Viscous Fluid Flow

Prof. Amaresh Dalal
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 01
Introduction
Lecture - 01
Preliminary concepts

Hello everyone. Welcome to this course entitled Viscous Fluid Flow, I am Professor Amaresh Dalal, from the Department of Mechanical Engineering at Indian Institute of Technology, Guwahati.

Viscous fluid flow is a fluid mechanics course as an advanced point of view, in which we will discuss more about the viscous fluid flows. As a prerequisite, you need to have credited the basic fluid mechanics course in your undergraduate level.

(Refer Slide Time: 01:10)

So, first let us discuss about the course outline. So, in module 1, we will discuss the preliminary concepts that you have already studied in the basic fluid mechanics course; Lagrangian and Eulerian approach, Reynolds transport theorem and from here, we will derive the mass conservation equation and momentum conservation equation which is known as Navier-Stokes equations.

In week 2, we will have the exact solution of the Navier-stoke equations for Steady one-dimensional rectilinear flows. Here, we will consider Plane Couette flow which is the shear driven flow, then we will consider Plane Poiseuille flow which is purely pressure-driven flow, then Plane Poiseuille flow with slip thin film flow and we will discuss about combined Couette and Poiseuille flow. That means, it is a combination of shear and pressure-driven flow.

In module 3, we will study the Steady Axisymmetric flows. First, we will consider the pipe flow and the exact solution of fully developed pipe flow is known as Hagen-Poiseuille flow. Then, we will consider thin-film annular flow, then steady flow between rotating cylinders.

In module 4, we will consider Transient One-dimensional Unidirectional Flow. Here, we will discuss Flow near a plate suddenly set in motion which is known as Stokes first problem, then we will consider flow due to an oscillating plate which is known as Stokes second problem, then we will consider transient plane Couette flow and transient axisymmetric Poiseuille flow.

In module 5, we will solve steady two-dimensional rectilinear flows. Here we will solve flow through the rectangular duct, flow through an equilateral triangular duct and flow through elliptical duct.

In week 6, we will discuss about the Lubrication Theory which is a kind of creeping flow. Here, we will discuss the Reynolds equation of lubrication, slipper bearing, journal bearing, piston-ring lubrication and two-dimensional lubrication.

(Refer Slide Time: 03:34)

In modules 7 and 8, we will discuss Laminar Boundary Layers, where first we will introduce the boundary layer equations. Then, we will derive the equations for flow over a flat plate and using a suitable similarity variable approach, we will derive the Blasius equation, then we will discuss about the momentum integral equation.

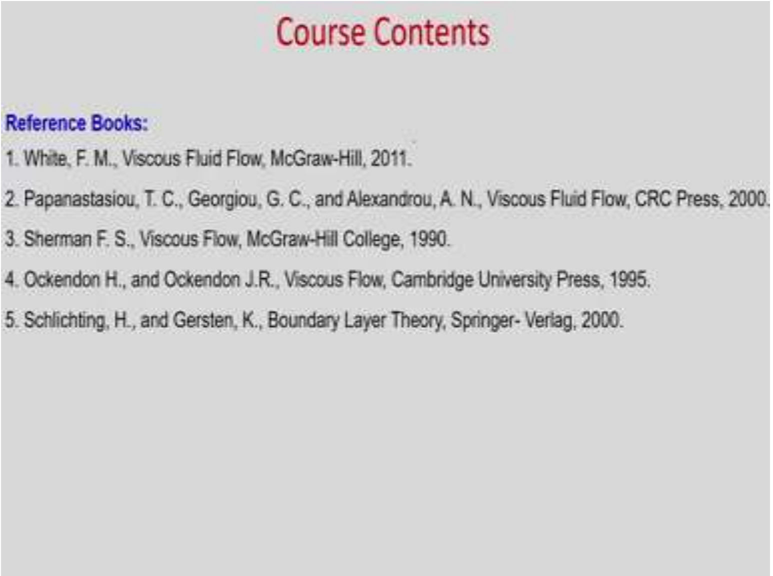
In module 8, we will consider non-zero pressure gradient for a flow over a curved plate or wedge and we will derive the Falkner-Skan equation and we will show the solution of this Falkner-Skan equation, then Karman Pohlhausen approximation, we will discuss; separation of boundary layer and wake behind the circular cylinder and we will discuss about vortex shedding.

