Viscous Fluid Flow

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Module - 08
Laminar Boundary Layers - II
Lecture - 04
Example Problems

Hello everyone, so in these 2 modules we have studied Laminar Boundary Layers. So, today we will solve some Example Problems on these laminar boundary layers.

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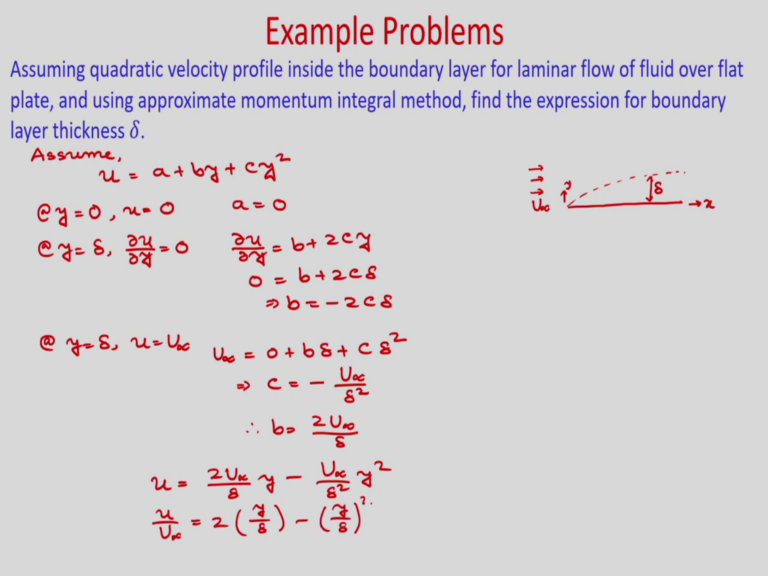

So, let us take the first problem. Assuming quadratic velocity profile inside the boundary layer for laminar flow of fluid over flat plate and using approximate momentum integral method, find the expression for boundary layer thickness delta. So, you can see that we have already used the integral method which is approximate method to solve the boundary layer equations.

So, here we need to assume the velocity profile as quadratic. So, let us consider flow over a flat plate, so this is your edge of the boundary layer and at any location x this is the boundary layer thickness delta. So, this is your x and this is y and we have free stream velocity U infinity.

So, if you assume the velocity profile let us say u which is function of x and y as quadratic; that means, we will assume a plus b y plus c y square ok, assume velocity profile u. So, you can see here we have three coefficients which are unknown. So, we need three boundary conditions to find these coefficients.

So, we have the boundary conditions at y is equal to 0, we have u is equal to 0, so if you put it here you will get coefficient a is equal to 0. Then we have at y is equal to delta at the edge of the boundary layer we have the velocity gradient is 0. That means, del u by del y is equal to 0, from this expression you can find del u by del y, so it will be b plus twice c y.

So, if you put at y is equal to delta del u by del y is equal to 0 then you will get 0 is equal to b plus 2 c delta; that means, b will be minus 2 c delta. And we have another boundary condition at y is equal to delta we have free stream velocity U infinity ok. So, we can put U infinity is equal to a is 0 plus b at y is equal to delta so, b delta plus c delta square. So, b is equal to minus 2 c delta.

So, if you put it here and if you rearrange you will get c is equal to minus U infinity by delta square and you will get b after putting here you will get twice U infinity by delta. So, now from here so we are getting velocity u is equal to b y; that means, 2U infinity by delta y and c is minus U infinity by delta square y square. So, we can write u by U infinity is equal to 2 y by delta minus y by delta square. So, now this velocity profile will put in the momentum integral equation and find the boundary layer thickness delta.

