Applied Thermodynamics
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Steam Power System
Lecture - 16
Steam Nozzle: Analysis and Efficiency

I welcome you all to the session of Applied Thermodynamics. And today, we shall start discussing on the Steam Nozzle. And, we will see that though in the schematic diagram we did not show this particular device or equipment, but it is again an important device for the efficient and smooth operation of the steam power plant.

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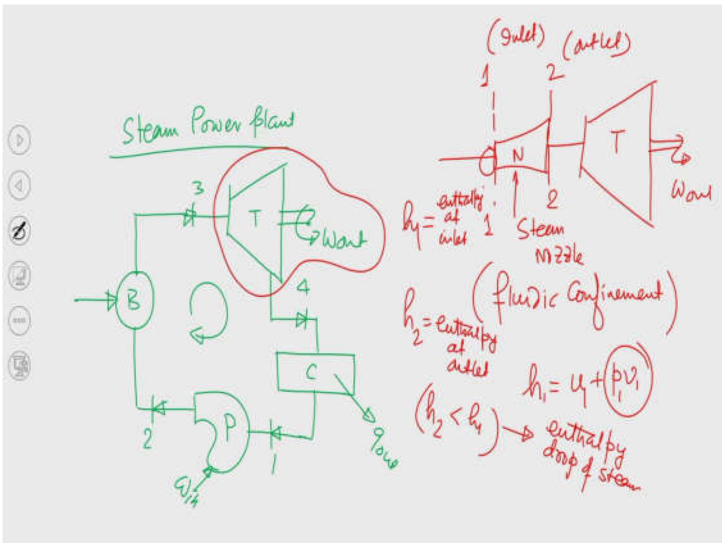

So, if we try to draw the schematic diagram of the steam power plant; there is a boiler and this is turbine, and after doing work steam leaves from the turbine, it is taken to this heat sink wherein upon rejecting heat, the working fluid is again taken for the next cycle.

See, as we have discussed that there are two different types of steam turbine, impulse and reaction. In impulse turbine, the steam before it enters into the turbine, entire pressure drop takes place in the nozzle, while for the reaction turbine pressure drop takes place both in the nozzle or the fixed blades as well as in the moving blades.

So, question is when steam is getting generated inside the boiler, that steam is not directly allowed to go into the turbine for the work output, that is steam will be allowed to go into the turbine, but before entering into the turbine steam will be taken through the steam nozzles.

So, now question is, if we try to draw this particular device, so steam will be taken through the nozzle, and then it is allowed to go to the steam turbine, W_{out}. So that means, we have seen that steam is allowed to pass through the nozzle before it is allowed to strike the blade of the turbine.

And while here it is striking the turbine blade, it should have high kinetic energy. To ensure that the steam before it strike the turbine blades, it will be having high kinetic energy.

Now, and to ensure that the kinetic energy will be high it is essential that the steam will pass through this fluidic confinement and that is nozzle. So, this is an important or special fluidic confinement through which when steam is allowed to pass, thermodynamic properties will be altered, will be changed, right. So this is inlet, so this is 1-1 and this is 2-2, outlet. Now, question is, at the inlet thermodynamic property of steam is not equal to the thermodynamic properties at the outlet of the nozzle.

So, we have studied that there is enthalpy drops of steam while passing through the flow nozzle. So, enthalpy is h_1 at the inlet and this is h_2 at the outlet. We have studied $h = u+pv$.

So, this is nothing, but the internal energy and this is the flow energy that is when steam is allowed to flow through the nozzle, the energy required to maintain the flow in presence of pressure at section 1 that is nothing but this flow work or energy. We have studied first law of thermodynamics, so heat and work are the two different forms of energy.

So, when steam is entering into the nozzle, we need to maintain the flow, and to maintain the flow in presence of pressure, we need to invest energy and that is nothing but work, so that is pv. See, so you know that there is a enthalpy drop. So, $h_2 < h_1$ signifying the enthalpy drop of steam.