In week 9, we will discuss about the Free-Shear flows, self-similar solution of free shear flows, flow in the wake of a flat plate, free shear layer between two different streams. In week 10, we will introduce the Stability Theory; where first we will derive the Orr-Summerfeld equation. We will discuss the Rayleigh's theorem, stability envelope, Squires theorem and we will also solve some stability problem for some simple parallel flows.

In modules 11 and 12, we will discuss about the Turbulent Flows. First, we will derive the Reynolds average Navier-Stokes equations, then we will discuss about the external and internal turbulent flows, Prandtl mixing length hypothesis and we will discuss about the universal velocity profile on flat plate.

Then, in last module, we will have the Integral solution of turbulent boundary layer flow and we will derive this k-epsilon model and we will discuss other two-equation turbulence model.

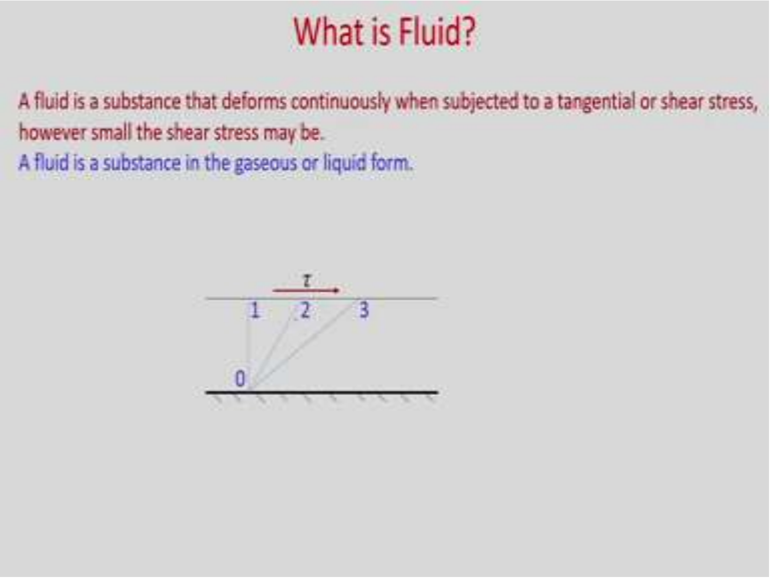
(Refer Slide Time: 05:56)



So, for this course mostly, we will follow the Viscous Fluid Flow book by F. M. White, you can see here. So, this book you can have as the textbook. In addition, you can follow these books as reference books; Papanastasiou, Georgiou and Alexandrou, Viscous Fluid Flow, CRC Press; Sherman, Viscous Flow; Ockendon and Ockendon, Viscous Flow; Schlichting and Gersten, this is especially for Boundary Layer Theory. In addition, you can have other basic fluid mechanics books which you have already studied at your undergraduate level.

So, in today's class, we will first discuss about some preliminary concepts which you have already studied. First, let us discuss about what is fluid. So, first, let us define what stress is. So, if any force is acting on in some elemental area, then stress is defined as the force per unit area. So, a normal component of this stress is known as normal stress and the tangential component of this stress is known as tangential or shear stress.