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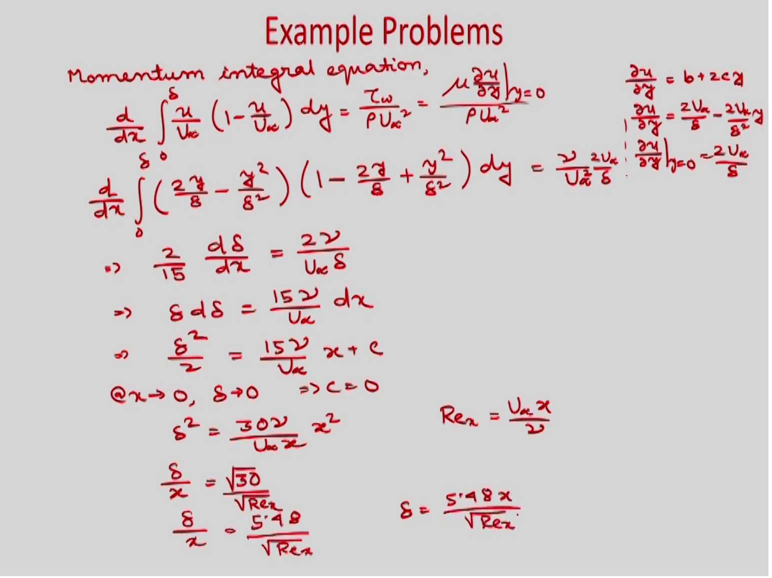

So, momentum integral equation we have. So, you can write d of dx integral 0 to delta u by U infinitely 1 minus u by U infinity dy is equal to tau w by rho U infinity square and tau w is nothing but mu del u by del y at y is equal to 0 divided by rho U infinity square ok. So, we know the velocity distribution u by U infinity in terms of delta. So, we can find the unknown parameter delta from here.

So, let us put d by dx integral 0 to delta u by infinity we have 2y by delta minus y square by delta square and 1 minus 2y by delta plus y square by delta square dy is equal to, so mu by rho will get nu by U infinity and del u by del y. So, we have del u by del y this velocity gradient as b plus twice cy ok.

So, if you put the value then you will get del u by del y b is twice U infinity by delta and plus 2c. So, c we have minus U infinity by delta square. So, we will write minus 2U infinity by delta square y ok. So, del u by del y at y is equal to 0 we will get twice U infinity by delta.

So, you can see we can write twice U infinity by delta ok. So, you evaluate this integral ok, so if you evaluate this integral we will get 2 by 15 d delta by dx is equal to we will get here twice nu by U infinity into delta. So, we can write delta d delta is equal to 15 nu by U infinity dx, ok. So now, you integrate it so you will get delta square by 2 is equal to 15 nu by U infinity x plus c.

So, as x tends to 0 that means at the leading edge we have delta tends to 0. So, we will get c is equal to 0. So, from here we can find delta square is equal to 30 nu by U infinity let us write here x and we can write x square c is equal to 0. So, from here you can see you can define Reynolds number based on x as U infinity x by nu.

So, from here you can write delta by x is equal to root 30 by root Re x ok. So, you can write delta by x is equal to 5.48 divided by root Re x. So, delta you can write as 5.48 x by root Re x. So now let us consider the next problem where velocity distribution is different.

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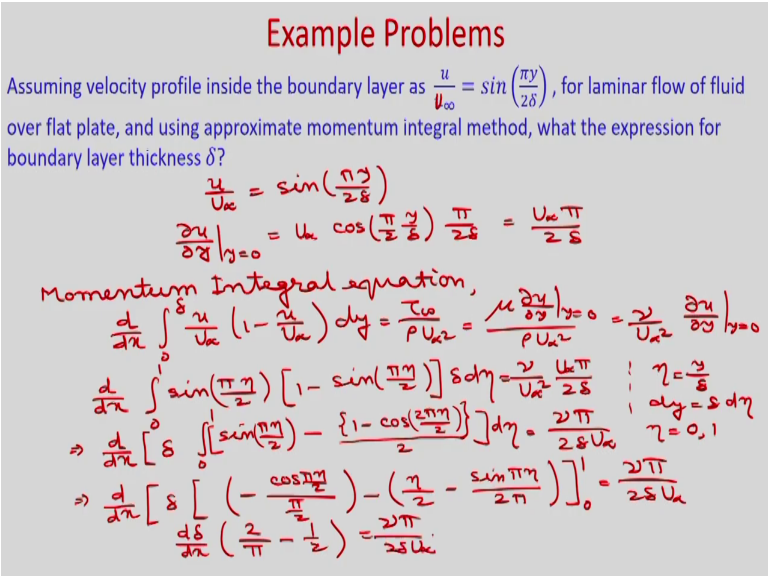

Assuming velocity profile inside the boundary layer as u by U infinity is equal to sin pi y by 2 delta or laminar flow of fluid over flat plate and using approximate momentum integral method what is the expression for boundary layer thickness delta. So we will follow the same procedure, but here velocity distribution is given u by U infinity as sin pi y by 2 delta ok.

