Applied Thermodynamics
Prof. Niranjan Sahoo
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 04
Gas Turbine Engines
Lecture - 35
Aircraft Propulsion Cycle - I

Dear learners, greetings from IIT Guwahati. We are again in this course Applied Thermodynamics, Module 4; Gas Turbine Engines.

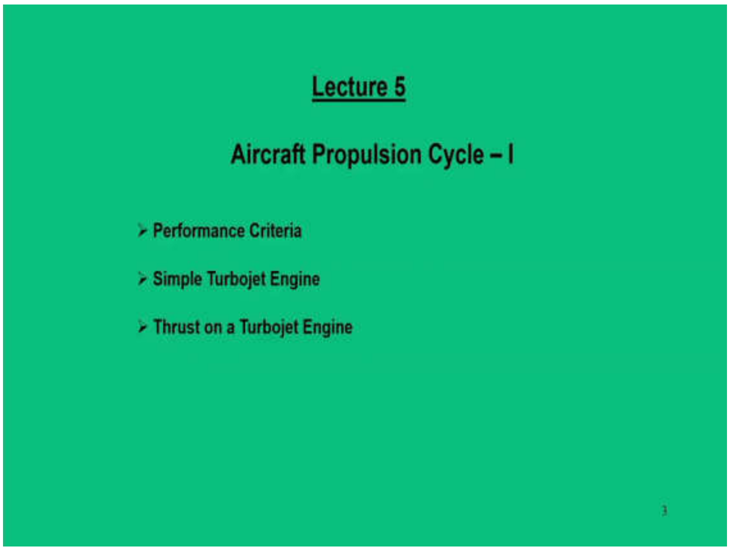
(Refer Slide Time: 00:57)

So, prior to this lecture, in this module on Gas Turbine Engines, we have covered 4 lectures first one is Gas Turbine Engine-Components, Thermal Circuits. So, lecture 2 and 3 were devoted to Gas Turbine Performance Cycle and mainly these two lectures were intended for power generations.

Then in the last lecture, that is lecture number 4, we discussed about some real performance cycle by defining some parameters typically known as stagnation properties. And in fact, this parameter takes care of velocity changes of working fluid in the cycle. Now, in today's lecture we are going to start aircraft propulsion cycle.

So, in the beginning of the lecture we have made it clear that gas turbine engines are used for two purposes, one for power generation other for thrust generations. In earlier lectures we mainly concentrated on gas turbine engines that were used for power generations. Now onwards we will see that how this gas turbine engines makes use of the propulsion cycle to generate thrust and its main intention and application is the aircraft engines.

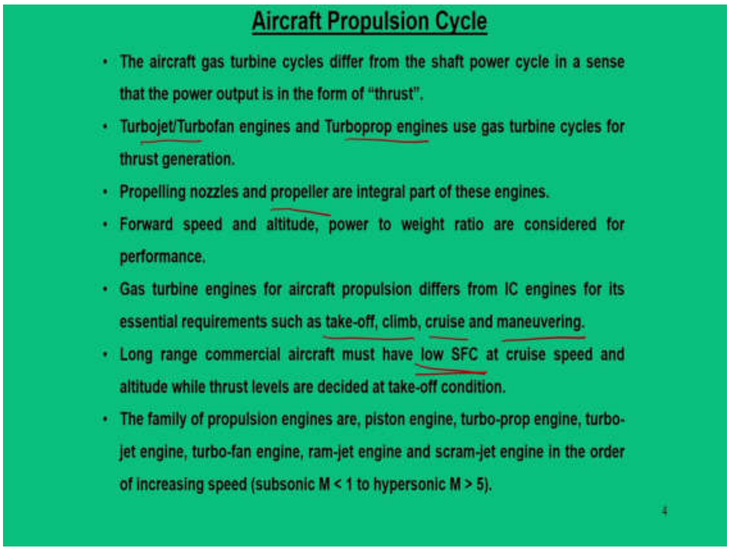
(Refer Slide Time: 02:38)



So, in this aircraft propulsion cycle we will discuss about three important topics. First is performance criteria because the conventional gas turbine performance and aircraft performance are different. So, based on the requirement of aircraft, the performance criteria was decided.

Now, with respect to a particular simple turbojet engines using gas turbine cycle, we will see that how thrust is going to generate. So, these three things we are going to cover. In fact, while talking about this turbojet engines, we will briefly discussed about different components of a gas turbine cycle which has used for thrust generation in aircrafts.

(Refer Slide Time: 03:29)



So, let me start with brief introduction that how a Propulsion Cycle is different from Conventional Gas Turbine power generation. How this propulsion cycle is different from conventional IC engines. Normally in IC engines the combustion process is cyclic in nature and we do not use the word propulsion although the vehicle moves, but it is mainly concentrated towards power.

But in this case, specific word propulsion comes into pictures because these engines are thrust specific, and this is one aspect. Second aspect is that concept of this aircraft cycle is different to that of conventional gas type turbine approach. Where the attention was focused to power generations and all those gas turbine systems, they are on ground based systems and power generation does not require any vehicle that should fly.

But, for aircraft propulsion system, we mainly require that the aircraft must fly, when it is supposed to fly, the power to weight ratio is a one of the concerns. And secondary approach is that you also have to use those materials which have sufficient strength, at the same time they should be light enough.

So, these are some of the major concerns and another aerodynamic approach would be that, when you move along with the altitude what happens the pressure and temperature drops. So, the vehicle can fly faster. So, taking that advantage different engines have been developed.

So, just to give the brief introduction on this aircraft cycle, first thing that the aircraft gas turbine systems differs from shaft power cycle, in a sense that the power output is in the form of thrust. And to get this thrust typical engines that we are going to use are turbojet, turbofan engines; so, these are kind of a jet engines.

Other category is the turbo-prop engines where propellers are used. So, these are two category that is jet engines and propeller type engines. And for turbojet engines the propelling nozzles are the very important component for thrust whereas, turboprop engines it is the propeller that gives the major thrust content.

Another aspect is the forward speed altitude, power to weight ratio are considered for the aircraft performance. And when the gas turbine engines are used for aircraft systems, their essential requirement is take-off, climb, cruise, manoeuvring. So, take-off means during the take-off, it requires very high initial thrust.

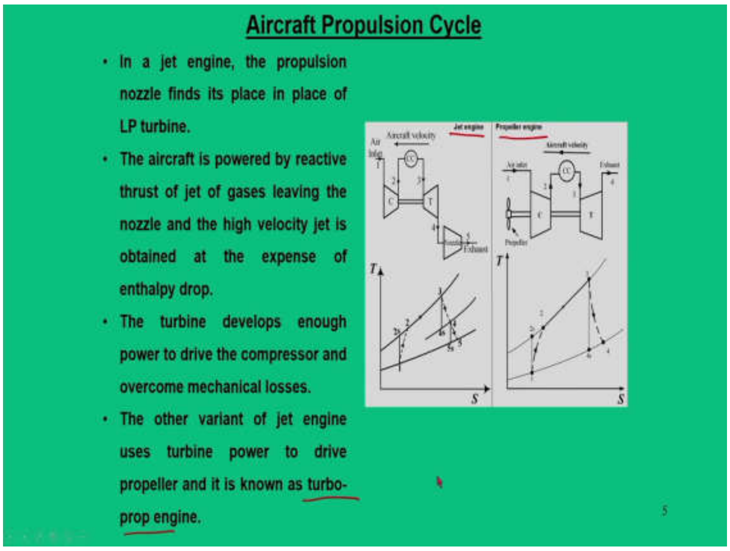
So, it has to climb to a particular altitude and when it is just a cruising, it is just a kind of providing necessary thrust just to overcome drag and manoeuvring in the higher altitude. During manoeuvring the aircraft makes a turn and in fact, they do it very for a limited time or limited requirement. So, you call this as a manoeuvring.

While talking about long range commercial aircraft, another concept that drops in fuel consumption. So, low specific fuel consumption at high cruise speeds. So, while cruising speeds at a particular altitude, it must consume very less fuel and while it moves; the thrust levels are very high when the engine takes off.

