

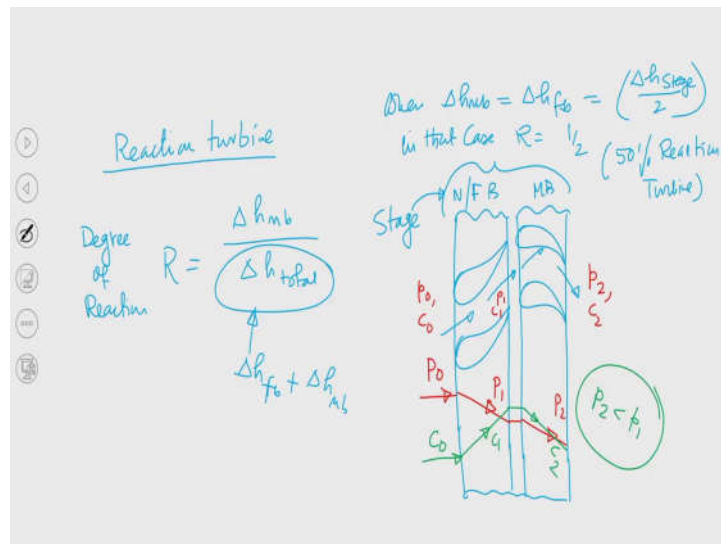
Applied Thermodynamics
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Steam Power System
Lecture - 15

Reaction Steam Turbine: Velocity Diagram, Work Transfer, Blade Efficiency

I welcome you all to this session of Applied Thermodynamics and today, we shall discuss about the Velocity Diagrams, Work Transfer and then finally, discuss about the Efficiency of a Reaction Steam Turbine. So, if you try to recall in the last class, we have discussed about reaction turbine and we have tried to discuss the difference between the reaction turbine and impulse turbine.

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And we have also introduced one term that is degree of reaction. And from there, the relative magnitude of the enthalpy drop in the moving blade leads to the classification of different types of reaction turbine.

So, now, you know that we had defined this degree of reaction $R = \frac{\Delta h_{mb}}{\Delta h_{total}}$. So, in a way

this quantity signifies the fraction of the enthalpy drop in the moving blades.

So, this $\Delta h_{total} = \Delta h_{mb} + \Delta h_{fb}$. So, basically when $\Delta h_{mb} = \Delta h_{fb}$, in that case $R = 0.5$.

So, that is 50 percent reaction turbine. Now, today, we shall try to understand that what will be the velocity triangle at the inlet and velocity triangle at the outlet for the 50 percent reaction turbine.

Now, let us draw the blading, So, this is moving blades and this is fixed blades and these two constitutes together to form a stage. Now, in case of the reaction turbine, the pressure drop takes place both in the fixed blades and as well as in the moving blades.

So, if we try to draw the velocity and pressure distribution. So, pressure falls, then again slight pressure drop. So, this is p_0 , this is p_1 and this is p_2 . So, basically this is p_0, C_0 . So, this is p_2, C_2 and this is p_1, C_1 . So, that is at the inlet of the moving blades or exit of the fixed blades.

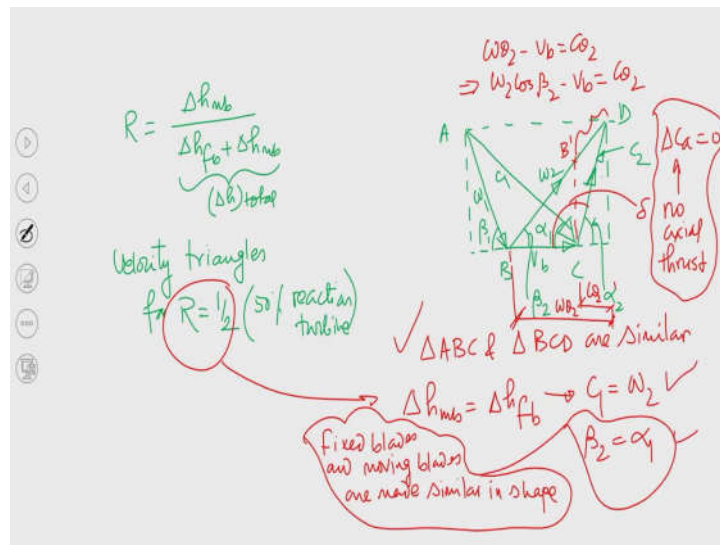
Now, question is if we try to draw the velocity distribution using different color, say this is C_0 and then, this is C_1 and then, we will be having pressure drop. So, this is C_2 . So, try to understand that there is a pressure drop both in the fixed blades as well as in the moving blades right.

