Applied Thermodynamics

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Module - 06
Reciprocating Air Compressor
Lecture - 44
Multistage Compression - Analysis and Modelling

Dear learners, greetings from IIT Guwahati, we are in the course Applied Thermodynamics module 6, Reciprocating Air Compressors.

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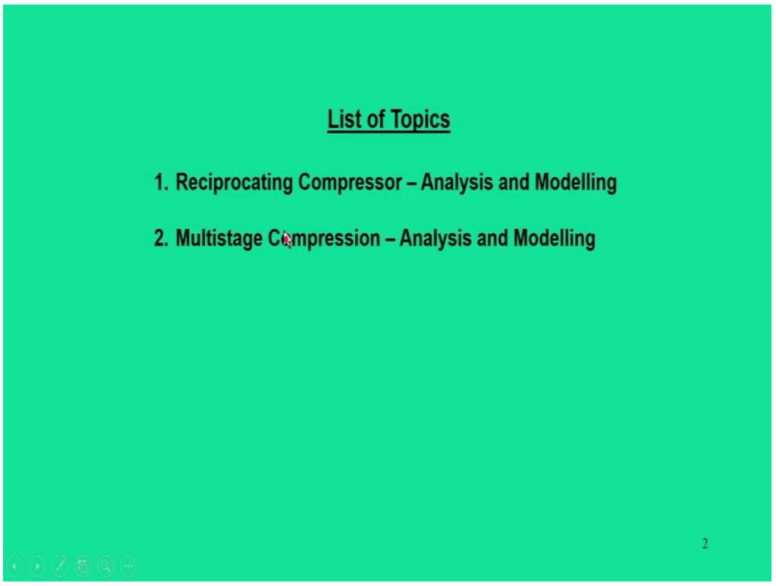

There are two lectures on this module 1st one is reciprocating compressor mostly in the single stage compressor, its analysis and modelling, which you have covered and in today’s lecture we will be discussing about Multistage Compressions how you are going to do and its Analysis and Modelling.

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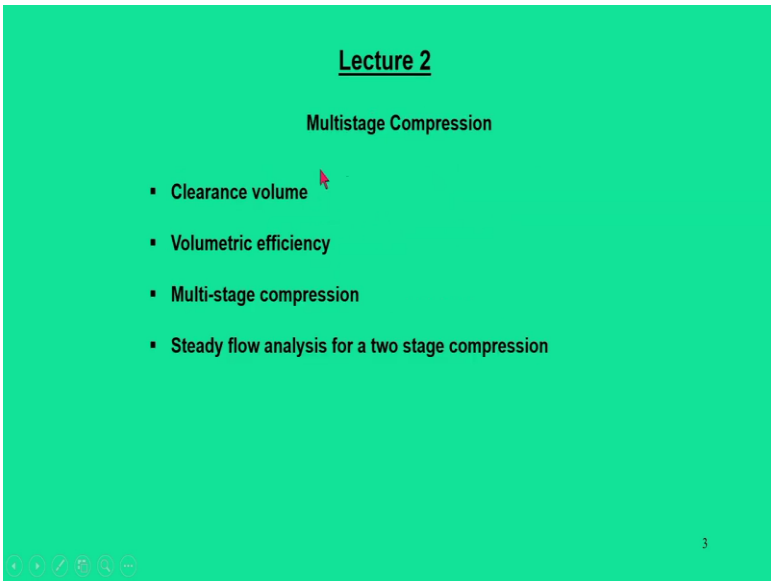

Now, before you go for this multistage compression mode, we also have to know some certain basic concepts for the compressor and what we call as a clearance volume and volumetric efficiency. And in fact, these two things are very much required when you deal with a multi compression mode maybe involving number of stages. Then we will try to see what is the modelling for this multistage compressions and finally, I will end this topic through a thermodynamic steady state analysis for a two stage compression.

