Viscous Fluid Flow

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Module - 11
Turbulent Flows - I
Lecture - 01
Introduction to Turbulent flows

Hello everyone. So, till now we studied the laminar flows, today we will start with Turbulent Flows most flows encountered in engineering application and nature are turbulent. You can see there are many applications of turbulent flows in engineering like flows in jet, flows in mixing, then flow over a aerofoil section.

In nature also you will get different turbulent flows like flow in river or the flow of wind over a earth surface; so these are some applications of turbulent flows. Turbulent flows are very difficult to study because you know it has disordered motion and velocity and other variables fluctuates with time.

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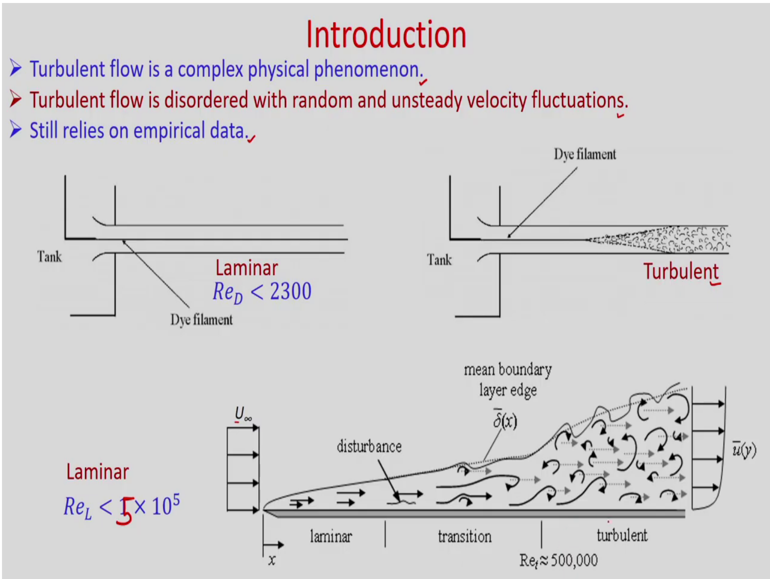

So, turbulent flow is a complex physical phenomena, turbulent flow is disordered with random and unsteady velocity fluctuations and it still relies on empirical data based on experiments. You are very familiar with this Reynolds experiment. So, we have a tank and this is the circular pipe say here this liquid is flowing through this circular pipe and you are injecting the ink here ok at the centre of the pipe.

So, you can see that if flow velocity is very small, then obviously you will see that the dye is going in a straight line ok. So obviously, you know that this is called laminar flow and for flow inside circular pipe if Reynolds number based on diameter is less than 2300, then the flow will be laminar.

But, if you increase the velocity what will happen this dye filament will start oscillating in downward direction and you can see that this dye will no longer will be flowing in a straight line; that means, the flow has become turbulent. Similarly, if you consider flow over a flat plate where you have free stream velocity U infinity then you can see after a certain distance from the leading edge, there will be fluctuation of the velocity starting from the near to the wall; and it will lead to turbulent flows.

And we know for flow over flat plate if we define Reynolds number based on the length then if it is less than 5 into 10 to the power 5, then it will be laminar flow ok; otherwise it will become turbulent flow. And if Reynolds number is greater than 5 into 10 to the power 5 then flow will become turbulent. So, you can see that in both the cases we have defined Reynolds number. So, why this Reynolds number is so important in finding the onset of turbulence?

