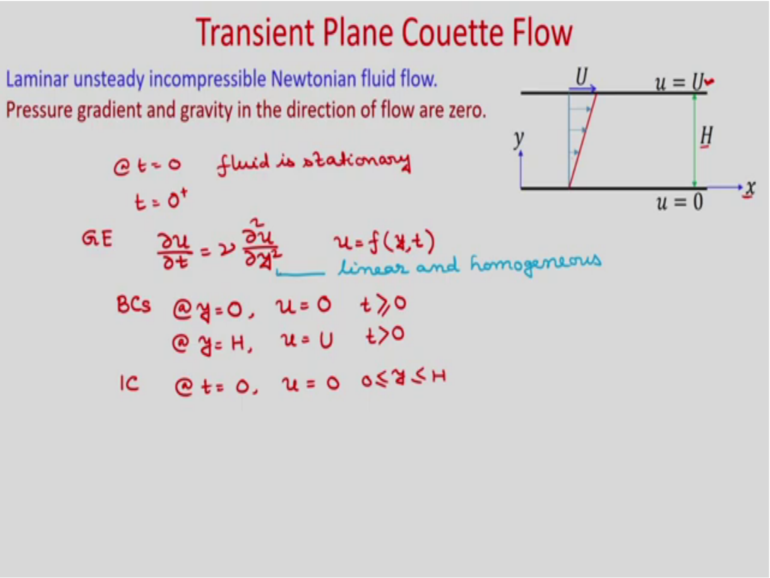
Viscous Fluid Flow
Prof. Amaresh Dalal
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 04
Transient One-dimensional Unidirectional Flow
Lecture - 03
Transient Plane Couette Flow

Hello everyone. So, today we will solve another tangent problem. Today we will consider Tangent Plane Couette Flow. You know the steady plane Couette flow results where the velocity distribution is linear, but if the fluid is stationary at t is equal to 0 bounded by two infinite parallel plates.

And suddenly, the upper plate starts moving with a constant velocity u then; obviously, inside the fluid domain from 0 velocity to the linear profile at larger time t tends to infinity, where it becomes linear this velocity profile will change. So, we want to find this velocity distribution which is tangent with time.

(Refer Slide Time: 01:30)



So, let us consider a Newtonian liquid is bounded by these two parallel plates separated by a distance H. x is the axial direction and y is measured from the bottom plate. And you know that at steady state, this is the velocity profile which is linear. Here obviously, it is purely shear driven flow and we are neglecting the pressure gradient and the gravity.

So, now we want to find that at t is equal to 0 fluid is stationary and at t is equal to 0 plus, this plates starts moving with a constant velocity U. So, what will be the temporal evolution of the velocity inside the fluid domain, we want to find. So, you know that in this case pressure gradient and the gravity are 0, so; obviously, you know the governing differential equation.

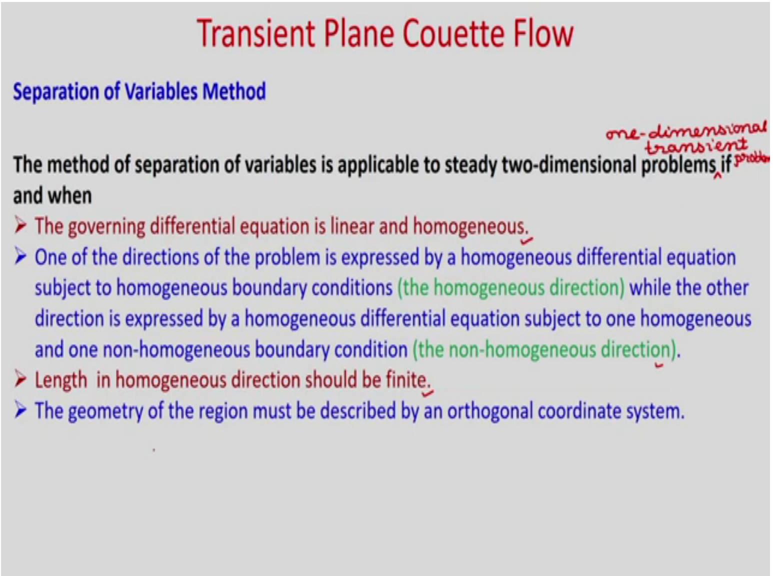
So, we can write the governing equation as del u by del t is equal to nu; which is your fluid kinematic viscosity del 2 u by del y square; where u is function of y and t. So, in this case we cannot have the similarity solution. So, we need to use some other method. So, first let us see the boundary conditions.

So, boundary conditions are at y is equal to 0 which is your bottom plate it is stationary u is equal to 0, for t greater than equal to 0 and upper plate at y is equal to H; it is moving with a constant velocity U, for t greater than 0 ok. So; obviously, you can see that once it starts moving, the velocity inside the fluid domain will start changing from 0 to some finite value.

So, now what is the initial condition? Obviously, at t is equal to 0 everywhere the fluid is stationary. So, u is equal to 0 in the range of y 0 to H. So, if you see the governing equation, this governing equation is linear and homogeneous; this is linear and homogeneous.

So, in this case actually we can use separation of variables methods. So, another way to solve this partial differential equation using separation of variables method where we can convert this partial differential equation to set up ordinary differential equations. So, now we have to know when can we use separation of variables method.

(Refer Slide Time: 04:49)

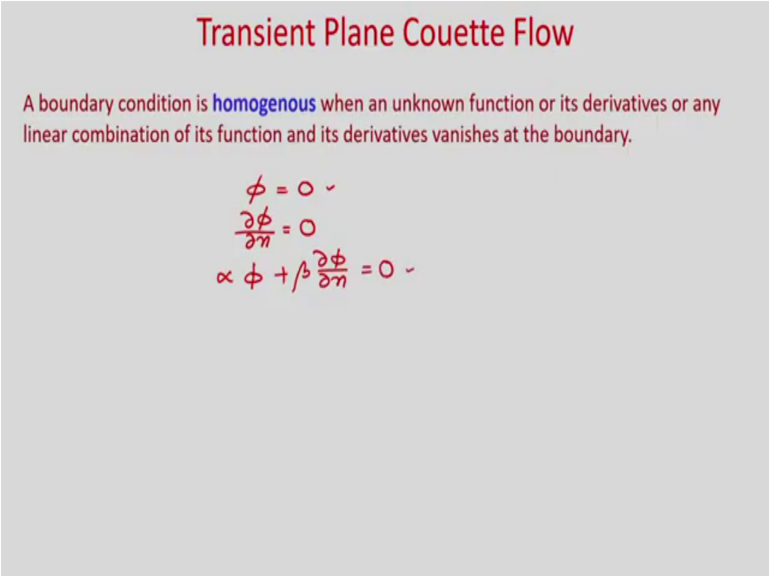


So, you can see here the method of separation of variables is applicable to steady two-dimensional problems or one-dimensional transient problem. Transient problem or three-dimensional problem as well; if and when the governing differential equation is linear and homogeneous. In our case governing equation is linear and homogeneous.

