Applied Thermodynamics
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Steam Power System
Lecture - 17
Steam Nozzle: Analysis and Efficiency (Contd)

I welcome you all to the session of Applied Thermodynamics. And the topic of our today's discussion is the Steam Nozzle. In fact, in continuation of our last discussion on the analysis of steam nozzle today we shall see how we can express the mass flow rate of steam when it is passing through the nozzle.

It is very important to know at this point of time that when steam is coming out from the flow nozzle before it enters into the turbine, kinetic energy of steam jet is very important. So, depending on the magnitude of the kinetic energy, the power rather the work that will be getting from the turbine depend.

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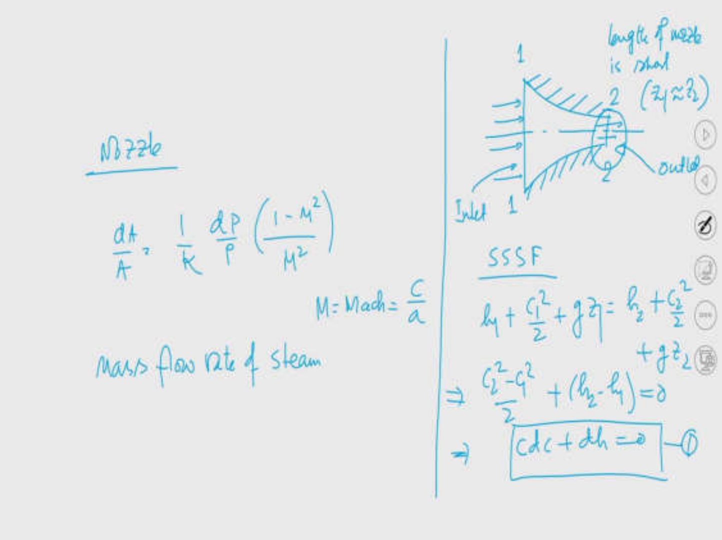

Nozzle

$$\frac{dA}{A} = \frac{1}{k}\frac{dP}{P}\left(\frac{1-M^2}{M^2}\right)$$

$M = \text{Mach} = \frac{C}{a}$

mass flow rate of steam

SSSF

$$h_1 + \frac{C_1^2}{2} + gz_1 = h_2 + \frac{C_2^2}{2} + gz_2$$

$$\Rightarrow \frac{C_2^2 - C_1^2}{2} + (h_2 - h_1) = 0$$

$$\Rightarrow \boxed{CdC + dh = 0} \text{ — ①}$$

So, now, question is if we try to recall for the flow nozzle we have seen that the area velocity relation can be written like this. I am not going to describe again the significance of individual term rather we have seen in the last class. We have also discussed that depending on the magnitude of flow velocity I mean if you try to compare that M which is nothing but Mach number and which is nothing but *C/ a*.

So, depending on the magnitude of Mach number whether the flow is sonic or subsonic or supersonic; we have different flow configuration. That is whether the flow is occurring through a gradually decreasing area or it is flowing through a fluidic confinement having equal cross sectional area or it is flowing through a duct having gradually increasing area.

So, today we shall see that mass flow rate of steam is another important quantity, essentially from the perspective of the kinetic energy of steam at the exit of the nozzle. So, we will try to find out what is mass flow rate of steam? We are going to have this analysis with few assumptions, which are those?

First of all we have consider that is the flow of a compressible fluid and the flow is isentropic that is it is reversible as well as adiabatic. So, there is no heat loss from the system to the surroundings as well as the fluid friction both internal as well as external is absent.

So, if you try to find out the expression of mass flow rate through the flow nozzle; again let us define the geometry and say this is section 2-2 and this is section 1-1. And this is inlet and this is outlet. If we apply steady state steady flow equation, we can write $h_1 + \frac{C_1^2}{2} + gz_1 = h_2 + \frac{C_2^2}{2} + gz_2$. This equation is obtained essentially from first law of thermodynamics applied to a control volume system.

So, since the outer surface of the flow nozzle is insulated and by doing so we are ensuring that there is no heat transfer from the system to the surroundings. So, the q term is 0 and the control volume is not doing any work. So, there is no physical displacement of the control volume at a given instant of time. If that is the case this is this steady flow energy equation in which we have consider both the internal energy as well as the flow work.