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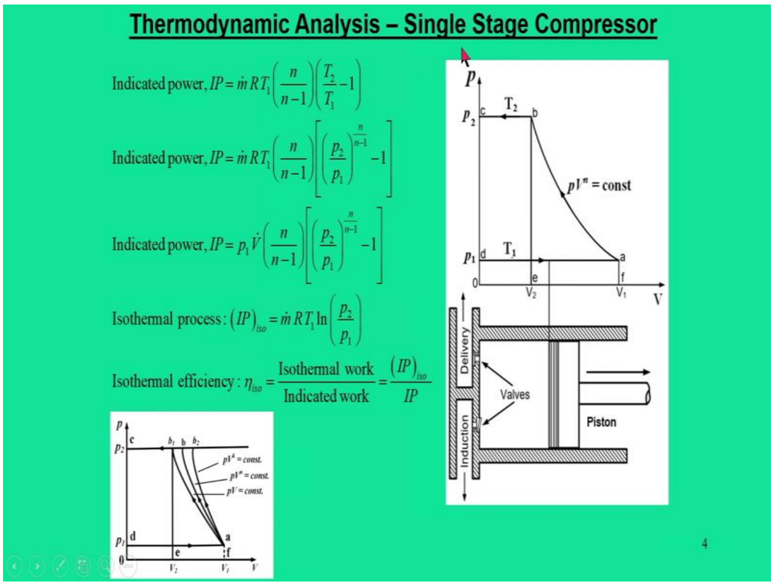

Now, in the last lecture that is when you dealt with the single stage compression. So, what was the main intention is that, we have piston and that piston moves in a cylinder and in fact, there is zero clearance; that means, it is assume that piston position is just end of this cylinder face and where this induction or inlet or exit valves are placed. So, through this induction the air enters into the system or in to compressors and to delivery with finally, the pressure that goes out is at higher pressure p_2. So, the very basic bottom line is that at the end of our compression process the gas undergoes a pressure change from p_1 to p_2 by following a thermodynamic compression process polytropic process that is $pv^n = C$.

And for this compression process we derive that how much indicated power we require. And this indicated power is expressed in many versions, one is in the form of temperature ratios, other is in the form of pressure ratios and where in some cases we have used mass flow rate in some cases we have used volume flow rate. So, depending on the nature of the problem we can use this expression.

Another important aspect is that an ideal compressor would be the one which consumes minimum work and from this curve we have seen there are two extremes; that means, if a compression process goes from a to b which is a normal process and that follows the law $pv^n = C$ and we have seen is that the only possibilities that we can minimize the work is only through changing the index of compressions.

Because p_1 and p_2 is desired by the user, volume is fixed by the given size of the compressors. So, only possibilities to minimize the work is based on the value of n. Ideally for isothermal process n is equal to 1. So, this equation becomes $pv = C$ and when n is equal to k or gamma that is for air the value is 1.4.

So, the other extreme is the $pv^k = C$. So, in that sense that we can think of two extreme processes one is isothermal process a-b_1 other is the isentropic process a-b_2. And through this processes, we can say that isothermal process undergoes the minimal work because from the area under p-v diagram will show you that isothermal process will give you the minimum work whereas, the isentropic process will give you the highest work.

So, isentropic process is not advisable and isothermal process, although it goes for minimum work and it is not a realistic version because when you do this compression temperature is going to increase and it is very difficult to maintain a constant temperature or isothermal process while undergoing the compressions.

So, ideal nature of this compression process is normally dealt with $pv^n = C$. And typically, n value is kept in the range of 1.2 to 1.3 and based on this isothermal comparisons we can find the ratio that what is the isothermal efficiency that is minimum work for the isothermal process to the indicated work.

