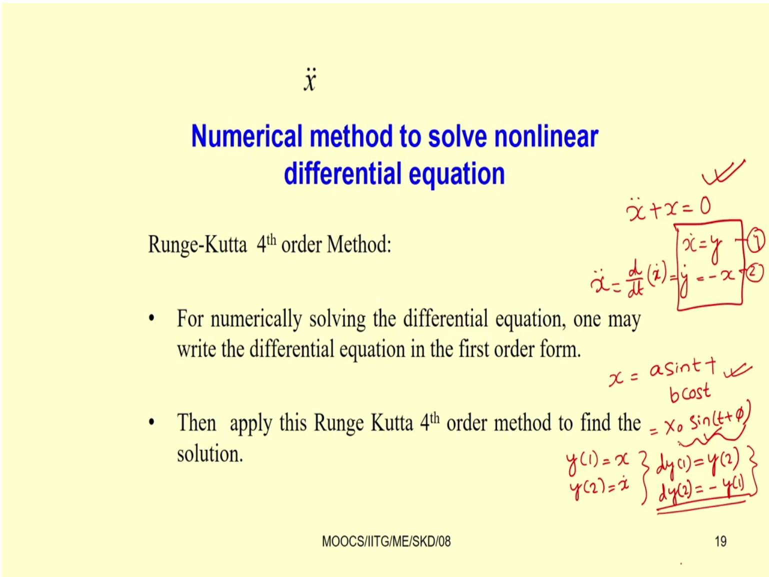
(Refer Slide Time: 05:51)



So, you can use this potential function of a simple pendulum that is F versus theta, then this displacement that is theta versus theta dot. So, this V is nothing but theta dot. So, by plotting this thing, so we can know how the response is varying along; so, how theta dot is varying v theta. So, we can plot. So, you can clearly observe that you have a periodic motion. So, already we have seen the example where we have taken y equal to for example sin t and y dot equal to.

So, in that case it will be equal to 5 cos t and if you plot y versus y dot in this case, so you can see this is a form like this. So, this curve and this curve are similar. So, that means, so these represent a periodic response. So, you can find this periodic response here. So, the response is periodic and these point these points are known as saddle points. So, these things we will study in a later date.

(Refer Slide Time: 06:55)



So, numerically how we can solve these differential equation? So, here we have taken a qualitative method to solve that equation. So, we can use conveniently this Runge-Kutta 4rth order method, 4rth or 5th order Runge Kutta method also. So, for numerical solving the differential equation we may write the differential equation in the first order form.

For example, let us have let us take a simple example x double dot plus x equal to 0. So, in this case already you know the solution of this system. So, your x you can write the solution of x will be equal to a sin here omega equal to 1. So, a sin t plus b cos t or you can write it also can be written equal to x 0 sin t plus phi.

So, either you can write using this way or you can write both a sin t plus b cos t where a and b can be obtained from the initial conditions. For example, at t equal to 0 displacement equal to a 0 and velocity equal to 0. So, you can substitute it here and you can get what is a and b, similarly also you can substitute in this second equation and you can get the value of x 0 and phi.

So, as this is a second order differential equation, so you required two constant to express the motion of the system so, but numerically how you can solve this thing by using this Runge Kutta method to get the response. So, in that case, so first you have to write this equation using this first order 2 first order differential equation.

So, for example, you may write let x dot equal to y. So, this is the this is one equation. The second equation you can write this x dot equal to y. So, this x double dot x double dot is nothing but; x double dot is nothing but d by dt of x dot; so, d by dt of x dot. So, this is nothing but this is equal to y dot. So, y dot equal to, so minus x.

So, our second equation becomes. So, first equation x dot equal to y and second equation becomes y dot equal to minus x. So, this way you can convert any differential any second order differential equation to that of a 2 first order differential equation. So, if you have a third order differential equation, then you can convert that that thing to 3 first order differential equation.

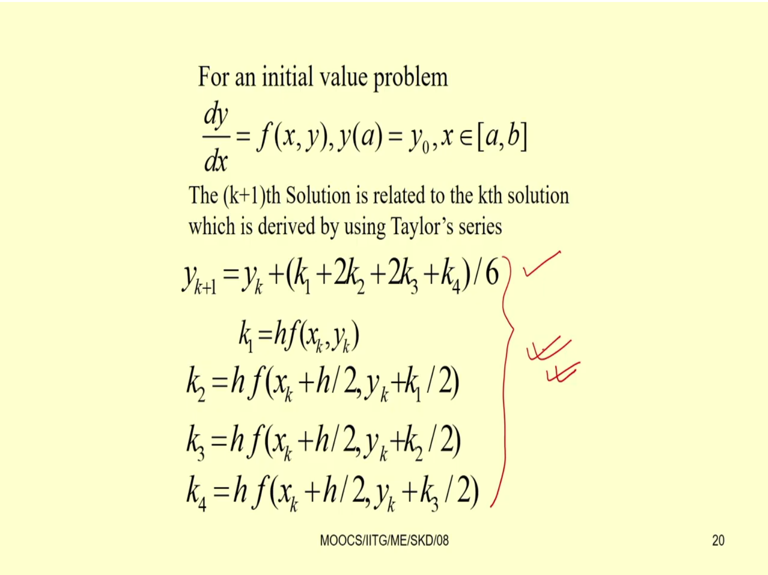
So, first you just put x dot equal to y. So, that is your equation 1, then y dot which is nothing, but x double dot can be obtained from the governing equation this equation because this x double dot equal to minus x in this case. So, x double dot equal to minus x, so this y dot equal to minus x. So, we have 2 equation; so one equation is this and the second equation is this and while writing the code, so you can write this way also by using.