One of the directions of the problem is expressed by a homogeneous differential equation subject to homogeneous boundary conditions that is known as the homogeneous direction while the other direction is expressed by a homogeneous differential equation subject to one homogeneous and one non-homogeneous boundary condition that is known as the non-homogeneous direction.

We will discuss about what is homogeneous boundary condition and length in homogeneous directions should be finite and the geometry of the region must be described by an orthogonal coordinate system. So, in our case you can see that we have Cartesian coordinate system and also in the length in y direction is finite. And now, we need to discuss about the boundary conditions.

(Refer Slide Time: 06:32)



So, a boundary condition is homogeneous when an unknown function or its derivatives or any linear combination of its function and its derivatives vanishes at the boundary; that means, say you can have for any variable. Let us say general variable phi. If phi is equal to 0 at the boundary or del phi by del n normal to this boundary this is 0.

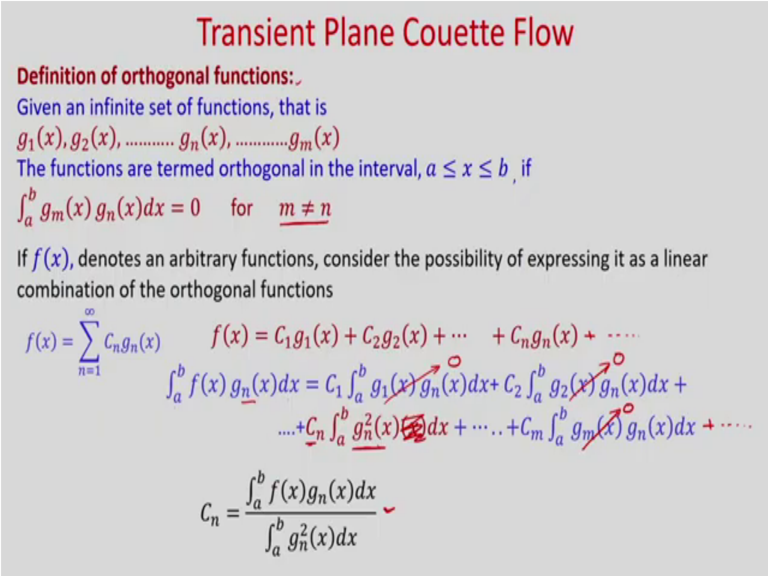
So, you can see this is your Dirichlet condition this is; obviously, Neumann condition which is homogeneous; that means, the gradient is 0 or linear combination of these two; that means, alpha phi plus beta del phi by del n is equal to 0. So, this is kind of a mixed a boundary condition. So, you can see these boundary conditions are known as homogeneous boundary condition.

Now, before using separation of variables method first; we need to check whether the governing equation is linear and homogeneous. Second, we have to check the boundary conditions. So, in one direction it should be homogeneous boundary condition and another direction, it may be non-homogeneous boundary condition.

And another thing is to check the in homogeneous direction, the lens should be finite. If we consider our boundary conditions you can see that in y direction, u is equal to 0 and u is equal to U; that means, this is your non-homogeneous boundary condition right. So, y direction is not homogeneous direction. And in t, it is marching so; obviously, you cannot consider as a homogeneous direction.

So, we have to choose y direction as homogeneous direction with suitable transformation of the boundary condition or by using some superposition technique of the problem.

(Refer Slide Time: 08:34)



Now, let us discuss about another important property that is known as orthogonal property. So, this we will use while solving this problem using separation of variables method. So, you can see definition of orthogonal function. Given an infinite set of functions that is a g 1 g 2 g n g m; maybe, it is function of x.

The functions are termed orthogonal in the interval a to b, if integral a to b g m x g n x d x is equal to 0 for m not equal to n ok; for m not equal to n, this integral will give 0 then, you can say that g m and g n are orthogonal functions. If f x, denotes an arbitrary functions, considered the possibility of expressing it as a linear combination of the orthogonal functions.

Let us say f x; we can represent as summation of n is equal to 1 to infinity c n g n x where c n is the coefficient. So, if you expand it, you can write f x is equal to c 1 g 1 x for n is equal to

1; for n is equal to 2, you can write c 2 g 2 x. Similarly, c 3 g 3 and if you put n is n. So, c n g n x and plus it will continue to infinity ok.

So, now what we will do? Now, you multiply both sides with g n x and integrate over a to b. So, you can see now in the left-hand side, we are multiplying with g n x and we are integrating a to b. Similarly, in the right-hand side also you multiply and integrate. So, C 1 is coefficient it is not function of x. So, you can take it outside. Integral a to b g 1 x d x plus C 2.

Now, integral a to b g 2 x g n x d x plus C n a to b g n x or d x plus it will continue and let us say is for a m C m a to b g m x g n x and plus it will continue to infinity. So, now if you look here that if you g m x and g n x are orthogonal function then; obviously, for m not equal to n, this will become 0.

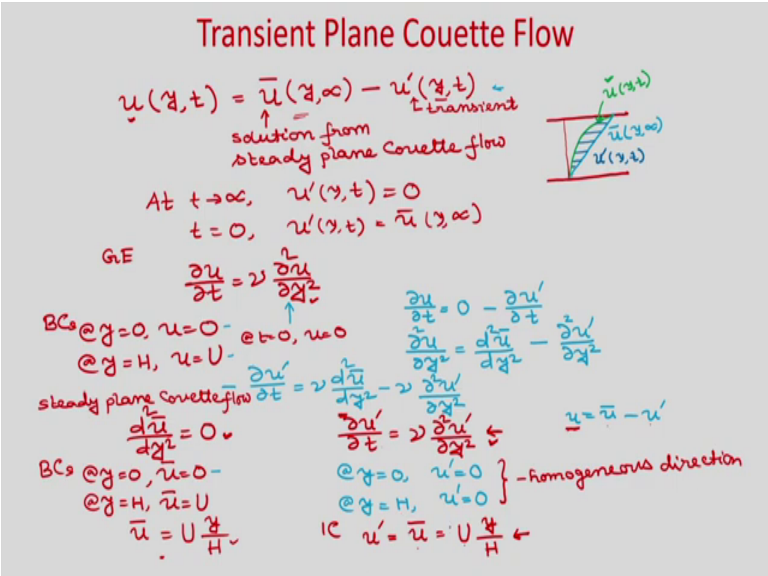
So, in the right-hand side you can see that g 1 x and g n x, this will become 0, because it is m not equal to n right m not equal to n, because m is equal to 1. So, this will become 0. Similarly, this term will become 0, but here as m is equal to n. So, it will be g n squared. So, this term will remain and this term also will become 0 and all other terms except this term will become 0 in the right-hand side ok.