Also, velocity increases while steam is passing through the fixed blades and that is what is needed because steam eventually will strike the moving blades in the form of a jet. Also, there is a change in pressure drop as a result of which there will be slight increase in the kinetic energy.

So, that is what is very important to understand, when steam is passing through the passage between two consecutive blades in the moving blade row, then pressure falls that is seen from this pressure distribution. So, p_2 is less than p_1 .

So, accounting for this pressure drop there will be an increase in the kinetic energy of steam as it passes through the row of moving blades and that is why the shape of the moving blades are in the nozzle shape. Since, blades are of the nozzle shape, so there will be a slight increase in the kinetic energy of the steam even when the steam is passing through the moving blades. So, now, let us draw the velocity triangles and then, it will be clear.

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So, $R = \frac{\Delta h_{mb}}{\Delta h_{total}}$. Now, we are trying to draw velocity triangles for $R = 0.5$. So, this is v_b

that is blade velocity and this is the inlet velocity diagram. Now, this angle is α_1 and this angle is β_1 that is what we have seen in the last class. Now, this is inlet triangle. What about the outlet velocity triangles? This angle is α_2 and this angle is β_2 and say this is D. So, this angle is δ .

If you try to recall for the impulse turbine, the relative velocity was less say this is v'_b . But now the relative velocity is higher that is from B' to D.

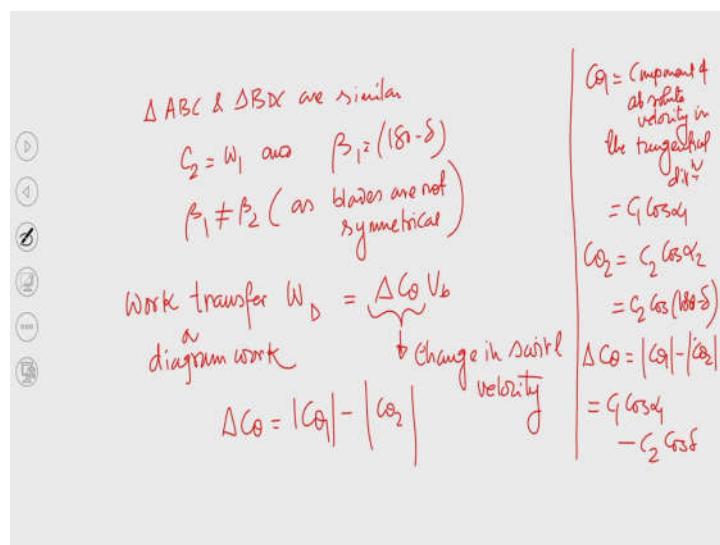
See in case of a reaction turbine, impulse action is there because of this change in momentum as the jet is having a change in its direction.

So, when steam in the form of a jet is striking the moving blades, it is getting deflected. And because of this change in direction, momentum change will be there. So, impulse effect is there. On top of that the absolute velocity leaving the first row of moving blades will be even higher than the absolute velocity which is entering into the moving first row of fixed blades. And since, their absolute velocity is still higher and blade velocity is fixed, so the resultant effect relative velocity will be higher. So, because of the relative increment in the kinetic energy as the steam passes through the row of moving blades a reaction force is there. So, when the steam is coming out from the moving blades, a reaction force is impressed by the jets on the blades in the opposite direction.

So, you can see these two velocity triangles are similar. So, ΔABC and ΔBCD are similar. We are having 50 percent reaction turbine, we can write $\Delta h_{mb} = \Delta h_{fb}$; $C_1 = w_2$ relative velocity will be higher.