So, as I told that when steam will be entering into the flow nozzle to maintain the flow in presence of work; there is a work which is known as flow work. So, the work associated with the flow so that to maintain the flow both at inlet and outlet in presence of pressure is known as flow energy or flow work. So, as I told that the length of the flow nozzle is very small.

If that is the case we can trivially say that $z_1 = z_2$. If that is the case we can write that; $h_2 - h_1 + \frac{C_2^{\,2} - C_1^{\,2}}{2} = 0$. So, we can write it in differential form that is $\Rightarrow dh + CdC = 0\,(1)$.

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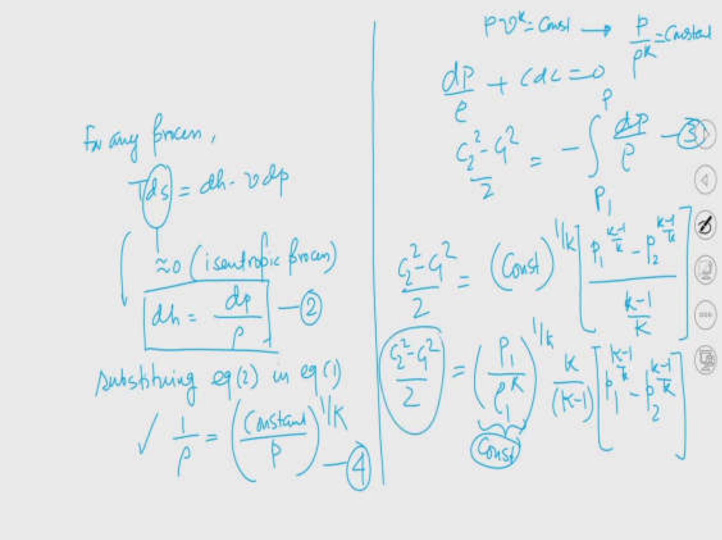

So, for any process we can write $Tds = dh - vdp$. As I told you that in thermodynamics there are a few quantities which cannot be directly measured on the contrary a few properties are there which can be directly measured. So, by measuring those properties which can be directly measured we also can evaluate the non measurable or directly non measurable quantities like entropy and enthalpy.

So as the process is isentropic, so, ds is almost equal to 0 and from there we can write $dh = vdp = \frac{dp}{\rho}$ (2). In thermodynamics we are always try to write in terms of specific volume because specific volume is easily obtainable from property chart. Here instead of writing specific volume; I will be writing in terms of density. Now, if we substitute this equation 2 in equation 1 and after some algebraic manipulations we can write $\frac{1}{\rho} = \left(\frac{const}{p}\right)^{1/k}$ (4). k is index of expansion.

So, I can plug in the value of dh from equation 2 over equation 1 and eventually I can write $\frac{dp}{\rho} + CdC = 0$ So, now from this equation I can write $\frac{C_2^{\,2} - C_1^{\,2}}{2} = -\int_{p_1}^{p} \frac{dp}{\rho}$ (3).

So, if I plug in the value of a 1/ρ from equation 4 over in equation 3 then I can easily get $\frac{C_2^{\,2}-C_1^{\,2}}{2}=(const)^{1/k}\left[\frac{p_1^{\frac{k-1}{k}}-p_2^{\frac{k-1}{k}}}{\frac{k-1}{k}}\right]$. So, this equation pv^k = constant is the isentropic process that I have discussed in the last class. I need to know the mass flow rate at the exit of the nozzle. And now, if I would like to calculate mass flow rate I should know the cross sectional area of the section then if I multiply with ρ_2 and also C_2.

So, now what I can do? I can express this constant per *1/k* in terms of $\left(\frac{p_1}{\rho_1^{\,k}}\right)^{1/k}$; so this is basically constant. So, $\frac{C_2^{\,2}-C_1^{\,2}}{2}=\left(\frac{p_1}{\rho_1^{\,k}}\right)^{1/k}\frac{k}{k-1}\left[p_1^{\frac{k-1}{k}}-p_2^{\frac{k-1}{k}}\right]$.

So, next step is very important step that if we do some algebraic manipulation this term; then I can write this fellow in different form.