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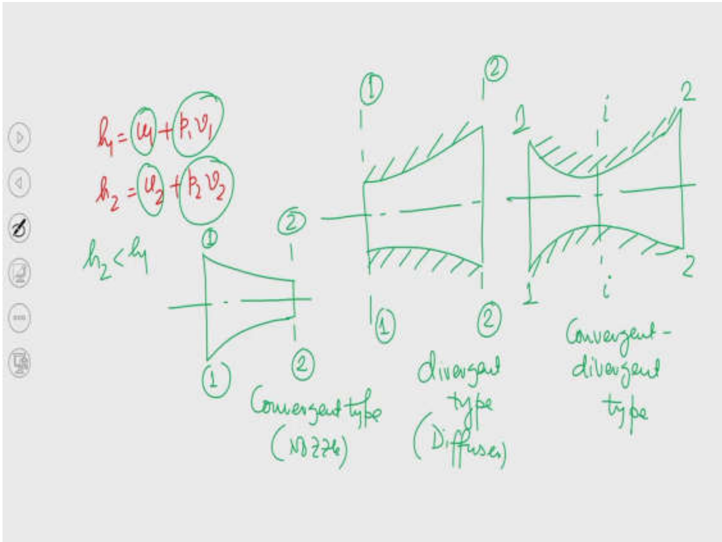

So $h_1 = u_1 + p_1v_1; h_2 = u_2 + p_2v_2$. See, $h_2 < h_1$, but, we cannot change u_2. Since temperature is an important quantity to indicate the internal energy and in fact we can assume that when steam is flowing through the nozzle; we should not allow temperature to fall. So, we should not allow u_1 and u_2, since our objective should be not to reduce temperature, but we can play with this quantity, pv. So, you have studied from fluid mechanics, so this is a gradually increasing cross sectional area. It may be also like gradually decreasing area, then, constant and then gradually increasing area.

So, we can play with the pressure and velocity such that $h_2 < h_1$, but keeping $u_1 = u_2$. So, when steam is passing through this section, the area is reducing which in turn will allow velocity to increase at the cost of the reduction in pressure.

Now, by reducing the pressure, we can increase velocity and as I told you objective should be not to reduce temperature, and that is why steam nozzles are insulated. But, in reality it is very difficult that there will not be any temperature or heat transfer from the nozzle surface to the ambient.

So, we can see from this is that we have to play with the pressure and velocity in such that the kinetic energy of steam leaving the nozzles should be high, so that it can strike the steam blade you know steam turbine blades and there will be change in momentum.

Now, commonly divergent type fluidic confinement is known as diffuser, while the convergent type fluidic confinement is nozzle, so in the direction of flow if area gradually decreases that is commonly known as nozzle. In the direction of flow, if area gradually increases it is commonly known as diffuser.

And we may have another fluidic confinement that is purely convergent type and it is commonly known as nozzle, but still we may have convergent divergent type nozzle. So, you can see our objective is to increase velocity and we will be playing with p and v. So, why we need to go for the nozzle analysis? So, what could be the length of this fluidic confinement? What could we the area at the outlet of the fluidic confinement such that, the velocity at the exit of the nozzle will be the desired one.

So, basically we need high kinetic energy at the exit of the nozzle, so to ensure that the velocity of steam at the exit of the nozzle will be sufficient for the optimum work output. So, that is why we need to go for the analysis from the design point of view.

So, at the exit as we are not interested in losing temperature, so pressure and velocity will change. And at the outlet what could be the pressure, and since at the cost of the reduction in pressure velocity will increase, that is important. So, we need to go for the analysis, so that we can understand what will be the cone angle, what will be the cross sectional area at the outlet and inlet, so on. So, now, we will go for the flow through the steam nozzle.

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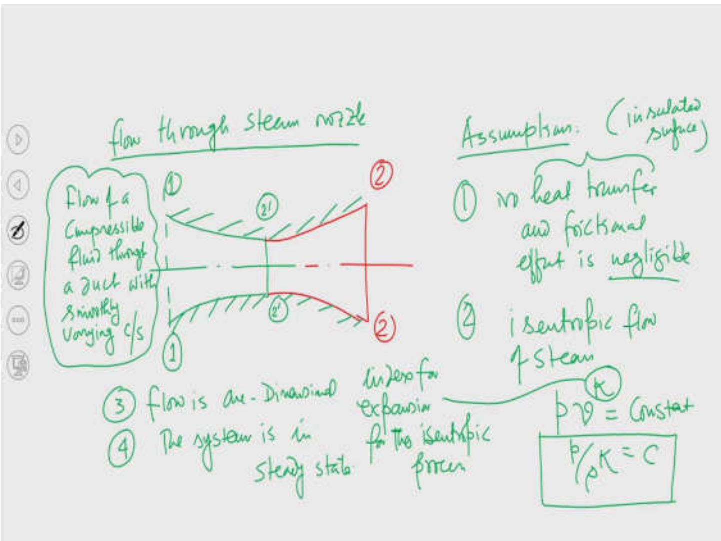

Say, while we are trying to understand the variation of pressure and velocity, when steam is passing through this fluidic confinement essential to understand the magnitude of exit velocity, we need to take a few assumptions.