So, for example, let me take y 1. So, let y 1 equal to x and y 2 equal to x dot. So, the equations can be written this way. So, dy 1 that is derivative of y 1, so, that is equal to x dot. So, x dot is nothing, but this is equal to y 2. So, the first equation becomes dy 1 equal to y 2. The second equation becomes dy 2 dy 2 that is differentiation of this x double dot x dot. So, this becomes x double dot dy 2 is nothing but x double dot.

So, this is equal to. So, in this case this is equal to minus x, but minus x is nothing but minus y 1. So, by using this y 1 y 2 also by using in a vector form, also you can write this dy 1 equal to y 2 and dy 2 equal to minus y 1. So, this way you can convert a second order differential equation with set of first order differential equation and for your programming purpose, so you can write it in this form.

So, if it is written in matrix form also, you can write in the same way. So, you have to replace this by in the vector form and you can write down this equation. So, if you have a nth order equation, so you can write the equation write 2 n number of first order differential equation. So, then one can apply this fourth order Runge Kutta method to find the solution numerically.

(Refer Slide Time: 11:47)



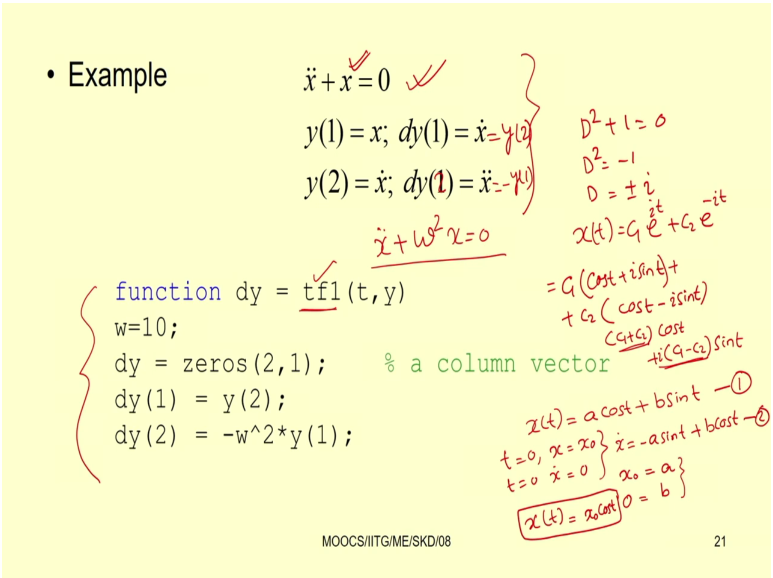
So, this is the most widely used method to solve this differential equation. So, there are several forms of the differential equations; so, for example, this initial value problem. So, you can write this dy by dx equal to f x y where y a equal to y 0 and x belongs to a and b. So, x is known to you and y also is known. So, the range of x and y are known to us. So, the k plus 1h solution is related to the kth solution. So, the k plus 1th solution is related to the kth solution and it can be derived actually from by using this Taylor series which can be written in this form.

So, it can be written y k plus 1 equal to y k plus k 1 plus 2 k 2 plus 2 k 3 plus k 4 by 6 where k 1 equal to h into f x k, y k, then k 2 equal to h f x k plus h by 2 y k plus k 1 by 2 k 3 equal to h f x k plus h 2 y k plus k by 2 k 2 by 2 and k 4 equal to h f x k plus h by 2 and y k plus k 3 by 2. So, where h is the increment you are taking or the time interval you are taking, time increment you are taking in this case. So or time step you are taking in this analysis, the time step should be very very small in order to get a accurate solution.

So, you can write a code conveniently by using these equations. So, given a differential equation first order differential equation dy by dx equal to f x, y where the initial conditions are given to you and knowing the initial condition, so you can start this y k plus 1 equal to y k plus k 1 plus 2 k 2 plus 2 k 3 plus k 4 by 6, where k 1, k 2, k 3, k 4, you can write down by using this expression.

So, this way by using this. So, this is the 4rth order Runge Kutta method as you are using four constraints here. So, by using this method, so you can solve any differential equation conveniently.