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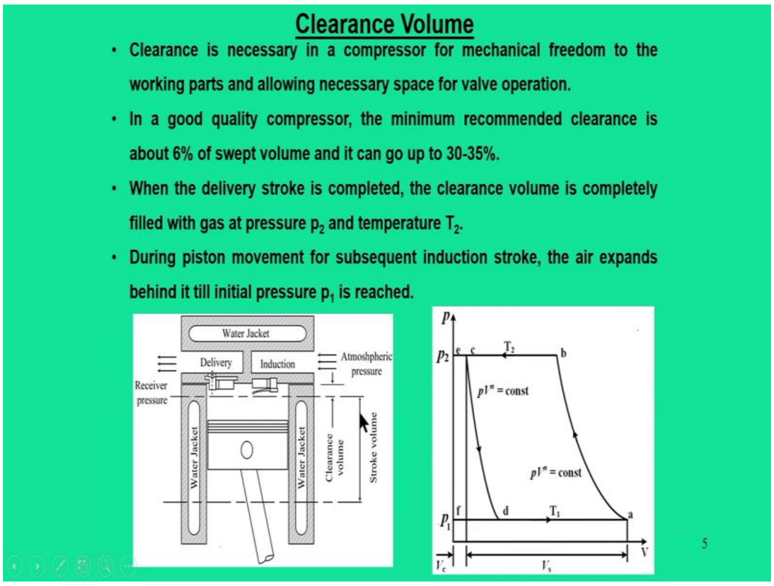

Next thing is that we are going to dig into this another concept, which is clearance volume. So, it is a realistic version of a single stage compression process, a realistic version can be realised only when we can have a clearance volume to be maintained in a piston cylinder arrangement.

That is mainly because since piston is a mechanical device and it has a finite length of travel within the cylinder and we call this as a swept volume. There are upper limit and lower limit and at the same time we also expect that piston should not go and hit the bottom part of the cylinder.

So, for that thing, we must expect that there has to be a clearance volume and for the free movement of the piston within the cylinder. There will be also water jackets in and around that because we have to keep this temperature close to isothermal so that during the compression process we can take out this heat through the water jackets.

So, that is the reason that we expect that two things; one is we have to find a realistic version to maintain a close to isothermal process just by releasing the temperature of heat to the water jacket, second thing we have to maintain a clearance volume. And typically for a good quality of compressors the minimum recommended clearance is about 6 % but it can go up to 30 to 35% maybe for a large scale.

But when the delivery stroke is completed, the clearance volume is completely filled with the gas p_2 and T_2 and during the piston movement of subsequent motion in induction stroke the gas expands behind it till the final pressure is reached. What it means is that after the compression process that starts from a and goes up to b and thereby this delivery of air that goes out, but side by side there are some gas that left out within this clearance volume.

And the during the pistons backward movement these gases tries to expand. So, when they expand, the volume which was supposed to be at c, now expands and finally, the pressure comes down to p_1 and from actual volume that gets induced that is from d to a, that is the amount of gas that is required for this volume that is V_a - V_d.

Ideally if there is no clearance, it would have started taken this swept volume as Vs and since, the gas expands, the induced volume gets reduced that is from original value Vs to V_a - V_d. So, that is the effect of clearance.

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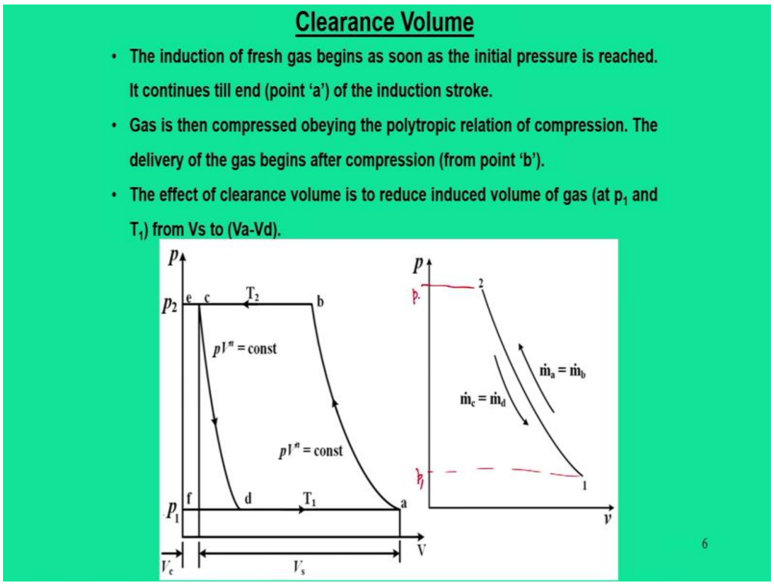

So, though the induction of fresh air begins as soon as the initial pressure is reached and it continues to till the end of the point a for the induction stroke. So, gas is then compressed by obeying the polytropic compressions and delivery of gas begins from the point b.

