Viscous Fluid Flow

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Module - 06
Lubrication Theory
Lecture - 04
Journal Bearing and Piston-ring Lubrication

Hello everyone. So, in today's class, we will discuss about the Journal Bearing and Piston-ring Lubrication then we will solve two example problems. So, you know that the journal that means, the shaft when it rotates inside the bearing then there will be fluid flow due to the presence of these lubrication between the surfaces of these journal and the bearing.

This bearing is also known as bushing. So, in no load condition, obviously the centers of journal and bearing will coincide but, if load is applied, then there will be a distance between the centers of this journal and bearing and that is known as eccentricity. So, first let us consider this hydrodynamic analysis of this journal bearing.

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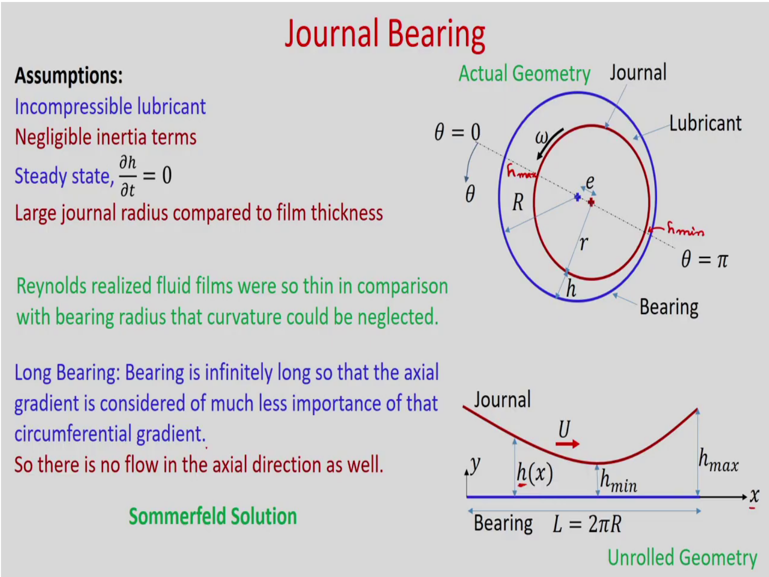

So, you can see that, this is the bearing which is known as also bushing and this is the journal, which is also known as shaft and in between we have the lubricant. So, when this journal rotates, obviously due to the shear of this journal surface, there will be flow of this lubricant.

When it rotates in anti-clockwise under the effect of load, this minimum thickness will occurs slightly in the right direction. And you can see, we have given a dotted line, where we are telling that theta is 0 here, where we have maximum film thickness h max and here at theta is equal to pi, we have h min. So, this is the minimum film thickness.

And r is the radius of this journal and h is the height of the film thickness and capital R is the radius of the bearing and theta is measured in anti-clockwise. These are the assumptions we are taking that we have incompressible lubricant and with negligible inertia terms and it is a steady state phenomena.

And we are considering that the journal radius is very large compared to the film thickness. So obviously you can see that, Reynolds realized fluid films were so thin in comparison with bearing radius that curvature could be neglected. So in that case, if we unroll the geometry; then you can see this will be the journal and this is the bearing and curvature can be neglected.

So, we can give the velocity U in the axial direction x which is your theta direction and this is the h minimum and this is the h maximum and h is function of x. And obviously, if you unfold, then the length of this bearing will be twice pi R. In this case, we will consider long bearing; what does it mean by long bearing?

Long bearing means, the length of the bearing is infinite, so that the gradient of any quantity in this direction is negligible and the velocity in that direction is also 0. So with this simplification, we will make this analysis and Sommerfeld first did this analysis; that is why it is known as Sommerfeld solution. So, as you are considering long bearing, so there is no flow in the axial direction.