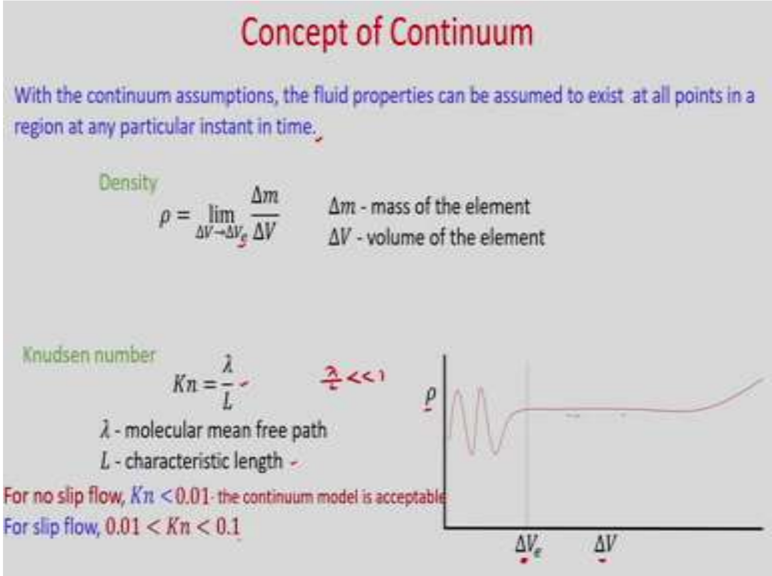
(Refer Slide Time: 07:24)



So, now we will define the fluid. So, if you can see that let us say one stationary fluid is there. So, this is the stationary plate and this is the fluid layer and if some tangential stress is imposed on this fluid layer, then you can see it will continuously deform. So, at time t is equal to 0, if this is a vertical line 0, 1. So, if some shear stress is applied on this fluid layer, then obviously, it will continuously deform and this line vertical line initially was vertical line. So, it will deform as 0, 2 and 0, 3.

So, we can define the fluid as a substance that deforms continuously when subjected to tangential or shear stress; however, small the shear stress may be. So, a fluid is a substance in a gaseous or liquid form. Next, we will discuss about the concept of the continuum. In a study of fluid mechanics, it is convenient to assume that the gases and liquids are continuously distributed throughout a region of interest; that means, the fluid is treated as a continuum.

(Refer Slide Time: 08:56)



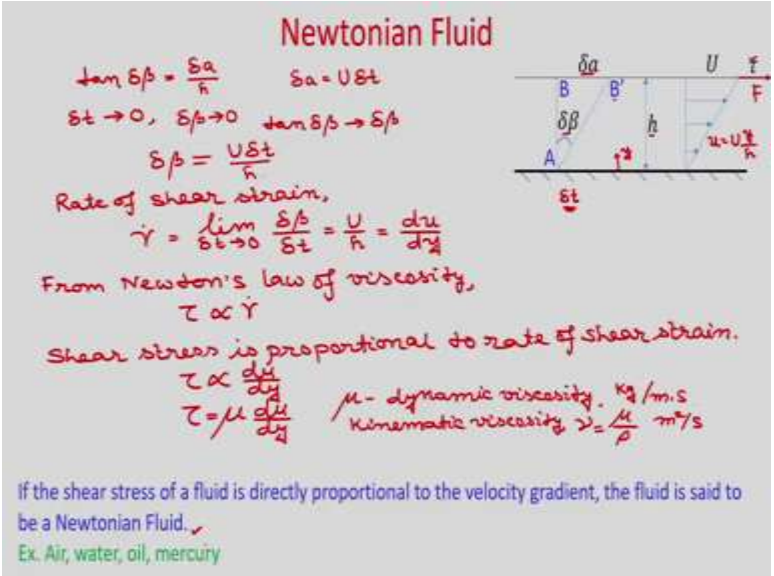
So, with the continuum assumptions, the fluid properties can be assumed to exist at all points in a region at any particular instant in time. So, consider the variation of density as a function of the size of any element ΔV. So, ρ is the density of the fluid; ΔV is the volume.

So, if you see that at larger ΔV, the density is affected by the inhomogeneities in the fluid itself arising from varying composition and temperature distribution and if ΔV becomes smaller in this region if you consider that it is almost a constant and it is uniformly distributed and if ΔV is very small, so you can consider in this region, then there will be random fluctuation of density ok.