Now while talking about this the family of propulsion engine developments are piston engines, turbo-prop engines, turbo-jet engines, turbo-fan engines, ram-jet engine, scram-jet engines. So over the period of time, these engines have been developed to travel from subsonic Mach number to very high hypersonic Mach number.

So, that is the reason that this propulsion engines are required and these engines are mainly considered for their operation at different altitude level.

(Refer Slide Time: 08:31)



Now, as I already mentioned that if you look broadly the thermal circuits, one for jet engine other for propeller type engines, the very simple schematic diagram can be shown here that, it is a simple gas turbine engines. For a jet engine what happens the in a similar concept; in a simple gas turbine engines turbine power is sufficient to drive the compressors. So, we have a combustion chamber.

And the gas that comes out from the turbine exhaust, so instead of they release it to atmosphere, it has to be used as a jet. To use its power as a jet, we use a nozzle. And main purpose of this nozzle when the exhaust comes out it gives the thrust. So, essentially the power that is available in the exhaust or through jet, the aircraft gets necessary thrust.

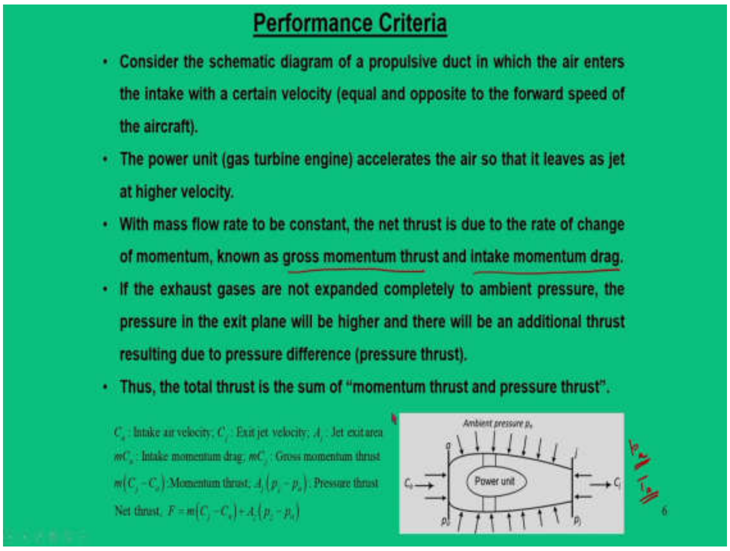
So, if you look at very basic Ts diagram so, compressor part there is it is a continuous compression and its gives necessary compression ratio. And while expanding it expands in two parts; one is in a turbine, second one is in the nozzle. And the turbine part is mainly used to drive the compressor and the nozzle expansion that is used mainly for thrust.

And this is what happens in a jet engines whereas, in a propeller engines it goes in a similar manner as in a conventional gas turbine cycles. But another difference that remains at as it is that in a conventional gas turbine engine the turbine just drives the compressors.

So, in addition to this we have a propeller attached to it. So, power for the propeller is also achieved from the turbines. So, in a sense that the turbine drives the propeller as well as the compressor. And when it gets the thrust, major thrust comes from the propeller and some partial thrust that comes from the exhaust.

So, if you look at the this Ts diagram, so there is no nozzle here. So, the expansion is only in the turbine. And this variant we call this as a turbo-prop engines means it is a propeller type engine. And we will try to discuss both the cycles in our lecture.

(Refer Slide Time: 11:17)



So, let me go to next part the performance criteria. So, while talking about this performance criteria we must emphasize that here while dealing with the aircraft cycle our main attention is to generate thrust. And this thrust we get it through jets that comes out from the exhaust.

Now, for that we can simply talk about a schematic diagram and we call this as a power unit or propulsive duct. We can call this as a propulsive duct. So, what does this propulsive duct, it contains a power unit and on this duct there is ambient pressure that acts on it.

So, that is air that enters at certain velocity C_a and air that goes as exist from the duct as velocity C_j, how this C_j velocity it gets it get to the power unit and this power unit is nothing but our gas turbine engine. So, we consider a propulsive duct in which air enters to the intake at certain velocity and this velocity is equal and opposite to that of forward speed of the aircraft. So, as if that, at same speed this duct is also moving

The power unit accelerates the air, so that air leaves as jet with higher velocity. And with mass flow rate assuming to be constant, the net thrust is mainly due to rate of change of momentum and simply this follows the Newton's Law. And this momentum, we call this as a gross momentum thrust. And apart from this there is also a part called as intake momentum drag.

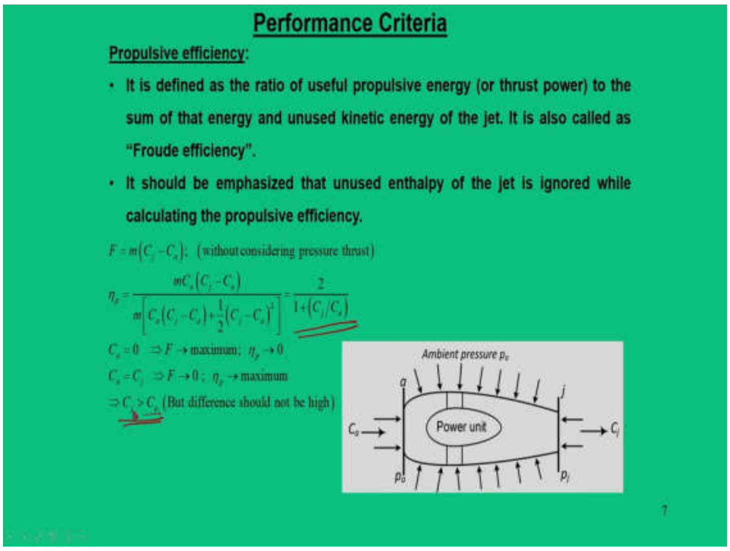
So, this net thrust is mainly due to this two difference. And if the exhaust gases are not expanded completely to atmosphere, later on we will see that when the gas comes out of this duct, it has a certain pressure p_j. But whereas, the atmospheric pressure or ambient pressure can be at different pressure p_a or temperature could be T_a.

So that means, we are not able to expand the jet to this ambient pressure. So, in such cases there will be an additional thrust that will result from due to pressure difference and we call this as a pressure thrust. So, the total thrust would be some of this momentum thrust that comes from this Newton's law and also from the pressure thrust.

So typically, mathematically, we can represent that we can say that:

C_a : Intake air velocity; C_j : Exit jet velocity; A_j : Jet exit area
mC_a : Intake momentum drag; mC_j : Gross momentum thrust
$m\left(C_j - C_a\right)$:Momentum thrust; $A_j\left(p_j - p_a\right)$: Pressure thrust
Net thrust, $F = m\left(C_j - C_a\right) + A_j\left(p_j - p_a\right)$

(Refer Slide Time: 14:47)



And next criteria that we are going to define is propulsive efficiency. This is the word that you are going to use that how the conventional engine developments has happened, based on these performance criteria. So, one performance criteria is known as a propulsive efficiency.

So, it is defined as the ratio of useful propulsive energy or thrust power to the sum of t that energy and unused kinetic energy of the jet. So, first we have to calculate useful propulsive energy and in the denominator we have to keep it the sum of this propulsive energy plus the unused kinetic energy. So, this is also known as Froude efficiency or propulsive efficiency.

Now, here we should emphasize that the unused enthalpy of jet is ignored in our calculation, because we do not talk about temperature here and that is what the unused enthalpy is ignored here.

So, based on this what we can do is that, in a simple sense if you assume that the jet is completely expanded to ambient. So, that there is no pressure thrust, then we can have the total thrust is $F = m\left(C_j - C_a\right)$, which you call this as a thrust power. And the denominator is nothing but this thrust power plus this unused kinetic energy.