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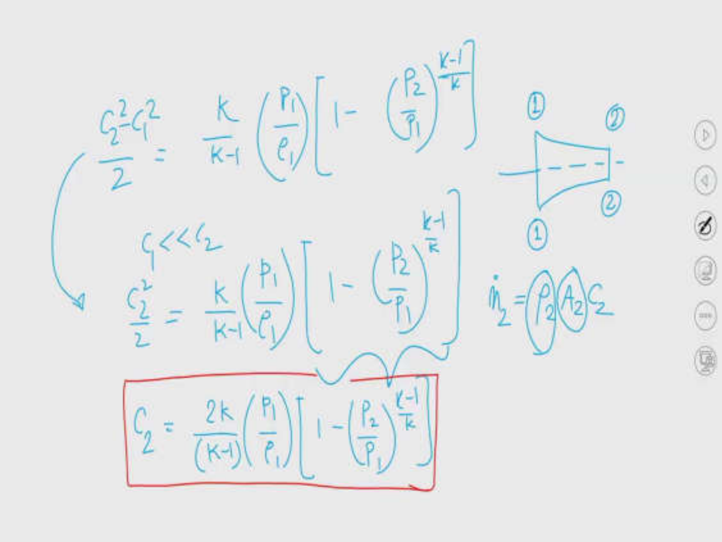

I can write $\frac{C_2^{\,2}-C_1^{\,2}}{2}=\frac{k}{k-1}\frac{p_1}{\rho_1}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]$. So, if we now look at the schematic depiction, if we say a fluid is flowing through this fluidic confinement. Now, mass conservation will be satisfied; if I now look at the area of you know section 1-1 and area of section 2-2; we can easily say that; area of section 2-2 is very less than area of section 1-1. If that is the case, it is the shape of the nozzle.

Since this area is significantly larger than the area at section 2-2 we can write that; $C_1 << C_2$. If that is the case; we can write $\frac{C_2^2}{2}=\frac{k}{k-1}\frac{p_1}{\rho_1}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]$. So, this is nothing but pressure ratio that is ratio of pressure to the outlet of the nozzle and that at the inlet of the nozzle. This particular ratio has consequence on the mass flow rate that we are going to study today.

Now, So, mass flow rate $\dot{m}_2=\rho_2 A_2 C_2$. We know A_2. ρ_2 we have to relate because it is not the incompressible flow that the density will remain constant.

So, as the steam flows from section 1-1 to section 2-2 at any special location, we will be having special variation of density. So, we have to calculate ρ_2. And now C_2 already we can have from this expression $C_2^2=\frac{2k}{k-1}\frac{p_1}{\rho_1}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]$.

Now, we need to calculate the density at section 2-2. Because it is the flow of a compressible fluid. So, the density will change with the change in pressure. So, let me briefly reiterate compressible fluid is a fluid in which there is a significant change in density with the change in pressure.

While if the density of the fluid does not change significantly with the change in pressure that is called incompressible fluid. So, since we are dealing here with the compressible fluid we must calculate density at section 2-2 as it differs from the density which the fluid had at section 1-1.

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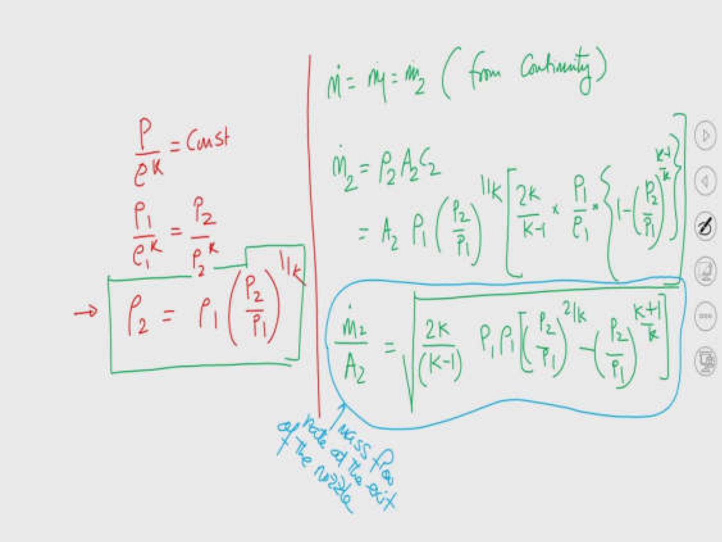