So, you can see that we can define a density ρ in the limit of this ΔV_e because below this ΔV_e, so there will be a fluctuation and due to this fluctuation, there may be a change in mass So, obviously, you can see that it will not be continuously distributed in the region of interest. So, density, we can define as the limit, ΔV tends to ΔV_e which is the limiting volume $\Delta m/\Delta V$. So, Δm is the mass of the element and ΔV is the volume of the element.

So, if λ is the molecular mean free path of the molecules and L is the characteristic length, then this continuum approximation is valid when λ/L is much much smaller than 1. So, if we define the Knudsen number as the ratio of λ/L, then the continuum model is acceptable if the Knudsen number is less than 0.01 and no-slip flow will be valid in this range of Knudsen number less than 0.01 and for slip flow, this Knudsen number range is between 0.01 and 0.1.

(Refer Slide Time: 11:48)



So, let us discuss the Newtonian fluid. So, again, we will consider one stationary fluid element over a flat plate. So, you can see, so this is the flat plate which is stationary and this is the fluid layer and the upper layers some force, you applied in the tangential direction and due to that, there will be shear stress.

Now, if you consider initially one vertical line A, B. So, after time delta t;, due to these applied tangential force, this A, B will come to a position A, B prime and it will make one angle δβ and the distance B to B prime is δa. So, obviously, in δt time this B travels to B'.

For this case, if the height of the fluid is h, then the velocity profile may look like this linear profile and u will be just uy/h, where y is measured from the bottom plate. So, thus, you can see that a velocity gradient is developed in the fluid.

So, in a small-time δt, δt increments an imaginary vertical line A B in the fluid would rotate through an angle δβ and it will have the position A B'. So, with this if you consider tan δβ; then, you can write from here you see, it will be δa/h; δa/h and what is δa?

So, δa is the distance traveled from this B to B' in time δt. So, if u is the velocity of this upper layer, then u. δt. So, obviously, if δt tends to 0, then you can see delta beta also will be tending to 0 because it will make very small angle and tan δβ, you can write as δβ. So, from here, you can see δβ, you can write as; so, δa is u δt/h.

So, from here, we can define a rate of shear strain $\dot{\gamma}$ is equal to

$$\dot{\gamma} = \lim_{\delta t \to 0} \frac{\delta\beta}{\delta t} = \frac{U}{h} = \frac{du}{dy}$$

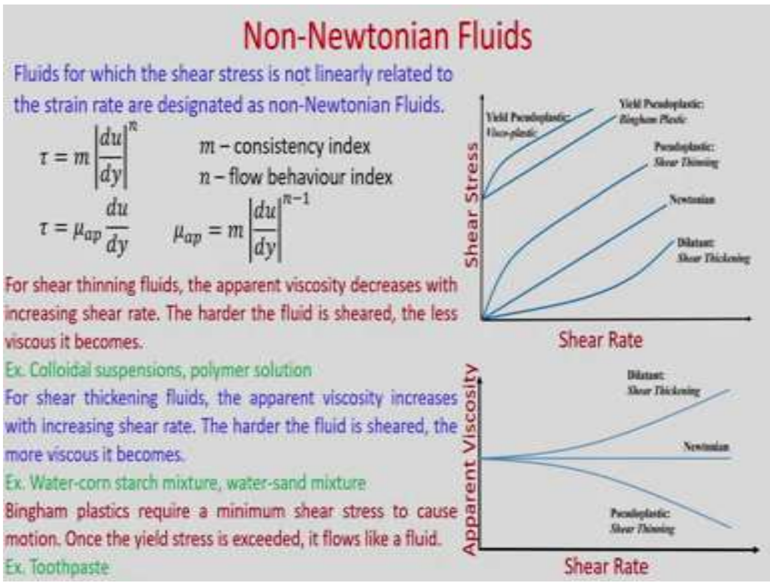
So, from Newton's law of viscosity, we can say that the shear stress is the applied shear stress. The applied shear stress τ is proportional to the rate of shear strain $\dot{\gamma}$. So, that means, shear stress is proportional to the rate of shear strain.