First one is we can consider that there is no heat transfer, and frictional effect is negligible. It is very difficult to ensure that frictional effect will not be there, but it is negligible. So, there is no heat transfer, so it is insulated. Outer surface of the nozzle is insulated.

So, internal friction that is friction between the fluid layers, and external friction that is the friction between solid surface and the fluid is negligible.

We can assume this is isentropic flow of steam and it can be represent $pv^k = C$. So, k is known as index of expansion for the isentropic process. In thermodynamics, typically we consider specific volume, it is easily obtainable from the property chart, but still we can write $p/\rho^k = C$. So, we are considering the flow of steam through the nozzle is isentropic. It is not bad assumptions as long as assumption 1 is justified. So, that there is no heat transfer, frictional effect is negligible, so we can assume that the flow is internally reversible. So, as long as there is no heat transfer from the nozzle surface to the ambience, so external irreversibility is not there. And if we assume that the frictional effect is negligibly small, so it can be assumed that the flow is internally reversible.

A flow attaining both internally reversible and externally reversible, and no heat transfer we can assume that is isentropic flow, and it can be you know mathematically described by this equation. As I told you that in thermodynamics, we are interested in this specific volume, but I can write this $p/\rho^k = C$ because as I told you we are much more interested in the variation of pressure and velocity with a change in area of the cross section.

Since, it is thermodynamic analysis it is compressible fluid flow, so we need to know equation of state. So, I am trying to relate all those with ρ. And another important is we can assume flow is one-dimensional. And the system is at steady state.

So, we consider the flow of the compressible fluid through a duct which is having initially decreasing area then increasing area. So, instead of writing decreasing initially, then the increasing later, I can write with variable area.

So, now we will write continuity equation because you have studied in fluid mechanics, continuity will be satisfied between section 1-1 and section 2-2.

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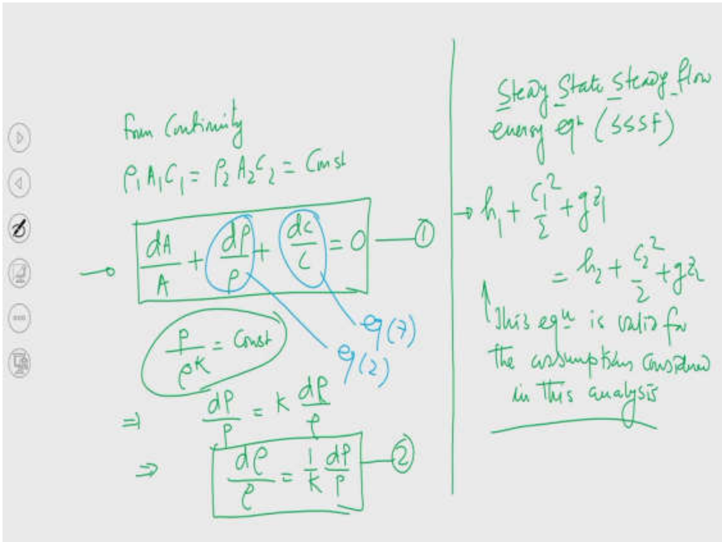

Then, we can write from continuity that $\rho_1 A_1 C_1 = \rho_2 A_2 C_2 = const$.

Now, if we take log, and differentiate we can write $\frac{dA}{A}+\frac{d\rho}{\rho}+\frac{dC}{C}=0$ (1).Now, since $\frac{p}{\rho^k}=const$, just by differentiating it we can write $\frac{dp}{p}=k\frac{d\rho}{\rho} \Rightarrow \frac{d\rho}{\rho}=\frac{1}{k}\frac{dp}{p}$ (2). Here we need to do write ρ in terms of P. We are essentially trying to find out the area velocity relationship. As I told you, our objective is to find out the velocity at section 2-2. So, I have taken varying fluidic confinement which is having varying cross sectional area. Now, since we are interested in the velocity and change in pressure. So, as I told you that internal energy, we are going to consider that will remain almost same.