So, now from here you can see you can find this constant C n. So, C n you can write now, integral a to b f x g n x d x divided by integral a to b g n square x d x. So, here we could find the constant C n using this orthogonal property. So, when we will solve this problem using separation of variables method this expression will use if those functions are orthogonal to each other.

Now, we have seen that our governing equation is linear and homogeneous, but boundary conditions are not homogeneous in y direction. So, we need to use some suitable superposition technique such that boundary conditions becomes homogeneous in y direction. So, whichever we will do it. You can see in this particular case, you know the steady state solution steady plane Couette flow solution that is linear profile.

Now, if you subtract a tangent velocity profile from this steady state velocity then, you can get tangent solution of this plane Couette flow.

(Refer Slide Time: 13:04)



In this case, you can see that if you have velocity profile. Let us say this is your final steady state velocity profile and it is let us say u bar y and t infinity at a larger time; obviously, it will become steady state. So, this is your velocity profile. Now, we will subtract one velocity from here; such that you will get at any time that tangent velocity profile.

So, let us say this is your velocity profile at any t; so, u y t ok. So, this is we want to find tangent velocity profile and this is the steady state velocity profile u bar. So, you can see that from the steady state profile, if you subtract this portion then; obviously, you will get the velocity profile at any time t and this is your tangent velocity ok.

And this tangent velocity; obviously, you have to subtract it ok. So, let us say this is your u prime y t ok. So, u prime y t will become 0 when t tends to infinity; so that you get the linear velocity profile. And you can see that; obviously, this u prime will be equal to u bar at t is equal to 0; so that you will get 0 velocity profile at time t is equal to 0.

So, now, we are representing now the velocity profile which you are interested to find tangent velocity profile as plane Couette flow velocity profile u bar y infinity t tends to infinity; that means, at larger time. So, this is the steady state solution or plane Couette flow minus u prime y t and this is the transient some velocity profile which you want to subtract with the steady plane Couette flow velocity profile.

So, you can see this is your solution from steady plane Couette flow. So, please try to understand what we are doing. So, we are interested to find this velocity profile u y t. Now, we know the steady state plane Couette flow velocity profile which is linear ok. So, that is your u bar y infinity and this is the velocity profile.

Now, we are subtracting one tangent velocity profile from this steady state profile and we want to get the final velocity profile. So, this u prime; obviously, at steady state or at t tends to infinity; obviously, u prime y t will be 0, right. If it becomes 0 then, you will get linear velocity profile ok.

And you can see at t is equal to 0; obviously, u bar y t will be u prime y infinity right. So, that you will get 0 right u is equal to 0. So, with this now, you can see the solution already you know. Now, we need to find the solution of this velocity profile u prime using separation of variables method.

So, now let us write down the boundary conditions for each individual problem. So, as the governing equation is linear, you can write this velocity profile using this superposition of two velocity profile ok; one is steady state velocity profile minus one tangent velocity profile and that you can write as the governing equation is linear ok.

So, now, let us write the governing equation. So, del u by delta t is equal to nu del 2 u by del y square. It is our original governing equation with boundary conditions at y is equal to 0 u is equal to 0 and at y is equal to H u is equal to U ok. Now, you know the problem of steady plane Couette flow ok. What will be the boundary condition for that?

You can see from here if you write del u by del t left-hand side. Right-hand side; what will be that? So; obviously, you can see this is steady state right. So, it will be 0 minus del u prime by del t and del 2 u by del y square. So, in the right-hand side, you can see you can write d 2 u bar, because this is function of y only right divided by d y square minus del u prime del 2 u prime by del y square ok. This is we know that it is solution from the steady plane Couette flow.

So, we are writing the ordinary differential. Now, if you put it in this equation. So, what you will get? So, you will get minus del u bar del u prime by delta t is equal to nu d 2 u bar by d y square and you will get minus nu del 2 u prime by del y square ok. So, now, as the governing equation in the linear, so, we will split into two problems; one is this equal to 0 d 2 u bar by d y square is equal to 0, because this is the governing equation for this problem; we know steady plane Couette flow.

And if you put it 0 then, another equation you will get del u prime by del t is equal to nu del 2 u prime by del y square ok. Now, we have splitted into two problems; one is steady plane Couette flow and this is your transient problem. So, now, if you write the boundary conditions for these; so, you know at y is equal to 0 u is equal to 0 and at y is equal to H u is equal to U.

So; obviously, this is the solution you know that u bar will be U y by H. It is a linear profile, right the solution we know we have already solved ok. So, now what will be the boundary condition for this tangent problem? So, if you see that this is your u is equal to u bar minus u prime right.

So, already you know that y is equal to 0 u is equal to 0. So, if u is equal to 0 and here, these are u bar is equal to 0, because this is the steady velocity profile. Now, if u bar is equal to 0 so; obviously, you can see that at y is equal to 0 u prime will be 0. So, from here you can see u is equal to 0 from here u bar is equal to 0 from here so; obviously, u prime will be 0.