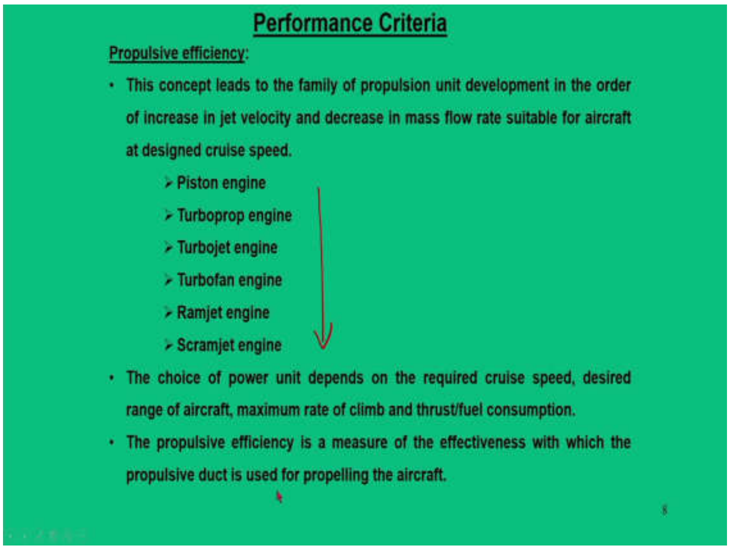
$$\eta_p = \frac{mC_a\left(C_j - C_a\right)}{m\left[C_a\left(C_j - C_a\right) + \frac{1}{2}\left(C_j - C_a\right)^2\right]}$$

Now, after simplifying this expression we can find that propulsive efficiency takes the following simplified form that is equal to $\frac{2}{1+\left(C_j / C_a\right)}$. Now a close look of this expression will tell you that when $C_a = 0$ means that there is no air that is rushing into the duct. And F is maximum that means, whatever you get from this power unit that gives the maximum thrust, but that point of time we do not get propulsion, that mean propulsive efficiencies goes to 0.

But at the other extreme that when you have jet velocity and the air velocity that is same, $C_a = C_j$, F goes to 0, that means, there is no thrust. But that point of time you get maximum propulsive efficiency, but these are two contradicting in nature, because we need both of them we need thrust, we need also good amount of propulsive efficiency.

So, the major target will be that we must have a $C_j > C_a$, but this difference should not be too high that which will make this thrust to be 0. So, you will have some substantial means from optimum difference between C_j and C_a.

(Refer Slide Time: 17:59)

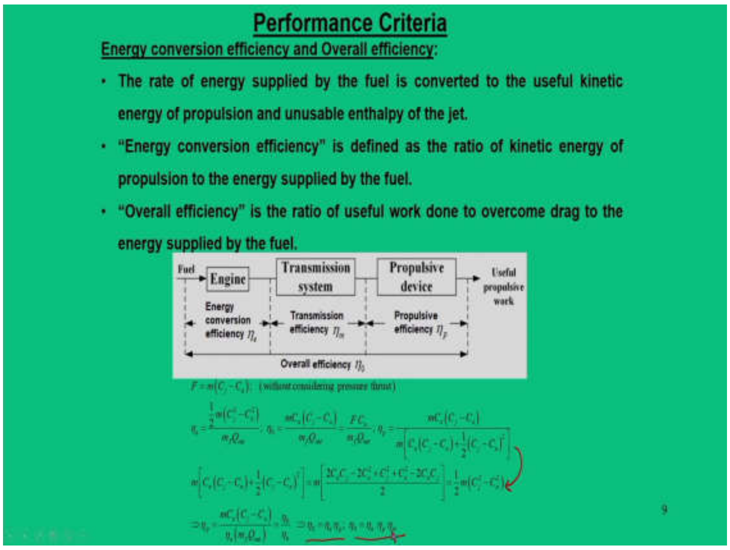


And this particular concept of propulsive efficiency gives a many family of propulsion units in which is in the order of jet velocity and decrease in the mass flow rate that are suitable for design aircraft or cruise speeds. And these engines are piston engines, turbo-prop engines, turbo-jet engines, turbo-fan engine, ram-jet engine and scram-jet engine.

So, these are the over the period of time this engine development. We will not go all these engines because that is a different topic. But we will mainly concentrate one simple turbo-jet engines out of which we will do all our understandings clear as far as this course is concerned. And our main intention would be that when we have to look for a power unit for a cruise speeds, desired range of the aircraft and maximum rate of climb and thrust and fuel consumption.

So, for this reason the propulsive efficiency is a measure of effectiveness with which the propulsive duct is used to propel the aircraft.

(Refer Slide Time: 19:09)



That is in addition to the propulsive efficiency another performance criteria is nothing but the energy conversion efficiency and overall efficiency. I mentioned here that we did not take care about unused enthalpy. Let us see how you are going to take care these unused enthalpy.

So, this is taken care through energy conversion efficiency or η_e; that means, if you look at this energy conversion efficiency, it is defined as the ratio of kinetic energy of the propulsion to the energy supplied by the fuel.

$$\eta_e = \frac{\frac{1}{2}m\left(C_j^2 - C_a^2\right)}{m_f Q_{net}}$$

So, Q_{net} is nothing but is calorific value of the fuel. And we also use the another word that is overall efficiency which is the ratio of useful work done to overcome the drag to the energy supplied by the fuel.

$$\eta_0 = \frac{mC_a\left(C_j - C_a\right)}{m_f Q_{net}} = \frac{F\,C_a}{m_f Q_{net}}$$

So, for the same energy supply the useful work done could be $mC_a\left(C_j - C_a\right)$. And in a word that you can say the thrust times the $F\,C_a$. So, here we define three efficiency; one is energy conversion efficiency, overall efficiency and prior to this we have propulsive efficiency. But however, looking at these expressions, we can find there is some similarity among them.

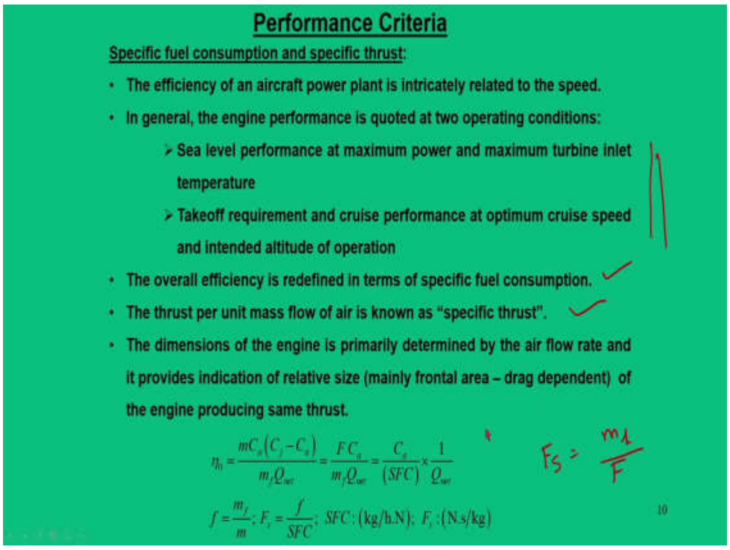
So, if you look at the denominator part of this propulsive efficiency this term that is $m\left[C_a\left(C_j - C_a\right) + \frac{1}{2}\left(C_j - C_a\right)^2\right]$, if you simplify this, and we can find out this dominator part turns out to this expression nothing but $\frac{1}{2}m\left(C_j^2 - C_a^2\right)$. And this term is appearing in energy conversion efficiency.

So, by putting these things one, can find a relation that how overall efficiency is related to propulsive efficiency and energy conversion efficiency. $\eta_0 = \eta_e\,\eta_p$. That is overall efficiency is equal to energy conversion efficiency multiplied by propulsion efficiency.

Now, if you look at this particular figure, we start with a fuel and end is useful propulsive work. So, in between we have the engines that converts the energy from the fuel and the propulsive duct that gives the propulsive power and side by side while running this particular things, there is a transmission system because turbine has to drive the compressor.

So, there is a transmission efficiency. However, if you include this transmission efficiency, the overall efficiency we can rewrite it as overall efficiency is equal to energy conversion efficiency multiplied by propulsion efficiency multiplied by transmission efficiency. $\eta_0 = \eta_e \, \eta_p \, \eta_m$

(Refer Slide Time: 22:24)



The next important criteria we are going to discuss is about Specific Fuel Consumption and Specific Thrust. Since we are talking about efficiency, useful power from the fuel then we must take into account that how much fuel we are going to consume. So, this is in similar sense the specific fuel consumption we are going to use for our requirement and now this fuel is used for thrust.