$$\frac{P}{\rho^k} = \text{Const}$$

$$\frac{P_1}{\rho_1^k} = \frac{P_2}{\rho_2^k}$$

$$\rightarrow \boxed{\rho_2 = \rho_1 \left(\frac{P_2}{P_1}\right)^{1/k}}$$

$$\dot{m} = \dot{m}_1 = \dot{m}_2 \text{ (from Continuity)}$$

$$\dot{m}_2 = \rho_2 A_2 C_2$$

$$= A_2 \rho_1 \left(\frac{P_2}{P_1}\right)^{1/k} \left[\frac{2k}{k-1} \times \frac{P_1}{\rho_1} \times \left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right\}\right]$$

$$\frac{\dot{m}_2}{A_2} = \sqrt{\frac{2k}{(k-1)} P_1 \rho_1 \left[\left(\frac{P_2}{P_1}\right)^{2/k} - \left(\frac{P_2}{P_1}\right)^{\frac{k+1}{k}}\right]}$$

mass flow rate at the exit of the nozzle

So, $\frac{p}{\rho^k} = const \Rightarrow \frac{p_1}{\rho_1^{\,k}} = \frac{p_2}{\rho_2^{\,k}} \Rightarrow \rho_2 = \rho_1 \left(\frac{p_2}{p_1}\right)^{1/k}$. Now $\dot{m}_2 = \rho_2 A_2 C_2$; just if we plug in the value of all those quantities we will get $\dot{m}_2 = A_2 \rho_1 \left(\frac{p_2}{p_1}\right)^{1/k} \left[\frac{2k}{k-1} \frac{p_1}{\rho_1} \left\{1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right\}\right]$. So, mass flow rate, $\dot{m} = \dot{m}_1 = \dot{m}_2$. So, this is obtained from continuity. So, if we have this expression of mass flow rate, I can try to write this expression in a differential form.

But I am writing $\frac{\dot{m}_2}{A_2} = \sqrt{\frac{2k}{k-1} p_1 \rho_1 \left[\left(\frac{p_2}{p_1}\right)^{2/k} - \left(\frac{p_2}{p_1}\right)^{\frac{k+1}{k}}\right]}$. So, this is the expression of mass flow rate at the exit of the nozzle. We have been able to express this quantity in terms of the pressure ratio $\frac{p_2}{p_1}$.

Definitely this pressure ratio is an important ratio in the context of the analysis of flow through a nozzle and perhaps we are going to discuss the significance of this pressure ratio on the flow phenomenon through a nozzle.

So, I mean our objective should be to increase the mass flow rate. If I can, then perhaps our objective should be to increase mass flow rate in a way that it indicates that if A_2 is remaining fixed then perhaps you are trying to increase the velocity of steam at the exit of the nozzle, as if we can increase the velocity of steam at the exit of the nozzle kinetic energy will increase.

If we can increase the kinetic energy of steam before it tries the turbine blades we can expect the work output from the turbine will be higher. If we act to increase this quantity, k is fixed, also as we know the quality of steam at inlet so pressure and density is constant.

Then this quantity $\frac{\dot{m}_2}{A_2}$ is a function of the pressure ratio $\frac{p_2}{p_1}$. So, what we can say that mass flow rate of at the exit of the nozzle can be increased by tuning the pressure ratio.

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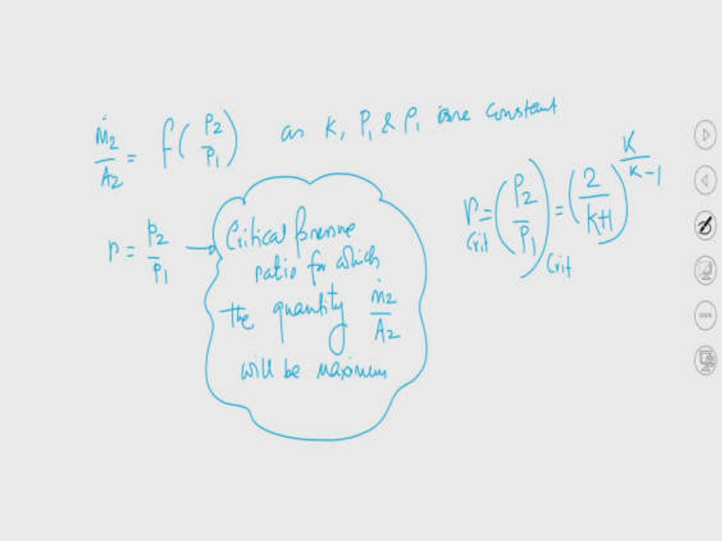