The effect of clearance is to reduce the induced volume at point p_1 and T_1 from Vs to V_a - V_d. Another interesting point is that whatever mass that is available at point a ($\dot{m}_a$) and that entire mass gets compressed which becomes ($\dot{m}_b$). So, $\dot{m}_a = \dot{m}_b$ and what are the left out gas that is at point c, $\dot{m}_c = \dot{m}_d$ because this gas actually expands.

And due to presence of the gas, induced volume into the cylinder gets reduced. But if you take the final thermodynamic states from 1 and 2 you will find that in the forward cycle $\dot{m}_a = \dot{m}_b$, in the backward cycle $\dot{m}_c = \dot{m}_d$. At the end the pressure rises from p_1 to p_2.

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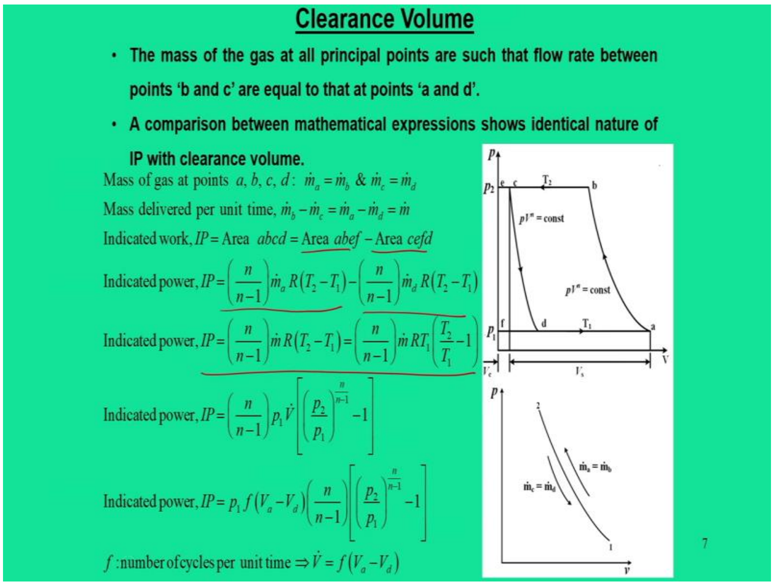

Now, let us see some mathematical insights what happens if there is a clearance volume what happens to our indicated power calculations? So, first thing we have to calculate at the mass of the gas at important points a b c d. And we have already proved that $\dot{m}_a = \dot{m}_b$ & $\dot{m}_c = \dot{m}_d$. So, this mass delivered per unit time will be $\dot{m}_b - \dot{m}_c = \dot{m}_a - \dot{m}_d = \dot{m}$.

So, $\dot{m}$ is the mass that is actually delivered per unit time. And now from this p-V diagram we can easily calculate what is the value of indicator work and that is nothing but your area a-b-c-d and this area is nothing but the difference in the area that is $\text{Area } abef - \text{Area } cefd$. So, if you take these things and if you recall our earlier expressions the indicated power can be calculated by two expressions.

$$IP = \left(\frac{n}{n-1}\right)\dot{m}_a R\left(T_2 - T_1\right) - \left(\frac{n}{n-1}\right)\dot{m}_d R\left(T_2 - T_1\right)$$

And ultimately since $\dot{m}_a - \dot{m}_d = \dot{m}$. So, we can rewrite this expression as indicated power is $IP = \left(\frac{n}{n-1}\right)\dot{m} R\left(T_2 - T_1\right) = \left(\frac{n}{n-1}\right)\dot{m} RT_1\left(\frac{T_2}{T_1} - 1\right)$. So, this comparison for earlier mathematical expression for single stage compression without clearance volume, we have also same expression. So, this means clearance has no impact on the indicated power, but it changes the volumetric efficiency.

Then other expressions for these things we can write in same way that IP in the form of volume flow rate $IP = \left(\frac{n}{n-1}\right) p_1 \dot{V}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$ and if there are f number of cycles per unit time this volume will be $\dot{V} = f\left(V_a - V_d\right)$ that is the actual swept volume that enters into the cylinder.