Now, you have studied an important equation that is called steady state steady flow equations, SSSF.

So, I am writing that between section 1 and 2, $h_1+\frac{C_1^{\,2}}{2}+gz_1=h_2+\frac{C_2^{\,2}}{2}+gz_2$. See, there is no heat interaction and work transfer and also there is change in internal energy.

So, now this fluidic confinement is a plane and there is no elevation defines between these two sections. So, now, if we do so, then we can say that the length of the fluidic duct or confinement is small, such that z_1 almost equal to z_2.

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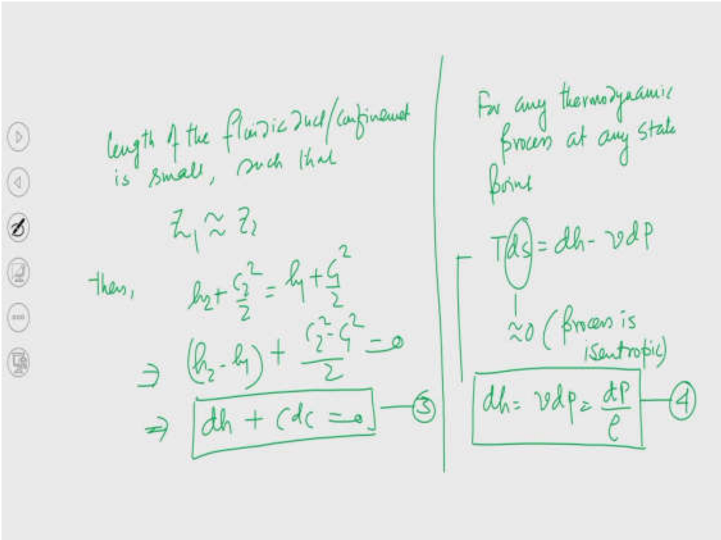

And in that case we can write $h_2 + \frac{C_2^{\,2}}{2} = h_1 + \frac{C_1^{\,2}}{2} \Rightarrow h_2 - h_1 + \frac{C_2^{\,2} - C_1^{\,2}}{2} = 0$. So, we can write this in the differential form, $\Rightarrow dh + CdC = 0$ (3).

In previous equation I am trying to relate $\frac{dA}{A}$ in terms of pressure and velocity. I can relate $\frac{d\rho}{\rho}$ with the pressure that is equation 2. I have to relate $\frac{dC}{C}$, again with in terms of pressure and velocity. Since, A is the cross sectional area, so we can have an information which will be very much useful to the designer to know what will be the length of the duct, area at the inlet and outlet of the duct, so that the velocity of steam at the exit of the nozzle will be sufficient to ensure that it will strike the turbine blades at a design value of the kinetic energy.

So, try to understand, can I relate dh in terms of any other known quantities? So, basically enthalpy can be measured from the measurable quantities. So, these are not the directly measurable quantity, but this can be measured by knowing the value of other measurable quantities.

So, again I am telling enthalpy, entropy these are not the directly measurable quantities, wherein you can directly measure pressure temperature etc. So, by measuring pressure temperature we can quantify the value of enthalpy. So, you know that for any thermodynamic process you have studied, we can write the property relation, $Tds = dh - vdp$. You have studied the property relation $Tds = du + pdv$, this is first T ds and second equation $Tds = dh - vdp$. This equation can be written for any process at state point. So, now since the process is isentropic, $ds = 0$.

So, here employing the assumptions that the process is isentropic, we can write $dh = vdp = \frac{dp}{\rho}$ (4). See, our objectives should be to relate dh in terms of known quantities and the kwon quantity is pressure and velocity.