So, from here you can find del u by del y at y is equal to 0 as U infinity cos pi by 2 y by delta and we have pi by 2 delta. So, cos pi by 2 y del y by delta, so at y is equal to 0, cos 0 is 1, so you will get U infinity pi by 2 delta. Now, we will use the similar procedure. So, we will put this velocity distribution in the momentum integral equation and find the unknown parameter delta.

So, Momentum Integral equation so we have d of dx integral 0 to delta u by U infinity 1 minus u by U infinity dy is equal to tau w by rho U infinitely square. That means, del u by del y at y is equal to 0 by rho U infinity square; that means, nu by U infinity square del u by del y at y is equal to 0.

So, you put these values, so you will get d of dx integral 0 to. So now, we will put eta is equal to y by delta ok. So that means you can write dy is equal to delta d eta and limit we can see that it will change from eta is equal to 0 to 1 ok. So, 0 to 1 sin pi eta by 2 into 1 minus sin pi eta by 2 and dy is delta d eta is equal to nu by U infinity square and del u by del y at y is equal to 0 is U infinity pi by 2 delta.

We can write d of dx delta and now you can write 0 to 1. So, it will be sin pi eta by 2 minus it is sin square pi eta by 2 and sin square pi eta by 2 you can write 1 minus cos 2 pi eta by 2 divided by 2 ok. So, it will be d eta and it will be nu pi by 2 delta U infinity. So now you evaluate this integral ok.

So, you will get d of the dx delta. So, if you find it you will get minus cos pi eta by 2 divided by pi by 2 minus it will be eta by 2 minus it will be sin pi eta, sin pi eta divided by it will be pi and denominator we have 2. So, it will be 2 pi and the limit 0 to 1 is equal to nu pi by 2 delta U infinity. So, now you can see that if you put the limits ok. So, you will get d delta by dx 2 by pi minus half is equal to nu pi by 2 delta U infinity.

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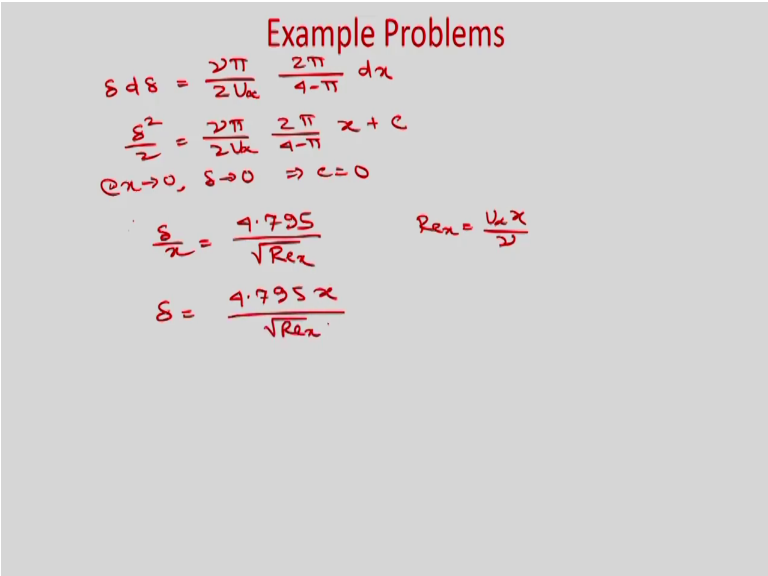

So, now, if you rearrange it you will get delta d delta is called to nu pi by 2 U infinity twice pi divided by 4 minus pi dx ok. So now, you integrate it you will get del square by 2 nu pi by 2 U infinity 2 pi by 4 minus pi x plus c. So, again at x tends to 0, we have delta tends to 0 that will give c is equal to 0. So, if you rearrange it we will get delta by x is equal to 4.795 divided by root Re x, where Reynolds number based on x is U infinity x by nu. So, you can write delta is equal to 4.795 x by root Re x.