So, because of these higher C_2 , this amount of relative velocity should be higher and since, the degree of reaction is such that enthalpy drop in the moving blade is equal to enthalpy drop in the fixed blade. So, $C_1 = w_2$ and also, $\beta_2 = \alpha_1$. This is coming from the fact that fixed blades and moving blades are made similar in shape. As I told that moving blades and fixed blades are geometrically similar in shape ok. So, $\beta_2 = \alpha_1$,

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Again, I am writing ΔABC and ΔBDC are similar. So, $C_2 = w_1$ and $\beta_1 = \alpha_2 = 180 - \delta$. Now, question is this $\beta_1 \neq \beta_2$ as blades are not symmetrical. So, had the relative velocity been like in case of an impulse turbine, we would not have this extra component. But here, we are having this additional you know amount additional magnitude of this velocity, as a result of which you can see the $\Delta C_a = 0$. So, there is no any axial thrust. So, it implies no axial thrust.

We have seen in the context of impulse turbine that because of the difference in the magnitude of the relative velocity components, we had axial thrust. But here, we really do not have axial thrust because relative velocity at the exit of the moving blade is equal to the relative velocity at the inlet of the moving blade. So, there is no axial thrust, but there will be axial thrust that we really cannot ignore. .

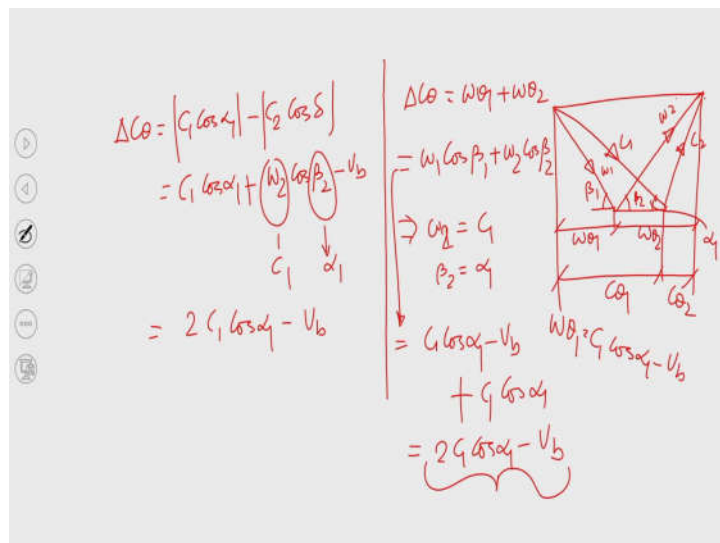
Why this axial thrust will be there? So, when steam is passing through the row of moving blades, pressure will slightly change and that you can see from this diagram. So, because of this reduction in pressure, slight axial thrust will be there.

So, though, theoretically there is no axial thrust; but in real practice, as the steam passes through the row of moving blades, there is a reduction in pressure and because of that we will be having slight thrust.

Now, we need to understand what is work transfer? So, work transfer $w_D = \Delta C_\theta v_b$. So, this ΔC_θ is nothing but the change in swirl velocity and $\Delta C_\theta = |C_{\theta 1}| - |C_{\theta 2}|$; $C_{\theta 1}$ is nothing but the component of absolute velocity in the tangential direction. So, that is $C_1 \cos \alpha_1$.

Similarly, $C_{\theta 2} = C_2 \cos \delta$. So, if we plug in the value, $\Delta C_\theta = C_1 \cos \alpha_1 - C_2 \cos \delta$. So, now, question is what is $C_2 \cos \delta$?

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So, $\Delta C_\theta = C_1 \cos \alpha_1 - C_2 \cos \delta$. So, $C_2 \cos \delta = w_2 \cos \beta_2 - v_b$. This is from $w_{\theta 2} - v_b = C_{\theta 2}$.

So, this is $w_2 \cos \beta_2 - v_b = C_{\theta 2}$.

So, now, if we plug in the value over there, then $\Delta C_\theta = |C_1 \cos \alpha_1| - |C_2 \cos \delta| = C_1 \cos \alpha_1 + w_2 \cos \beta_2 - v_b$. So, we know that $C_1 = w_2$ and $\beta_2 = \alpha_1$.