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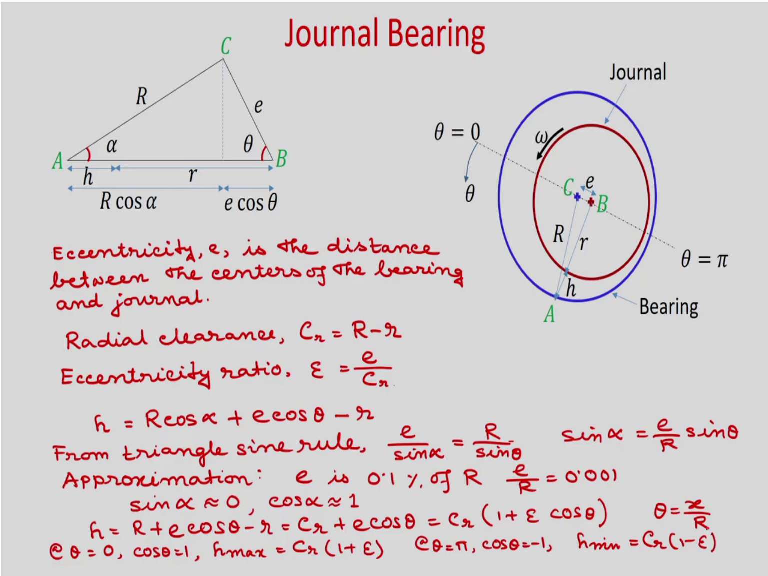

Now, let us consider this triangle A, B, C where you know that AB is r plus h and AC is the radius of this bearing and that is capital R. And we define the eccentricity as the distance between the self centers of these bearing and journal. So first let us define the eccentricity.

So, eccentricity e is the distance between the centers of the bearing and journal. And the difference between R and small r is known as radial clearance; so we are defining radial clearance, C r as R minus r, ok. So, we will define eccentricity ratio, epsilon as e which is your eccentricity divided by C r.

So, now, if you consider this triangle ABC; so obviously you can see that this is AB is the length r plus h and AC is the length capital R and BC is the eccentricity e and this angle is theta, ok. So, at any angle we are considering, so this is theta and let us say this is alpha.

So you can see that as it is theta, so the component of BC on these AB will be just e cos theta. Similarly, the component of AC along AB will be R cos alpha. So, from here we can say that, h is the film thickness, which is function of x is equal to R cos alpha plus e cos theta and minus this r, so minus r. So, this is h, ok.

And from this triangle you can see that, we can write the triangle sine rule as. So, from triangle sine rule, we can write e by sin alpha is equal to R by sin theta, ok. So, we can write sin alpha is equal to e by R sin theta; sin 0 will be 0 and sin pi will be it is 0, but sin pi by 2 will be 1.

So, obviously it varies between 0 to 1 and in this case, we will approximate that the maximum clearance is 0.1 percent of R. So, we will assume or approximate. So, this approximation will take e, which is your eccentricity is 0.1 percent of R, ok. So, that you can see that, e by R will be 0.001. So, you can see from this relation that, sin alpha is e by R sin theta and sin theta varies between 0 and 1 and e by R is 0.001; so obviously sin alpha is close to 0.

So if sin alpha is close to 0 ok, so cos alpha will be close to 1, ok. So, from here you can see that, if cos alpha is close to 1; then in this relation if you put, h will become R plus e cos theta minus r and R minus r is nothing but the radial clearance C r. So, you can write C r plus e cos theta and C r if you take outside; then it will be e by C r and e by C r is nothing, but the eccentricity ratio. So, you can write 1 plus epsilon cos theta.

So, you can see that, from here we have found the film thickness h as C r into 1 plus epsilon cos theta. And also you can see from here that, theta you can write as x by R ok; because at any distance if you find x, x will be just R into the angle theta. So, that will be x equal to R theta, so theta also can represent as x by R.

And now, you can see from here that, at theta is equal to 0 ok, cos theta is 1 right; cos theta is equal to 1. So, h max you can write as C r into 1 plus epsilon, ok. So, at theta is equal to pi obviously cos theta will be minus 1. So, you can write h min as C r into 1 minus epsilon. So, assuming that there is no flow in the axial direction that means, the bearing is very long.

So, Sommerfeld first solved the Reynolds equation and found the pressure distribution inside this journal bearing and that is known as Sommerfeld solution.