And now, we can write τ as proportional to the velocity gradient du/dy and we can write τ is equal to $\mu \frac{du}{dy}$. So, μ is the proportionality constant and this proportionality constant μ is known as dynamic viscosity ok.

Its unit is kg per meter second and we can define the kinematic viscosity ν as dynamic viscosity by the density of the fluid ρ and its unit is meter square per second ok. So, you can see if the shear stress of a fluid is directly proportional to the velocity gradient, then the fluid is said to be a Newtonian fluid.

So, many common fluids like air, water, oil, mercury are all Newtonian fluids. So, we have seen that if shear stress is directly proportional to the shear strain rate. Then, those fluids which obey this law are known as Newtonian fluid and the other fluids which does not obey this linear relation, then those are known as non-Newtonian fluid.

(Refer Slide Time: 17:51)



So, fluids for which the shear stress is not linearly related to the strain rate are designated as non-Newtonian fluids ok. So, in these cases shear stress, we can define as

$$\tau = m\left|\frac{du}{dy}\right|^{n}$$

So, where m is the consistency index and n is the flow behavior index.

So, in a general way if we define that τ is equal to $\mu\frac{du}{dy}$, then this is known as apparent viscosity and this apparent viscosity now from these two relations, we can write as

$$\tau = \mu_{ap}\frac{du}{dy}$$

$$\mu_{ap} = m\left|\frac{du}{dy}\right|^{n-1}$$

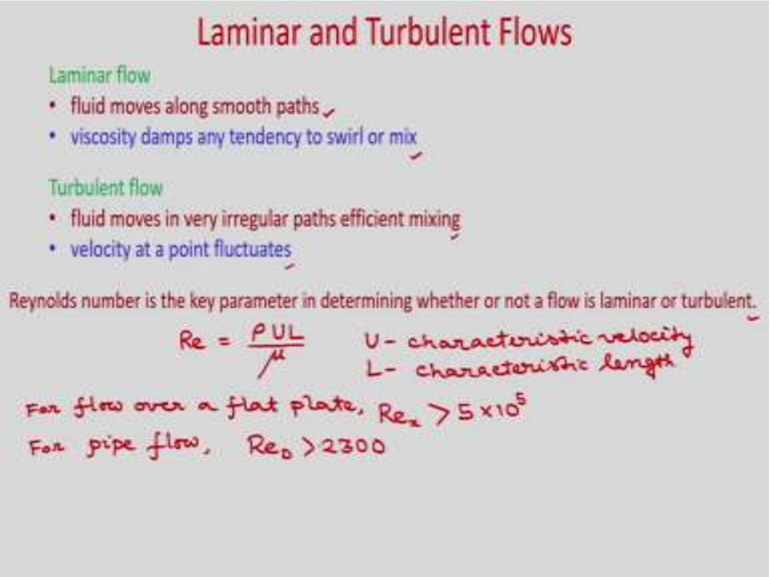
So, you can see obviously, for Newtonian fluid, the shear stress is linearly varying with strain rate or shear rate ok.

So, this is the Newtonian fluid and if you can see this apparent viscosity in this case, obviously, it is mu. It does not vary with the shear rate. So, you can see it is constant for Newtonian fluid. Now, for non-Newtonian fluids, there are different kinds of fluids. So, one is shear thinning fluid ok.

So, this is the shear thinning fluid. So, for shear thinning fluids, the apparent viscosity decreases with increasing shear rate. So, the harder the fluid is sheared, the less viscous it becomes ok. So, for the example of shear thinning fluid is the colloidal suspensions, polymer solutions.