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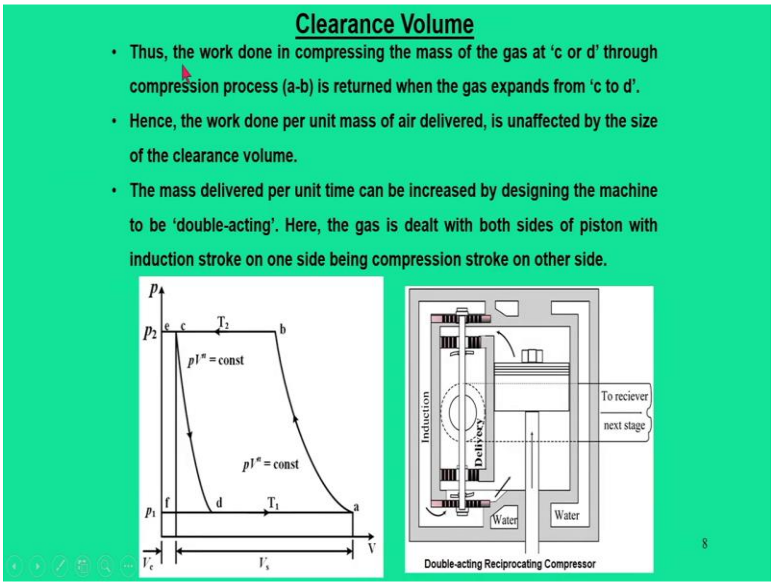

Thus, we can derive that the work done in compressing the mass of the gas at c or d through this compression process a-b is returned when the gas expands from the c to d. So, the work done per unit mass delivered is unaffected by the size of the clearance volume, but what happens is that this is there is a limitation of mass that is going to be inducted, this mass delivered per unit time can be increased by designing the machine to be a double acting.

So, the other way of looking at the same problem is that we can say that it is single stage compression, but double acting. Double acting basically means that during compression stroke, other side would be a expansion and vice versa. So, basically the induction and delivery valves are arranged in such a manner when the piston gets compressed, so, this part of the gas gets compressed and the other part of the gas gets expanded. So, for this compression stroke, this is the delivery and vice versa and when the piston moves backward the bottom part of the gas gets compression top part of the gas gets expanded.

In a sense that for same movement of the pistons we can get a double mass flow rate with some change in the arrangement. So, that particular machine we call is a double acting machine. Another version of the indicator diagram we also call as actual indicator diagram.

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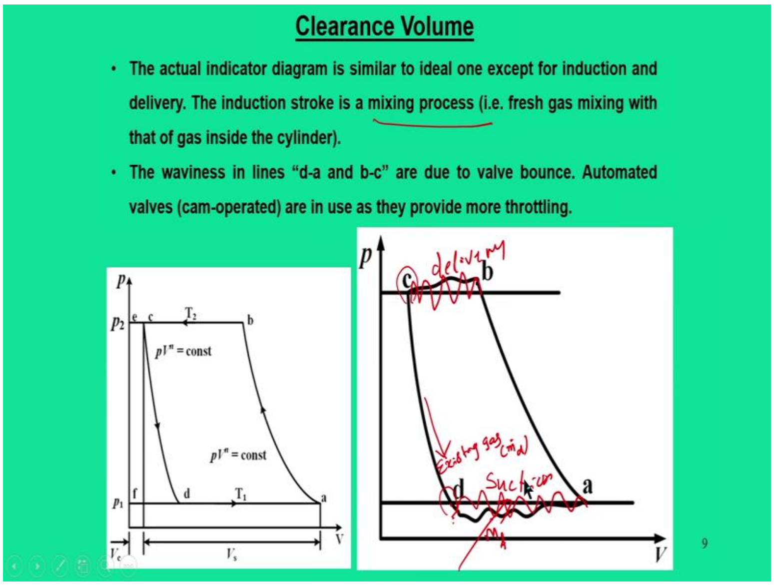

So, what we have seen is that, either induction or delivery of air we have the inlet valve or delivery valves. So, these valves are like a cam operated valve. So, cam operated valve means they suddenly opens and suddenly closes. Although they are automated, but these types of continuous opening and closing leads took some kind of waviness in the indicator diagrams.