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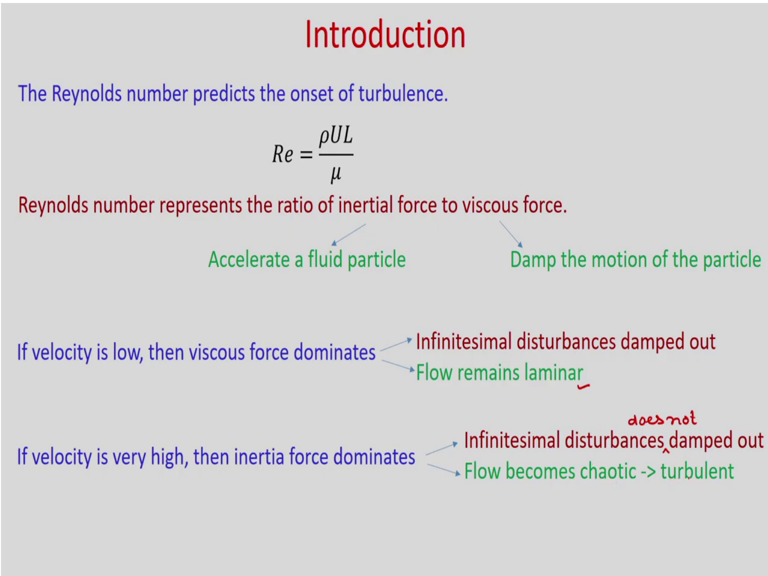

So, you can see that Reynolds number we define with rho UL by mu where rho is density of the fluid, mu is viscosity of the fluid, U is the flow velocity and L is the characteristic length. Now, you can see this Reynolds number is the ratio of inertia force to the viscous force.

So, this inertia force you can see that it accelerate a fluid particle right, but on the other hand this viscous force damps the motion of the particle. So you can see these are the two forces acting in opposite direction. You can see that if velocity is low then viscous force dominates and in that case infinitesimal disturbances damp out due to this viscosity effect and flow remains laminar.

But if velocity is very high then inertia force dominates. So, infinitesimal disturbances does not damp out and flow becomes chaotic, so then the flow becomes turbulent. So, you know that the in turbulent flow the velocity fluctuation will be there and due to that there will be some formation of small swelling motion, which is known as eddies.

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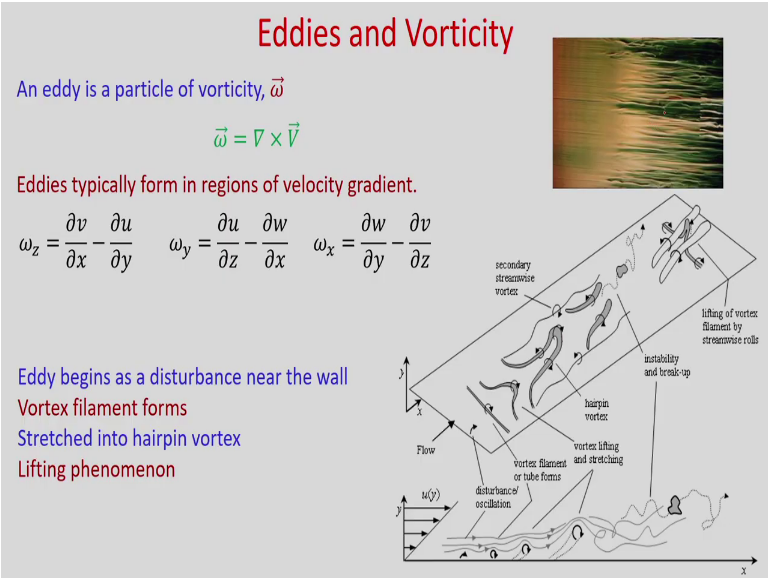

So, you can see that an eddy is a particle of vorticity omega and we know that vorticity we define as a curl of velocity vector. So obviously, you can see that this vorticity obviously we can represent in terms of the velocity gradient ok. So, when we define the eddy; that means, eddy will contain high velocity gradient ok. So, eddy is typically formed in the regions of velocity gradient and these are the three components of the vorticity and if you see that flow over a flat plate case say when flows is flowing over the flat plate ok.

So, you can see that eddy begins as a disturbance near to the wall and there will be some vortex filament formation on the or tube forms and due to these disturbances or fluctuating velocity components, these vortex will lift and it will stretch like this and there will be formation of this hairpin vortex and there will be some instability and breakup of this hairpin vortex.

And further if you go in the downward direction then you can see that these team wise roles actually will try to lift this vortex filament and the flow becomes turbulent. In this figure you can see that these are some top view of the flow over a flat plate and you can see that there are some disturbances appearing on the surfaces; so this flow becomes turbulent.

Now, you can see that there will be the formation of eddies due to this vorticity and these eddies the largest eddies size will be the size of the domain ok. And you can see that there will be formation of smaller eddies breaking these larger eddies, so this is known as energy cascade proposed by scientist Richardson; so let us discuss this phenomena first.