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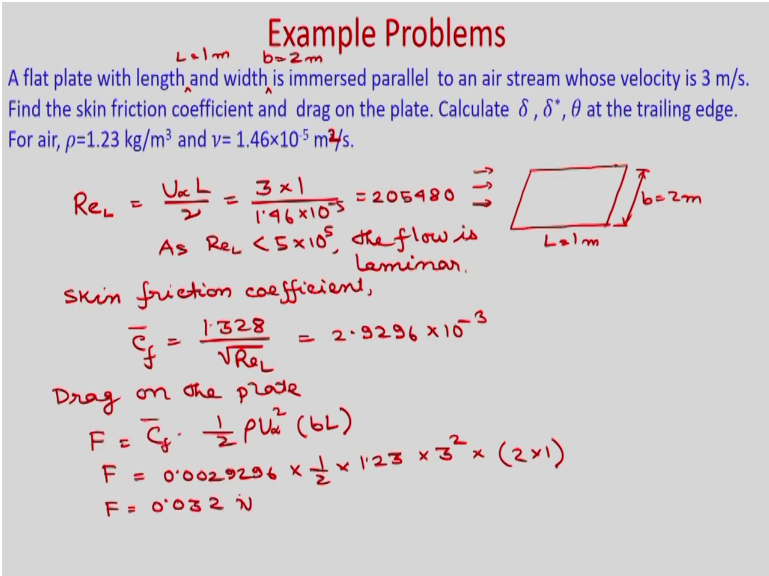

Now, let us consider one numerical problem. A flat plate with length and width is immersed parallel to an air stream whose velocity is 3 meter per second. Find the skin friction coefficient and drag on the plate. Calculate delta, delta star, theta. That means, boundary layer thickness then displacement thickness and momentum thickness at the trailing edge and density and kinematic viscosity are given.

So, we have one flat plate right. So, this here length actually given as 1 meter and width is given as 2 meter ok. So, this is your L 1 meter flow is taking place over this plate and the width is b is equal to 2 meter. So now, let us calculate at the trailing edge what will be the Reynolds number.

So, Reynolds number at the trailing edge will be U infinitely L by nu, where infinity is given as 3 meter per second, L is 1 divided by 1.46 into 10 to the power minus 5 sorry it will be meter square per second. And the Reynolds number if you calculate you will get 205480 ok.

So, you can see that this is in the so as Reynolds number is less than 5 into 10 to the power 5 the flow is laminar ok. So now, you will be able to calculate the skin friction coefficient. So, what is skin friction coefficient? We have already evaluated, so c f average is 1.328 divided by root Re L. So, Re L if you put it here you will get 2.9296 into 10 to the power minus 3.

So, total drag on the plate, so f will be just c f into half rho U infinity square into area, so area is b into L ok. So, F will be just c f is 0.0029296 into half into density is 1.23 kg per meter cube into U infinity square into b into L so 2 into 1 ok. So, F will be 0.032 Newton. Next we need to calculate the boundary layer thickness, displacement thickness and momentum thickness.

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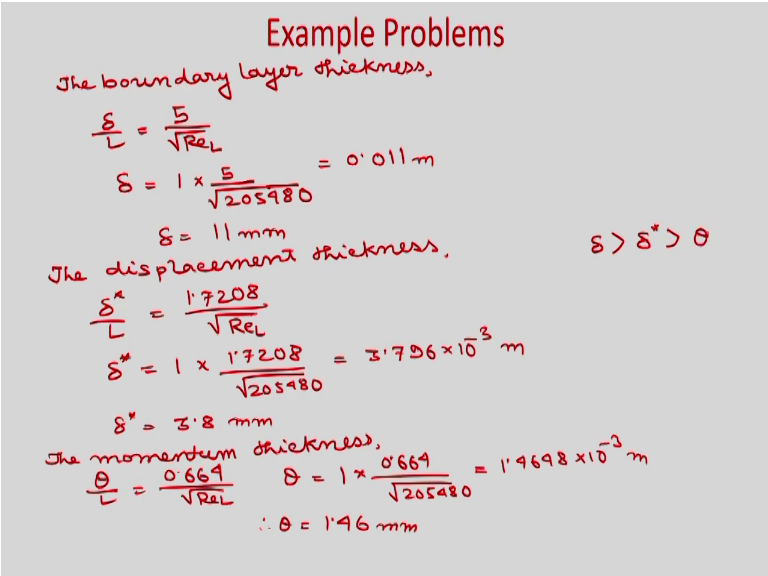

So, we know the expression now let us evaluate the boundary layer thickness. So, delta by L is 5 by root Re L. Now we know that delta is equal to. So, L is 1 into 5 divided by root Re L. So, root Re L means root 205480 ok. So, if you calculate it you will get 0.011 meter. So that means, delta will be 11 millimeter and next let us calculate the displacement thickness.

So, delta star by L is equal to 1.7208 divided by root Re L ok. So, to calculate this using the exact solution delta star by L you will get 1.7208 divided by root Re L. So, if you put it the values then you will get L is equal to 1 into 1.7208 divided by root 205480. So, this will come as 3.796 into 10 to the power minus 3 meter. So, you will get delta star as 3.0 approximately 3.8 millimeter. And the momentum thickness so you will get theta by L as 0.664 divided by root Re L ok.

So, theta will get as L as 1.664 divided by root 205480. So, it will come as 1.4648 into 10 to the power minus 3 meter. So, theta you will get 1.46 millimeter ok. So, you can see from this expression that obviously delta is 11 millimeter delta star is 3.8 millimeter and theta is 1.46 millimeter. So that means, delta is greater than delta star and greater than theta.

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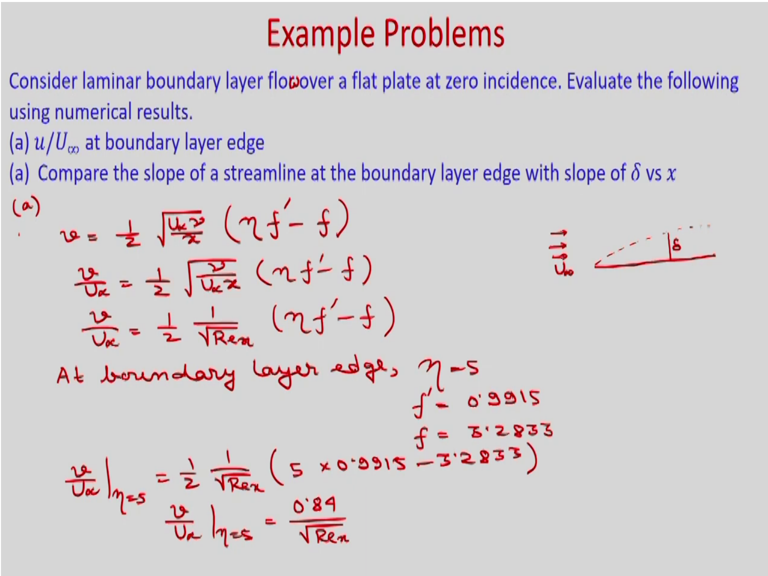

So, let us consider the next problem consider laminar boundary layer flow over a flat plate at zero incidents, evaluate the following using numerical results u by U infinity at boundary layer edge and compare the slope of a streamline at the boundary layer edge with slope of delta verses x ok.

So, we are considering flow over a flat plate at 0 incidence, so that means, free stream is parallel to the flat plate and this is the boundary layer and at any x this is the boundary layer thickness delta. So, you have to compare the slope of streamline with the slope of delta verses x and u by U infinity at boundary layer edge.

If you remember that from the solution of the Blasius equation we have found that v as half root U infinity nu by x into eta f prime minus f right. So, this we have found from this analytical method when we are solving the Blasius equation. So now, v by U infinity you can write as half root nu by U infinity x ok. So, U infinity we have divided here also we have divided so it will be in the root in the denominator into eta f prime minus f.

So, we can write v by U infinity is equal to half one by root Re x eta f prime minus f ok. So, at the edge of boundary layer we know eta is equal to 5 right at boundary layer edge eta is equal to 5 ok. So, you can see that eta is equal to 5 and you see at the numerical results from the table that at eta is equal to 5 f prime will be 0.9915 and f will be 3.2833 ok.

So, from the numerical results just we are writing f prime and f value at eta is equal to 5. So now, we can see that at eta is equal is 5 it is the boundary layer edge. So, if you put it here so you will get v by U infinity at eta is equal to 5 half 1 by root Re x and eta is 5 f prime is 0.9915 minus f 3.2833 ok. So, v by U infinity at eta is equal to 5 it will be 0.84 divided by root Re x ok. So, this is the part a we have evaluated sorry this is d by U infinity.