So, if you look at the actual p-V diagram which is constitutes a-b-c-d and this is how we did our analysis and if you look at these things, there will be some waviness in the cycle and mainly not in the compression and delivery part, but only during the suction part and the delivery part. So, during compression and expansion, we will see these waviness is not there.

So, that thing we call is as a actual indicator diagram and also another important aspect is that during the induction process which is nothing but a mixing process so; that means, during the induction stroke and the gas which was at point c, now that gas has expanded and at point d.

So, we have this is as a existing gas that is at $\dot{m}_d$ and this is the fresh gas that during the suction stroke that enters into this cylinder and this gas comes from the clearance volume. So, these two gas mixes. So, this induction stroke is now a mixing process and this mixing process also has some boost for this kind of waviness on the indicator diagram curves.

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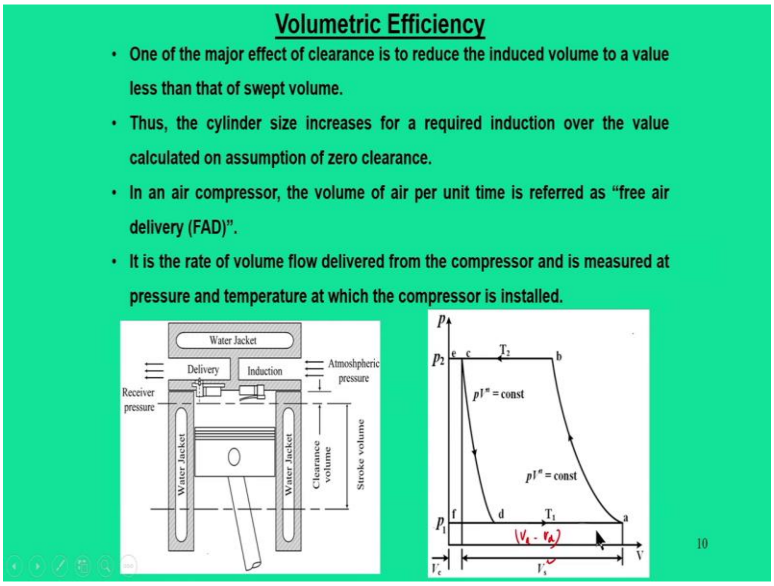

So, let us talk about another terminology what we call as a volumetric efficiency. So, what we have seen is that indicated power remains same irrespective of putting the clearance. Then the question will arise that there is no limit for the clearance, the other parameter that happens is that volumetric efficiency.

Now if you want to have a larger volume to be handled by the same compressors. then although we may not have a significant contribution of input power in indicated power requirement, but you have to compromise on the volumetric efficiency or volume handle by the compressors.

So, how do you quantify this volumetric efficiency and what are the significance and all these things we are going to discuss here. So, one of the major effect of clearance is to reduce the induced volume to a value less than that of swept volume that we can see that initially this volume was Vs, now this volume is now reduced to V_a -V_d.

So, the cylinder size increases for a required induction over a value and calculated under zero clearance. So, if this is the case then cylinder size must increase, another important aspect that we frequently use the term what is a FAD. So, that is called as free air delivery and that is for an air compressor we say is that volume of air per unit time that we call this as a FAD for a compressor.

So, it is the rate of the volume delivered from the compression and it is measured at pressure and temperature in which compressor is installed. So, what happens is that when the compression is placed in atmospheric conditions. So, atmospheric air enters, now for air to enter into the system we must create a pressure difference.

So, normally the suction pressure for the compressor is kept lower than atmospheric. So, the suction pressure is different from the atmospheric pressure, but the gas that enters into the system it is an atmospheric. So, all this mass flow rate which was calculated is based on the atmospheric conditions.

So that is what it says that the free air delivery is the rate of the volume delivered from the compressor and this free air delivery is measured at pressure and temperature in which compressor is installed and typically it is atmospheric pressure and temperature.