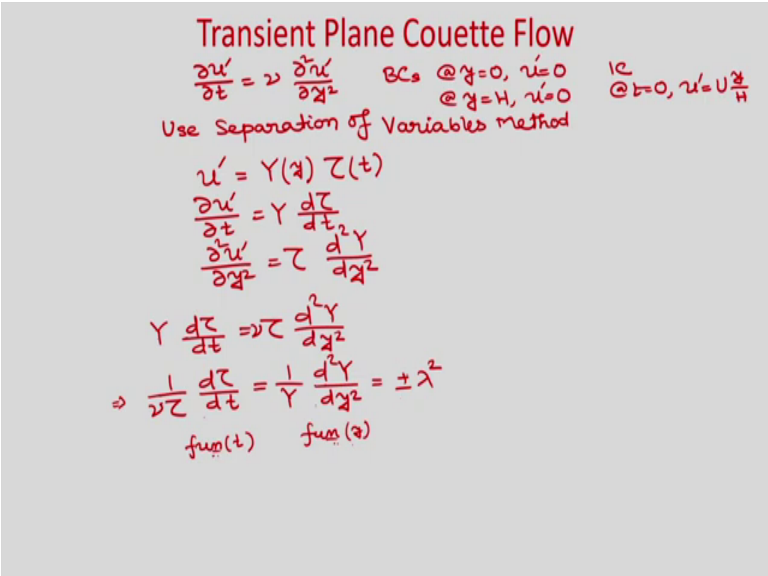
And at y is equal to H, what will be the u prime? You can see here from here. You can see left-hand side u is equal to u and here, u bar is equal to U. So, u prime is equal to 0. Now, let us discuss about the initial condition. So, this is the steady state problem right. So, we do not have any initial condition. For this problem original problem, we have at t is equal to 0 u is equal to 0. What will be the initial condition for this problem ok?

So, initial condition from this you can see; at t is equal to 0 u is equal to 0. So, what will be the u prime? u prime will be just u bar you can see u prime will be u bar. So, at that is the initial condition and that u bar is U into y by H ok. So, these we have to see that from this expression as it is a steady state problem. So, u bar is U y by H. So, that we are putting here.

So, the initial condition for this transient problem; it will be u prime is equal to u bar which is nothing, but u y by H ok. So, that is the initial condition for this tangent problem. Now, you check. For this problem we have the solution only we need to solve this problem using separation of variables method now you check. The governing equation is linear and homogeneous and now, boundary conditions we have also made homogeneous. So, this is your homogeneous direction ok.

Because u prime is 0 at y is equal to 0 and y is equal to H. So, this is your homogeneous direction. So, we can use separation of variables method. Now, let us solve this tangent problem and find the velocity profile u prime and we have seen that the governing equation is linear and homogeneous and boundary conditions we have also made homogeneous with this superposition method.

(Refer Slide Time: 23:46)



So, now our problem is del u prime by delta t is equal to nu del 2 u prime by del y square subjected to boundary conditions at y is equal to 0 u is equal to 0 and at y is called to H u is equal to 0 sorry these were prime. And initial condition at t is equal to 0 u prime is equal to u y by H. Now, we will use separation of variables method.

So, now we can use separation of variables method, as the governing equation is linear and homogeneous and boundary conditions are homogeneous in y direction and it is finite direction. So, we will find the solution u prime as product of two solutions as y which is a function of y only and tau which is function of t only. So, you know that in separation of variables method we write the solution as product of two individual solution and where Y is function of y only and tau is function of t only.

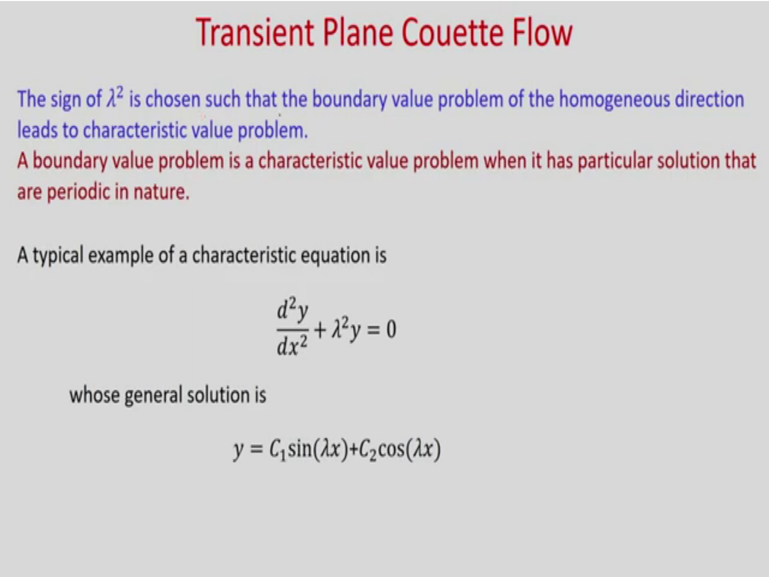
Now, we will write the partial differential equation to two ordinary differential equation using separation of variables method. So, now, you can see that you can write del u prime by delta t is equal to Y d tau by d t, because tau is function of t. So, we can write ordinary derivative.

And similarly, del 2 u prime by del y square you can write tau d 2 Y by d y square right. So, this is also ordinary derivative we can write as well as function of Y. So, if you put it in this governing equation then, you can write Y d tau by d t is equal to nu tau d 2 Y by d y square.

Now, write left-hand side, where it is function of tau only d tau by d t and right-hand side will write as function of Y only. So, now, we have separated the variables right. You can see that left-hand side is function of t only and this is the function of y only ok. So, we have separated the variables.

Now, you can see this is function of t only and this is function of y only. So, it will be equal to some constant ok, but what will be the sign of this constant ok, because the left-hand side depends on only t right hand side depends on only y. So, it should be equal to some constant ok. So, let us say lambda square and what will be the sign of this lambda square plus or minus to find what should be the sign of this lambda square. There are some guidelines.

(Refer Slide Time: 27:23)

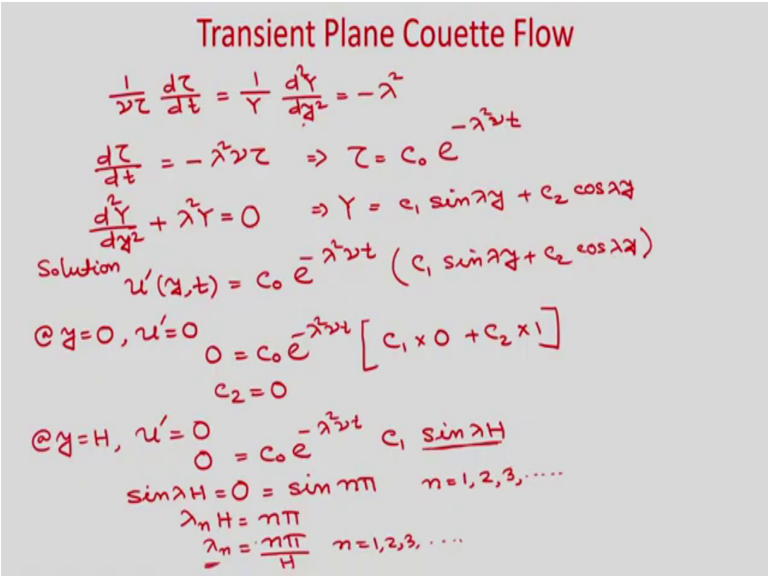


So, you can see the sign of lambda square is chosen such that the boundary value problem of the homogeneous direction leads to characteristic value problem. A boundary value problem is a characteristic value problem when it has particular solution that are periodic in nature. So, you can see one example of characteristic equation d 2 y by d x square plus lambda square y is equal to 0.

So, this is the characteristic equation and the solution of this you know y is equal to C 1 sin lambda x plus C 2 cos lambda x where C 1 C 2 are integration constants. So, you can see that we have to choose the value of lambda square such a way that in the homogeneous direction we should get a characteristic equation in this form, so that we should have a solution which is periodic in nature ok. So, for this problem now, see that we have to choose lambda square such a way that in the y direction, because y direction is homogeneous direction.

So, in the y direction we should get characteristic value problem; that means, the solution of that ordinary differential equation should give a period solution as periodic in nature ok. So, now, you can see that we have to choose minus lambda square then, only we will get the characteristic value problem in homogeneous direction which is y in this case.

(Refer Slide Time: 28:53)



So, now let us write again 1 by nu tau d tau by dt is equal to 1 by Y d 2 Y by d square. Now, we are choosing minus value right, because minus we are choosing. So, that in y direction we will get the characteristic value problem like d tau by dt. We will write as minus lambda square nu tau and what is the solution of this?

Solution you know right, tau will be some constant C naught e to the power minus lambda square nu t ok. So, this is the one solution and in the homogeneous direction now, we are getting the ordinary differential equation as d 2 Y by d y square. If you take in the minus lambda square where in the right-hand side then, we will get lambda square y is equal to 0 ok.

So, you can see that this is the characteristic value problem. So, this is this solution will give you the periodic solution so; that means, we can write Y is equal to C 1 sin lambda y plus C 2 cos lambda y ok. So, you can see now we got the solutions tau and Y we need to find C naught C 1 and C 2 which you can find from boundary conditions and initial condition.

Now, can write the solution u prime which is function of y and t as product of two solutions; C naught e to the power minus lambda square nu t and C 1 sin lambda y plus C 2 cos lambda y ok. So, now, apply the boundary conditions ok. So, if we apply the first boundary condition at y is equal to 0 u prime is equal to 0 ok. So, if u prime is 0.

So, you can see at y scale to 0. So, left hand side will be 0. So, it will be C naught e to the power minus lambda square nu t this cannot be 0 and we have C 1. Now, sin lambda 0 is 0 right. So, it will be 0 plus cos 0 is 1 so, C 2 into 1. So, you can see from here, left-hand side is 0 C naught cannot be 0 so, C 2 must be 0.

So, C 2 is 0. And apply another boundary condition at y is equal to H u prime is equal to 0. So, if we apply this boundary condition then, you will get left hand side is 0. Here, you will get C naught e to the power minus lambda square nu t and you will get C 1 into sin lambda H.

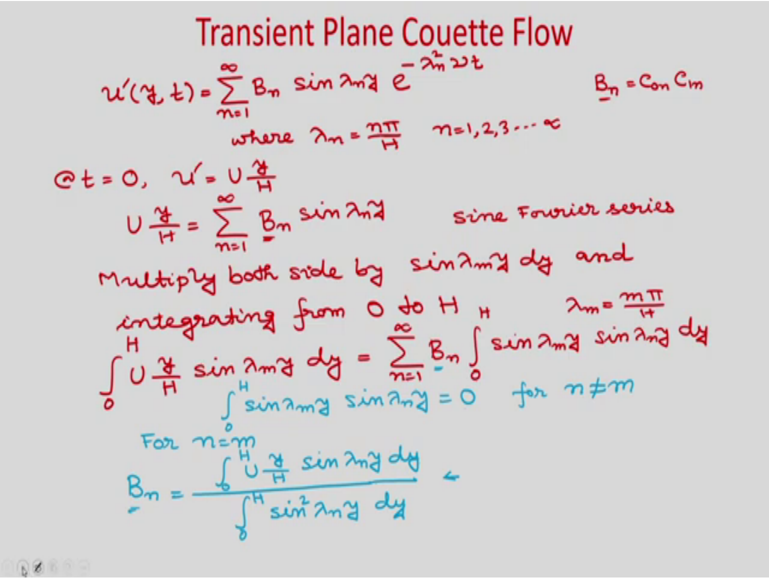
And from here, you can see that C naught cannot be 0 C 1 cannot be 0 then, you will get 0 solution of this right. So, this is not physical. So, sin lambda H must be 0. So, sin lambda H is equal to 0. Now, here you can see that for different values of lambda, you might get this sin lambda H is equal to 0 right. So, we can write that sin lambda H is equal to sin n pi where n varies from 1 to infinity.

So, you can see 0 you will get for sin n pi right; n is equal to 0 anyway it is 0 ok. So, for the n is equal to 1 2 all you will get right sin 0 is 0. So, for n is equal to 0. So, for the other also you will get. So, n is equal to 1 2 3 and so on ok, but n will not write as 0, because if n is 0 then, you can see that lambda will be 0 and if lambda is 0 then, you will not get any solution; that means, for n is equal to 0 u prime is 0 right for n is equal to 0 u prime is 0. So, we will not write n is equal to 0. So, we will put n is equal to 1 2 3.

For other values you will get and you can write now, lambda n H is equal to n pi, because for different values of n, you will get different values of lambda n and n is equal to 1 2 3 and so on. So, you can see the value of lambda n now we have found which is your n pi by H ok.

So, you can see that it is a linear equation and for different values of n, you are going to get different solutions. So, now, we can write that final solution u prime as summation of all the solutions for n is equal to 1 to infinity right and that we can write. Because we can just summing up all the solutions and that we can write as the governing equation is linear ok.

(Refer Slide Time: 34:45)



So, now we can write that u prime y t is equal to. Now, we are writing summation of n is equal to 1 to infinity that we can write. Now, we have C naught and C 1 so that we will write B n as C naught n into C 1 n. Two constants we are together writing B n. So, B n sin lambda n y and we have e to the power minus lambda n square nu t where lambda n is equal to n pi by H and n is 1 to infinity.

So, now, we need to find the other constant B n and that we will find applying the initial condition and applying the orthogonal property ok. So, now, from here if you put the initial condition that at t is equal to 0 u prime is equal to U y by H right. So, if we put it here. So, left-hand side you will get U y by H is equal to summation of n is equal to 1 to infinity B n sin lambda n y and as t is equal to 0.