So, we call this as a specific thrust whereas, in power generation we say it is a specific power. And now this thrust requirement has two important criteria, one is at sea level performance for maximum power and maximum turbine inlet temperatures, other is the take off requirement and cruise performance at optimum cruise speed and intended altitude operations.

So, based on these two criteria, we have to find the specific thrust requirement for an aircrafts. And because of these reasons we have to redefine this overall efficiency in the form of specific fuel consumptions. This one aspect.

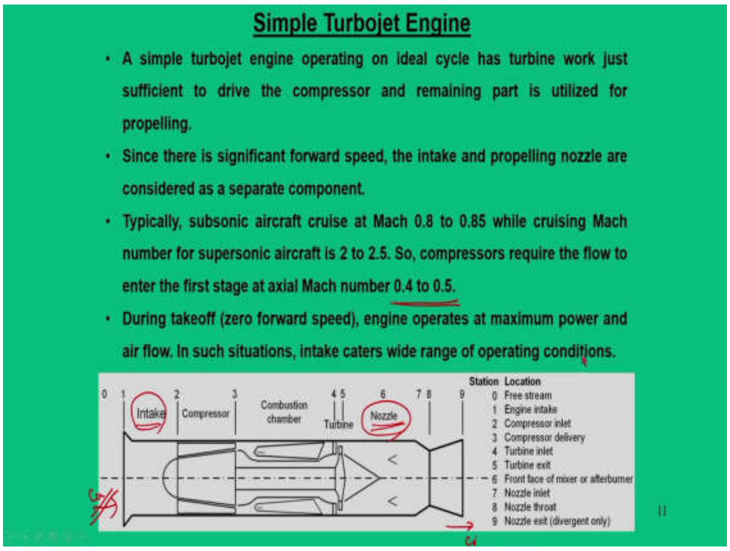
And second thing is that we have to define another word which we call as a specific thrust and that is defined as a thrust for unit mass of the flow rate. That is $F_s = \frac{m_f}{F}$, thrust per unit mass flow rate of air is nothing but the specific thrust. And this specific thrust we are normally going to represent in the form of a non-dimensional form we call this as a fuel air ratio.

And let us see that how you are going to get this expression that we start with overall efficiency $\eta_0 = \frac{mC_a\left(C_j - C_a\right)}{m_f Q_{net}} = \frac{F\,C_a}{m_f Q_{net}}$.

Now, here what you do is that we define this fuel consumption efficiency SFC which is nothing but $\frac{m_f}{F}$. And Q net is nothing but the calorific value of the fuel.

Now if you recall that your fuel air ratio f that is nothing but mass flow rate of fuel divided by mass flow rate of air. And from this we can define a term F_s that is specific thrust in the form of non-dimensional way that is fuel air ratio to the SFC $F_s = \frac{f}{SFC}$. And here the SFC is normally represented in kg/hN and specific thrust is defined in the term of Ns/kg.

(Refer Slide Time: 25:32)



Now, let us move on to simple turbojet engine based on which we are going to study this performance criteria. So, if you look at a turbojet engines. So, this is a kind of a duct systems consisting of many components like intake, compressor, combustion chamber, turbine, nozzle and then from this nozzle you get this thrust.

And every component does its job. So, compared to our previous gas turbine cycle and this cycle the main difference is that, since they are thrust specifics two additional components that drops in one is intake system other is the nozzle systems. And other components like compressor, turbine, combustion chamber, they are more or less same.

But only thing is that when you deal with the earlier gas turbine cycles for power generations, there the approach was that when you calculate these temperatures or pressures, it could be a static pressure or temperature because we do not have a significant change in the kinetic energy when the flow takes goes from one component to other. But here since they are thrust specific and it is not possible to ignore this kinetic energy.

At the same time since we are also looking for the altitude specific potential energy also is taken into account here. So, anyway whatever may be the things, but the very common components we have compressor, turbine and combustion chamber they are common, but what is the difference intake and nozzle systems.

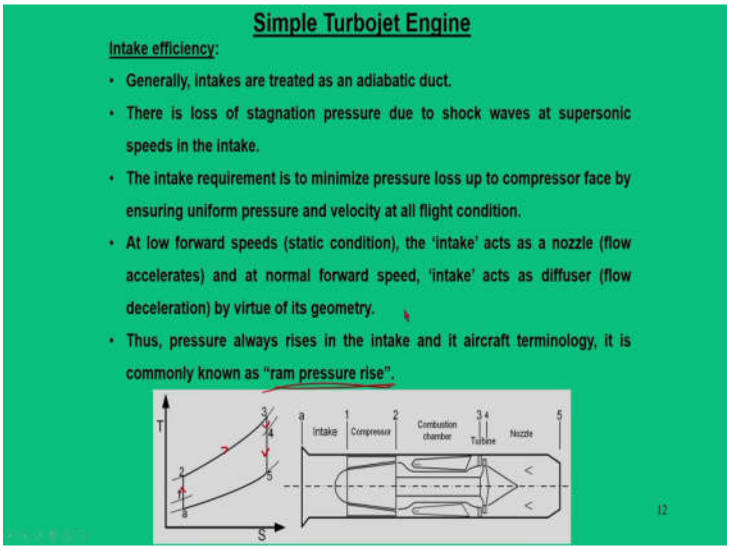
So, in using this simple turbojet engines, in our study we mainly concentrate on two things, one is this intake systems other is the nozzle system and we call this nozzle as a propelling nozzle. And now next question is that why this intake requirement is there? So, we must say that we must have a C_j, at the same time we must have air that should enter with C_a.

Now, we cannot make this C_a to be 0 because there is no meaning of this thrust. So, we must have C_a. Now for the continuous supply of this C_a at a fixed rate, we require a unit which we call as intake and this nozzle gives the thrust.

So, this intake system why we require because when we are operating a subsonic aircraft which is cruising at Mach number 0.8 to 0.85 and for a supersonic aircraft when it cruise at 2 to 2.5, we expect that the axial Mach number to the compressor should be in the range of 0.4 to 0.5.

Now, how we will ensure that the flow Mach number at the entry to compression should be 0.4 to 0.5; that is what this intake system that does its job. So, what it does during takeoff, the engine operates at maximum power and airflow, in such situation intake cater the wide range of operating conditions.

(Refer Slide Time: 29:10)



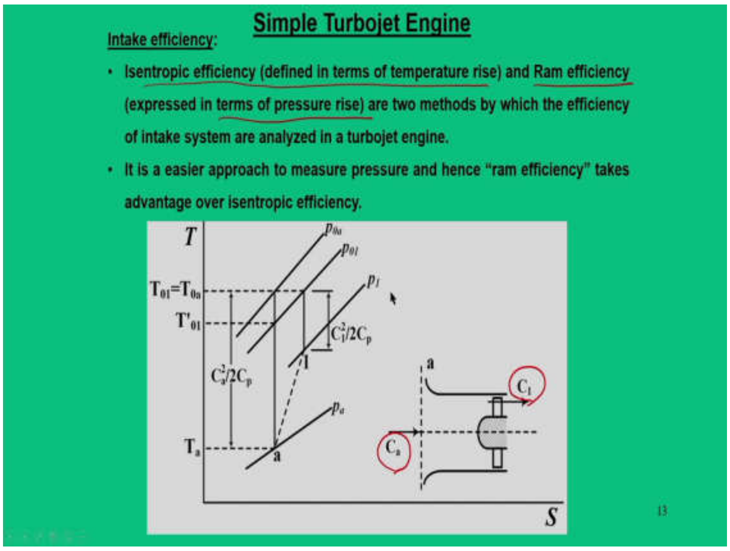
And what does this mean by this intake, this intake is typically nothing but a adiabatic duct. And in this adiabatic duct, the intake requirement is to minimise the pressure loss up to compressor phase by ensuring uniform pressure and velocity at all flight conditions. At low forward speeds that is static conditions, the intake acts as a nozzle which means its geometry is such that at low speeds it acts as a nozzle that means, flow accelerates when it tries to reach the compressor.

Now, when the the flight reaches at a very normal speed, the intake acts as a diffuser; that means, flow deceleration takes place. That means, it reduces the flow of speed by virtue of its geometry, because you will find that when the geometry is such that in subsonic flow, the shape of nozzle and for supersonic flow, the concept of nozzle are geometric specific.

Now, either you accelerate the flow or decelerate the flow, one thing that happens is that pressure always rises in the intake. So, when the pressure always rises and this pressure we call this as a “ram pressure rise”.

So, if you look at the Ts diagrams. So, 1 to 2 is actual for the compression stage, but to reach 1 we have to go from ambient a to 1. So, this particular part happens in the intake system. And similarly from the exhaust side what happens, from the turbine the expansion takes place from 3 to 4 that is in the turbine and rest of the expansion takes place in the nozzle that is from 4 to 5.

(Refer Slide Time: 31:10)



So, based on this we are going to define some other parameters and we call this as a Intake efficiency. So, intake efficiencies are defined in two ways. One is conventionally we call as Isentropic efficiency, which defined in the form of temperature rise, other we express in the form of ram efficiency and that is expressed in the form of pressure rise. So, these two methods we use to analyse the intake system for a turbojet engine.

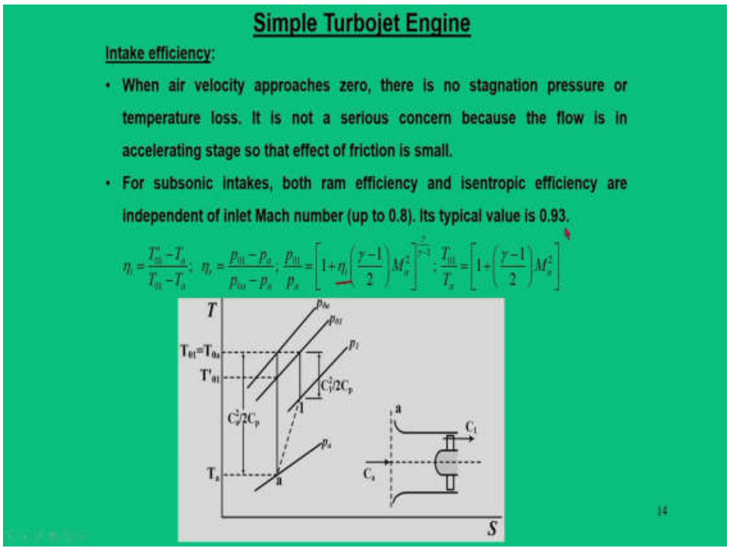
Now, if you look at a TS diagram and C_a is your airspeed that enters in the intake and C_1 is the speed that which air enters to the compressor. Now if you look at the TS diagram. So, we draw the constant pressure line p_a which is ambient and we have to reach the condition 1 that is a to 1 is your actual process that happens.

On this diagram we are going to superimpose that what should be the stagnation pressure values. So, corresponding to this p_a we have stagnation pressure p_{0a}, and corresponding to this p_1 that is static pressure condition 1, we have stagnation pressure p_{01}.

So, if you drop normal to this temperature axis, then we get the temperature values depending on situation. Like if you drop this value from a constant total pressure line or stagnation pressure line, we get stagnation temperature, the vertical line and here we will also get this static temperature value if you start from the static pressure line.

So, T_a is a static pressure whether T_{0a} is your stagnation pressure because this point refers to a stagnation pressure value.

(Refer Slide Time: 33:13)



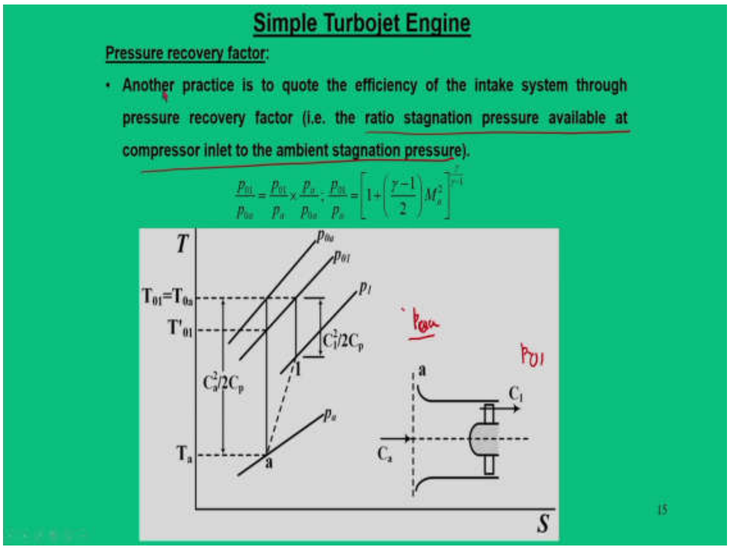
Now let us see that we are going to define this intake efficiency. So, first form is defined in the form of isentropic efficiency $\eta_i = \frac{T'_{01} - T_a}{T_{01} - T_a}$; that means, the actual temperature rise to the isentropic temperature rise.

And similarly for ram efficiency can be calculated as $\eta_r = \frac{p_{01} - p_a}{p_{0a} - p_a}$. In the last class I have explained about the ratio of stagnation pressure to static pressure and that relation with respect to Mach number.

So, one can find out the relations $\frac{p_{01}}{p_a} = \left[1 + \eta_i \left(\frac{\gamma - 1}{2}\right) M_a^2\right]^{\frac{\gamma}{\gamma-1}}; \frac{T_{01}}{T_a} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M_a^2\right]$.

So, here we are going to introduce this $\frac{p_{01}}{p_a}$ as a function of intake efficiency η_i. And intake efficiency value is normally taken as 0.93 for aircraft cycle.

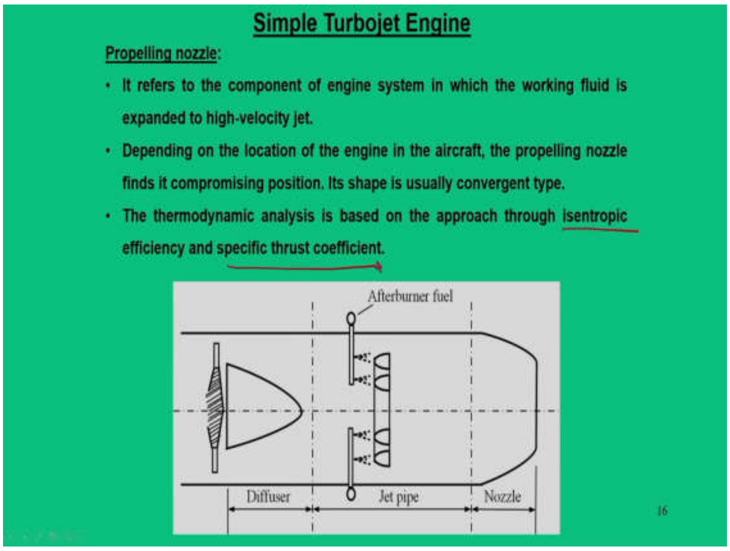
(Refer Slide Time: 34:31)



Apart from this intake efficiency there is another term that is widely used we call this as a pressure recovery factor. So, in order to quote the efficiency of intake system through pressure recovery factor, what we must know is that that the ratio of stagnation pressure available at the compressor inlet to the ambient stagnation pressure.

So, if you just want to calculate that this ratio pressure recovery factor, we can simply write that $\frac{p_{01}}{p_{0a}} = \frac{p_{01}}{p_a} \times \frac{p_a}{p_{0a}}$. And this $\frac{p_{01}}{p_a} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M_a^2\right]^{\frac{\gamma}{\gamma - 1}}$. So, these are the working formula that are used to calculate the pressure recovery factor.

(Refer Slide Time: 35:37)

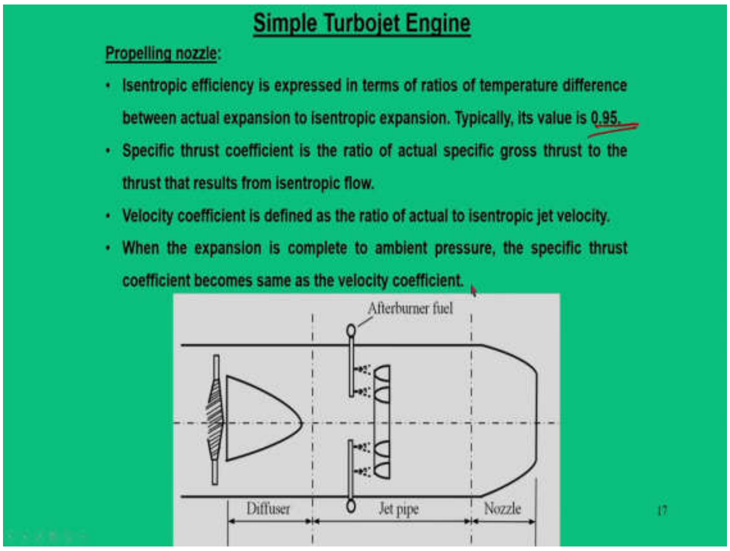


So, this is how we want to analyse the intake part. Now let us see how we are going to analyse the propelling systems.

So, propelling system is attached towards the end of the gas turbine engine. So, here what we see is when the flow goes out of a nozzle to exhaust. So, after expansion the flow has to pass through a nozzle. So, nozzle has to do this job to take care about the flow expansion. So, the propelling nozzle refers to the component of engine systems in which the working fluid is expanded to high velocity jet.

So, depending on the location of engine on the aircraft, the propelling nozzle finds a compromising positions or engine should be in a balanced positions and typically it is a convergent type or many situation which also can have divergent type or many case we can have a variable nozzle also. And the thermodynamic analysis or approach is based on through isentropic efficiency that is one part and second part is calculation of specific thrust coefficients.

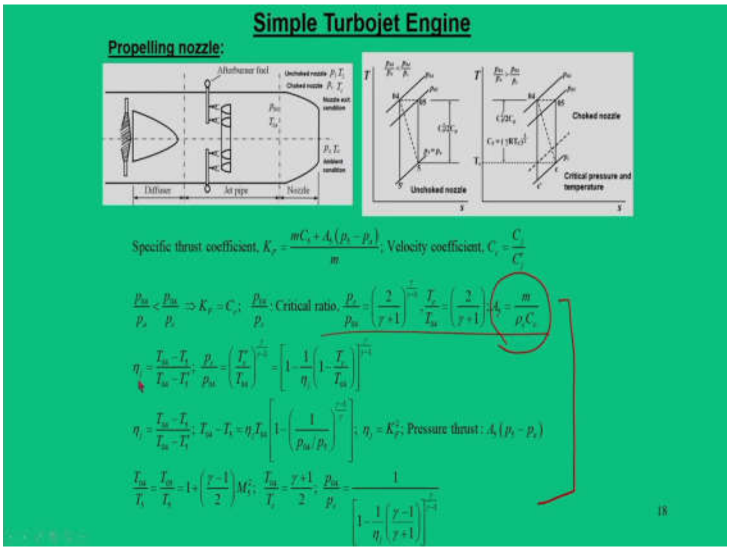
(Refer Slide Time: 36:56)



Let us see how you are going to get. This first the isentropic efficiency is expressed as the ratio of temperature difference between actual expansion to isentropic expansion and its value is typically 0.95. Whereas, specific thrust coefficient is nothing but the ratio of actual gross thrust to the thrust that is resultant from isentropic flow. Had the flow have been isentropic what would have been the thrust.

Some other parameter of importance is the velocity coefficient which is defined as the ratio of actual to isentropic jet velocity. Now, when the expansion is complete to ambient pressure, the specific thrust coefficient becomes same value as velocity coefficients. So, we will see that how you are going to analyse this propelling nozzle.

(Refer Slide Time: 37:47)



Now, let us see that what does this propelling nozzle do.

So, main intention that flow has to expand and we get a jet out of it. Now when the initial condition that comes out for this nozzle is nothing but your stagnation condition at the nozzle inlet. So, you start this stagnation condition either p_{04} and T_{04}, and from this value it has to expand to ambient conditions.

Now, while expanding that there could be two possibilities that the nozzle is sufficient enough to expand into ambient conditions or the nozzle is not doing its job. It could not expand properly to it ambient conditions. So, when it is unable to expand to its ambient conditions we call this as a unchoked nozzle. So, outlet condition on those situation will be p_5 and T_5. So, it is not expanded properly.

Now, when the nozzle becomes choked; that means, at particular conditions nozzle becomes choked and we get another pressure ratio instead of p_5 we call this as a p_c. So, basically there are three possibilities nozzle expands completely to ambient pressure or it will be unchoked conditions after the nozzle that will be p_5 and T_5, other thing is that third condition is nozzle will be choked that condition will be p_c and T_c.

So, these three conditions we can have. So, if you look at this particular situation when you have unchoked nozzle, the pressure ratio that is $\frac{p_{04}}{p_a} < \frac{p_{04}}{p_c}$. So, we will have ambient pressure still below 5, but still we are not getting that ambient condition through this nozzle. So, it is a unchoked nozzle.

Now, when the nozzle is choked; that means, nozzle cannot expand further mass flow rate is freezed, those condition is decided by the critical pressure and temperature conditions. Now, when the mass flow rate is freezed, so, we call this as a sonic flow at the nozzle and those conditions are decided by the critical value known as critical pressure or temperatures. So, we call this as a choked nozzle.

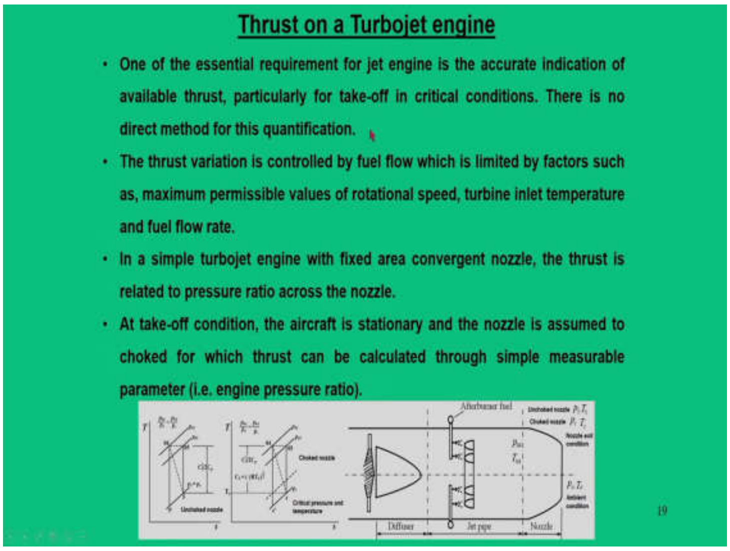
And there instead of 5, we will write them as a condition C or C`. So, expansion goes from 4 to C. So, this is how the basic philosophy of nozzle process in a turbojet engines.

So, based on that we have calculated different terms first term is a non-dimensional term we call as a specific thrust coefficient $K_F = \frac{mC_5 + A_5\left(p_5 - p_a\right)}{m}$. Other term is velocity coefficient $C_c = \frac{C_j}{C_j'}$ and we also define this critical pressure value when the flow is choked. So, when the flow is choked, $A_5 = \frac{m}{\rho_c C_c}$.

Apart from this we have the jet efficiency, isentropic efficiency of nozzle $\eta_j = \frac{T_{04} - T_5}{T_{04} - T_5'}$ can be also written in the form of $T_{04} - T_5 = \eta_j T_{04}\left[1 - \left(\frac{1}{p_{04}/p_5}\right)^{\frac{\gamma-1}{\gamma}}\right]$.

And typically these expressions are used for as a working formula and they are derived based on the gas dynamic principle of the conditions of the how the static conditions and stagnation conditions are related in terms of pressure and in terms of temperature. Apart from that we have introduced terms like efficiency term.

(Refer Slide Time: 42:33)



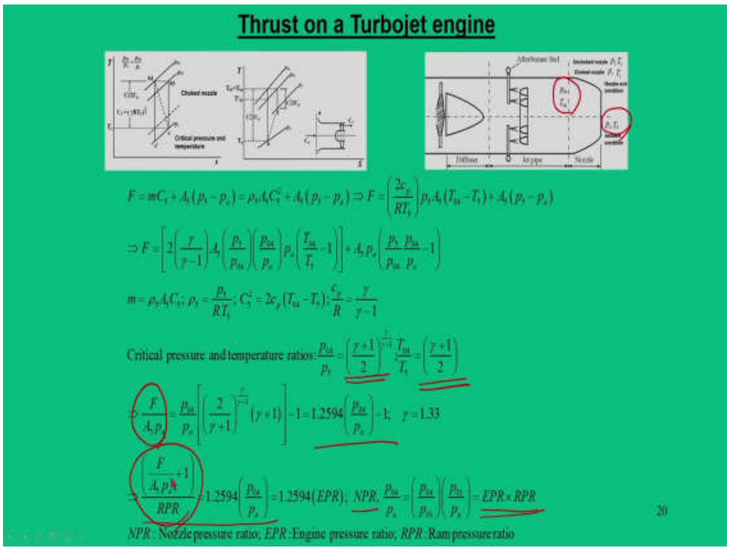
And next aspect is that after getting the requisite jet, now let us say that from that jet how you are going to evaluate the thrust and that we call as a thrust on a turbojet engines. So, as I mentioned, we can have a situation that nozzle exit condition can be a choked or unchoked condition and mostly it is choked conditions. And when it is choked condition; that means, nozzles are designed that it has to do its job till choking conditions.

So, based on those conditions, we are going to evaluate the exit parameter. And with this exit parameter we are going to calculate the thrust. So, accurate indication of thrust on a jet engine is very vital for takeoff conditions. So, but there is no direct method of its quantifications. And this thrust variation is controlled by the fuel flow which is limited by factors such as permissible values of rotational speed, turbine inlet temperature and flow rate.

So, in a simple turbojet engines with fixed area convergent nozzle we are going to calculate how the thrust is related to the pressure ratio in the nozzle. So, means that we are going to give an estimate in a Layman sense that how by regulating the pressure ratio across the nozzle we can control the thrust. So, this is the basic principle.

So, at a takeoff condition the aircraft is stationary and the nozzle is assumed to be choked for which thrust can be calculated through simple measurable parameters.

(Refer Slide Time: 44:17)



So, how it is calculated. So, thrust is basically two parts: jet velocity plus the pressure drag that is $F = mC_5 + A_5\left(p_5 - p_a\right)$. Now, putting this value mass flow rate as ρAC, we can get this particular expressions $\rho_5 A_5 C_5^2 + A_5\left(p_5 - p_a\right)$.

So, we also know that you have to use the terms like density is calculated based on $\rho_5 = \dfrac{p_5}{RT_5}$. And this p_5 and T_5 conditions are limited with respect to its critical pressure values. And that critical pressure values is $\left(\dfrac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}$ and $\dfrac{T_{04}}{T_5} = \left(\dfrac{\gamma+1}{2}\right)$.

So, these expressions, we are going to use here. How? Now from this equations here we are going to replace $m = \rho_5 A_5 C_5;\ \rho_5 = \dfrac{p_5}{RT_5};\ C_5^2 = 2c_p\left(T_{04} - T_5\right);\dfrac{c_p}{R} = \dfrac{\gamma}{\gamma-1}$.

So, when we simplify this particular equations we get a term which is a non-dimensional term $\dfrac{F}{A_5 p_a}$; A_5 is the exit area at condition 5 and p_a is your ambient pressure. And this becomes a function of a parameter which is nothing but $\dfrac{p_{04}}{p_a}$. So, the non-dimensional term $\dfrac{F}{A_5 p_a} = 1.2594\left(\dfrac{p_{04}}{p_a}\right) - 1$.

Now see here, the importance of this expressions that, now if you simplify this equation, we get another relations.

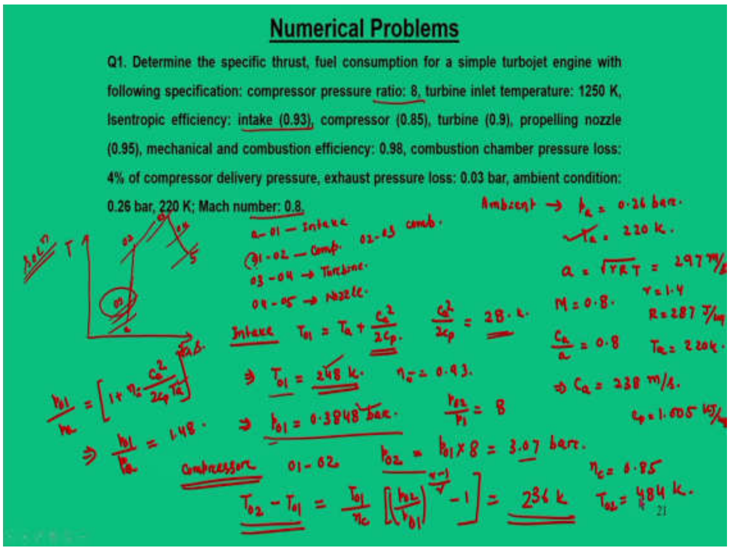
$$\frac{\left(\dfrac{F}{A_5 p_a}+1\right)}{RPR}=1.2594\left(\frac{p_{04}}{p_a}\right)=1.2594\left(EPR\right);\;\; NPR,\ \frac{p_{04}}{p_a}=\left(\frac{p_{04}}{p_{01}}\right)\left(\frac{p_{01}}{p_a}\right)=EPR\times RPR$$

NPR: Nozzle pressure ratio; EPR: Engine pressure ratio; RPR: Ram pressure ratio

So that means, nozzle pressure ratio is equal to EPR multiplied by RPR. And this left hand side of this expression is the non-dimensional number for the thrust indicator.

So, this non-dimensional number can be regulated by this one particular parameter which is known as a nozzle pressure ratio. And in fact, we started with a big equations and ultimately we end up in a very simplified empirical relations just by regulating the nozzle pressure ratio. So, this is how the thrust is calculated for a turbojet engine.

(Refer Slide Time: 48:54)



So, now we are going to solve a numerical problem for a simple turbojet engine and with a main intention is that here we have to look for thrust, not the power output. So, the problem is that for a given turbojet engine we have specifications as compression pressure ratio, turbine inlet temperature 1050 K, we have isentropic efficiency for intake 0.93, compressor 0.85, turbine 0.9.

And we have propelling nozzle efficiency 0.95. Mechanical and combustion efficiency this is required as a transmission efficiency and combustion efficiency 0.98, combustion pressure loss is 4 percent of delivery pressure, exhaust pressure loss is fixed at 0.3 bar, ambient condition is 0.26 bar. That means, it is at some altitude conditions where ambient pressure is 0.26 bar and temperature is 220 K. And the flight is moving at Mach number of 0.8.

So, to solve this problem the first thing that we have to do is that we have to draw the circuit diagram; Ts diagram. So, TS diagram we will just imply some of the important processes, first is intake to compressor. We have two pressure line intake pressure and compression pressure, and from this we have turbine and the nozzle.

So, we say ambient condition 'a' and we have compressor inlet condition p01 and it goes this is compressor exit condition 02, then we have turbine that is 3-4 process then 4-5 is nothing but ambient pressure. So, 0 ambient to 01 intake, 01 to 02 compression, 03 to 04 turbine of course, prior to this 02 to 03 combustion and 04 to 05 nozzle. So, here I have used 0 because this refers to we have to use the stagnation value.

So, ambient is given for which we have p_a 0.26 bar and T_a is 220 K.

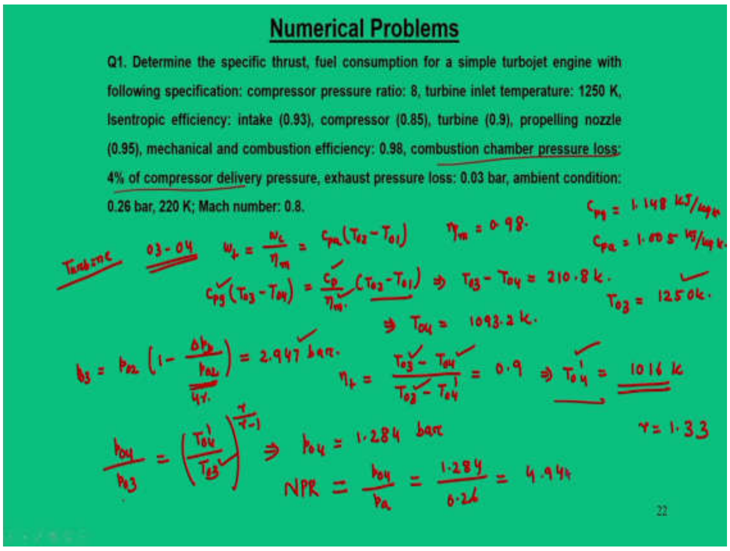
$$a = \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 220} = 297\text{m/s}; M = 0.8 \Rightarrow C_a = 0.8 \times 297 = 238\text{m/s}$$

Intake: $T_{01} = T_a + \dfrac{C_a^{\,2}}{2c_p} = 220 + 28 = 248K; \dfrac{p_{01}}{p_a} = \left[1 + \eta_i \dfrac{C_a^{\,2}}{2c_p T_a}\right]^{\frac{\gamma}{\gamma-1}} = 1.48$

$\Rightarrow p_{01} = 0.3848\text{bar}$

Compressor: $p_{02} = p_{01} \times 8 = 3.07\text{bar}; T_{02} - T_{01} = \dfrac{T_{01}}{\eta_c}\left[\left(\dfrac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] = 236K \Rightarrow T_{02} = 484K$

(Refer Slide Time: 58:24)



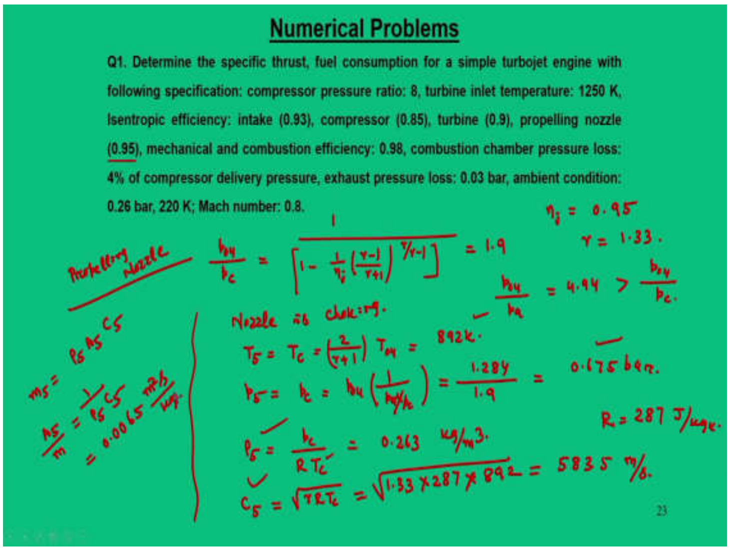
$$\text{Turbine: } w_t = \frac{w_c}{\eta_m} \Rightarrow C_{pg}\left(T_{03} - T_{04}\right) = \frac{C_{pa}\left(T_{02} - T_{01}\right)}{\eta_m} \Rightarrow T_{04} = 1250 - 210.8 = 1093.2K$$

$$p_{03} = p_{02}\left(1 - \frac{\Delta p_b}{p_{02}}\right); \frac{\Delta p_b}{p_{02}} = 4\% \Rightarrow p_{03} = 2.947\text{bar}$$

$$\eta_t = \frac{T_{03} - T_{04}}{T_{03} - T'_{04}} = 0.9 \Rightarrow T'_{04} = 1016K; \frac{p_{04}}{p_{03}} = \left(\frac{T'_{04}}{T_{03}}\right)^{\frac{\gamma}{\gamma-1}}, \gamma = 1.33 \Rightarrow p_{04} = 1.284\text{bar}$$

$$\text{NPR} = \frac{p_{04}}{p_a} = 4.94$$

(Refer Slide Time: 63:48)



Once you have this then we will move to propelling nozzle.

$$\frac{p_{04}}{p_c}=\frac{1}{\left[1-\frac{1}{\eta_j}\left(\frac{\gamma-1}{\gamma+1}\right)\right]^{\frac{\gamma}{\gamma-1}}};\eta_j=0.95,\gamma=1.33\Rightarrow\frac{p_{04}}{p_c}=1.9$$

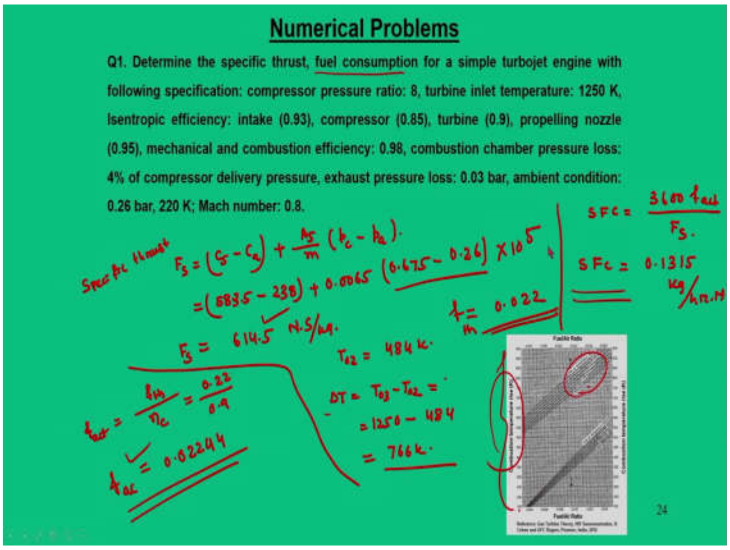
$$\because\frac{p_{04}}{p_a}=4.94>\frac{p_{04}}{p_c}\Rightarrow\text{Nozzle is choking}$$

$$\Rightarrow \mathrm{T}_5=T_c=\frac{2}{\gamma+1}T_{04}=892K,\,p_5=p_c=p_{04}\left(\frac{1}{p_{04}/p_c}\right)=0.675bar$$

$$\rho_5=\frac{p_c}{RT_c}=0.263kg/m^3;\therefore\text{ As choked, }M=1\Rightarrow C_5=\sqrt{\gamma RT_c}=583.5\text{m/s}$$

$$\because m_5=\rho_5A_5C_5\Rightarrow\frac{A_5}{m_5}=0.0065\text{m}^2\text{s/kg}$$

(Refer Slide Time: 68:02)



Then we can find out specific thrust.

$$F_s=m\left(C_5-C_a\right)+\frac{A_5}{m}\left(p_c-p_a\right)=\left(583.5-238\right)+0.0065\left(0.675-0.26\right)\times10^5=614.5\text{Ns/kg}$$

Then once you have these things we have to use the fuel consumption. For the fuel consumption you have to use this particular chart. So, for that we require two parameters T_{02}; that is compressor delivery temperature 484 K, $\Delta T = T_{03} - T_{02} = 766$ K. And from this we can calculate this fuel air ratio as 0.022.

So, when you have theoretical fuel air value, then you can find actual value that is a theoretical divided by combustion efficiency that is 0.22 by 0.98. So, actual value would be 0.02244. Then we can calculate SFC, $SFC = \frac{3600 f_{act}}{F_s} = 0.1315\text{kg/hr.N}$

So, this particular problem will give you a complete understanding how you start a turbojet engine from its intake side and you end with your values or calculations in the propelling nozzle side. Now when you end with this your main intention should be calculation of thrus, fuel consumption.

So, with this we conclude for the aircraft propulsion part 1 lecture. So in my next lecture I will continue aircraft performance cycle part 2.

Thank you for your attention.