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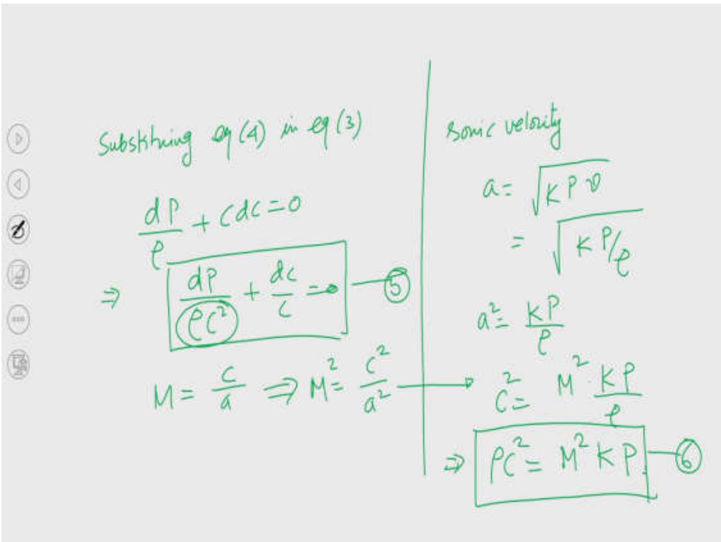

So, now, if we try to substitute equation 4 in equation 3, we can write $\frac{dp}{\rho} + CdC = 0$.

$\Rightarrow \frac{dp}{\rho C^2} + \frac{dC}{C} = 0\,(5)$.

Now, we know Mach number is nothing but $M = C/a$; velocity of fluid passing through any section at a particular temperature and pressure to the local sonic velocity of the fluid at the same section. So, you can write $M^2 = C^2/a^2$. We can write sonic velocity $a = \sqrt{kpv} = \sqrt{\frac{kp}{\rho}}$. So, $a^2 = \frac{kp}{\rho}$.

So, we can write $C^2 = M^2 \frac{kp}{\rho} \Rightarrow \rho C^2 = M^2 kp$ (6). So, k is nothing but the index of the expansion for the isentropic process.

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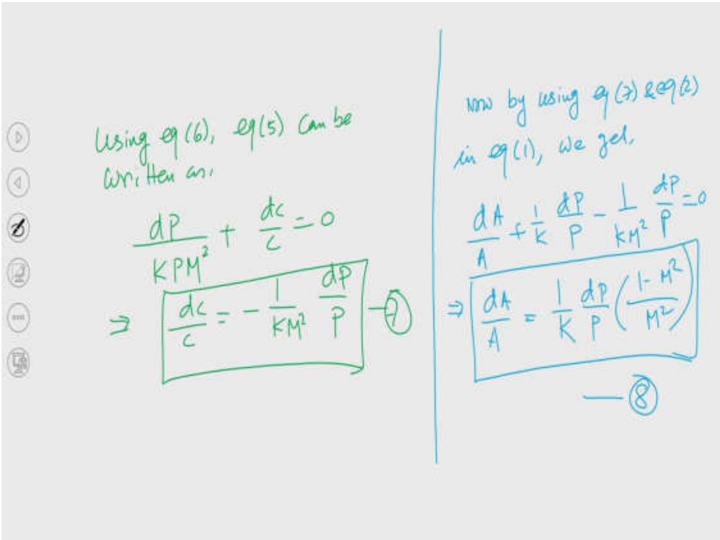

So, if we try substituting equation 6 in equation 5, $\frac{dp}{kpM^2} + \frac{dC}{C} = 0 \Rightarrow \frac{dC}{C} = -\frac{1}{kM^2}\frac{dp}{p}$ (7). So, now, if we use *dC/C* from equation 7, *dρ/ρ* from equation 2; what will be the final expression of equation 1?

So, we will be getting $\frac{dA}{A} + \frac{1}{k}\frac{dp}{p} - \frac{1}{kM^2}\frac{dp}{p} \Rightarrow \frac{dA}{A} = \frac{1}{k}\frac{dp}{p}\left(\frac{1-M^2}{M^2}\right)$(8).

In fact, this equation is known as area velocity equation for the nozzle. So, basically this equation will give you an indication that by suitably designing the nozzle; that means, by changing the area at different sections, we can play with pressure and velocity of any particular section essentially to ensure that the velocity of steam at the exit of the nozzles should be sufficient to ensure desired work output from the turbine.