And for shear thickening fluid, you can see the apparent viscosity this is the apparent viscosity increases with increasing shear rate ok. The harder the fluid is sheared, the more viscous it becomes. So, the example of shear thickening fluids are water-corn starch mixture, water-sand mixture. And for Bingham plastic, you can see it requires a minimum shear stress to cause the motion ok. After that, it linearly varies. So, once the yield stress is exceeded, it flows like a fluid. So, the example of Bingham plastic is toothpaste. So, next we will discuss about the laminar and turbulent flows. So, you can see the laminar flows are very ordered flows with smooth streamline.

(Refer Slide Time: 20:28)



So, in laminar flow fluid moves along smooth paths and viscosity damps any tendency to swirl or mix; whereas, the turbulent flow is a highly disordered fluid motion, characterized by velocity fluctuation and eddies. So, this fluid moves in a very irregular path and it is having efficient mixing and velocity at a point that fluctuates.

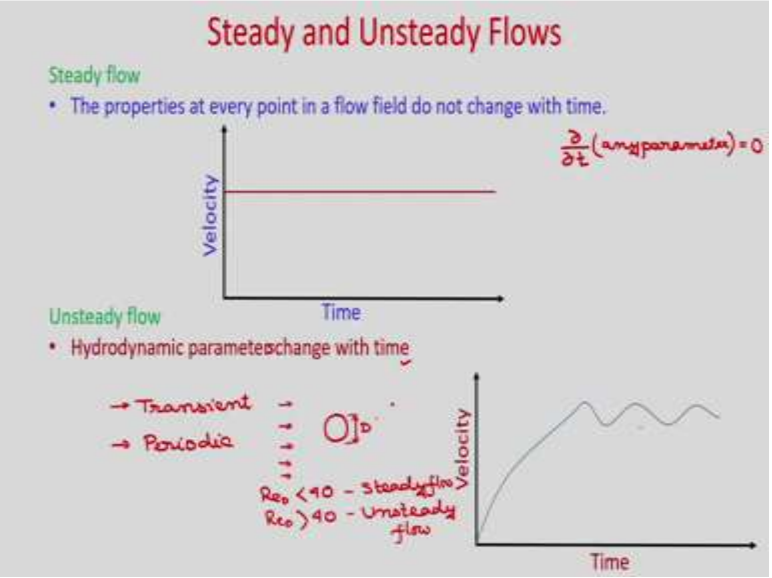
So, Reynold's number is the key parameter in determining whether or not a flow is laminar or turbulent. So, you know the Reynolds number, we define as the density of the fluid, some characteristic velocity U and characteristic length L divided by μ which is the dynamic viscosity of the fluid; where, U is characteristic velocity and L is characteristic length.

So, the characteristic length and the characteristic velocity depend on the type of flow you consider. So, if you consider external flow, let us say flow over a flat plate, then obviously, the characteristic length will be the length of the flat plate and the characteristic velocity will be the free steam velocity U_∞.

If you consider internal flows, let us say flow inside a circular pipe, then generally we consider characteristic length as the diameter of the pipe and average velocity as the characteristic velocity. So, for the external flow, if you define this Reynolds number based on this free stream velocity and the length of the plate, then the flow becomes turbulent if the Reynolds number is greater than 5×10^5.

So, you can see that for flow over a flat plate, this Reynolds number based on any x, so it will be greater than $5x10^5$; then, the flow becomes turbulent and for pipe flow, Reynolds number based on the diameter of the pipe and the average velocity, if it becomes greater than 2300, then the flow becomes turbulent.

(Refer Slide Time: 23:23)



Now, let us discuss about the steady and unsteady flows. So, obviously, in a steady flow, the flow parameters does not vary with time ok. So, the gradient of any flow parameter with respect to time will be 0.

So, in this case, the properties at every point in a flow field do not change with time. So, that means, del of delta t of any parameter will be 0. So, if you consider let us say a flow inside a pipe and in laminar regime, then you can see that the velocity any quantity U or V, if you measure with time, then it you can see that it will be a flat curve, it does not vary with time.