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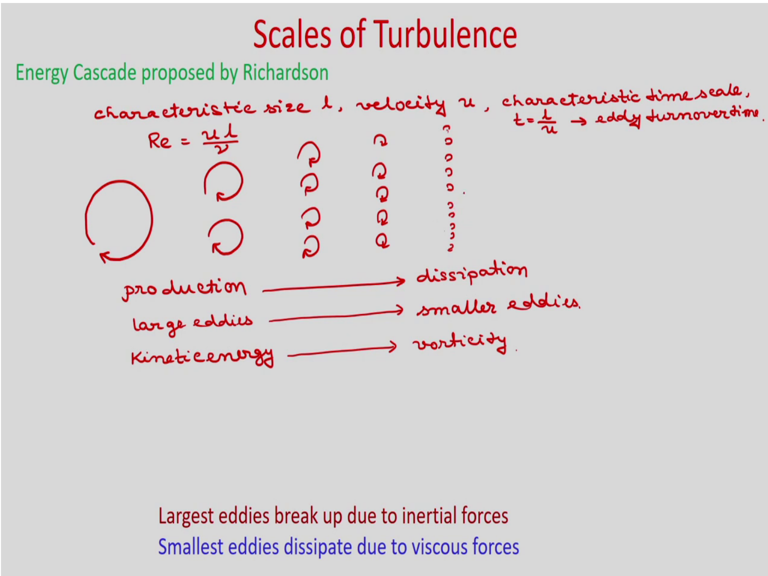

So, let us say that the characteristic size is l and velocity is u and characteristic time scale is t which is found as l by u ok, which is known as turn overtime eddy turn over time. So, we can define the Reynolds number for this eddy as u l by nu, where nu is the kinematic viscosity of the fluid.

So obviously, the largest eddy will be as large as the characteristic length of the main flow and let us say that this is the largest eddy ok. And this largest eddy, obviously will contain most of the kinetic energy and it will have the highest Reynolds number. This large Reynolds number implies that inertial force are important and viscous effects are very small.

Now, you can see that these eddies will be actually splitted into smaller eddies ok and because these larger eddies will be unstable and break up into smaller eddies by inertial force and this smaller eddies split the kinetic energy of this larger eddy. And further these eddies will be also splitted like this.

So, and these smaller eddies again will be further splitted into smaller eddies having the less kinetic energy. And after that you can find that there will be very very small eddies like this ok. And this actually are the smallest eddies possible and this will actually dissipate the kinetic energy to the heat.

So, you can see that in the smaller eddies viscosity will be the important and these smallest eddies actually due to viscous effect these eddies will convert this kinetic energy to heat. So, you can see that we have production of these eddies and there will be dissipation to heat from the smaller eddies and we have large eddies.

And this kinetic energies due to the inertial effect this will be breaking up into smaller eddies which will have the lower kinetic energy and it will become smaller eddies. And you can see that we will have highest kinetic energy in the larger eddy and further it will become actually vorticity.

So, you can see that largest eddies breakup due to the inertial forces and smallest eddies dissipate due to viscous forces. So, in the process of this energy cascade you have seen that the largest eddy size will be the as big as of the domain size, but what will be the size of the smallest eddies? Ok; so, that we can actually find using the Kolmogorov scale.

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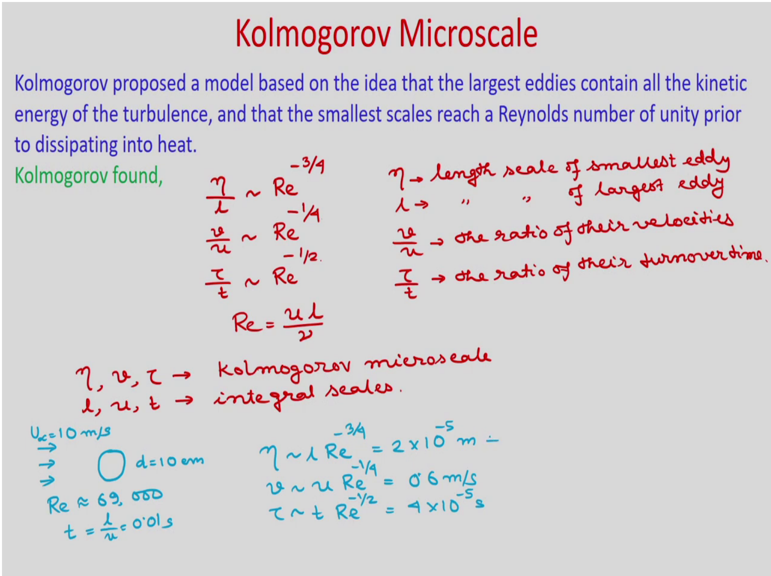