So, this will become 1 ok. So, B n sin lambda n y. So, you can see that this is the sine Fourier series this is sine Fourier series and now, we will take the advantage of the orthogonality property and we will find the constant B n. If you can see that multiply both sides by sin lambda m y d y and integrating from 0 to H where lambda m is nothing, but m pi by H ok.

And now, multiply and you can see that sin lambda n y and sin lambda m y are orthogonal to each other. So, now, in the left-hand side, you will get 0 to H U y by H sin lambda m y d y is equal to summation of n is equal to 1 to infinity B n integral 0 to H sin lambda m y sin lambda n y d y ok, because for different values of n you will get.

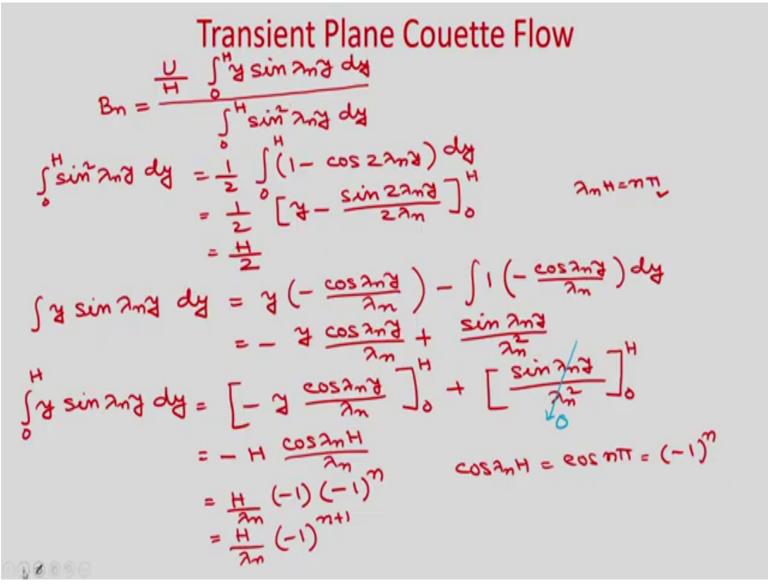
So, we have already discussed this while discussing about the orthogonal property. So, now, you can see that this sin lambda m y and sin lambda n y are orthogonal to each other. So, you will get 0 to H sin lambda m y sin lambda n y is equal to 0 for n not equal to m ok. So, for n is equal to m we will only get the solution. So, for n is equal to m; now, all other terms in the right-hand side will become 0 invoking this orthogonal property.

So, what you will get in the right-hand side and now, you can find the B n. So, you can write now, n is equal to m you can write. So, you can write B n; it is already this we have seen that

B n will be just integral 0 to H U y by H and this we will write lambda n, because n is equal to m. So, we can write sin lambda n y d y and in the denominator, we will have 0 to H.

So, it will be sin square lambda n y d y ok. So, now we have to find these integrals in the numerator and denominator then, we can find the constant B n and once you know the B n, you will be able to find u prime and once you know the prime, you can find the transient velocity profile.

(Refer Slide Time: 39:46)



So, we can right now B n ok. So, you can write that U by H is constant. So, you can take outside the integral 0 to H y sin lambda n y d y divided by 0 to H sin square lambda n y d y ok. So, first find this denominator this integral. So, you can see 0 to H sin square lambda n y d y; we can write half into 2 sin square lambda n y and 2 sin square lambda n y we can write as 1 minus cos 2 lambda n y d y and if you find the integrals it will be half y minus sin 2 lambda n y divided by 2 lambda n limits 0 to H.

So, if you put y is equal to 0 so; obviously, this will become 0 and y is equal to H; you can see 2 lambda n H and lambda n H is equal to n pi and n 1 2 3 to infinity. So, if you put it. So, y is equal to H also, this term will become 0. So, hence you will get H by 2. So, here you are going to get H by 2.

And in the numerator now, we have to use integration by parts integral y sin lambda n y d y. So, if we use integration by parts. So, what you will get? So, first we will not write the limits first find the integral. So, you can see that this will be y then, integral sin lambda y d y that will be minus cos lambda n y divided by lambda n minus integral.

Now, the derivative of this y. So, d y by d y. So, it will be 1 and this you will get again minus cos lambda n y by lambda n, because integral sin lambda n y d y ok. So, we can see this we will write minus y cos lambda n y divided by lambda n and this. Now, again if you take the integral of cos lambda n y you will get sin lambda n y by lambda n. So, this minus minus it will become a plus sin lambda n y divided by lambda n square.

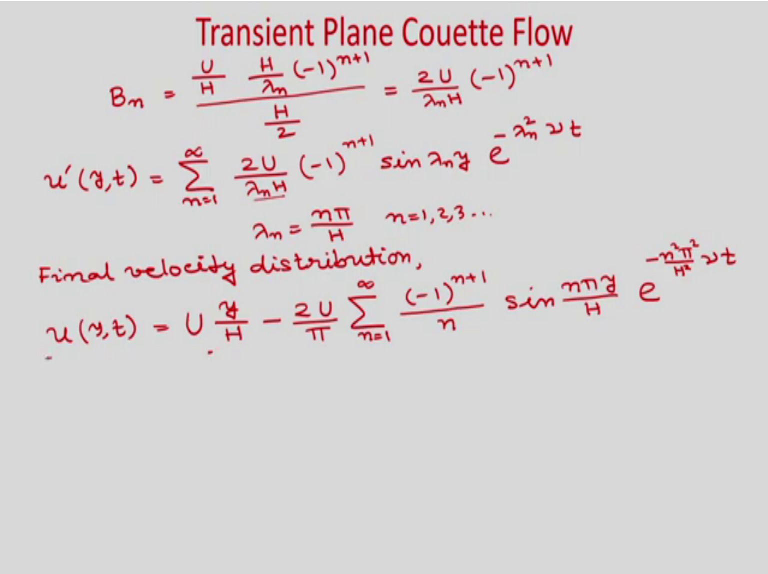
So, now if you find putting the limits. So, you will get y sin lambda n y d y. So, if you put the limits minus y cos lambda n y by lambda n from 0 to H. So, these are the limits plus sin lambda n y by lambda n square limit 0 to H. First, let us see the second term in the right-hand side. So, you can see at y is equal to 0; obviously, sin 0 is 0 and y is equal to H lambda n H. So, sin lambda h is 0.

So, this term will become 0 and in the only this term will remain and you can see that you will get at y is equal to H. So, it will be minus H cos lambda n H by lambda n and if you put minus this limit y is equal to 0. So, y is equal to 0. So, this term will become 0. So, this you will get now you can see H by lambda n ok. Here, it is 1 minus 1 and cos lambda n H ok. So, what is cos lambda n H ok? Cos lambda n H is equal to cos n y.

Because lambda n H is equal to n pi. So, lambda cos n pi. So, cos n pi. So, if n is equal to 1 cos pi minus 1 then, n is equal to 2 cos 2 pi so, it will be 1. Again, 3 pi minus 1 so, you can see this will be minus 1 to the power n, because for n is equal to 1, it is minus 1 n is equal to 2, it will become plus 1 n is equal to 3, it will become minus 1. So, this we are writing minus 1 to the power n.

Hence, you can write H by lambda n H by lambda n minus 1 to the power n plus 1. So, now, we have found the numerator and denominator this integral. Now, you put and find the value of B n.

(Refer Slide Time: 44:46)



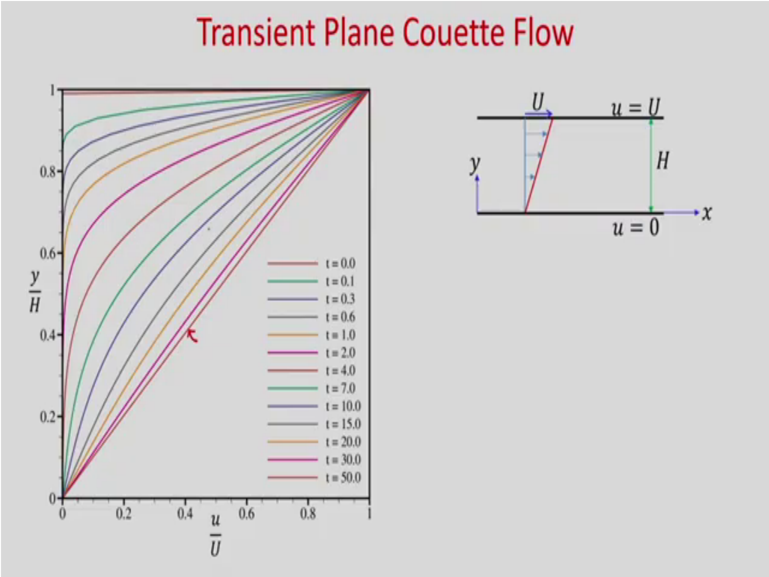
So, now B n will be U by H ok. So, it is U by H this integral in the numerator we have H by lambda n minus 1 to the power n plus 1 H by lambda n minus 1 to the power n plus 1 divided by in the numerator we have H by 2; H by 2. So, from here you can see that we will get B n as twice U by lambda n H minus 1 to the power n plus 1 ok.

So, now, once you know B n now, you can find the velocity u prime y t is equal to summation of n is equal to 1 to infinity put the value of B n. So, it will be twice U by lambda n H minus 1 to the power n plus 1 and we have sin lambda n y then, e to the power minus lambda n square nu t where lambda n is equal to n pi by H and n varies from 1 to infinity.

So, now we can see we have found the transient part of this velocity u prime y t and now, if you put it in the final expression then, we will be able to find the solution part u as function of y and t. So, final velocity distribution. So, what we will get? u y t as U y by H. So, this is the solution from the steady part and from here, you will get minus twice U by lambda n if you put lambda n H if you put n pi ok. So, you will get pi here lambda n H we are putting n pi. So, it will be pi here, summation of n is equal to 1 to infinity.

So, we will get minus 1 to the power n plus 1 and divided by n ok. From n pi we will write we are writing n here then, sin n pi y by H and e to the power minus n square pi square by H square and nu t ok. So, you can see this is the final velocity distribution and as t tends to infinity this right-hand side term will become 0 and you will get a steady solution. So, this is the tangent velocity profile for tangent plane Couette flow.

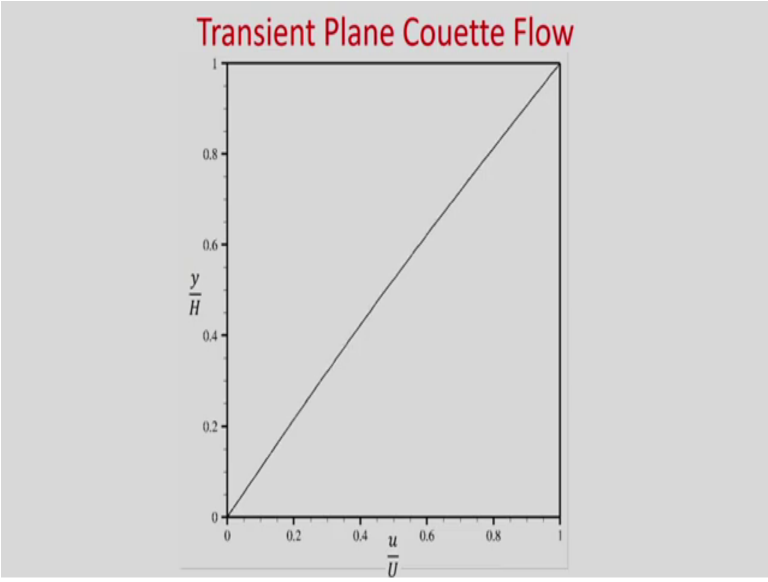
(Refer Slide Time: 48:13)



So, if you solve it and plot u by U versus y by H at different time instances. So, you can see at t is equal to 0 only 0 velocity everywhere as t progresses this velocity you can see how it is evolving with time ok. So, inside the domain, this velocity profile is evolving and at t tends to infinity you will get these velocity profile which is your linear velocity profile.

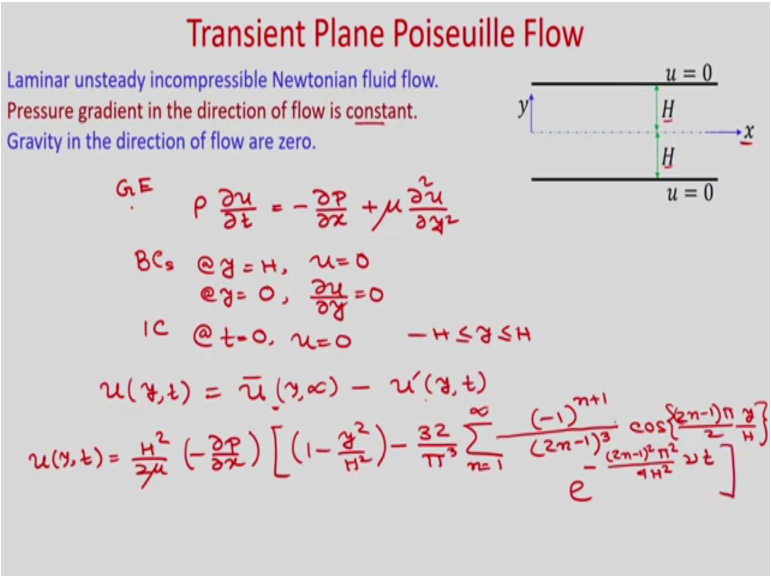
So, this is the solution for steady plane Couette flow, but at t tends to infinity. Solving this problem, you will get linear velocity profile and if you go beyond this t; obviously, there will be no change in the velocity profile ok.