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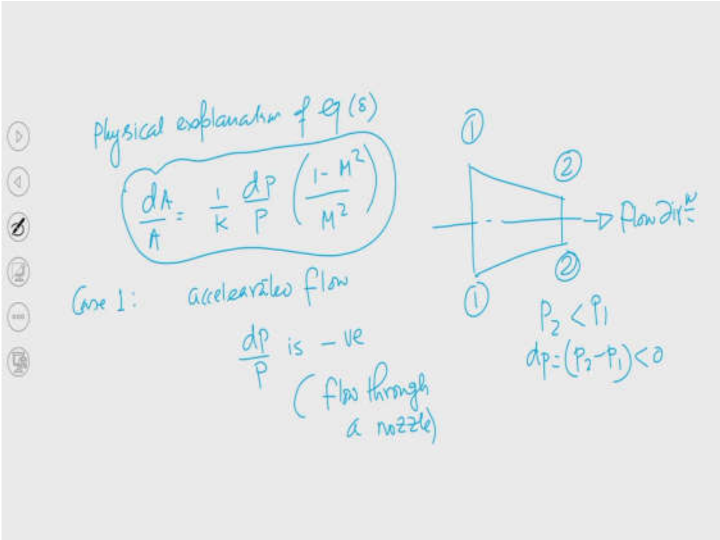

And now, I will be discussing you know two important cases. So, let us discuss the physical explanation of equation $\frac{dA}{A}=\frac{1}{k}\frac{dp}{p}\left(\frac{1-M^2}{M^2}\right)$.

So, the equation that we have established today that is generic equation that can be applied between any two sections when a compressible fluid is flowing through a duct having smoothly varying cross section.

So, If we now consider case one that is accelerated flow that means in the direction of flow velocity will increase, if velocity increases pressure will fall so, that is $p_2 < p_1$, therefore, $dp = p_2 - p_1 < 0$. So, if it is accelerated flow, you can see that dp/p is negative, so basically that is called flow through a nozzle.

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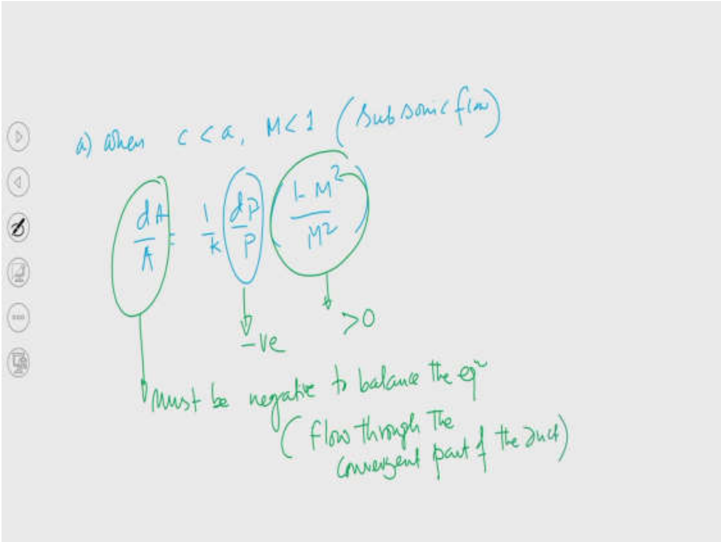

So, I will be discussing a few sub cases. Say, case a, when C is less than a, that is Mach number less than 1, either subsonic or it may be incompressible flow also, so maybe subsonic flow.

In that case you know if we try to write the equation $\frac{dA}{A}=\frac{1}{k}\frac{dp}{p}\left(\frac{1-M^2}{M^2}\right)$. So, this fellow *dp/p* is negative. If M less than 1, then $\frac{1-M^2}{M^2}$ will be greater than 0. So, to balance this equation *dA/A* must be negative. It indicates flow through a nozzle.

So, if we consider generic cross section for the present analysis when *dA/A* less than equal to 0 that is it indicates flow through the convergent part of this fluidic confinement. So, this is this indicates flow through the convergent part of the duct.