$\frac{\dot{m}_2}{A_2} = f\left(\frac{P_2}{P_1}\right)$ as K, P_1 & ρ_1 are constant

$r = \frac{P_2}{P_1} \rightarrow$ Critical pressure ratio for which the quantity $\frac{\dot{m}_2}{A_2}$ will be maximum

$r_{crit} = \left(\frac{P_2}{P_1}\right)_{crit} = \left(\frac{2}{K+1}\right)^{\frac{K}{K-1}}$

So, now if we try to find out a particular pressure ratio for which flow rate through the nozzle will be maximum. So, we need to find out the critical pressure ratio that is the pressure ratio for which the quantity $\frac{\dot{m}_2}{A_2}$ will be maximum. And that critical pressure ratio $r = \left(\frac{p_2}{p_1}\right)_{crit} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$.

So, now, this is very important at this point of time let me discuss about few issues. So, what do you mean by critical pressure ratio? Let me tell you though it is not included in this course, but still I am telling you since I have discussed this part in detail.

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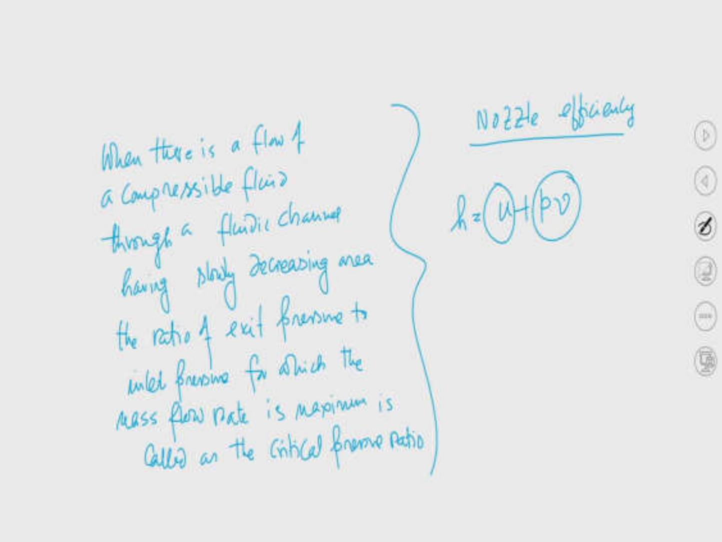

When there is a flow of
a compressible fluid
through a fluidic channel
having slowly decreasing area
the ratio of exit pressure to
inlet pressure for which the
mass flow rate is maximum is
called as the critical pressure ratio

Nozzle efficiency

$h = \boxed{u} + \boxed{pv}$

So, when there is a flow of a compressible fluid through a fluidic channel having slowly decreasing area, the ratio of exit pressure to the inlet pressure for which right mass flow rate is maximum is called as the critical pressure ratio. So, also when the pressure at which the area is minimum and discharge is maximum is also termed as critical pressure ratio. Also the pressure ratio for which the discharge area is minimum and discharge per unit area is maximum is known as critical pressure ratio.

In the last class we have been able to establish the expression relating area and the velocity that is in terms of Mach number. Today we have derived the expression of critical pressure ratio. To be precise we have also discussed about the significance of critical pressure ratio.

So, a nozzle is said to have the critical pressure ratio when the flow rate at the exit of the nozzle is maximum. So, now I will be discussing about one important term that is called nozzle efficiency. See we have discussed about this particular component which is an essential component for this smooth operation of the power plant.

As we have discussed many times that steam is not allowed to directly go into the turbine after it leaves steam generator or boiler. It should be allowed to pass through the flow nozzle essentially it will increase the kinetic energy. From the discussion of reaction turbine and impulse turbine we have seen; that steam is allowed to pass through different rows of nozzles.