(Refer Slide Time: 14:22)

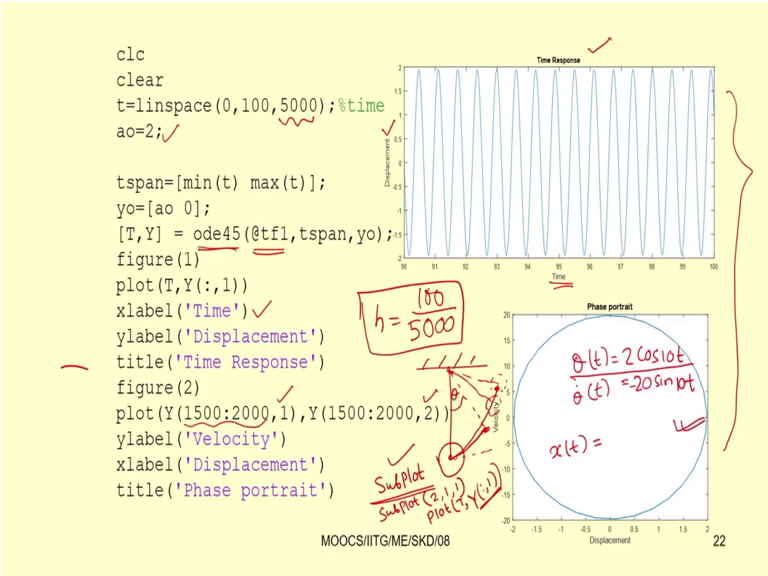


So, for example, let us take these things. So, x dot plus x equal to 0; so, the solution already I have written that this the solution of this equation you can find for example, this auxiliary equation becomes D square plus 1 equal to 0.

So, this becomes D square becomes minus 1 or D become plus minus i. So, the roots of the equation auxiliary equation is plus minus i. So, the solution becomes. So, that is your solution x t equal to c 1 e to the power i t plus c 2 e to the power minus i t. So, now, if you expand these things, so this becomes e 1.

So, e to the power i t is nothing but cos t plus i sin t. Similarly plus here you can get this is this, so c 2. So, the next part becomes c 2 plus c 2 into cos. So, cos minus t equal to cos t itself. So, minus i sin t. So, now you can write this thing in this form taking this cos t common. So, this become c 1 plus c 2 cos t plus. So, if you take the sin t common this becomes i c 1 minus c 2 sin t.

(Refer Slide Time: 15:58)



Here you can take this part. So, this c 1 plus c 2, you can take equal to a and this you can take equal to b. So, you can write the solution x t. So, the solution x t is nothing, but a cos t plus b sin t.

So, previously we have written the same equation here also you can see. So, we have taken a sin t plus b cos t. So, here you can reverse this order also you can write this as a and this as b that is i c 1 minus c 2 as a and c 1 plus c 2 as b, so that to get the same equation what you have written there that is a sin t plus b cos t where this a and b depends on the initial condition.

For example, let t equal to take one example let t equal to 0, x equal to x 0 and t equal to 0 let x dot equal to 0. So, taking this example, so you can find a and b. So, you can write this x dot expression also. So, x dot equal to so a cos t, so minus a sin t. So, this becomes a sine t plus b cos t.

So, in the first equation if you put the initial condition at t equal to 0 x equal to x 0, so you got x 0 equal to. So, x 0 equal to. So, now t equal to 0. So, sin t becomes 0 and cos 0 equal to 1, so, x 0 equal to 1 and in the second equation now substituting t equal to 0, so x dot equal to 0.

So, 0 becomes minus a sin t, so a part has 1. So, this equal to 0 and b cos t cos t equal to 1, so, this is equal to 0. So, in this case we got b equal to 0 and x 0 equal to a. So, our solution becomes x t, x t equal to x t equal to x 0 cos t; so, this way by using initial conditions.

So, analytically we can find the solution of the system, but numerically if you want to do, so this is a simple equation. So, we can easily find the analytical solution. So, when the equation becomes complex like the equations, you have seen in case of the duffing equation or Van der Pol equation or Mathieu equation or a combination of these equations. So, that time is simplified solutions is not possible.

So, for that purpose one has to use these numerical methods. So, here the this equation can be written for example, already I have written this x 1 equal to x y 1 equal to x. So, dy 1 equal to x dot. So, dy 1 equal to x dot means, so already you know this x dot we are written equal to y 2. So, this is y 2, this is y 2 and then this dy 1 y 2 equal to. So, this becomes dy 2; so dy 2 equal to x double dot. So, that is nothing but so from the first equation it can be written minus x and minus x is nothing but minus y 1.

So, this way we can write this equation. So, in the MATLAB, so you can write a function, so you can use a function file. So, there are options of writing these things already last class I told you. So, where you have the script file; in the script file, you can have a function file also. So, you can develop a function file for example, this is a function file.

So, in this function file you can write dy equal to, so this is the filename. So, tf1 I have put the file name. So, tf1 t comma y t comma y means this time t and y equal to. So, this y vector contain both y 1 and y 2 that is displacement and velocity. So, for this equation, so you can write this equation also this form x double dot plus omega square x equal to 0. So, let this is the equation which is solved where omega square is put 1 to get this special case.

So, here for example, omega is taken. So, w is taken 2. So, this is nothing, but omega. So, here as this is this dy is a 2 rows. So, we can write this is 2 is to 1 dy 2 is to 1, then dy 1 this is the equation first equation we have written. So, dy 1 equal to y 2 and dy 2 equal to minus omega square y 1. So, this way you can write in your MATLAB the function file and you can generate the response. So, this is the main code written to plot these things. So, the simple code also can be used for writing any other function.

So, for example so here first you divide this time. So, for example, we want to plot from 0 to 100 or we want to find the response starting from 0 to 100 second. So, let us divide that thing into 5000 points. So, as many points you want you divide, so that you can get a very smooth curve. That means, your increment here h will be equal to. So, you have divided 100 by 5000. So, h in this case will be 100 by 5000 h increment equal to. So, your time you have taken 100 and that is divided into 5000 steps ok.

So, this way you can find h or you can take h this way. So, this is the increment you are taking or time steps you are taking in your analysis, but here directly I am using a built in function in MATLAB. So, for example, this ode45 is a built in function in MATLAB otherwise you can write the your own code by using this equation.

So, you write down use this equation instead of writing ode45 directly you can use this expression to find write down your own code. So, this, so here T, Y equal to ode45. So, this is the filename already I have shown you the filename tf1, then so tspan already, so for minimum t to maximum t.

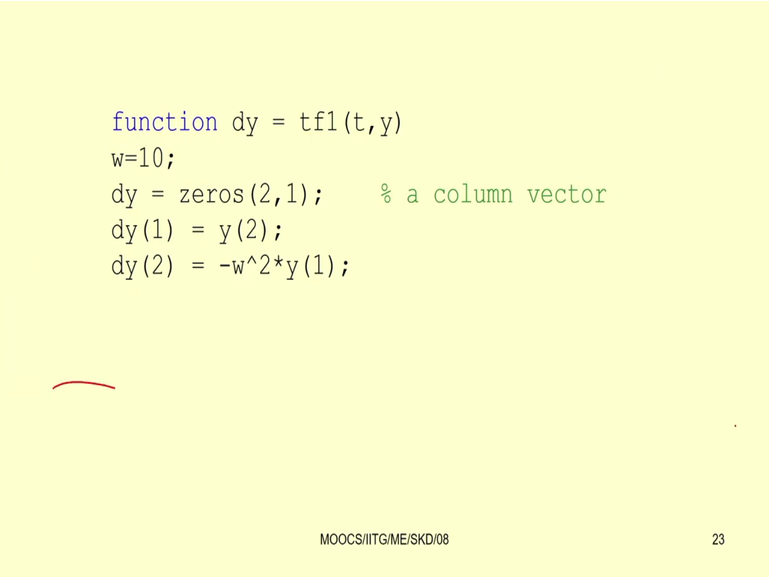
So, minimum t equal to 0 and maximum t equal to so your 100. So, tspan already you know and y 0; y 0 is nothing but the initial conditions. So, initial condition for example, let us start a equal to 2, so you can take any function any number also. So, this is the initial condition. So, from at what displacement it is starting, so you can start with 0 also.

So, it depends on u depending on the conditions for example, this simple pendulum. So, if you are hanging from, so the simple pendulum case. So, in case of the simple pendulum, so what do you mean by initial condition? So, here initially, so when you are starting both the things are 0, but you are taking to some position and you are leaving from this position.

So, it if it has not reached its maximum position, so here your displacement equal to theta and velocity also you can find corresponding to this theta;. so, you can find what is the velocity because you know the total energy and if you know the, so corresponding to this point it has some energy and here you can find what is the velocity.

So, knowing this velocity and theta, so you can write down your initial condition. So, initial condition means, so whether you are releasing from this position or releasing from this position or releasing from some other position.

(Refer Slide Time: 23:59)



So, depending on the position from where you are releasing. So, your the initial conditions will be different. So, that means, at t equal to 0 from which positions you are releasing the system. So, by taking this a 0 equal to 2. So, this way also we can a 0 equal to 2 and theta and a 0 equal to 2 and this velocity equal to 0; that means, velocity equal to 0 means, so we are releasing it from some position here where the velocity equal to 0.

So, we are releasing from this position this is at initial position, then this T, Y equal to ode45, this tf1 is the function file already I have shown you. So, this is the function file. So, you have name of the function file you have written this. So, this is tf1 dot m and this file name you can give any file name.

You can save with this file name. So, you can plot different figures. So, here I have plotted two figures. You can plot both the figures using a single figure, also by using the subplot command you can use the subplot command also to plot both the figures in a single figure by using the subplot command subplot.

So, in subplot, so you have to write subplot. For example, if you are writing subplot, so you have to divide the space into number of rows and column. For example, if you are writing 2 comma 1 comma 1 and then plot. So, you can plot the first one that is your that is t comma. So, here you just see, so it is T comma Y 1. So, here Y contains 2. So, the first one is the displacement and the second one is the velocity, so T Y comma 1 y. So, this is capital Y we have written. So, t capital Y colon comma 1, so colon comma 1. So, this is; so this gives the displacement and similarly now you can write the subplot 2 1 2. So, 2 1 means, so two rows and one column; so, it can plot using two rows and one column in that case.

So, here this code gives figure 1 and figure 2. So, in two figures also we can plot. So, you can write this x label. So, for example, here the time is written as the x label y label also you can write. So, y label that is the Displacement, here displacement is written in y label you can put a title. So, time response is we have put the time response us the title of these thing similarly in figure 2.

So, you can see we have not plotted. So, in this figure 2, so we have taken only from 1500 to 2000 the time we have taken. So, the time step you have taken 1500 to 2000 because this thing is divided into 5000 points. So, out of that thing, so we have taken. So, from 1500 to 2000 to plot this plot the phase portrait.

So, in phase portrait, so you are plotting between this displacement and velocity that is theta and theta dot. So, you just see the first column of y contain this displacement and the second column of y contain this velocity. So, you can write this y level velocity and x level displacement and this is phase portrait and clearly you can see the response to be periodic which you have already observed from your analytical work.

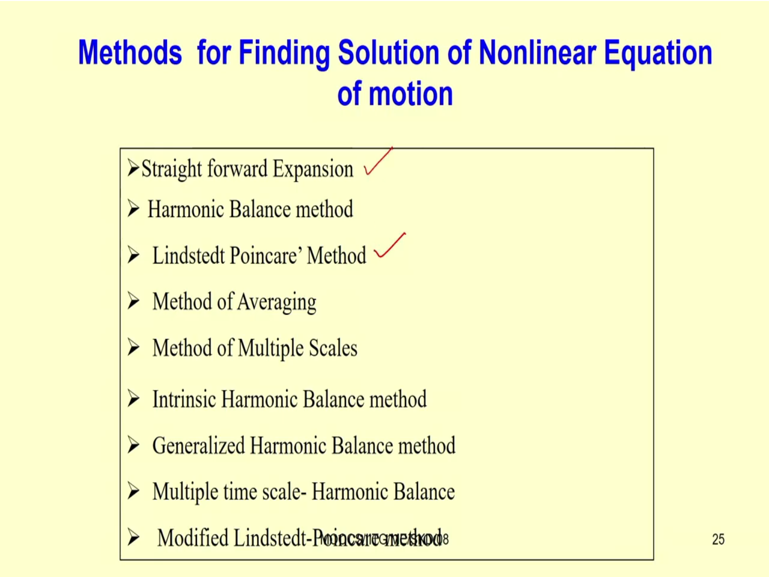
So, from your analytical things also you know the response equal to Y equal to or x t equal to just now we have seen. So, when you have put this is equal to 0, we have taken the same example for t equal to 0 x equal to x 0 and t equal to 0 x dot equal to 0. So, here x equal to that is x 0 equal to you have taken 2 and x dot equal to 0.

So, the solution become x t equal to a cos t. So, x 0 if you are taking this is 2, so then this becomes 2 cos t and so this response that means, this theta equal to. So, in this case we have the response theta t equal to 2 cos t. So, in this case we have taken omega, so here omega equal to taken omegas equal to taken. So, you can see in this case in the function file we have written omega equal to 10. So, the solution will be 2 cos 10 t.

Similarly, the velocity expression also can be written. So, theta dot t will be equal to theta dot t will be equal to. So, 10 into 2 that is 20. So, you can write this is equal to 20 minus 20 sin t minus 20 sin 10 t. So, if you plot this x theta versus theta dot; theta versus theta dot if you plot, then you can get the same plot same figure you can get. So, you can see.

So, to verify this thing to cos 10 t are not, so you just see this plot is clearly that of 2 cos 10 t. So, it is plotted from t. So, total plot is not plotted. So, from 90 second to 100 second is plotted to. So, clearly the periodic nature of this response. So, this way given any function or given any differential equation by using this numerical method that is Runge Kutta method. So, you can find the response of the system.

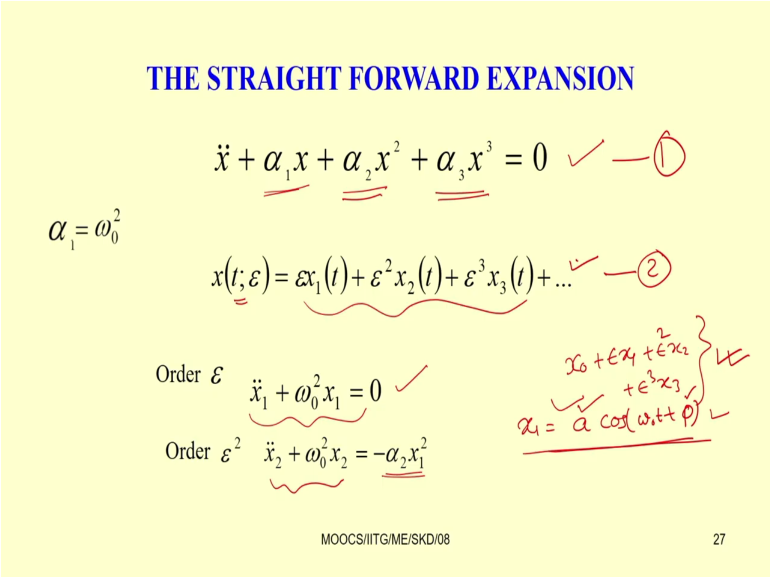
(Refer Slide Time: 30:02)



So, after knowing this numerical methods there are several other solution methods approximate solution methods are available which we are going to study in this module. For example, we can start with the straightforward expansion, harmonic balance method, Lindstedt poincare method, method of averaging, method of multiple scale, intrinsic harmonic balance method, generalized harmonic balance method, multiple time scale harmonic balance method, modified lindstedt Poincare method and some homotopy method and many other methods are recently developed.

So, today just now we are going to see how we can use the straightforward expansion and the Lindstedt Poincare method to solve this equation.

(Refer Slide Time: 30:50)



So, in case of straight forward expansion, so let us take this Duffing equation with quadratic and cubic non-linearity. We can write this equation in this form x double dot plus alpha 1 x plus alpha 2 x square plus alpha 3 x cube equal to 0. So, here we can expand this thing x t semicolon. So, you just note, so we are putting the semicolon as this is a this epsilon is a parameter.

So, this is a book keeping parameter. So, you can write this x semicolon epsilon equal to epsilon x 1 t plus epsilon square x 2 t plus epsilon x epsilon cube x t. So, this way you can write or you can write also by using this x 0 plus epsilon x 1 plus epsilon x 2 plus epsilon square x 2 plus epsilon cube x 3. So, it depends on you how you can write this equation.

So, if you are writing this equation in this form and substituted in this equation first equation; so, this is your equation 2 and this is equation 1. So, now, you just see depending on the;

depending on the number of terms you are taking the complexity of expanding this thing will increase if you solve these things manually. So, you can use the symbolic software's or symbol platforms to conveniently solve these equations. I will show you how you can do the same thing by using MATLAB and also by using mathematical two examples I will show you.

So, by manually if you do this thing also you can do it. So, by substituting this in x double dot; x double dot will be nothing but epsilon into x 1 double dot plus epsilon square into x 2 double dot plus epsilon cube into x 3 double dot, then plus alpha x. So, for alpha x it will be alpha 1 into, so this is the x.

So, three times you have taken then plus alpha 2 x square. So, you can expand this thing by a plus b plus c whole square. So, in that formula you can use and you can do it, then plus alpha 3 x cube. So, doing the cubic of this thing you can do it. So, you just see so as you are going on increasing the order of this non-linearity, so the complexity of expansion by manually goes on increasing.

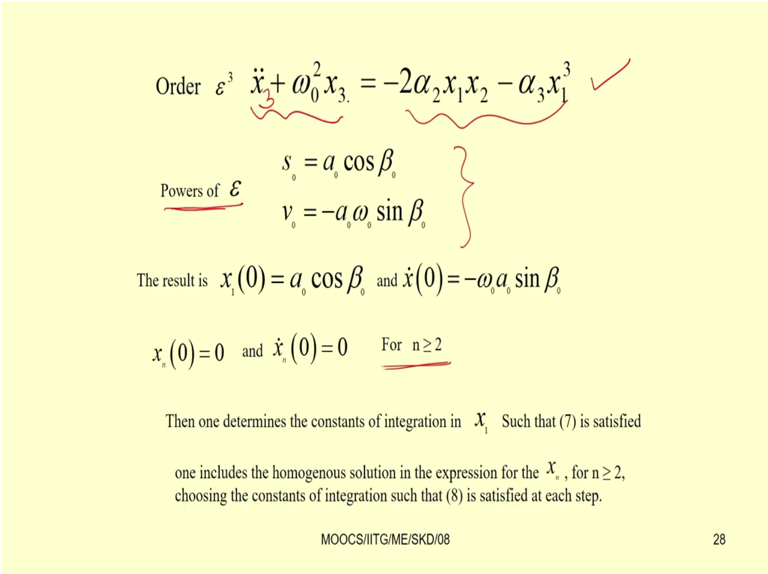
So, now if you order if you take or collate the terms with a order of epsilon order of epsilon square and order of epsilon cube, so you can get these equations. So, for example, this order of epsilon the solution become x 1 double dot plus omega 0 square x 1 equal to 0.

So, you know just now you have seen the solution of this equation is nothing but, so this is equal to. So, your x 1 you can write equal to a 1 or you can write in terms of c 1 e to the power i omega 0 t. So, that way you can write or using this cos and sin also you can write. So, for example, the solution you know, so this is equal to a cos omega 0 t plus phi.

So, you can write in many different ways the solution of the first term, then so you just see the order of epsilon square this is x square x double x 2 double dot plus omega 0 square x 2. You just note how you have to write it equal to. So, terms with x 2 you keep in the left side terms with x 2, you just keep it in left side and other terms you take it to right side. So, here in this case minus alpha 2 x 1 square you have taken to right side.

So, already you know the solution of x 1. So if you substitute it here, then you can find the solution of x 2, but till now you do not know what is your a and phi. So, till now you do not know what is a and phi. So, you can do it in two ways. So, initially also by putting this initial condition, so you can find x this a and phi and substitute it in this equation or at the end by finding all these solutions of all the parts that is x 1, x 2 x 3, you can finally apply the initial condition to find the solution.

(Refer Slide Time: 35:11)



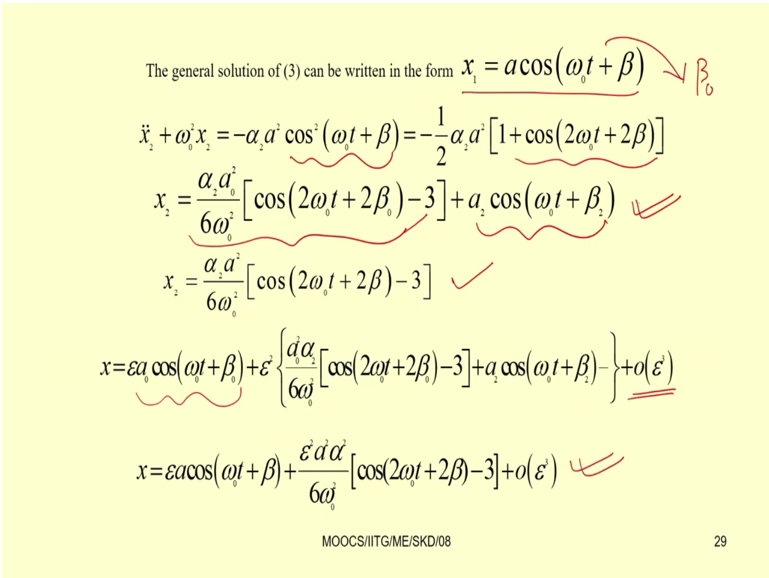
So, similarly for epsilon cube order, so this is the equation. Here also you have written in the same form, left side you keep the terms with x 3. So, this equation becomes x 3 double dot plus omega 0 square x 3 other terms you just keep it right hand side. So, which contain this x 1, x 2 and x 1 cube.

So, already you got this x 1 and expression for x 1 and x 2. So, by substituting these things, you can find. So, here so after writing initially we have written of different order. Now, the solution of this epsilon 0 already I have told this is can be written as a 0 cos beta 0.

So, beta 0 is nothing but your omega 0 t plus theta, so omega 0 t plus some other constant you can use. So, that way you can write or so, then velocity will be equal to minus a 0 omega 0 sin beta 0. So, the result is x 1 0 equal to a 0 cos beta 0 and x dot 0 equal to minus a 0 omega 0 a 0 sin beta 0.

So, here you can x n 0 equal to 0 and x n dot equal to 0; so, by substituting, so for n greater than equal to 0. So, because we have only up to we are keeping up to x to the power x 3, so then this for n greater than 2 greater than equal to 2. So, those terms will be equal to 0; so, then by substituting, so this is coming from the initial conditions. So, by substituting this initial condition we can expand and we can find the response.

(Refer Slide Time: 36:57)



So, here you can see the general solution can be written x 1 equal to a cos omega 0 t plus beta. So, this part in the previous slide it is written as beta 0 nothing but beta 0. So, then substituting this equation in the second equation x 2 double dot plus omega 0 square x 2, you can expand these thing and you can get this is equal to alpha to a square minus alpha to a square cos square omega 0 t plus beta. So, this cos square term you can expand by using this 2 cos square theta equal to 1 plus 2 cos theta.

So, this is 1 plus 2 cos theta equal to 2 cos square theta that is why this half is coming, so minus half alpha 2 a square 1 plus cos 2 omega 0 t plus 2 beta. So, now, the solution you just see. So, you have x 2 double dot plus omega 0 square x 2 equal to minus half alpha 2 a square into 1 plus cos 2 omega 0 t plus 2 beta. So, the solution of this thing can be written in this form. So, this is equal to alpha 2 a 0 square by 6 omega 0 square cos 2 omega 0 t plus 2 beta 0 by minus 3 plus a 2 cos omega 0 t plus beta 2.

So, this thing actually you can get by using. So, this part is due to your complimentary part and this part is due to your particular integral. So, due to your particular integral, the complementary part you can get by substituting this right hand side equal to 0 and the particular integral you can get by substituting. \

So, this right hand side divided by your d 0 square omega 0 square and for cos or sin. So, you know how to find this particular integral? So, simplifying these things, so you can get. So, your this expression can be written x 2 equal to in this form. So, now the total expression can be written as epsilon a 0 cos omega 0 t plus beta 0 plus epsilon square. So, this is the term and we can neglect the higher order term. So, x can be written in this form.

(Refer Slide Time: 39:31)

$$a = A + \varepsilon A_2 + \ldots\ldots, \qquad \beta = B_0 + \varepsilon B_1 + \ldots.$$

Then

$$\varepsilon a \cos(\omega_0 t + \beta) = (\varepsilon A_1 + \varepsilon^2 A_2 + \ldots.)\left[\cos(\omega_0 t + \beta_0)\cos(\varepsilon B_1 + \ldots) - \sin(\omega_0 t + B_0)\sin(\varepsilon B_1 + \ldots)\right]$$

$$= \varepsilon A_1 \cos(\omega_0 t + B_0) + \varepsilon^2\left[A_2 \cos(\omega_0 t + B_0) - A_1 B_1 \sin(\omega_0 t + \beta_0)\right] + o(\varepsilon^3)$$

$$= \varepsilon A_1 \cos(\omega_0 t + \beta_0) + \varepsilon^2 \left(A_2^2 + A_1^2 B_1^2\right)^{1/2} \cos(\omega_0 t + \theta_2) + O(\varepsilon^3)$$

Where $\theta_2 = B_0 + \tan^{-1}\left(\frac{A_1 B_1}{A_2}\right)$ We can choose $A_1 = a_0, B_0 = \beta_0$

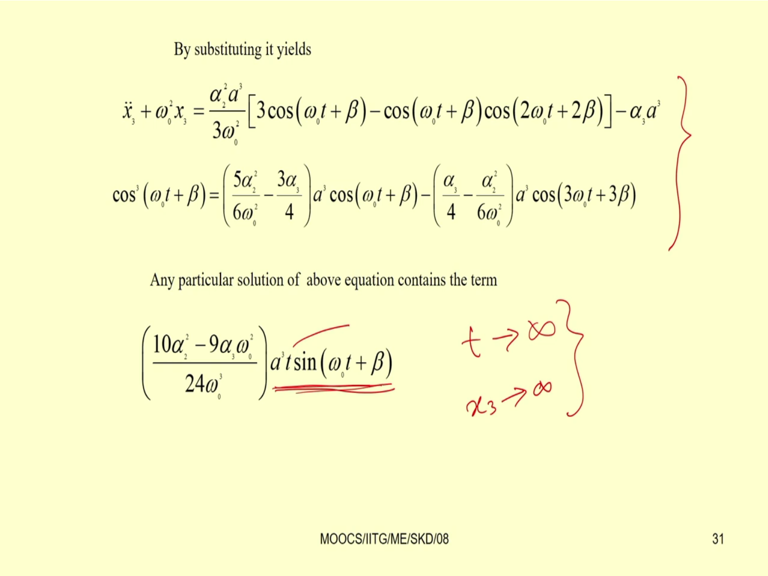
A_2 And B_1 Such that $\left(A_2^2 + A_1^2 B_1^2\right)^{1/2} = a_2$ and

$$\beta_0 + \tan^{-1}\left(\frac{A_1 B_1}{A_2}\right) = \beta_2$$

MOOCS/IITG/ME/SKD/08 30

So, now or x 2 can be. So, this is so by putting these initial conditions.

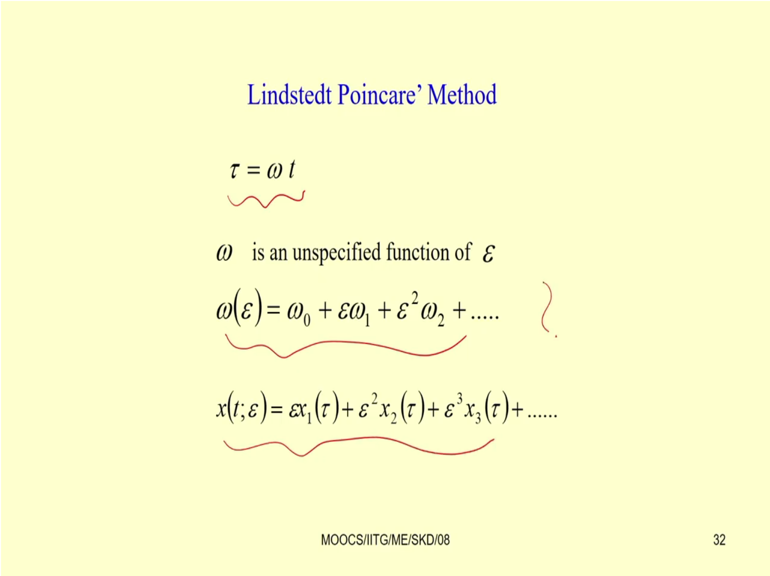
(Refer Slide Time: 39:36)



So, we can find the response of the system if those equations can be put in this third equation. So, you can see this x 3 term will contain a term x 3 will term solution of x 3 will contain a term like this. So, is that is a constant a cube t sin omega 0 t plus beta. So, here so if t tends to infinity t tends to infinity, you can see this x 3 term also tends to infinity, but in actual case the solution is bounded, but here we are getting the solution is unbounded.

So, what we are getting is not correct. So, that is why this straight forward method has to be modified to get the actual response of the system. So, these type of term what we are getting are known as the secular term. So, this term must be eliminated to get the proper response of the system. So, to modify this straight forward equation, so Lindstedt Poincare method is used.

(Refer Slide Time: 40:42)



So, in the next class we are going to study this Lindstedt Poincare method and Harmonic balance method. So, in this case in case of Lindstedt Poincare method briefly if I will tell, so we can write we can use a different time term tau. So, tau equal to omega t. So, then this omega 0 also omega we can write as a function of epsilon and we can expand that thing equal to omega 0 plus epsilon omega 1 plus epsilon square omega 2.

So, this way and same way we can write this x previous way we can write x now substituting this omega and x in the original equation and separating the terms with different order of epsilon and then solving one by one. So, we can get the proper solution of the non-linear differential equation.

So, we will see next class how we can use this Poincare method to solve the differential equation. Also we will briefly study regarding the Harmonic balance method also. So, with this so today class we will complete here, so.

Thank you very much.