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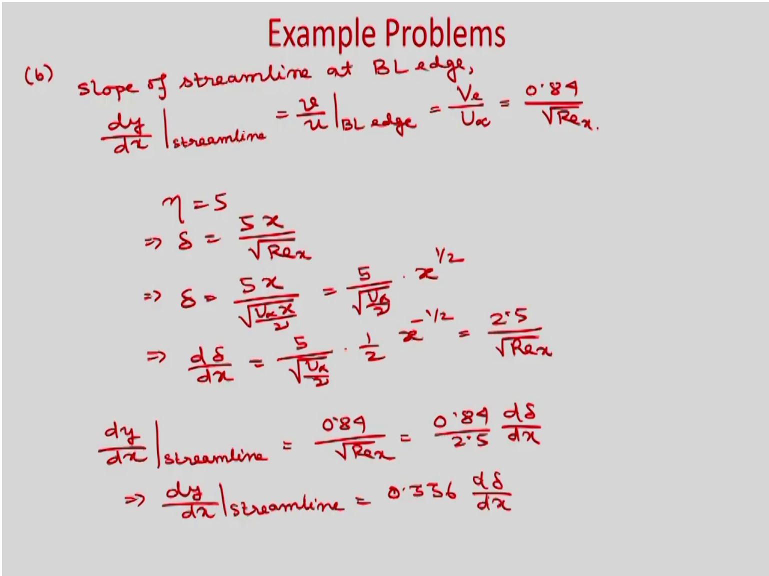

Now, next let us find the slope of stream line and delta versus x. So, we will get slope of streamline at boundary layer edge ok. So, how we will get we know the from the equation of streamline that dy by dx streamline should be v by u at boundary layer edge ok. So now, if we tell that at the boundary edge it is v e which we have already evaluated divided by at boundary layer edge u is equal to U infinity ok.

So, v u you have already evaluated. So, it will be 0.84 by root Re x and we know that eta is equal to 5 we will get that delta is equal to 5x by root Re x right. So, from here you can see that delta you can write as 5x divided by u infinity x by nu. So, from here you can write 5 by root u infinity by nu into x to the power. So, it will be 1 minus half so it will be half.

So now, we can write d delta by dx is equal to 5 by root U infinity by nu half x to the power minus half. So, this minus half if you put it here you can write 2.5 divided by root Re x. So, you can see that we have evaluated the slope of streamline in terms of Reynolds number and d delta by dx in terms of Reynolds number. So now, we can compare this 2 slope.

So, you can write dy by dx of streamline is equal to 0.84 by root Re x and this from here you can see that this you can write as 0.84 divided by 2.5 d delta by dx. So now, we can write dy by dx at streamline is equal to 0.336 d delta by dx ok. So, let us consider the last example problem which is the flow between 2 parallel plates and we will consider in the developing range.

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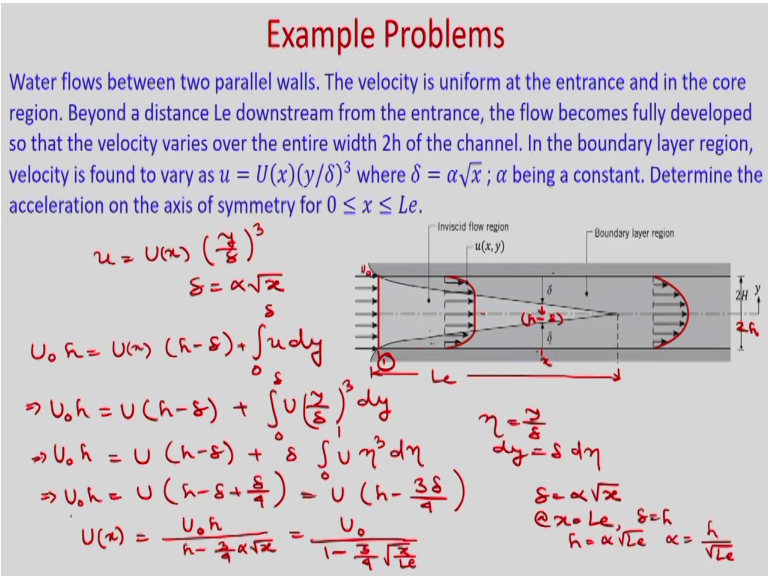

So, we can see water flows between two parallel walls, so these are two parallel walls, the velocity is uniform at the entrance. So, this is your velocity which is u and in the core region beyond a distance Le downstream from the entrance the flow becomes fully developed, so this length is Le.

That means, developing length after that it becomes fully developed and the distance between this plate is 2h, so it is 2h. Even the boundary layer region velocity is found to vary as u is equal to U which is function of x y by delta whole cube where delta is alpha into root x were alpha being a constant.

Determine the acceleration on the axis of symmetry for 0 less than x less than Le; that means, in the developing region you have to find the acceleration on the axis of symmetry. So, now let us do the mass balance so, at exit if it is 1 and at any location x ok. So, we can do the mass balance, so it will be at the inlet say have U naught so it will be U naught into the height.

So, we are considering half ok half of this. So, it will be h is equal to we have velocity U in the core region and this is the distance, so at x location the distance is this is h minus delta right. So, it will be U which is function of x h minus delta and in the boundary layer region whatever mass flow rate we have.

So, we can write 0 to delta u dy right. So, from here you can see you can write U naught h is equal to U h minus delta plus now this is u y by delta whole cube dy ok. So, if you evaluate it you will get U naught h is equal to U h minus delta plus if you write eta is equal to y by delta. So, dy is equal to delta d eta.

So, it is 0 to delta so now if you write it will be delta 0 to 1 U eta cube d eta ok. So, hence if you put this U naught h is equal to U it will be h minus delta and this if you evaluate you will get delta by 4, u into delta by 4. So, this will get as U h minus 3 delta by 4 and delta is alpha root x right.

So, if you put it here and at x is equal to Le ok, we have the delta as h right because this is the h at x equal to Le we have delta is equal to because this is the boundary layer thickness. So, it will be delta is equal to h. So, here if you put then you will get U x as U naught h divided by h minus 3 by 4 alpha root x.

So, from here you can write h is equal to alpha root Le, so alpha will be h by root Le ok. So, alpha if you put here h by root Le, so you will get U naught 1 minus 3 by 4 root x by Le h h will get cancelled here h and here we will get 1 h. So, it will get cancel and this is the U x.

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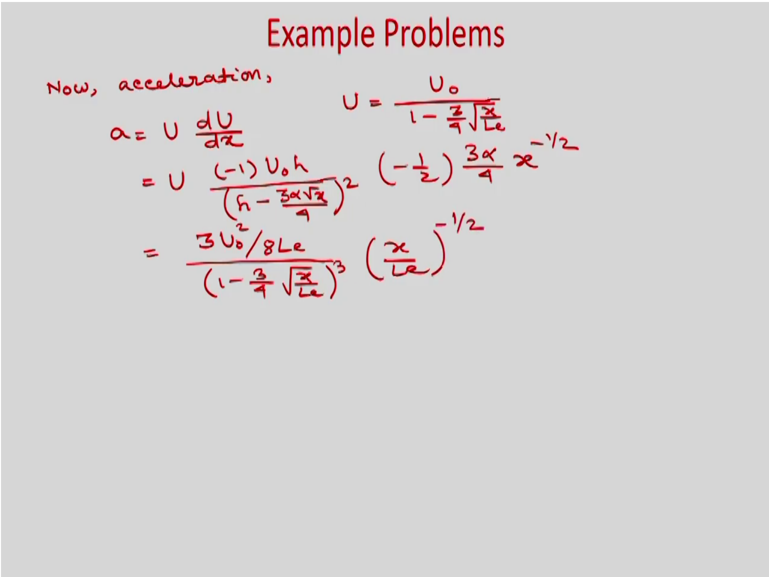

So now, acceleration we can write as a is equal to U dU by dx. So, if you put U, U expression we have as U naught divided by 1 minus 3 by 4 root x by Le ok. So, dU by dx will be just minus 1 U naught h divided by h minus 3 alpha root x by 4 whole square minus half 3 alpha by 4 x to the power minus half.

So, if you rearrange, you will get 3U naught square. So, if you put U value here, so you will get 3U naught square by 8 Le divided by 1 minus 3 by 4 root x by Le so it will be whole cube into x by Le to the power minus half. So, this is the acceleration. So, in today's class we solved several example problems and we have used the in some problems the numerical results of a Blasius solution.

And also we have found the boundary layer thickness, displacement thickness and momentum thickness in one problem. So, you can solve some example problems from any book or some exercise problem in the reference book which is mentioned in this course.

Thank you.