So, it will be coming $2C_1 \cos \alpha_1 - v_b$. We can simply write from this velocity triangle also, $\Delta C_\theta = w_{\theta_1} + w_{\theta_2} = w_1 \cos \beta_1 + w_2 \cos \beta_2$

So, I write here $C_1 = w_2$ and $\beta_2 = \alpha_1$; then, $\Delta C_\theta = C_1 \cos \alpha_1 - v_b + C_1 \cos \alpha_1 = 2C_1 \cos \alpha_1 - v_b$

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Work transfer $W_D = \Delta C_\theta v_b$
or
Diagram work $= (2C_1 \cos \alpha_1 - v_b) v_b$

Diagram efficiency }
or
Blading efficiency } $\eta_D = \frac{(2C_1 \cos \alpha_1 - v_b) v_b}{\text{Energy input}}$

So, now, if we quickly write work transfer or diagram work equal to $\Delta C_\theta v_b$. So, this is $(2C_1 \cos \alpha_1 - v_b) v_b$. Now, we need to find out diagram efficiency or blading efficiency that is η_D . So, that is $(2C_1 \cos \alpha_1 - v_b) v_b$ divided by energy input to the blades in the form of a steam jet. So, now, what is energy input?

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Energy input per kg steam flow (per unit mass flow of steam) = $\frac{C_1^2}{2} + \frac{1}{2}(w_2^2 - w_1^2)$ due to the change in relative velocity

Velocity triangle diagram showing w_1 , C_1 , v_b , α_1 , and β_1 .

$$= \frac{C_1^2}{2} + \frac{C_1^2}{2} - \frac{w_1^2}{2} \quad [w_2 = C_1] \text{ velocity}$$

$$w_1^2 = C_1^2 + v_b^2 - 2C_1v_b \cos \alpha_1$$

$$= \frac{C_1^2 - v_b^2 + 2C_1v_b \cos \alpha_1}{2}$$

So, energy input per unit mass flow of steam is nothing but $\frac{C_1^2}{2} +$ for the impulse turbine, we have seen this is only the energy input; but in case of a reaction turbine, on the top of that we are also having the reaction force and that reaction force is developed only because of the reduction in relative velocity.

So, basically when the steam is passing through the row of moving blade, they are changing kinetic energy and that change at the cost of the pressure drops. So, basically that leads to a reaction to the blades in the opposite direction.

So, that is nothing but $\frac{C_1^2}{2} + \frac{1}{2}(w_2^2 - w_1^2)$. So, if we look at the velocity triangles, here this additional amount of the relative velocity that was not there for the impulse turbine.

Why this increment relative velocity is there because C_2 will increase and as v_b is remaining same, for increase in C_2 the relative velocity will increase. So, as the shape of the moving blades are the nozzle shape, as a result of which when steam is passing through the row of moving blades kinetic energy will increase.

So, the w_2 will increase now since w_2 will be higher than w_1 , as a result of which will be getting reaction force.

So, when we are trying to obtain the input energy, we need to consider the change in relative velocity as well. As $C_1 = w_2$, so, it will be $C_1^2 - \frac{w_1^2}{2}$. From the inlet velocity triangle, $w_1^2 = C_1^2 + v_b^2 - 2C_1v_b \cos \alpha_1$

So, if we plug in the value over here, we will be getting $\frac{C_1^2 - v_b^2 + 2C_1v_b \cos \alpha_1}{2}$. So, this is the expression of the input energy. So, if we try to plug in the value there, then we will be getting efficiency.

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The image shows a handwritten derivation of the efficiency η_D and the optimum velocity ratio. The derivation starts with the efficiency formula:

$$\eta_D = \frac{2(2C_1 \cos \alpha_1 - v_b)v_b}{C_1^2 - v_b^2 + 2C_1v_b \cos \alpha_1}$$

This is then simplified to:

$$\eta_D = \frac{2v_b^2 \left(\frac{2C_1 \cos \alpha_1}{v_b} - 1 \right)}{C_1^2 \left(1 - \frac{v_b^2}{C_1^2} + 2 \frac{v_b}{C_1} \cos \alpha_1 \right)}$$

The velocity ratio is defined as $v_r = \frac{v_b}{C_1}$. Substituting this into the efficiency formula gives:

$$\eta_D = \frac{2 \left(\frac{2 \cos \alpha_1}{v_r} - 1 \right) v_r^2}{(1 - v_r^2 + 2v_r \cos \alpha_1)}$$