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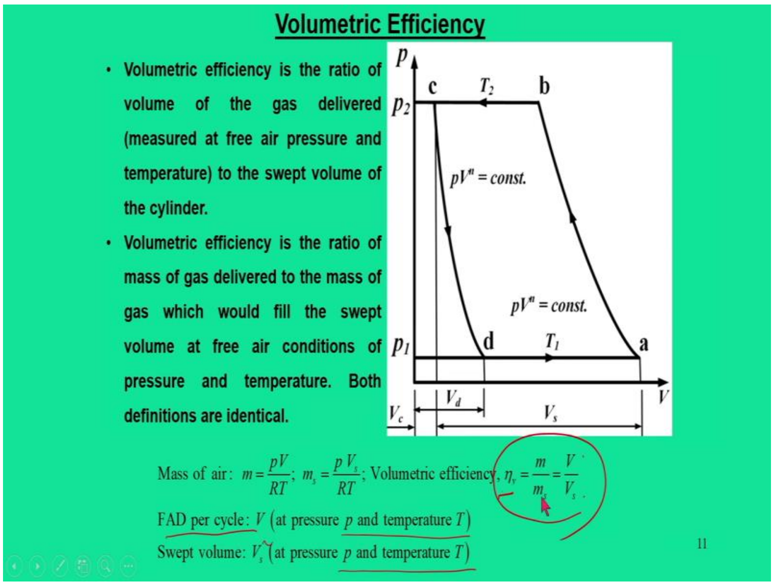

And now to quantify these clearance volume, we define a term what we call as a volumetric efficiency. In fact, there are two definitions one is in the form of mass other in the form of volume. So, the volumetric efficiency is ratio of the volume of the gas delivered that is measured at free air pressure and temperature to the swept volume of the cylinder.

So, cylinder swept volume is fixed and the volume of the gas that is delivered into the system that is measured at free air pressure and temperature (FAD). Other expression of volumetric efficiency is equal to $\frac{V}{V_s}$. The other expression we can also use that volumetric efficiency is the ratio of mass of gas delivered to the mass of gas that would fill the swept volume at free air conditions of pressure and temperatures.

So; that means, another expression we say $\eta_v = \frac{m}{m_s}$. Now we can see that both the expressions are identical. So, if you talk about this mass, we can calculate $m = \frac{pV}{RT}$ and m_s that is swept mass that is equal to $\frac{p\,V_s}{RT}$ and here there are two things one we have V. So, V is nothing but the free air delivery that is measured at pressure p and T.

And the V_s is the swept volume that is measured at pressure p and T. Now from the definition if you take $\eta_v = \dfrac{m}{m_s}$ and from this equation we can find volumetric efficiency in the function of mass ratio and as well as the volume ratio with respect to actual volume, with respect to swept volume and with respect to actual mass, with respect to swept volume mass.

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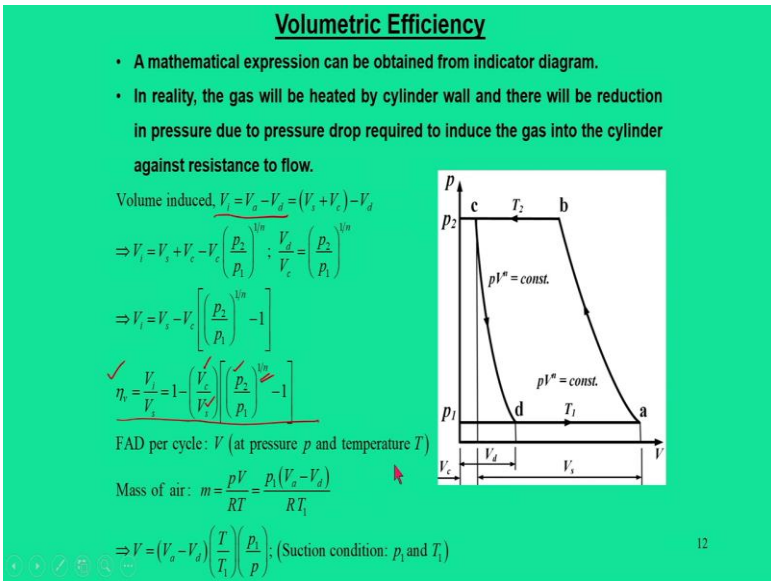

Now, let us find out more details into this mathematics based on this indicator diagram we can derive some working formula and what is that working formula? How much volume in this case induced if I say that is $V_i = V_a - V_d$. So, this V_a is nothing but $V_s + V_c$ and we also know that $\dfrac{V_d}{V_c} = \left(\dfrac{p_2}{p_1}\right)^{1/n}$. So, that is the index of compression, this equation holds good.