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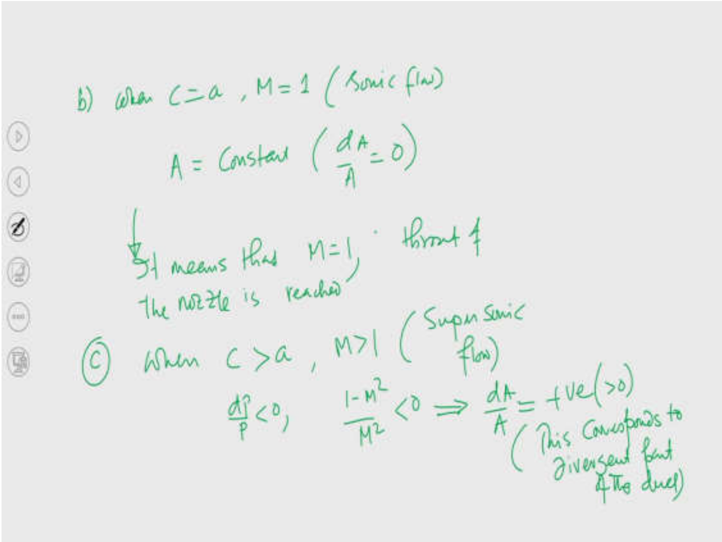

Now, let me discuss when C = a that is M = 1, sonic flow. If that is the case then A will be equal to constant as *dA/A* = 0. That means, when M = 1, throat of the nozzle is reached.

So, try to understand, when A is equal to constant that is a particular portion of the duct or the confinement, wherein there is no change in the area of the cross section. So, fluid flow is approaching or passing through the throat of the nozzle.

Now, next is when C > a that is Mach number > 1, say either supersonic or hypersonic flow. So, say this is supersonic flow.

In that case, *dp/p* < *0*, but when M > 1 then $\frac{1-M^2}{M^2}$ is also less than 0. So, it implies that *dA/A* must be positive. So, this is known as diverging part of the duct.

So, you can see this is an important equation. From this equation I have just tried to explain the physical significance of this equation in the context of the flow.

So, by doing this exercise, we have seen that for different local flow velocity we can see that which part will corresponds to which part of this fluidic confinement.

So, initially it was the convergent part of the duct, when M equal to 1 that is when local flow velocity is equal to the local sonic velocity, then this part corresponds to the throat of the nozzle. While, it is supersonic flow, in that case *dA/A* must accommodate in such a way that this part has to be the divergent part of the fluidic device or confinement.

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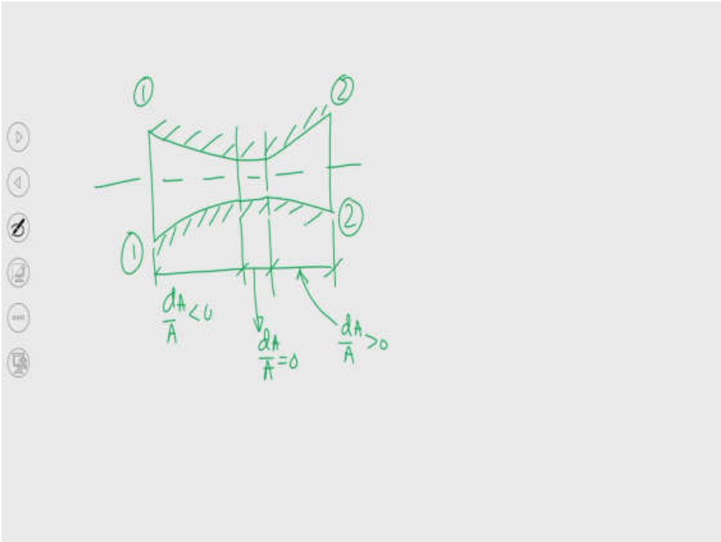

So, if we try to understand that initially we will be $dA/A < 0$, $dA/A = 0$, and this part $dA/A > 0$. So, this is very important equation which relates flow velocity with the area of the fluidic confinement, and it is very important to the designer to design the nozzle.

So, if we consider this nozzle is insulated. There is no heat loss to the surroundings from the flowing fluid. And if we consider internal friction and external friction are negligibly small, and if this is section 1-1 and this section is 2-2, by establishing this important equation we could establish that when there is a flow of compressible fluid through this duct having variable cross sectional area; so, when velocity is less than the local sonic velocity, in that case the flow velocity corresponds to the convergent part of the nozzle. When M is equal to 1, then we can say that throat of the nozzle is reached.

When flow velocity is higher than the local sonic velocity, then we can see that the flow velocity is passing through a duct which corresponds to the divergent part of this device.

So, this is what I wanted to discuss in today's class. And next class, I will try to discuss about the mass flow rate and from there we will try to quantify the nozzle efficiency which is very important. So, with this I stop here today.

Thank you.