So, Kolmogorov proposed a model based on the idea that the largest eddies contain all the kinetic energy of the turbulence, and that the smallest scales reach a Reynolds number of unity prior to dissipating into heat. So, Kolmogorov found that this eta by l is order of Reynolds number to the power minus 3 by 4 where eta is the length scale of smallest eddy and l is the length scale of largest eddy.

And this Reynolds number is based on the length scale of largest eddy. Then we have v by u of the order of Re to the power minus 1 by 4 ok, so this v by u is the ratio of their velocities; the ratio of their velocities. So obviously, u is the velocity of the larger largest eddy and v is the velocity of the smallest eddy.

And also we have tau by t of the order of Reynolds number to the power minus 1 by 2, where tau by t is the ratio of their turnover time. Here Reynolds number is defined as the velocity of the largest eddy, length scale of the largest eddy divided by the fluid viscosity nu. So, here you can see that this eta nu and tau are referred as the Kolmogorov micro scales.

So, this eta, v and tau are referred as Kolmogorov micro scale and l, u and t are called as integral scales. So, let us give some idea about this Kolmogorov scale and let us see that how small these scales are. So, let us consider a flow over a circular cylinder. So, let us say that this is a circular cylinder of diameter 10 centimeter and we have free stream velocity U infinity as 10 meter per second, ok.

So, this Reynolds number will be of the order of 69,000 and time scale this integral time scale you can see it will be l by u, so it will be around 0.01 second ok. So, from here you can see that the Kolmogorov scales eta will be order of l Reynolds number to the power minus 3 by 4 in this case it will be 2 into 10 to the power minus 5 meter and v will be u Re to the power minus 1 by 4 and it will be 0.6 meter per second.

And tau will be t into Reynolds number to the power minus half, so it will be 4 into 10 to the power minus 5 second. So, you can see that the length scale of this smallest eddy which is your Kolmogorov scale you can see it is 2 into 10 to the power minus 5 meter. So, you can imagine that how small it is, it is very difficult to define the turbulent flows, so the scientist actually they characterized the turbulent flows.

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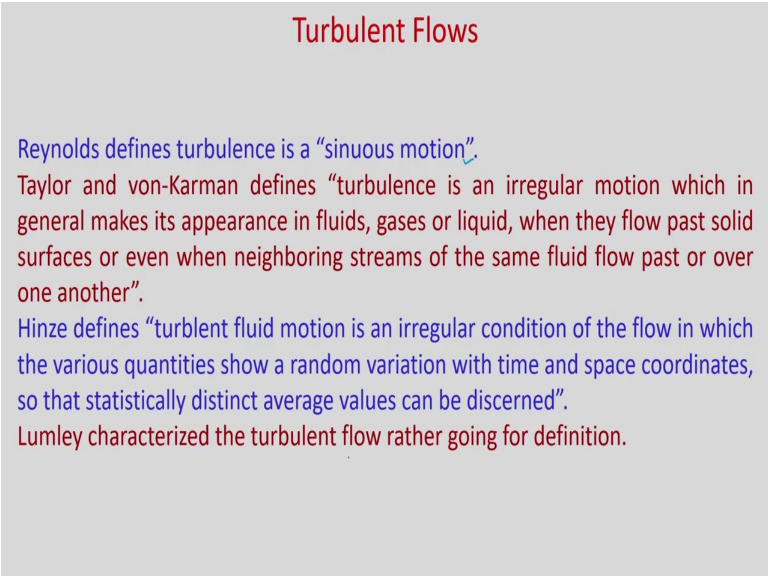

So, you can see that Reynolds defines turbulence is a sinuous motion, Taylor and von Karman defines “turbulence is an irregular motion which is general makes its appearance in fluids, gases or liquid, when they flow past solid surfaces or even when neighboring streams of the same fluid flow past or over one another”.