And from this we can simplify this expression; that means, induced volume now becomes $V_s - V_c\left[\left(\dfrac{p_2}{p_1}\right)^{1/n} - 1\right]$. So, by definition this volumetric efficiency we can write $\eta_v = \dfrac{V_i}{V_s}$. So, from this expression we can write $\eta_v = 1 - \left(\dfrac{V_c}{V_s}\right)\left[\left(\dfrac{p_2}{p_1}\right)^{1/n} - 1\right]$; as we can find a volumetric efficiency expressions and in the form of clearance volume *Vc* swept volume *Vs* and pressure ratio $\frac{p_2}{p_1}$; of course, there is a index of compression n.

So, another point is that in reality the gas will be heated by the cylinder wall and there will be reduction in the pressure due to the pressure drop to induce the gas into the cylinder against the resistance of the flow. So, what happens is that when you calculate actual mass flow rate $\frac{pV}{RT}$ and here volume is nothing but $V_a - V_d$

But the the volume that we are going to use which is volume induced that is $V_i = V_a - V_d$. But if such a case happens that suction and atmospheric conditions air different and that is when $V_a - V_d$ will be now replaced with $\left(\frac{T}{T_1}\right)\left(\frac{p_1}{p}\right)$.

Because this equation holds good that is mass flow rate remain same that is $pV = RT$ and that is also equal to $\frac{p_1(V_a - V_d)}{RT_1}$. So, '1' stands for the initial state in which the air is available to us.

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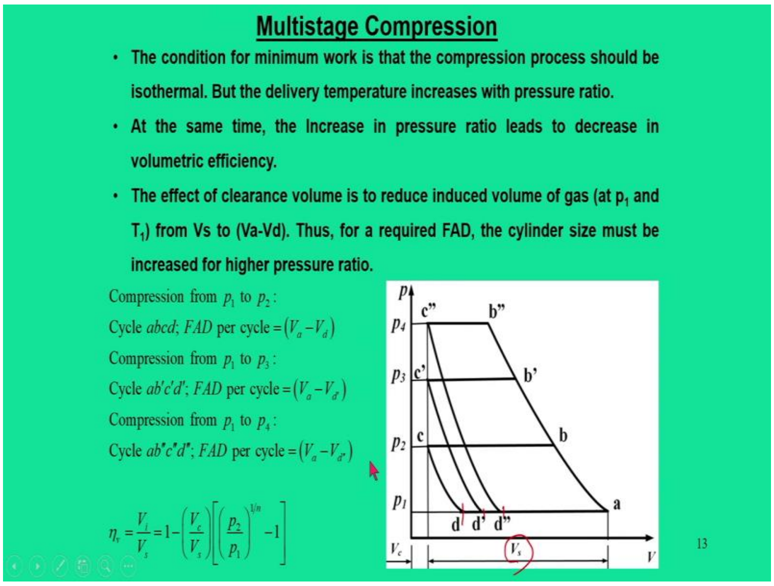

So, this is all about clearance volume and the volumetric efficiency. Now let us use this particular concept that how you are going for a multistage compressions. So, what we have already seen is that, the we have a condition of minimum work in a compression process for which the compression has to be isothermal, but delivery temperature increases with the pressure ratio.

So, maintaining isothermal condition is a difficult task side by side increased by through this compression process, but at the same time if we increase in the pressure ratio there is a decrease in the volumetric efficiency. So, these two things has to be compromised and the effect of clearance volume is to reduce the induced volume of the gas.

So, therefore, for a required free air delivery the cylinder size must be increased for higher pressure ratio; that means, if you want to go for a higher pressure ratio. So, in a single stage compression it is not a viable option. And we, your cylinder size must increase. