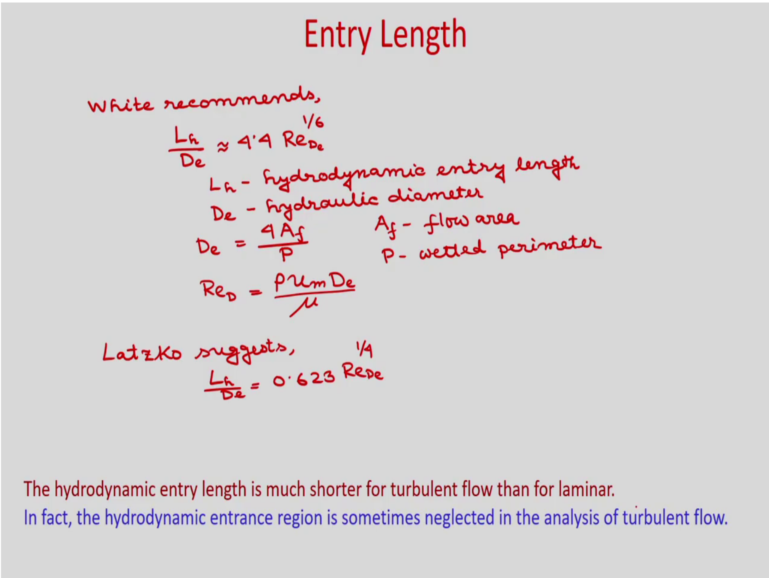
Viscous Fluid Flow
Prof. Amaresh Dalal
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 12
Turbulent Flows - II
Lecture - 02
Internal Turbulent Flow

Hello everyone, in last class, we derived the universal velocity profile for external turbulent flow. Today, we will consider Internal Turbulent Flows. And we will use the universal velocity profile which we derived in last class for these turbulent flows, and we will discuss about the mean velocity profile and the skin friction coefficient for pipe flow case.

(Refer Slide Time: 01:03)



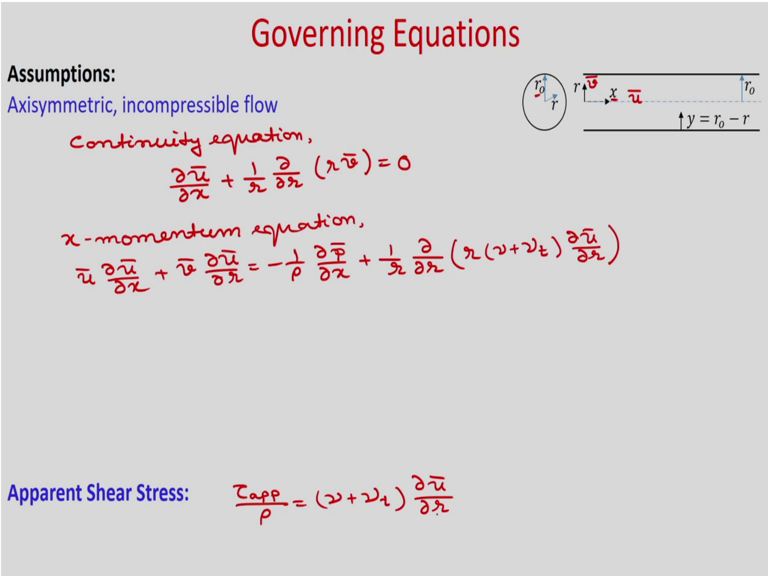
So, first let us discuss about the entry length. You know that we have already discussed the entry length for laminar flows. But for turbulent flows White recommends L h by D e, where L h is the hydrodynamic entry length is order of 4.4 into Re based on diameter hydraulic diameter to the power 1 by 6. So, L h is hydrodynamic entry length, and D e is hydraulic diameter.

And you know that how we calculate these hydraulic diameter D e is 4 into flow area divided by perimeter ok, where A f is flow area, and p is wetted perimeter. So, Reynolds number now defined based on mean velocity u m and diameter D divided by the fluid viscosity mu.

In addition Latzko suggested L h by D e as 0.623 R e D e to the power 1 by 4. So, you can see that the hydrodynamic entry length D e is much shorter for turbulent flow than for laminar. In fact, the hydrodynamic entrance region is sometimes neglected in the analysis of turbulent flow.

Now, let us write down the governing equations for pipe flow. We will consider the time average equations. In this case, in the axial direction, we will consider the velocity u; and in radial direction, we will consider the velocity v and axial direction is x, and r is the radial direction.

(Refer Slide Time: 03:42)

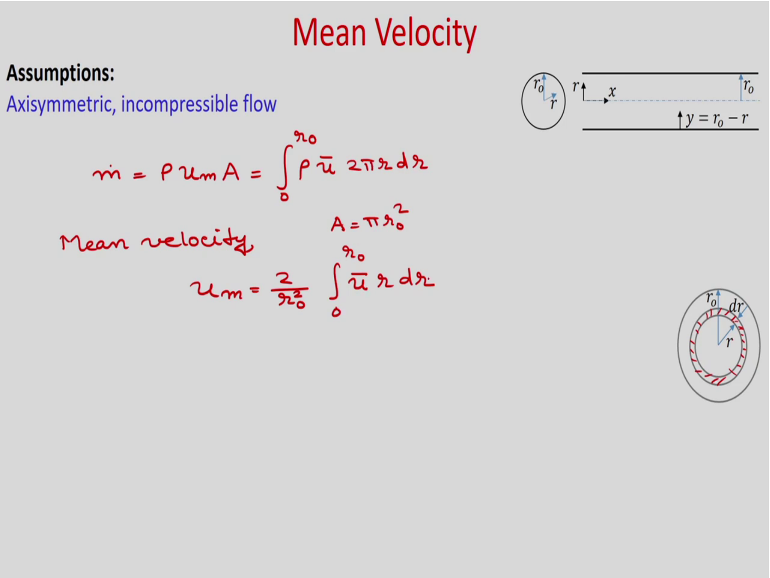


So, you can see this is the flow inside a circular pipe, where r naught is the radius of the pipe, r is measured from the central line, x is the axial direction. So, we are assuming axisymmetric incompressible flow. So, for that, we can write the continuity equation as del u bar by del x plus 1 by r del of del r r v bar is equal to 0. So, this is the time average continuity equation, where u bar is the velocity in axial direction, and v bar is the velocity in the radial direction.

Similarly, x-component of momentum equation we can write. So, this is the time average equation we are writing. So, we can write u bar del u bar by del x plus v bar del u bar by del r is equal to minus 1 by rho del p bar by del x plus 1 by r del of del r r nu plus nu t del u bar by del r. So, you can see that nu is the molecular viscosity, and nu t the eddy viscosity. And these are the time averaged velocity u bar v bar, and p bar is the time averaged pressure.

So, you can see that from this equation we have already written about the apparent shear stress we can write as tau app divided by rho is equal to nu plus nu t del u bar by del r ok. So, this is the total shear stress ok nu plus nu t del u bar by del r.

(Refer Slide Time: 05:59)



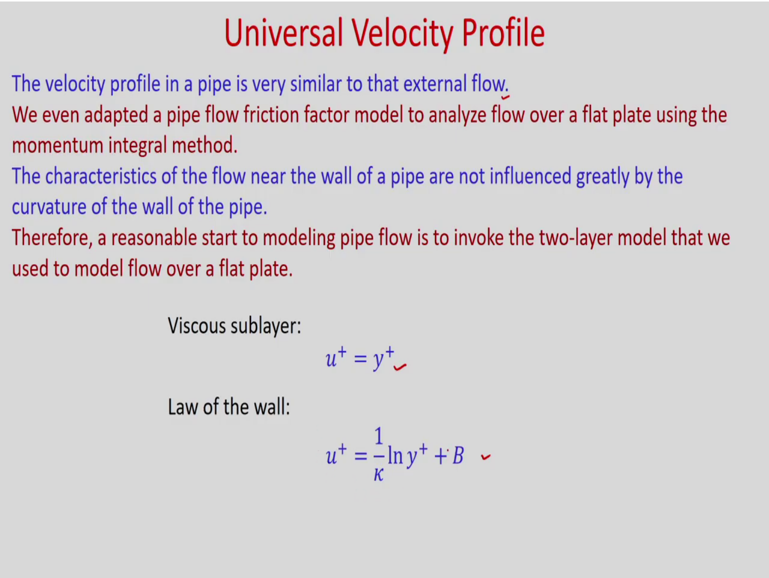
Now, let us write the expression for mean velocity. So, you know that at any cross section, we can actually integrate these velocity into area over the radius 0 to r. So, you can see for axisymmetric incompressible flow, we can write the mass flow rate m dot as rho mean velocity u m into flow area, and that we can equate with integral 0 to r naught rho u bar because u bar is varying right radially.

And at any radius r, if we consider a small radius dr, and this is your elemental flow area, then we can write this as twice pi r into dr ok. So, from here you can see that we can write the expression for mean velocity u m as 2 by r naught ok, because area is pi r naught square ok.

And we can write these integral 0 to r naught u bar r dr. So, it will be r naught square. So, 2 by r naught square integral 0 to r naught u bar r dr.

So, in last class we have already discussed about the different turbulent layers for flow over flat plate. So, we can see that very near to the wall, we have viscous sub layer; away from the wall, we have fully turbulent layer; and in between, we have buffer layer.

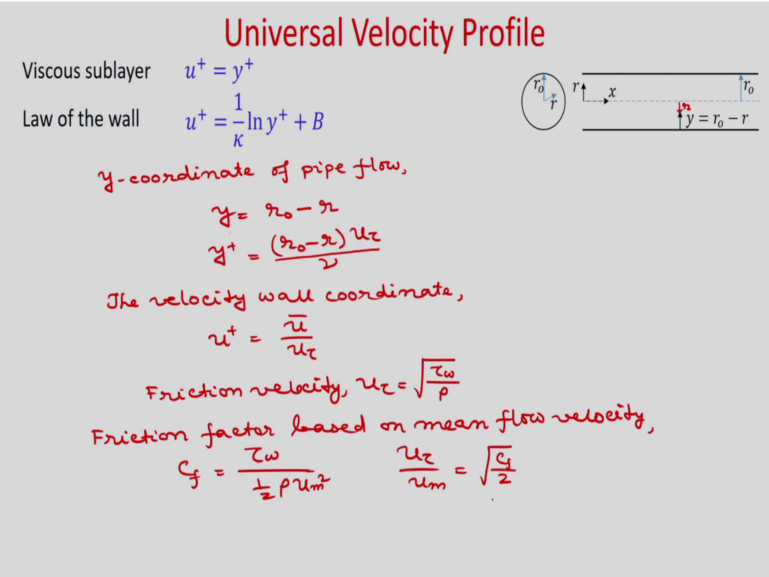
(Refer Slide Time: 08:08)



The velocity profile in a pipe is very similar to that external flow. And we even adapted a pipe flow friction factor model to analyze flow over a flat plate using the momentum integral method. The characteristic of the flow near the wall of a pipe are not influenced greatly by the curvature of the wall of the pipe.

Therefore, a reasonable start to modeling pipe flow is to invoke the two layer model that we used to model flow over a flat plate. So, you can see that for viscous sub layer, we have used this u plus is equal to y plus and law of the wall, we have used u plus is equal to 1 by kappa l n y plus plus B. So, using Prandtl's mixing layer hypothesis, we derived this law of the wall.

(Refer Slide Time: 08:57)



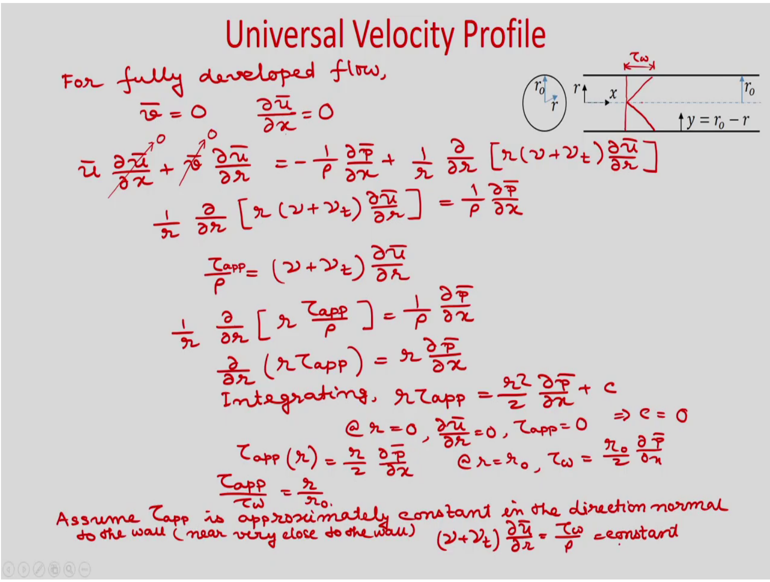
So, this let us use for the pipe flow. So, what we will now do here you can see that y is measured from the wall right. So, y, if we measure from the wall, then we can write y is equal to r naught minus r because you know that this is your radius r naught. So, at any radius r, so you can see this distance y, which is measured from the wall, it will be r naught minus r. So, now, we can write y coordinate pipe flow.

So, this we can write y is equal to r naught minus r ok. So, now, we can define y plus ok. So, we can define y plus as r naught minus r into the friction velocity u tau divided by the kinematic viscosity nu. So, and the velocity wall coordinate, we can write, so it will be u plus we know that it is u bar by u tau, where u tau is the friction velocity.

So, friction velocity u tau is equal to root tau w by rho, where tau w is the wall shear stress. So, friction factor based on mean flow velocity, we can write friction factor ok based on mean flow velocity. So, we can write c f as tau w by half rho u m square. So, from here you can see you can write u tau by u m as root c f pi 2, because tau w is u tau square into rho.

So, from there if you substitute it here, you will get u tau by u m is equal to root c f by 2. Now, whatever governing equation we have written for fully developed flow, we can simplify it.

(Refer Slide Time: 11:41)



Like say for fully developed flow, so for fully developed flow you know that v bar will be 0 right, the radial component of the velocity will be 0. And there will be no gradient of this axial velocity in the axial direction right. So, del u bar by del x will be 0 ok. So, for fully developed flow, we know that axial velocity gradient with respect to the axial direction it will be 0, so that means, the velocity profile does not vary in the axial direction.

So, if you invoke in the governing equation, what we wrote as del u bar by del x plus v bar del u bar by del r is equal to minus 1 by rho del p bar by del x plus 1 by r del of del r r nu plus nu t del u bar by del r ok. So, now, if you invoke the fully developed condition, then you can see this is 0, and this is also 0. So, from here you can see that we can write 1 by r del of del r r nu plus nu t del u bar by del r is equal to 1 by rho del p bar by del x ok.

So, now, you can see that this quantity actually we can substitute with the shear stress ok. So, because shear stress we know the tau by rho we can write as nu plus nu t del u bar by del r, which is your apparent shear stress right, this is your apparent shear stress. So, you can see that tau f. So, you can write apparent shear stress that we have written tau f by rho will be nu plus nu t del u bar by del r.

So, if you substitute it here, then what you will get we will get 1 by r del of del r. So, this quantity we are writing as r tau apparent divided by rho is equal to 1 by rho del p bar by del x. So, obviously, we are assuming incompressible flow. So, this rho rho will get cancelled.

So, we can write del of del r r tau app is equal to r del p bar by del x. Now, you can see that for a internal flows these pressure gradient del p bar by del x is constant. So, if you assume this constant, then you can easily integrate it. So, integrating this equation ok, so we can write r tau app is equal to r square by 2 del p bar by del x plus integration constant c ok.

So, now, you can see that at r is equal to 0, that means, at the central line at the central line we know that del u bar by del r is equal to 0 right, because maximum velocity will occur at the central line. So, del u bar by del r will be 0, that means, tau app will be 0, so that means, it will give c as 0 ok. So, from here you can see that you can write a tau app, which is function of r as r by 2 del p bar by del x.

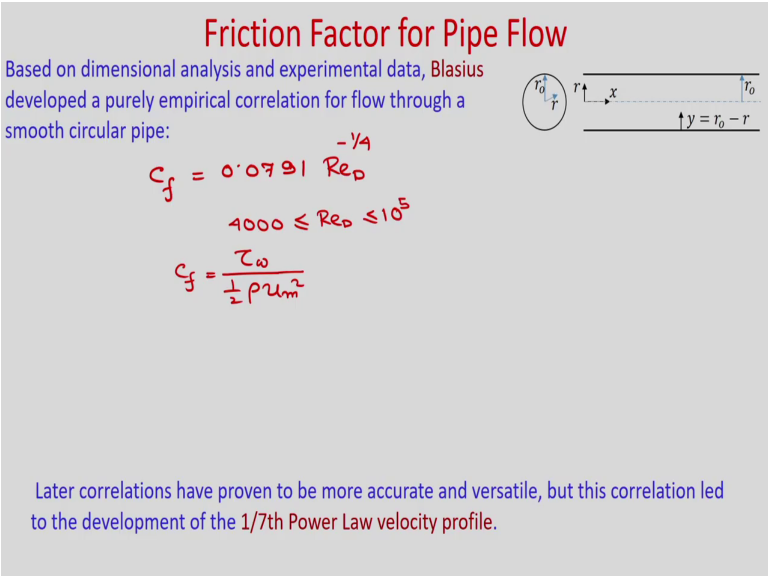
So, now, you can see that from this relation that your apparent shear stress varies linearly right with radius. So, if you see that, obviously, for laminar flow also you have seen that it will vary linearly. So, 0 will be at the central line and at the wall it will be maximum. So, if this is your tau w, then you can write at r is equal to 0 ok, at r is equal to 0 you can see it will be tau app and at r is equal to r naught, we will get tau wall as r naught by 2 del p bar by del x. So, here we can write tau f by tau w as r by r naught.

So, if you remember that when we considered flow over a flat plate near to the wall, we have assumed the viscous sub layer. And in this viscous sub layer, you know that wall shear stress remain constant ok, so that assumptions we have made when we consider flow over flat plate. And the same thing we can use for this flow over pipe flow and we can assume that very near to the surface ok in the normal direction this wall shear stress remain constant.

So, from the experimental results, it is already shown that if we assume it these velocity profile matches well with the experimental data. So, we can see that assume tau app is approximately constant in the direction normal to the wall ok. So, near to the very close to the wall ok, we can write that nu plus nu t del u bar by del r is equal to tau w by rho is equal to constant ok, so that means, the near wall behavior is not influenced by the outer flow.

So, in last class we have already discussed that based on some experimental data and dimensional analysis, Blasius proposed the correlation for the friction factor and that we used to find the hydrodynamic boundary layer thickness for the flow over flat plate.

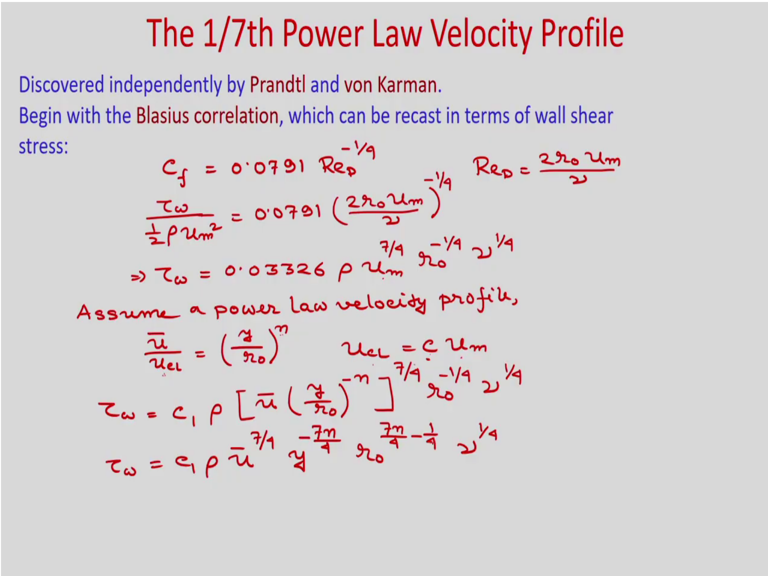
(Refer Slide Time: 19:15)



So, based on dimensional analysis and experimental data Blasius developed a purely empirical correlation for flow through a smooth circular pipe and that is your c f is equal to 0.0791 Reynolds number based on diameter to the power minus 1 by 4. And this is valid for the Reynolds number 4000 less than equal to Re D less than equal to 10 to the power 5 ok.

And we know that c f we have defined as wall shear stress divided by half into rho u m square, where u m is the mean velocity. So, later correlations have proven to be more accurate and versatile, but these correlation proposed by Blasius led to the development of the one-seventh power law velocity profile.

(Refer Slide Time: 20:13)



Discovered independently by Prandtl and von Karman, this one-seventh power law velocity profile. And begin with the Blasius correlation which can be recast in terms of wall shear stress. So, we have already written that Blasius proposed C f is equal to 0.0791 Re D to the power minus 1 by 4 ok. And Re d is 2 r naught into u m divided by nu ok. So, C f we can write tau w by half rho u m square is equal to 0.0791, and Reynolds number is twice r naught u m by nu to the power minus 1 by 4.

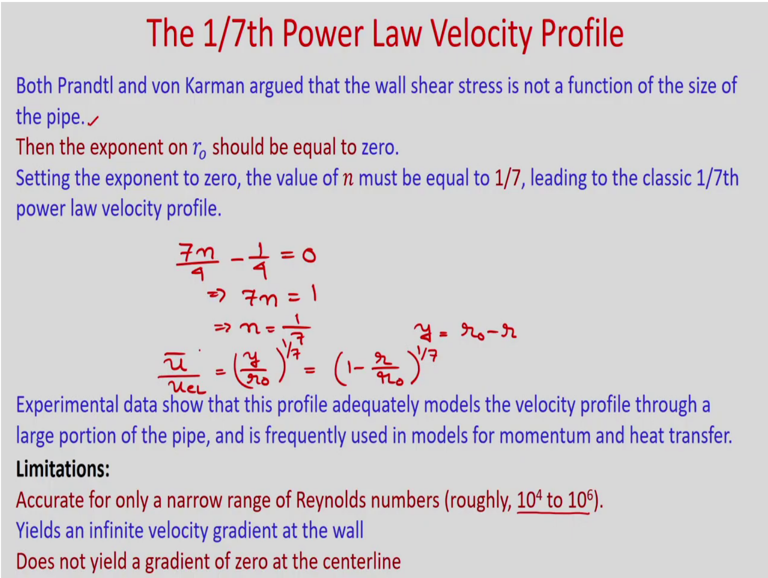
So, from here you can see tau w we can write as 0.03326 rho u m to the power 7 by 4 r naught to the power minus 1 by 4 nu to the power 1 by 4. So, let us assume a power law velocity profile. So, we can write u bar by u c l, where u C L is the velocity at the center line, and it is maximum velocity is equal to y by r naught to the power n ok.

So, now let us write that u C L is some constant c into the mean velocity u m, because we have seen that the maximum velocity or the center line velocity will be some constant into the mean velocity. So, for laminar flow you know that if it is a circular pipe, then the maximum velocity will be 2 times the mean velocity right. So, we are using some constant c here.

So, if you substitute here, then we can write you can see you u C L you write here c into u m and this u m you just substitute it here. So, you can see that tau w we can write as so these into constant c will be some C 1 rho into u bar y by r naught to the power minus n to the power 7 by 4 r naught to the power minus 1 by 4 and nu to the power 1 by 4 ok.

So, after rearranging, we can write tau w is equal to c one into rho u bar to the power 7 by 4 y to the power minus 7 n by 4, and r naught to the power 7 n by 4 minus 1 by 4, and nu to the power 1 by 4 ok. You can see whatever this exponent n we have retained, so it vary slightly with the Reynolds number, but for the correlation whatever we are using C f, so that we are writing this exponent n and now you can see that here we have this r naught to the power 7 n by 4 minus 1 by 4.

(Refer Slide Time: 24:10)



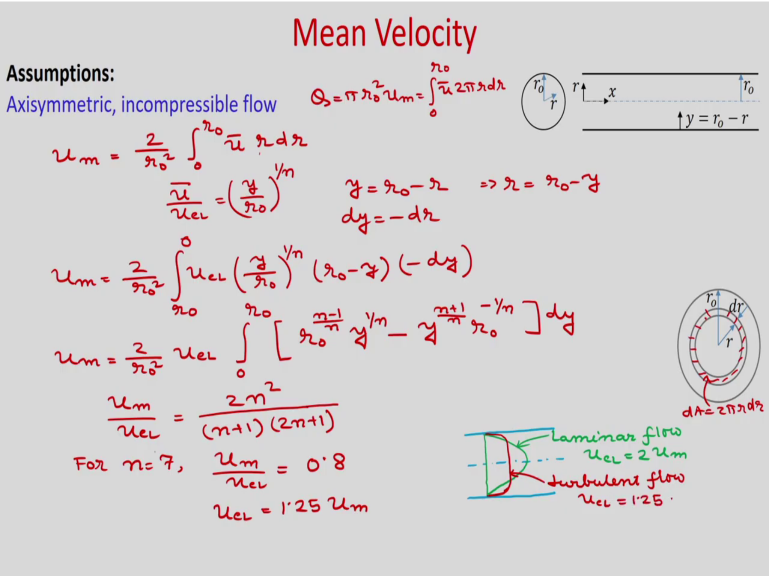
Both Prandtl and von Karman argued that the wall shear stress is not a function of the size of the pipe. So, you can see that whatever shear stress expression we have written we have the r naught. So, based on the experimental results, the shear stress is almost independent of the radius of the pipe, so that means, the exponent of r naught should be zero if this tau w is independent of r naught ok.

So, obviously, you can see that setting the exponent to 0, the value of n must be equal to 1 by 7 leading to the classic 1 7 power law velocity profile. So, you can see that 7 n by 4 minus 1 by 4 if you make it 0, then you will get 7 n is equal to 1, that means, n is equal to 1 by 7. So, now we are getting the velocity profile u bar by u C L as y by r naught to the power 1 by 7. And we know that y is r naught minus r. So, from here you can also write 1 minus r by r naught to the power 1 by 7.

Experimental data show this profile adequately models the velocity profile through a large portion of the pipe and is frequently used in models for momentum and heat transfer. But there are some limitations because it is accurate for only a narrow range of Reynolds number roughly 10 to the power 4 to 10 to the power 6 and it yields a infinite velocity gradient at the wall.

So, you can see that at y is equal to r naught actually the velocity gradient becomes infinity right. And it does not yield a gradient of 0 at the center line. And at the center line del u bar by del r should be 0, but these velocity profile actually does not yield these velocity gradient as zero. So, these are some limitations. Now, let us calculate the mean velocity based on this one-seventh power law of the velocity profile.

(Refer Slide Time: 26:33)



So, we will we have already written that u m is equal to 1 by r naught square integral 0 to r naught u bar, so it will be 2 r dr right. So, this we have already written this mean velocity right. So, now you can see that we have written that this elemental area as twice pi r into dr. And from there just equating Q is equal to pi r naught square u m is equal to 0 to r naught u bar twice pi r dr right.

With this we have written this expression. Now, we have this velocity profile u bar by u C L as y by r naught to the power 1 by n, and we have y is equal 2 r naught minus r ok. So, d y we can write as minus dr and this r from here we can write as r naught minus y ok.

So, if you write this u m, then we can write 2 by r naught square integral. So, u bar now you just replace with this u C L y by r naught to the power 1 by n ok. And this r, now, let us write r naught minus y and this dr is minus d y ok. And what will be the limits? So, you can see that at r is equal to 0 ok, r is equal to 0. So, it will be r naught, and r is equal to r naught this y limit will be 0 ok.

So, if you integrate it, you will get u m as, so u C L is constant. So, you can see that we can write 2 by r naught square u C L. So, now just we are changing the limit and this minus sign we are just involving it.

So, it will be 0 to r naught ok, and we will get r naught to the power n minus 1 by n y to the power 1 by n minus y to the power n plus 1 by n r naught to the power minus 1 by n d y ok. So, if you perform this integration and put the limits, we can write u m by u C L as twice n square divided by n plus 1 twice n plus 1 ok.

So, for n is equal to 7, one-seventh law velocity profile, this u m by u C L will be 0.8 ok. So, you can see that your center line velocity will be 1.25 times the mean velocity. So, you can see that for turbulent pipe flows your maximum velocity which is your central line velocity is 1.25 times the mean velocity. Whereas, in laminar flow, the maximum velocity or center line velocity is 2 times the mean velocity right. So, you can see that your velocity profile is flattened in the case of turbulent flows.

So, if you consider say this is the pipe, so this is the center line. So, if you have the velocity profile for laminar flow, so it will be like these ok. So, maximum velocity for laminar flow u C L will be two times the mean velocity. But in the case of turbulent flows, so it will be more flattened ok; and this is for turbulent flow, and maximum velocity will be 1.25 times the mean velocity.

In today's class, we considered the internal turbulent flows. We used the universal velocity profile which we derived for the flat plate. And just using the coordinate transformation, we have written those universal velocity profile for the turbulent pipe flow. In case of turbulent pipe flow, we have used the velocity profile which is your one-seventh law of the velocity profile.

And using the shear stress correlation proposed by Blasius, we have derived the one-seventh law of velocity profile in case of turbulent pipe flow. Then we have calculated the mean velocity for this pipe flow case in general. Then later for one-seventh law of velocity profile we have calculated the ratio of mean velocity to the center line velocity as 0.8. And you can see that the maximum velocity in the case of turbulent pipe flow is 1.25 times the mean velocity.

Thank you.