Hinze defines, “turbulent fluid motion is an irregular condition of the flow in which the various quantities show a random variation with time and space coordinates, so that statistically distinct average values can be discerned”. And Lumley characterized the turbulent flow rather going for definition.

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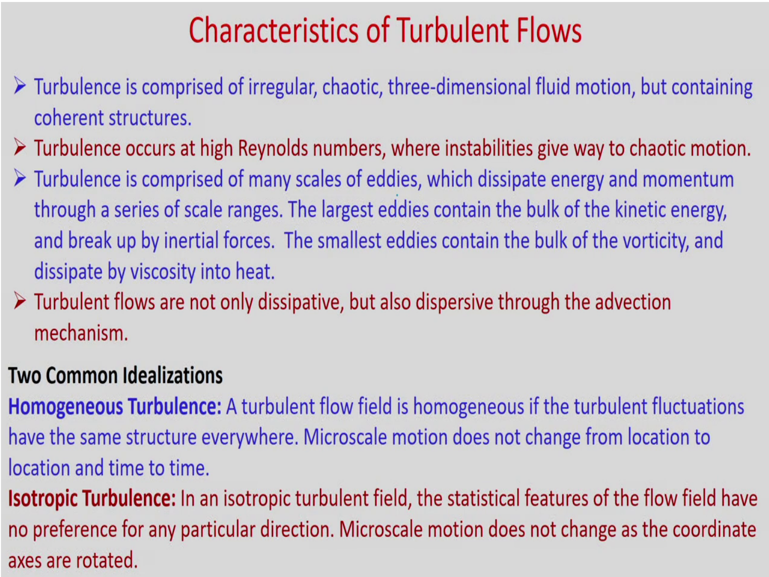

So, based on our discussion you can see these are the characteristic of the turbulent flows turbulence is comprised of irregular chaotic three-dimensional fluid motion but containing coherent structures. Turbulence occurs at high Reynolds numbers where instabilities give way to chaotic motion turbulence is comprised of many scales of eddies which dissipate energy and momentum through a series of scale ranges.

The largest eddies contain the bulk of the kinetic energy and breakup by inertial forces the smallest eddies contain the bulk of the vorticity and dissipate by viscosity into heat. Turbulent flows are not only dissipative, but also dispersive through the advection mechanism.

So, to study the turbulent flows we will make these two common idealization one is homogeneous turbulence another is isotopic turbulence. So, what is homogeneous turbulence? A turbulent flow field is homogeneous if the turbulent fluctuations have the same structure everywhere; what does it mean? That means, micro scale motion does not change from location to location and time to time.

So, and what is isotropic turbulence? In an isotropic turbulent field the statistical features of the flow field have no preference for any particular direction; that means, the micro scale motion does not change as the coordinate axis are rotated. So, when we study the turbulent flows as it is very complex fluid flow phenomena we make these two idealization like homogeneous turbulence and isotropic turbulence.

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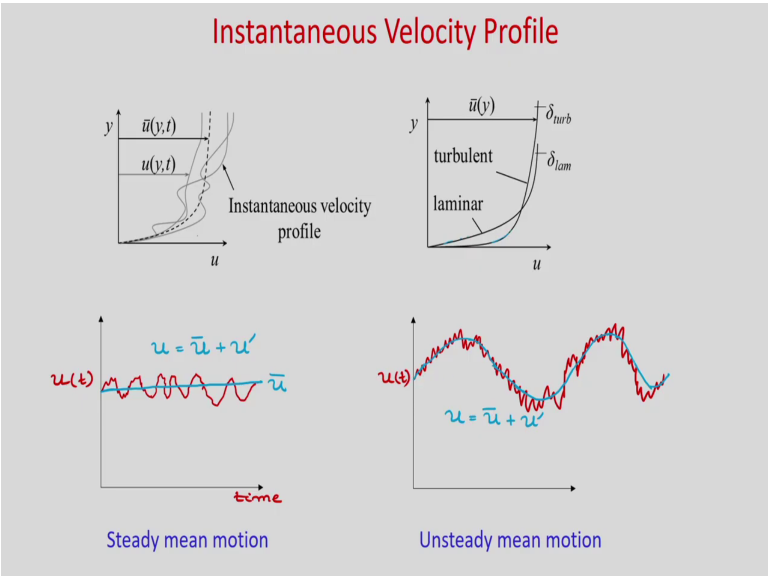