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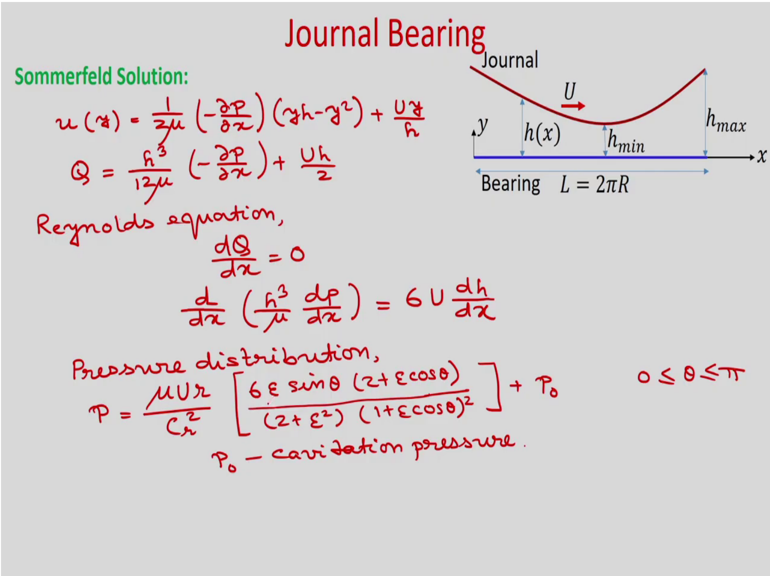

So, if you assume that this is the velocity U in the axial direction and curvature is very small; then we can find the velocity distribution u as function of y as 1 by twice mu minus del p by del x yh minus y square plus U y by h. So, from here you can find the volume flow rate Q as h cube by 12 mu minus del p by del x plus U h by 2.

So, from here you can write Reynolds equation. So, you know that d Q by d x will be 0 in steady state condition; so if it is so, then if you put it here and rearrange, you will get d of d x h cube by mu. So, p will be function of x, because there is we are assuming there is no variation of pressure in the y direction; so you can write dp by d x is equal to 6 U dh by dx,.

So now, if you integrate it and find the pressure distribution; then Sommerfeld actually solved this problem and found the pressure distribution. We are not going into detail of the derivation; but we are writing the pressure distribution. So, pressure distribution will be P is equal to mu U r by C r square 6 epsilon sin theta 2 plus epsilon cos theta divided by 2 plus epsilon square 1 plus epsilon cos theta square plus P naught in the range of theta between 0 and pi, ok.

So what is this P naught? So, when the shaft rotates inside this bearing; so there may be some portion, where pressure will be very low and that is known as cavitation pressure and this cavitation pressure is P naught and this P naught is known as cavitation pressure. So, this is the solution actually of this Reynolds equation and this solution is known as Sommerfeld solution.

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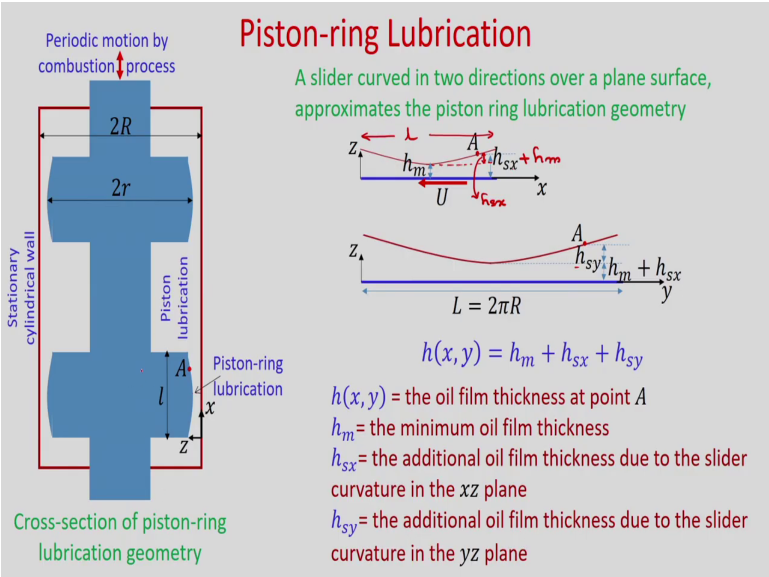

Now, let us consider piston ring lubrication. So, you know that in the combustion process, where inside the internal combustion engine; there will be piston moving around the cylinder surface and their lubrication will be there and that lubrication also we can find the analysis of this pressure using this lubrication theory.

So in this case you can see that, this is the stationary cylindrical wall and you inside we have the piston and piston is having this periodic motion by combustion process. And 2R is the diameter of this stationary cylindrical wall and with this piston, we have a ring and this radius is small r, so that diameter is 2r.

And we have inside this lubrication and this is piston ring lubrication; here ring is not there, so you have piston lubrication. And we are considering x along the cylindrical wall and z is perpendicular to this and y is in the circumferential direction, ok. And let us consider this l is the height of this ring.

Now, if you consider one point on this ring, let us say this is A; so obviously you can see that here if you consider the relative motion between this ring and the cylinder is U, ok. So, only we have the motion in the x direction, ok. So, in that case, if you consider this surface, curved surface; so you can see this is the curved surface and this is the z and this is the x and this is the cylinder surface.

So, obviously relative motion we are giving in here. So, this is your U in the negative x direction and the minimum thickness here you can see, here minimum thickness will be h m and this is the point A. So, you can see. So, there will be variation of this height and in the x direction that we are writing that it is h. So, this will be h m plus h sx.

So this portion is only h sx. So, you can see the total distance of this A from here it will be h sx plus the minimum thickness h m. Now, at this point you can see that A if you consider that, if you unfold the cylindrical surface ok; then you will get this as the ring surface and the point A is here and this is the cylindrical wall.

Now, you can see z obviously, will be in the perpendicular direction; but along these this is y, because we consider that y is the in the circumferential direction. So, this is the y. So, here also you can see. So, at this point we have the value h sx plus h m. So, this is the h m plus h sx and there will be variation of this height along y, so that is your h sy. So, this thickness is h sy

So total thickness is h m plus h sx plus h sy and the length of this, obviously we have unrolled it. So, it will be L is equal to twice pi R; at in this case this will be the ring length L, ok. So, we can write that h, which is the film thickness is function of x and y; this is equal to h m is the minimum thickness here plus h sx plus h sy.

And h m is the minimum oil film thickness, h sx is the additional oil film thickness due to the slider curvature in the xz plane and h sy is the additional oil film thickness due to the slider curvature in the yz plane. So basically you can see that is it is unsteady motion; because there is a periodic motion by the combustion process. So, now, first let us write down the Reynolds equation in 2 dimensional case.

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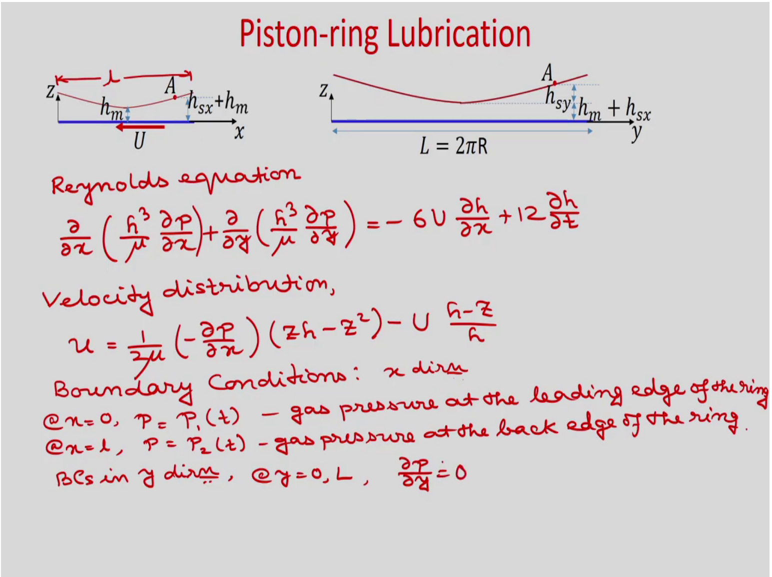

So, we have the this Reynolds equation ok; for hydrodynamic lubrication we have del of del x h cube by mu del p by del x plus del of del y h cube by mu del p by del y is equal to minus 6 U del h by del x plus 12 del h by del t, this we have already derived. So you can see this is in journal the Reynolds equation in 2 dimensional unsteady case.

And you can see in this case, obviously in the y direction if you consider, there is no motion; but only we have a motion in the vertical direction, that means, the relative motions were given in the cylindrical surface. So, this is the U in the negative x direction.

So, if you see the velocity distribution. So, in this case you can see that, due to this motion obviously there will be a pressure gradient and obviously, there is a velocity that is your U. So, due to that, you will get combined flow of this Couette and plane Poiseuille.

So, you can see that, we can write the solution u as 1 by twice mu minus del p by del x Z h minus Z square minus U h minus Z by h, ok. So, here velocity is in negative x direction, so it is minus U and we have the normal direction is Z. So, obviously this is the velocity profile. And what are the boundary conditions?

So, boundary conditions you can see, ok; so at x equal to 0 ok, we have P is equal to P 1, which is function of t and this is the gas pressure at the leading edge of the ring. And at x equal to l, so because this ring length is l; so at x equal to l, we have P is equal to P 2 t. So, this is the gas pressure at the back edge of the ring, ok.

And boundary condition in the y direction. So, you can see this is the boundary condition in the x direction. And boundary conditions in y direction; so you can see at y is equal to 0 and y is equal to L, we will have the pressure gradient del p by del y as 0. So, you can see if del p by del y 0; then obviously there will be no flow along the circumferential direction, because there is no relative motion between the cylindrical surface and the ring surface. So, obviously the you can assume that velocity is 0 in y direction.

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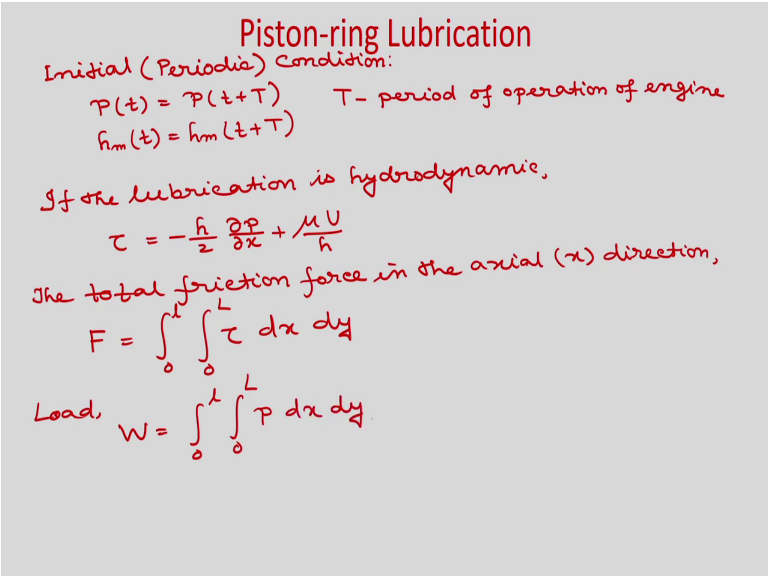

So, and we have the initial condition, we have initial condition, which is your periodic condition here. So, at, so, you can see that at any time t P will be just P t plus T, where T is the period of operation of the engine, period of operation of engine, ok. So, and also the thickness h m t will be h m t plus t; because in IC engine you can see that, the piston will periodically move, so you will get the periodic condition like this.

So, after solving the Reynolds equation along with these boundary conditions and the approximation, we can find the pressure distribution and also the shear stress distribution. So, shear stress you can see that, we have the velocity U; then you will get shear stress and from shear stress you can find, the total force acting on the surface.

So, obviously you can see that. So, if the lubrication is hydrodynamic; that means there will be always the lubricant between this ring surface and the cylindrical surface, then we can write. So, you can write the shear stress tau as minus h by 2 delta p by del x, this will be just mu del y del z.

So, you can write this and now you can write the total friction force, total friction force in the axial direction; that means x ok, because there is no motion in the y direction. So, we will get the total friction force in the axial direction F as just integral 0 to small l integral 0 to capital L tau dx dy, ok.

So you can write the load on the bearing that, W as integral 0 to l integral 0 to capital L; now you know the pressure distribution P into dx dy, ok. So, this detailed analysis we have not done in this class; but you can refer some journal papers and see how these pressure distribution is found and from there how you can find the load. Now, we will solve two example problems.

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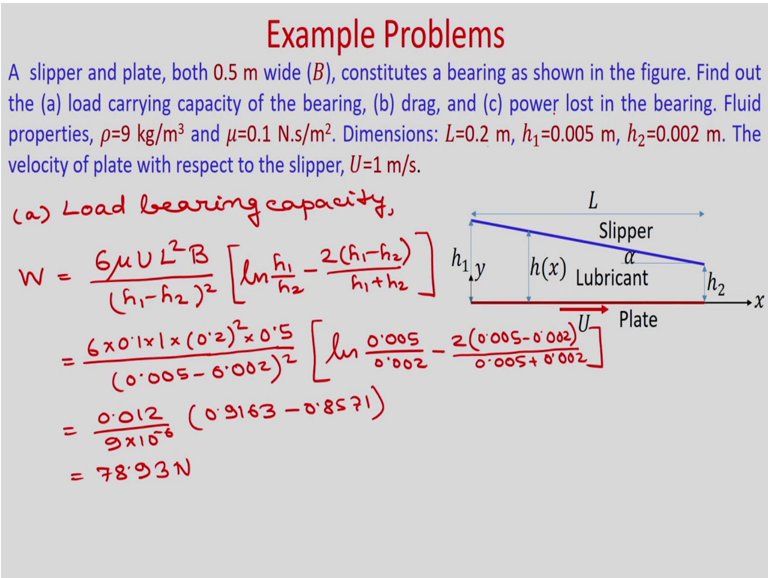

So, the first problem just I will read out first. A slipper and plate both 0.5 meter wide, constitutes a bearing as shown in the figure. Find out the a, load carrying capacity of the bearing, b drag and c power lost in the bearing. Fluid properties are given, dimensions are given and velocity of the plate with respect to the slipper is given.