And one row of nozzle and one row of moving blades; these two rows constitute together to form a stage. So, we have seen that if there are multistage turbine, so, we will be having multi rows of flow nozzles. So, that means, when steam is passing through the flow nozzle our essential objective is to increase the kinetic energy of steam before it enters into the turbine.

So, we can increase the kinetic energy of steam at the cost of the drop in the enthalpy. So, $h = u+pv$. So, this is internal energy what do you mean by that? As I told you in the last class the temperature is a good representative measure of the internal energy in addition to this because it is a flow process will be having potential energy as well as the kinetic energy.

So, this fellow will take care the potential energy kinetic energy as well as the temperature at that particular section. While this is known as flow energy or flow work that is when there is a flow through any fluidic confinement, to maintain the flow in presence of pressure we need to invest energy and that is nothing but the flow energy. So, these two terms together constitute one important property in thermodynamics that is enthalpy.

So, in a flow nozzle we have seen that essentially we are playing with pressure and velocity. So, at the cost of the reduction in pressure we are increasing velocity. So, you know that at the cost of that pressure reduction we are trying to achieve higher velocity; essentially you are playing with enthalpy. So, when there is a flow through a steam nozzle enthalpy will change.

If we try to look at the change in enthalpy, I mean entire analysis that we have carried out till now is based on an important assumption is that the flow is isentropic, that is reversible adiabatic. But in reality it is very difficult to have any process to be reversible.

You have studied in classical thermodynamics that attaining a reversible process is very very difficult. So, we can say the process is reversible essentially to have the analysis but in reality achieving such a process is really difficult. So, enthalpy will definitely drop from as steam passes through inlet to the outlet. But in the analysis since we have taken that important assumption is that the flow is isentropic flow. So, I mean we can assume that the actual drop of enthalpy will be definitely different than the ideal drop in enthalpy.

So, again we can do some mathematical calculation we can try to find out what will be the isentropic enthalpy drop. So, you have studied Mollier diagram that is very important diagram that help us to obtain different thermodynamic properties. So, the idea is that enthalpy which will drop as this steam passes through the nozzle between two sections that is 1-1 and 2-2. This actual enthalpy drop is not equal to the ideal enthalpy drop. So, accounting for that aspect we can define what is known as nozzle efficiency. As I told you that when steam is flowing through the nozzle we will be having internal friction I mean no fluid is inviscid.

So, viscosity will be there and if the fluid is having finite viscosity we really cannot ignore the internal friction. Also steam is moving through a fluidic confinement that is bounded by a solid surface. So, when the steam is passing over the solid surface we also cannot trivially ignored the external friction.

So, accounting for these two different sources of friction that is the internal friction and external friction the process will be highly reversible. And in that case though we can ensure that the external surface of the nozzle will be insulated. So, we can really minimize the heat loss, but we cannot make it completely 0.

So, still there will be some amount of heat transfer from the nozzle to the external ambience, but we can minimize that, but again we really cannot ignore the frictional losses when steam is passing through the flow nozzle.

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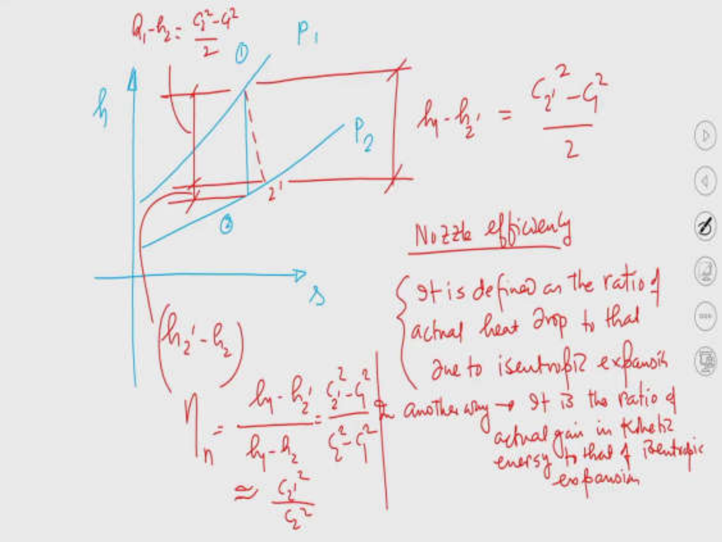