If you consider flow over a flat plate then you can see that if you plot the instantaneous velocity let us say u is the instantaneous velocity, then you can see that with time it will vary. Say this is your instantaneous velocity at some particular time some other time it will become like this.

And so you can see that instantaneous velocity profile will vary with time, but if we make some time average of these velocities then we will get u bar which you can see that it will look like this ok. If you compare the velocity boundary layer for a flow over flat plate for two for turbulent and laminar flows, then you can see that this is the velocity profile for the laminar flows and this is the velocity profile for the turbulent flows.

So, from here you can see that in case of turbulent flow the velocity gradient is very high near to the wall. So, you can see that in turbulent flows the fluid flow parameters fluctuates with time, so let us consider one flow variable u, which is function of space as well as time.

So, let us see that how it varies with time for a flow over a flat plate ok. So, if you consider that u ok at a particular location we are trying to find the velocity with time. So, you can see that if you are considering this case flow over a flat plate then your flow will fluctuate with time, so maybe you will get like this ok.

So obviously, you can see that if you make some time average quantity then this time average velocity may look like this. So, you can see that; obviously, this time average quantity is not varying with time, so this is known as steady mean motion. Now, if you consider flow over circular cylinder at a higher Reynolds number.

Then obviously, it will become unsteady because vortices will be shedding and if it is turbulent flows then there will be fluctuation in fluid velocity. So, let us consider one point behind the cylinder where wake is formed and obviously, it will vary with time, but it will also fluctuate. So, you can see that if you consider other case where you have unsteady motion.

Then say this u, so you can see this velocity will vary with time and also it will fluctuate like this ok. So, if you make a time average quantity of these then you can see that it will look like this, so this is your time average velocity. So, you can see that we can represent this velocity u as some time average velocity that is your u bar this one plus your fluctuating component ok.

So, plus u prime ok and in this case you can see that the velocity u ok it varies with time ok and this mean velocity also varies with time plus the fluctuating component u prime. So, this is known as u bar is known as unsteady mean motion because it varies with time. So, now, let us discuss about some averaging techniques in turbulent flows so; obviously, you can see that if it is a steady mean motion then your time average quantity will not vary with time.

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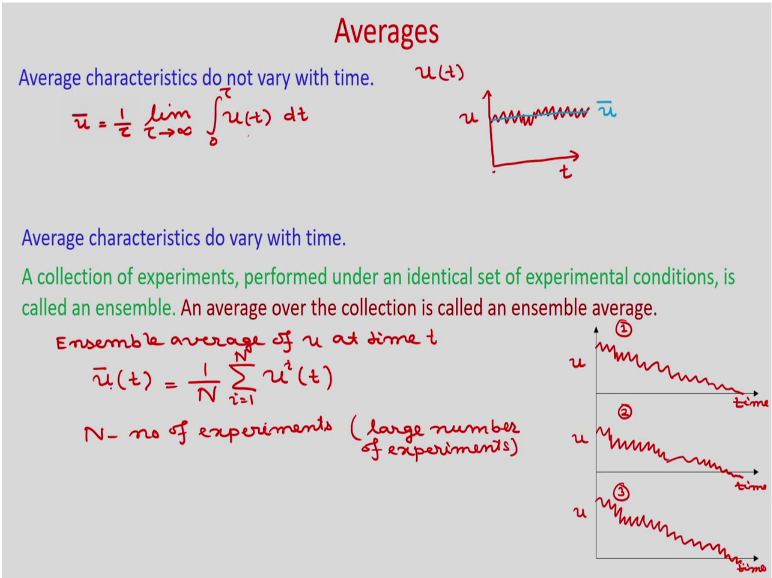

So, if average characteristic do not vary with time. Then we can write the any variable let us say u in the turbulent flow then we can define the average variable as the time mean u bar is equal to 1 by tau limit tau tends to infinity integral 0 to tau u t dt ok. So, we are taking some variable, flow variable which is function of t. So now, we are making the average quantity and this average quantity does not vary with time, so and we are just integrating over a large time 0 to tau ok over the time.