(Refer Slide Time: 49:08)



So, this is the temporal evolution of velocity profile you can see and if you see the animation. So, you can see how with time this velocity is evolving and at t tends to infinity it is becoming linear. So, it is becoming almost linear. You can see how the temporal evolution of this velocity profile is happening ok.

(Refer Slide Time: 49:26)



Now, quickly let us discuss the solution of transient plane Poiseuille flow. So, plane Poiseuille flow already you know the solution right and ag velocity profile is parabolic profile and that solution you know, but if the problem if we consider that at t is equal to 0, the fluid medium is stationary and suddenly, there is a imposed pressure gradient del P by del x at t 0 plus then; obviously, there will be temporal evolution of this velocity profile and at t tends to infinity it will become parabolic in nature.

So, similar way whatever we have discussed for plane Couette flow transient plane Couette flow. So, similar method you can use with proper boundary conditions you can use superposition technique then, you can use separation of variables method and convert the partial differential equation to ordinary differential equations then, find the solution.

So, here also we will see that we will take one part at a steady solution where t tends to infinity and that solution you know as it is a parabolic profile. And another tangent profile we will subtract and you will find the that velocity using separation of variables method.

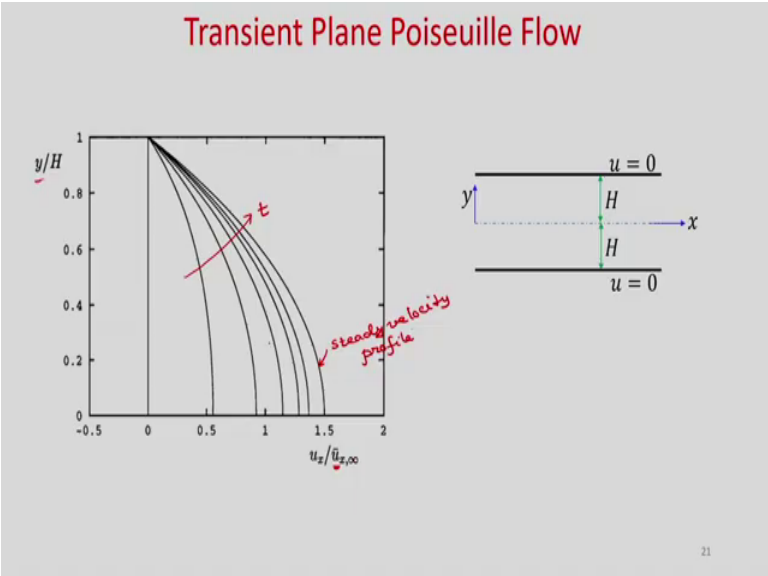
So, we will not go into detail just I will write the final solution. So, you can see our governing equation; obviously, yet there is a imposed pressure gradient and it is constant ok. So, we can write the governing equation as rho del u by delta t is equal to minus del P by del x; this pressure gradient is constant and we have this viscous term mu del 2 u by del y square. What are the boundary conditions at y is called to H u is equal to 0 and at y is equal to 0. So, you can see this is the coordinate we have taken. y is measured from the center line ok.

So, x is the axial direction and the distance between two plates is 2 H and plates that stationary ok. So, at y is equal to 0, you can see that this is the center line and we know that maximum velocity will occur at the center line. So, we can say that the gradient del u by del y will be 0 and initial condition is at t is equal to 0 the velocity is 0 everywhere ok.

So, now if you solve this problem then, we can write u the solution for this problem as solution of u bar y infinity which is your steady solution minus u prime y t whatever way we have solved the earlier problem that you can use and you know the solution you find the solution, final solution you will get u y t as H square by twice mu minus del P by del x steady solution 1 minus y square by H square.

This is the parabolic profile you know minus 32 by pi cube summation of n is equal to 1 to infinity minus 1 to the power n plus 1 divided by twice n minus 1 cube. Then, cos 2 n minus 1 pi by 2 y by H and e to the power minus twice n minus 1 square pi square divided by 4 H square nu t. So, this is the final solution and; obviously, at t tends to infinity this right-hand side term will become 0 and you will get a steady velocity profile.

(Refer Slide Time: 53:39)



Now, you can see the evolution of the velocity towards the parabolic steady state profile. So, you can see at different time instances, we have plotted this u x divided by u bar x. So, x means this velocity profile in x direction and this is the steady state velocity profile. So, and versus y by H this is the central line and center line; obviously, we are getting maximum velocity and you can see that as time increases.

How the temperature how the velocity profile is evolving towards the parabolic profile. So, this is the steady velocity profile which is parabolic ok. So, you can see how the velocity profile is evolving with time. So, in today's class we consider tangent plane Couette flow and we discussed about the separation of variables method.

We can use separation of variables method if it is if the governing equations are linear and homogeneous and boundary conditions in one direction it is homogeneous and in the other direction it may be non-homogeneous. And in homogeneous direction we should have finite length.

So, for our problem, we have splitted this transient plane Couette flow into two problems; one problem is the steady state plane Couette flow where you know the velocity profile is linear and minus we have subtracted one transient velocity profile and; obviously, as t tends to infinity that will become 0.

Then we have use the boundary condition for this transient part u prime and you know that boundary condition became homogeneous in y direction and we use the separation of variables method and we have chosen the values of lambda n square; such that in homogeneous direction the equation becomes a characteristic value problem and from there we have found the periodic solution in y direction.

So, after applying the boundary conditions and initial conditions using the orthogonality property, we have solved the constants and finally, we have shown the temporal evolution of the velocity profile for the transient plane Couette flow. Next, we discussed about the transient plane Poiseuille flow, similar way you can solve with proper boundary conditions. And we have also shown the velocity profile transient velocity profile for the problem transient plane Poiseuille flow.

Thank you.