So, obviously you can see this is the plate it is moving with velocity U which is 1 meter per second; this is the slipper, slipper length is L 0.2 meter and the maximum thickness h 1 is 0.005 meter and minimum thickness h 2 is 0.002 meter. And the width, perpendicular direction it is having the width 0.5 meter, which is capital D. So, now, first let us find the load bearing capacity.

So, we know the expression W is equal to 6 mu U L square B divided by h 1 minus h 2 whole square l n h 1 by h 2 minus 2 h 1 minus h 2 h 1 plus h 2, ok. So you know all the dimensions, you put it here. So, you can see 6 mu is 0.1; then we have velocity 1, L is 0.2, then the width B is 0.5 meter divided by 0.005 minus 0.002 square l n 0.005 divided by 0.002 minus 2 into 0.005 minus 0.002 divided by 0.005 plus 0.002.

So, now if you do the calculation you will get 0.012 divided by 9 into 10 to the power minus 6, 0.9163 minus 0.8571. So, you will get 78.93 Newton. So, next we need to calculate drag and power lost.

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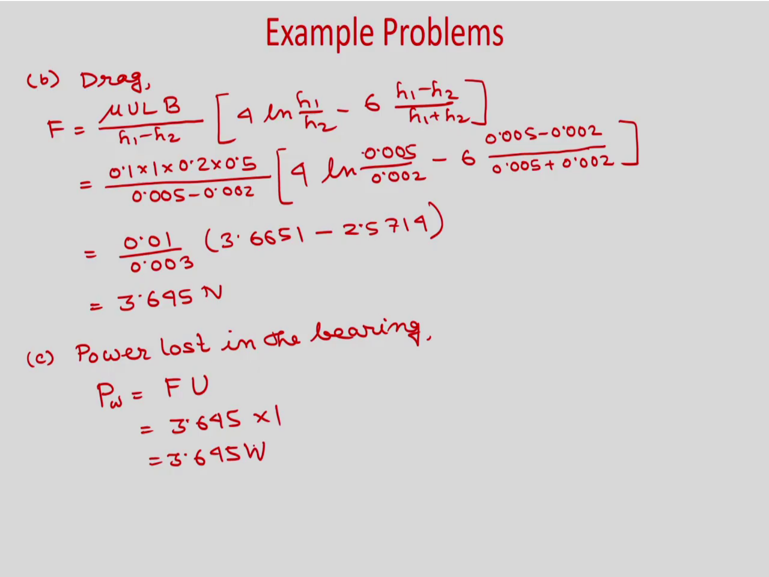

So, you know the expression of drag F as mu U L B h 1 minus h 2. So, when we found the F, actually we wrote it part unit width. So, in this case we know the width. So, we are multiplying by B into 4 l n h1 by h2 minus 6 h1 minus h2 divided by h1 plus h2. So, you put all the values. So, you will get 0.1 into 1 into l 0.2 width is 0.5 divided by 0.005 minus 0.002 into 4 l n 0.005 divided by 0.002 minus 6 0.005 minus 0.002 divided by 0.005 plus 0.002, ok.

So you will get 0.01 divided by 0.003, 3.6651 minus 2.5714. So, the drag will be 3.645 Newton. So, now, we can calculate the power lost in the bearing, ok. So, we know that it is moving with a velocity U, so P w will be just that drag force into the velocity U. So, it will be 3.645 into 1. So, it will be 3.645 watt.