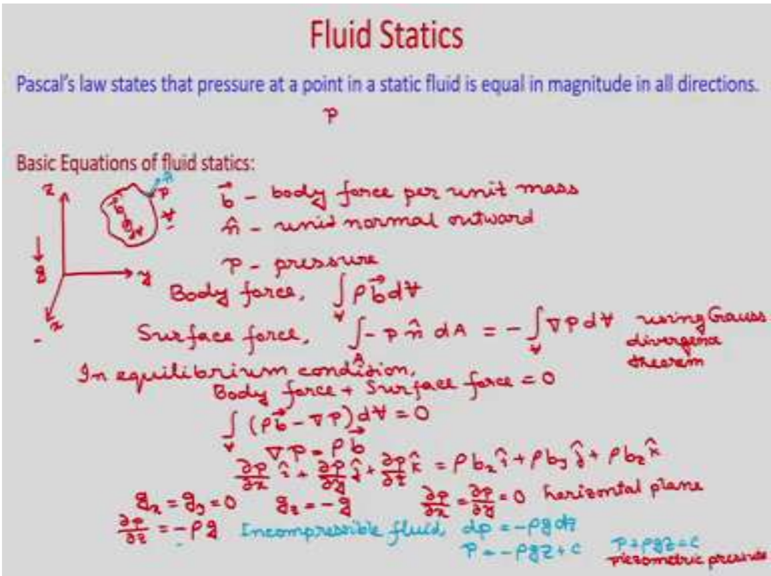
But if you consider unsteady flow, then obviously, the hydrodynamic parameters change with time. So, the unsteady flow we can have two types; one is transient, the other one is periodic ok. Say if you consider the flow over a circular cylinder of diameter D, then if Reynolds number based on diameter is less than 40, then generally it is a steady flow and if Reynolds number based on diameter is greater than 40, then it becomes unsteady.

So, if it becomes unsteady flow and if you are starting the solution from initial velocity as 0 inside the domain, then obviously, if you measure the velocity at any point in the domain, then initially it will vary or increase with time.

After a certain time, you will find that there will be vortices behind this cylinder, it will be shedded periodically which is known as von Karman vortex street and it will periodically said behind the cylinder. So, this velocity, if you measure with time, then you will notice that it will become periodic.

So, obviously, you can see the initial part is known as the tangent part because it is bearing with time after that, it repeats periodically ok. So, obviously, this is known as periodic flow. So, now, let us discuss about the fluid statics. So, fluid statics is the study of fluid at rest; that means, there is no relative motion between the fluids.

(Refer Slide Time: 26:41)



So, you know that Pascal law states that pressure at a point in a static fluid is equal in magnitude in all directions. So, you can see that whatever we consider the hydrodynamic pressure p at a fluid element, it is the same and also it acts at a point from all directions. So, now, let us discuss about the basic equations of fluid statics. So, you can see that if you have one fluid volume, let us say this is v and you have one elemental fluid volume that is d v and this is your y direction, this is x and this is z.

So, obviously, you can see that any body force acting on this elemental fluid by element db, let us say it is p ok. If you consider one elemental fluid area and the normal outward normal is n, then obviously, you can see that pressure will be acting on this fluid element in opposite direction, so it is p. Because p you know that is always compressive in nature.

Now, if we consider that b is the body force per unit mass and n is the unit normal outward and p is the pressure, then you know the body force. Obviously, it is acting on the elemental volume dv. So, ρdv is the elemental mass. So, $\rho\vec{b}d\forall$ is the body force acting on this elemental volume and if it is acting on this total volume $\forall$, then we have to integrate over the volume.