$$h_1 - h_2 = \frac{C_2^2 - C_1^2}{2}$$

$$h_1 - h_{2'} = \frac{C_{2'}^2 - C_1^2}{2}$$

Nozzle efficiency

It is defined as the ratio of actual heat drop to that due to isentropic expansion

$$\eta_n = \frac{h_1 - h_{2'}}{h_1 - h_2} = \frac{C_{2'}^2 - C_1^2}{C_2^2 - C_1^2} \simeq \frac{C_{2'}^2}{C_2^2}$$

another way → It is the ratio of actual gain in kinetic energy to that of isentropic expansion

So, this is known as Mollier diagram. So, I can say this is p_2 and this is p_1; say this is at the inlet to the nozzle and this is at the exit of the nozzle. So, the pressure lines diverge in h-s diagram. So, process 1-2 is ideal and process 1-2` is actual. So, the actual enthalpy drop is $h_1$ - $h_2$`. If we ignore the elevation of the flow nozzle, this will be equal to $\frac{C_{2'}^2 - C_1^2}{2}$ while theoretical enthalpy drop will be $h_1 - h_2 = \frac{C_2^2 - C_1^2}{2}$. So, you can see that the enthalpy drop will be from 1 to 2 that is ideal enthalpy drop, but actual drop will be from 1 to 2`; that is slightly less than the ideal one.

So, we can define the nozzle efficiency that is nothing but ratio of actual heat drop to that due to isentropic expansion. So, we can also say this as it is the ratio of actual gain in kinetic energy to that of isentropic expansion. So, basically we are expecting that the kinetic energy increment will be $\frac{C_2^2 - C_1^2}{2}$, but that is not the case. Rather we are having actual gain in kinetic energy to that of the isentropic expansion and this is known as nozzle efficiency.

Mathematically it can be expressed in terms of $\eta_n = \dfrac{h_1 - h_{2'}}{h_1 - h_2}$. In other way we can write $\eta_n = \dfrac{C_{2'}^2 - C_1^2}{C_2^2 - C_1^2}$. If I have C_1 is very very small as compared to C_2 then we can write this expression as $\eta_n = \dfrac{C_{2'}^2}{C_2^2}$.

So, now if we try to summarize today's discussion what we have discussed today that we have try to know the mass flow rate of steam at the exit of the nozzle, which is very important because our objective should be to design the nozzle in such a way that the mass flow rate of steam per unit area at the exit should be maximum.

For that how we can design? After establishing this expression we have found that the quantity $\dfrac{\dot{m}_2}{A_2}$ is a function of $\dfrac{p_2}{p_1}$. So, the ratio of pressure at the exit of the nozzle to the inlet of the nozzle plays an important role to dictate the maximum mass flow rate.

And we have also try to find out the critical pressure ratio for which mass flow rate will be maximum. And since it is not included in this course I did not discuss that when the nozzle is said to be choked. So, when the nozzle is operating at the critical pressure ratio if we try to reduce pressure at the exit further there will not be any increment in mass flow rate through the nozzle and the nozzle is said to be choked.

So, when the nozzle is operating at a critical pressure ratio for the reduction in pressure at the exit will not allow the mass flow rate to increase. So, the information at the exit that reduction pressure will not be sensed at the inlet so that additional amount of flow rate that we are looking for will not be achieved in practise and the nozzle is said to be choked.

And after that we have try to understand what do we mean by nozzle efficiency. Because whenever we are talking about any mechanical device mechanical component we need to know its efficiency. So, from this h-s diagram we have found that though our analysis is based on the assumptions that it is frictionless adiabatic flow, but still in reality we cannot ignore the fluid friction.

And on accounting for the fluid friction, the efficiency can be defined even when there is a flow through a nozzle. And we have tried to quantify the efficiency of nozzle mathematically. And we have seen that in it can be written in terms of the $\frac{C_{2'}^{2}}{C_{2}^{2}}$ that is the ratio of actual gain in kinetic energy that to that of the isentropic expansion.

So, with this I stop here today and we shall continue our discussion in the next class.

Thank you.