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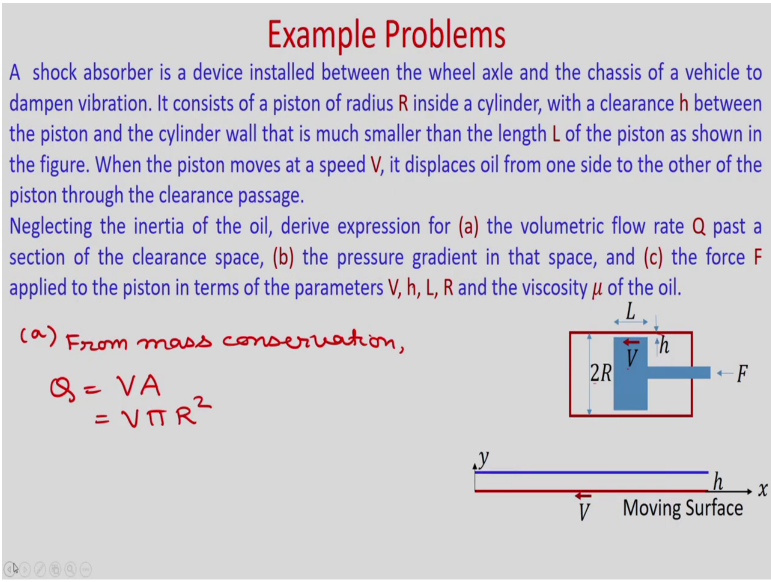

Now, let us consider the second problem. A shock absorber is a device installed between the wheel axle and the chassis of a vehicle dampen vibration. A shock absorber is a device installed between the wheel axle and the chassis of a vehicle to dampen vibration. It consists of a piston of radius R inside a cylinder with a clearance h between the piston and the cylinder wall that is much smaller than the length L of the piston as shown in the figure.

When the piston moves at a speed V, it displaces oil from one side to the other of the piston through the clearance passage. Neglecting the inertia of the oil, derived expression for a, the volumetric flow rate Q past a section of the clearing space; b, the pressure gradient in that space and c, the force F applied to the piston in terms of the parameters V, h, L, R and the viscosity mu of the oil.

So, you see. So, this is the cylinder and this is the piston. So, force is applied F. So, piston will move in this direction with a velocity V. And the radius of the cylinder is R and we have a small clearance between this piston and the cylinder. So, obviously when it is moving, the fluid will pass through the small clearance and this length of this piston is L.

So, now, in this direction, the velocity is V. So, this is the moving surface V and you can see that, this will be the length L and height is h. So, clearance is h. So, now, we need to calculate the volumetric flow rate. So, you can see whatever it is actually compressed. So, due to that movement of this piston V, whatever this fluid will be reduced, that will be going through this passage in the other direction, ok.

So, if a is the area, so b a will be the Q and that Q will obviously pass through this small clearance passage. So, we will get from mass conservation, ok. So, Q will be just V into A, A is the area. So, V into pi R square. So, next let us consider the pressure gradient in that space.

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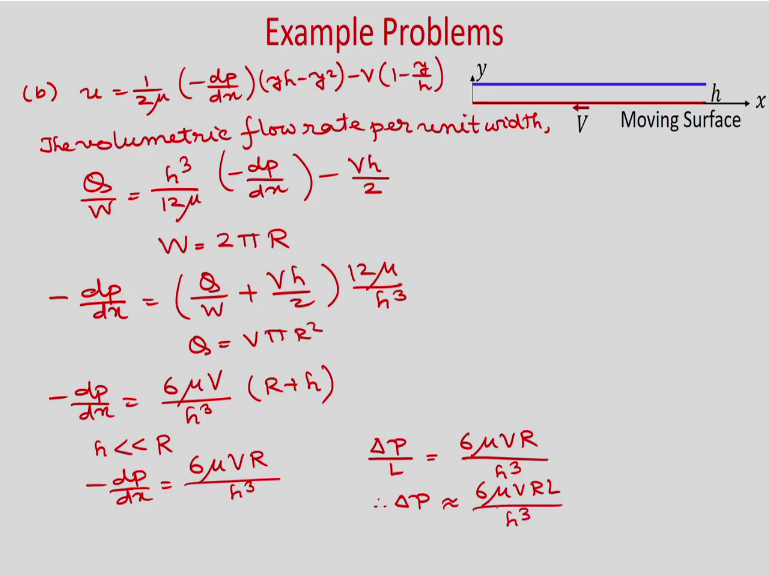

So, we know the velocity U is equal to 1 by twice mu minus dp by dx yh minus y square minus V 1 minus y by h. So, from here you can calculate the volumetric flow rate, volumetric flow rate per unit width, ok. So, Q by W will be just h cube by 12 mu minus dp by dx minus Vh by 2, ok. So, in this case the width W obviously, it is the circumferential direction. So, it will be twice pi R, ok.

So, the length in the circumferential direction, so it will be twice pi R. So, from here minus dp by dx we can express in terms of the known terms, so Vh by 2 12 mu by h cube. So, Q already we are found as V into pi R square and W we know. So, we can write minus dp by dx as 6 mu V divided by h cube R plus h, ok.

And in this case the clearance is very very small compared to the radius of the cylinder. So, if h is very very small to compared to R, so minus dp by dx. So, we can write just 6 mu V R by h cube, ok. So, this we can write as del P by L, L is the length of the piston as 6 mu V R by h cube. So, delta P we can write as 6 mu V R L by h cube.

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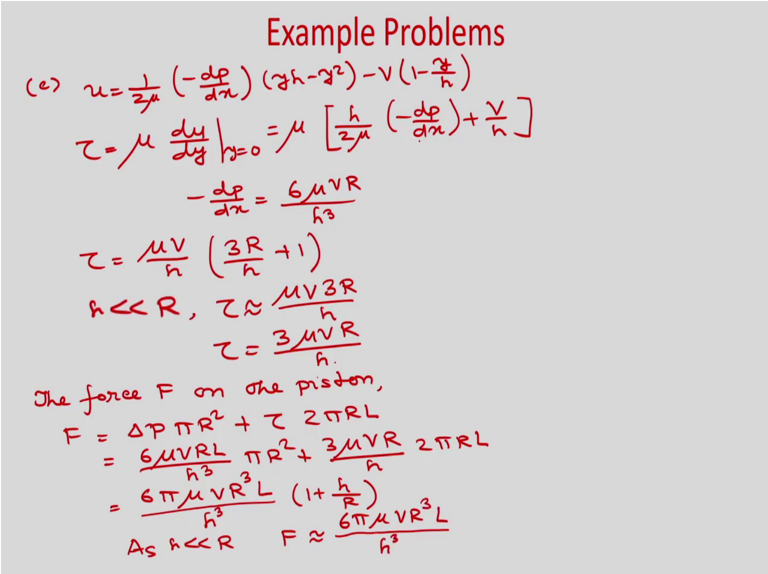

Next let us calculate first the shear stress and then from there we will calculate the total drag force. So, we know velocity as 1 by twice mu minus dp by dx yh minus y square minus V 1 minus y by h. So, tau will be just mu du by dy at y is equal to 0. So, it will be just mu h by twice mu minus dp by dx plus V by h, ok.

So, here you know that minus dp by dx already we have found. So, it is 6 mu VR by h cube. So, if you put in this expression, you will find tau as so, mu Vh3R by h plus 1, ok. H is much much smaller than R, so tau we can approximate as mu V3R by h; that means tau is equal to 3 mu VR by h.

So now, the force F on the piston we can calculate as. So, there is a pressure gradient, so due that you will get delta P pi R square and due to the shear stress, you will get tau into A. So, tau into A, so tau into A; A will be just twice pi R into L, ok. So, twice pi R is the perimeter into the length L.

So, you can write now delta P we have found a 6 mu VRL divided by h cube pi R square plus tau we have found as 3 mu VR by h into twice pi RL. So, from here you will get 6 pi mu VR cube L divided by h cube 1 plus h by R. So, as h is much much smaller than R; so F we can approximate as 6 pi mu VR cube L by h cube.

So, in today's class first we considered journal bearing and starting from the Reynolds equation, we found the pressure distribution, defining the clearance and the eccentricity ratio. This solution is known as Sommerfeld solution; because Sommerfeld first solve this equation in 1904 and assumed that there is no flow in the axial direction.

Next, we discuss about the piston ring lubrication. So, there will be a periodic motion of the piston due to the combustion process and we define the height as summation of the minimum thickness h m plus h sx plus h sy. Then we wrote the Reynolds equation in journal for 2 dimensional unsteady problem; from there we assume that del p by del y is 0, where y is the circumferential direction, because the piston is moving only in the axial direction.

So, from there we found the shear stress and the total force acting on the bearing and also the load bearing capacity we found. So, we just wrote the expression of this force as well as the load bearing capacity; but you need to solve the Reynolds equation with appropriate bounding condition and you can refer some journal paper for detailed solution. Then we solved two different problems; first problem we took as a slider bearing and next problem we considered as a piston ring lubrication.

Thank you.