To find the optimum velocity ratio, the derivative of efficiency with respect to the velocity ratio is set to zero:

$$\frac{d\eta_D}{dv_r} = 0$$

This leads to the optimum velocity ratio:

$$v_{r, \text{opt}} = C_1 \cos \alpha_1$$

Substituting this optimum velocity ratio back into the efficiency formula yields the maximum efficiency:

$$\eta_{D, \text{max}} = \frac{2 \cos^2 \alpha_1}{1 + \cos^2 \alpha_1}$$

$$\text{So, } \eta_D = \frac{2(2C_1 \cos \alpha_1 - v_b)v_b}{C_1^2 - v_b^2 + 2C_1v_b \cos \alpha_1} = \frac{2v_b^2 \left(\frac{2C_1 \cos \alpha_1}{v_b} - 1 \right)}{C_1^2 \left(1 - \frac{v_b^2}{C_1^2} + 2 \frac{v_b}{C_1} \cos \alpha_1 \right)}$$

So, we are defining velocity ratio, $v_r = \frac{v_b}{C_1}$. So, if we write this, then we can write

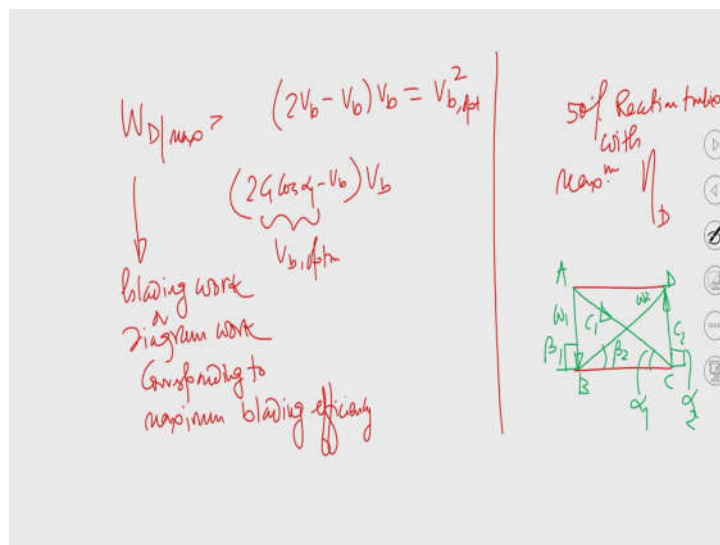
$$\eta_D = \frac{2v_r^2 \left(\frac{2 \cos \alpha_1}{v_r} - 1 \right)}{(1 - v_r^2 + 2v_r \cos \alpha_1)}$$

So, now, if we try to find out optimum speed ratio, optimum speed ratio or velocity ratio for which η_D is maximum. So, we need to do $\frac{d\eta_D}{dv_r} = 0 \Rightarrow v_{r,opt} = C_1 \cos \alpha_1$ and

$$\eta_{D,max} = \frac{2 \cos^2 \alpha_1}{1 + \cos^2 \alpha_1} .$$

So, objective is by defining velocity ratio, we can try to find out optimum velocity ratio for which the diagram efficiency or blading efficiency will be maximum and that is shown here and for that $V_{optimum}$, we can find out the maximum value efficiency.

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So, this is very important. Now, question is what will be w_D maximum? That will be $w_{D,max} = (2C_1 \cos \alpha_1 - v_b)v_b = v_{b,opt} \times v_b = v_b^2$. So, this is the blading work or diagram work corresponding to maximum blading efficiency. Now, if we try to draw the velocity triangles for this 50 percent reaction turbine, with maximum blade efficiency, then it will be like this; it will be like this. So, this is the velocity triangle at the inlet.

To summarize today's discussion, so we have discussed about the velocity triangles for the 50 percent reaction turbine, we have discussed why the extra relative velocity component is there.

From there, we have tried to quantify the you know work transfer or diagram work and finally, we have established the efficiency and again, from the efficiency by quantifying

the velocity ratio, we have expressed the optimum velocity ratio and finally, for the optimum velocity ratio, we have calculated maximum efficiency as well as the maximum work transfer. So, with this, I stop here today and we shall continue our discussion in the next class.

Thank you.