So, you can see that in last slide we have shown say if this is the velocity, this is the time you can see that it will vary with like this an average quantity will not vary with time ok. So, if you take a large time ok then if you integrate this u then you will get u bar. Now, if you take a unsteady case then; obviously, you can see average characteristic do vary with time.

So, in that case you can see that if we take time to be very large ok then in that case we may not get a local average and if we take time to be very small then we may not get a reliable average. So, for that we will just use ensemble average. So, what is ensemble average a collection of experiments performed under an identical set of experimental conditions is called an ensemble. An average over the collection is called the ensemble average.

What do we need to do when say we have a this mean velocity varies with time? Then we need to conduct a set of experiments under identical conditions, then at a particular point we need to take the average of these experiments. So, let us say here that your this u velocity ok at some experiment 1 it varies like this, like this, this is the first experiment with time.

The same experiment you are conducting under identical condition then if you measure this u, then maybe it will vary like this ok. So, this is your experiment 2. And let us say in third experiment this u varies with time like this ok. So, now, what you need to do that if you want to find the average characteristic then at a particular time you need to average the values of such n experiments.

So, in this case you can see that if you take a large time to take this average ok, then obviously you will not get a local average and if you take very small time period then; obviously, it will not be a reliable average. So, for that reason we are performing n experiments and we are finding the ensemble average of u at time t, so at a particular time t.

Then you can find u bar t as 1 by N, where N is number of experiments and it is large, summation of i is equal to 1 to N u i ok; where, N is number of experiments. And you need to take very large experiments ok, so large number of experiments you have to take ok. To find this ensemble average similarly in time derivative also you can use this ensemble average.

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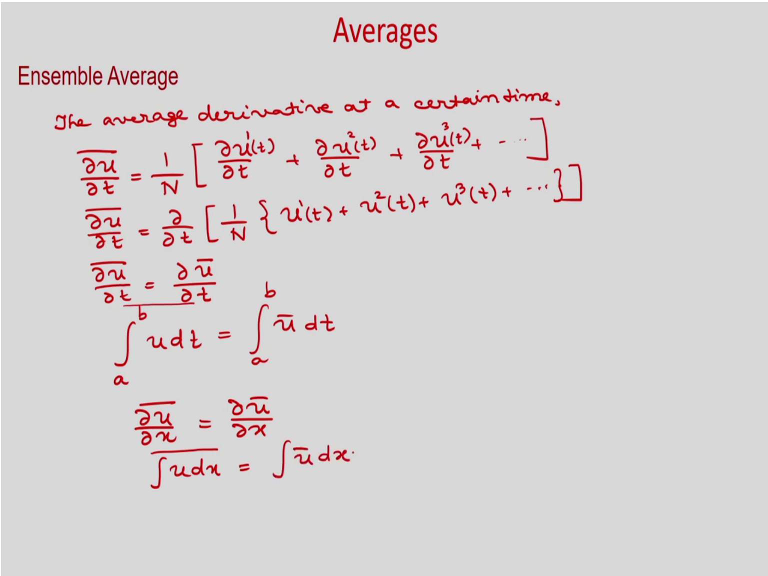

So, you can see that the average derivative at a certain time. So, if you want to make del u by del t this average then using ensemble average you can just write 1 by N where N is the number of experiments and del u by del t ok of experiment 1 ok plus del u by del t of experiment 2 then del u by del t of experiment 3 and so on ok.

So, now you can write del u bar by del t is equal to del of del t 1 by N, u 1 plus u 2 plus u 3 and so on ok. So, you can see that this del u bar by del t then we can write as del, so this actually represents the average of this u. So, u del u bar by del t ok. And similarly, if you make some integration let us say integral a to b u dt, that also you can write as integral a to b u bar dt.

So, this averaging we have done with respect to the time similar averaging also you can do when this flow variable is function of x function of space. So, you can see that if u is function of space then we can write del u by del x bar is equal to del u bar by del x. And similarly you can write the integral u dx ok, so this averaging you can write as integral u bar dx. So, now, let us discuss what are the techniques available to numerically solve the turbulent flows.

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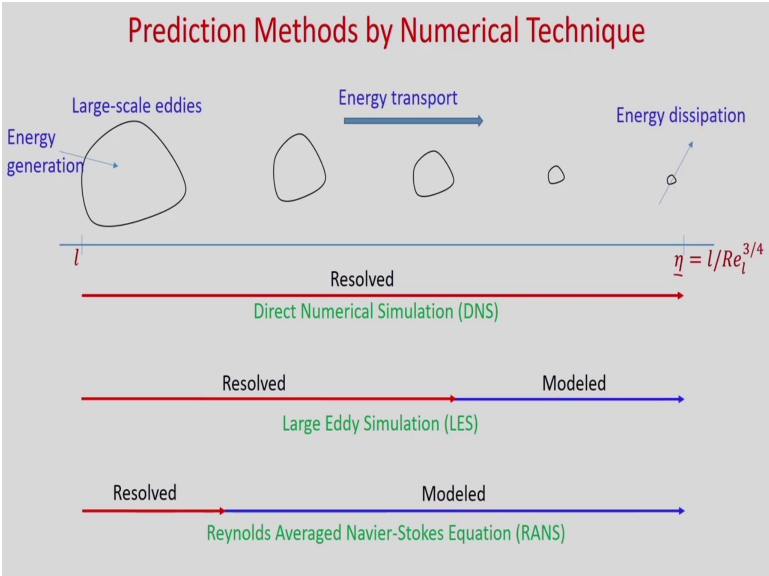

You can see as we discussed earlier the larger eddies are unstable and breakup into smaller eddies by inertial forces the smaller eddies, split the kinetic energy of the larger eddy. Those smaller eddies eventually break up due to the inertial forces and so on until the eddies reach a small enough that their Reynolds number approaches the order of unity.

Then this viscous forces quickly dissipate the energy of the smallest eddies into heat. So, that we know that this is your Kolmogorov scale eta. So, there are different ways to numerically model these turbulent flows, so one of this is Direct Numerical Simulation, which is commonly known as DNS.

So, in DNS we solved the continuity and Navier-Stoke equations without modeling, in DNS all the scales of turbulence motion are resolved up to Kolmogorov scales. However, due to its computational cost DNS is still restricted to the study of turbulent flow it needs very fine mess to resolve the eddies of sizes of Kolmogorov scales.

Another way to model is that large eddy simulation, which is known as LES, so LES is the approximate solution of continuity and Navier-Stoke equation on a course grid with some modeling. In LES filtering operation are applied in the Navier-Stoke equations and results explicitly the dynamics of the unsteady large scales of turbulence while modeling the small scales motions.

So obviously, you can see that in LES the computational cost is cheaper than the DNS. Another way we have already discussed that is the Reynolds average Navier-Stoke equations, so in this modeling it is you can see that we need to solve the time averaged these Navier-Stoke equations.

RANS modeling is the most common and widespread approach in industrial applications at as it does not require large CPU resources. RANS approach is based on ensemble or time averaging of the Navier-Stoke equations. RANS approach integrates the whole turbulence spectrum.

So, that turbulence modeling assumptions are required for the statistical closures; in this module actually we will derive this Reynolds average Navier-Stoke equation using the time averaging of the Navier-Stoke equations. So, in today's class we introduced with the turbulent flows, we discussed how the energy cascade takes place from larger eddies to the smaller eddies.

You can see that larger eddies contains the maximum kinetic energy and smaller eddies actually contains the highest vorticity. Then we discussed about the Kolmogorov scales and then we discussed about the steady mean motion and unsteady mean motion where the velocities we can decompose into two parts, one is mean quantity u bar and the fluctuating component u prime. So, we can write u is equal to u bar plus u prime, so any flow variable like velocities pressure, temperature we can write in this way.

Thank you.