And surface force you can see, here only the special force is acting on the surface. So, obviously, we can write as area integral $-p\hat{n}dA$. So, now in equilibrium condition when fluid is at rest; equilibrium condition fluid at rest, so this body force plus surface force is 0. So, obviously, from here you can see that if you use the Gauss divergence theorem, then you can convert this area integral to volume integral and you can write it as minus volume integral $\nabla p d\forall$ using Gauss divergence theorem.

So, you can see that it is

$$\int_{\forall}(\rho\vec{b} - \nabla P)d\forall = 0$$

$$\nabla P = \rho\vec{b}$$

So, obviously, this is you know that it is

$$\frac{\partial p}{\partial x}\hat{\imath} + \frac{\partial p}{\partial y}\hat{\jmath} + \frac{\partial p}{\partial z}\hat{k} = \rho b_x\hat{\imath} + \rho b_y\hat{\jmath} + \rho b_z\hat{k}$$

So, i, j, k are the unit normal in the direction x, y, z respectively.

So, if fluid is at rest and it does not undergo any acceleration, then obviously, the gravity if it is acting only in the z-direction, then obviously, g_x is equal to g_y will be 0 and g_z will be just minus g ok; where, g_x is the body force. So it is b_x is equal to b_y is equal to 0 and b_z which is your gravity acting in the negative z-direction, it is minus g ok.

So, now, from here you can see that you can write for horizontal surface

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$$

And or horizontal plane because there will be no change in the pressure because pressure acts in equal magnitude from all directions.

So, obviously, this will become 0 and you can see that

$$\frac{\partial p}{\partial z} = -\rho g$$

because g z is -g because you can see that dp is negative, if dz is positive; that means, the pressure decreases as we move up and they increase, if we move down. So, obviously, $\frac{\partial p}{\partial z}$ is equal to $-\rho g$.

If we assume incompressible fluid ok, so now if you integrate this, then you will get you can write

$$dp = -\rho g dz$$

and you will get

$$p = -\rho g dz + c$$

So, here p is the hydrodynamic pressure and p+ρgz; so, we can write

$$p + \rho g z = c$$

So, this $p + \rho g z$ is known as piezometric pressure. So, this is known as piezometric pressure.

So, in today's class, we first defined what is fluid; then we have discussed about Newtonian and non-Newtonian fluids. In a Newtonian fluid, we have shown that shear stress is directly proportional to the shear strain rate and we defined the fluid property viscosity. Next, we discussed about the non-Newtonian fluid, those fluids whose does not follow Newton's law of viscosity, those are known as non-Newtonian fluid and in non-Newtonian fluid, we discussed about shear thinning and shear thickening fluid.

Then, we discussed about laminar and turbulent flows. So, for external flows when we consider flow over flat plate; so, if Reynolds number based on the plate length, if it is greater than 5 into

10 to the power 5, then the flow becomes turbulent. For internal flows if we consider flow inside a pipe, then the Reynolds number based on diameter, if it is greater than 2300, then the flow becomes turbulent.

Next, we discussed about the steady and unsteady flows; so, obviously, if any parameter like velocity, pressure does not vary with time, then those are known as steady flow and for unsteady flow, obviously, if you measure any quantity like velocity or pressure at a region inside the domain, then obviously, it will vary with time.

We considered flow over a circular pipe of diameter d, for this particular case if the Reynolds number based on this diameter is less than 40, then the flow is steady and if it becomes greater than 40, then behind the cylinder the vortices will be set it periodically and the flow becomes unsteady. Then, we discussed the basic equation in fluid statics.

So, obviously, we know that p is the pressure that acts normal to the surface and it is compressive in nature and we have shown that if it is a horizontal plane, then in x and y direction, the gradient of pressure in these x and y-direction will be 0 and in the vertical direction, the gravity g will be acting in the negative z-direction and $\frac{\partial p}{\partial z}$ becomes -ρg and for incompressible fluid as ρ is constant, you can integrate it and you can write p+ρgz is equal to constant; where, p+ρgz is known as piezometric pressure. In the next class, we will discuss about the fluid kinematics.

Thank you.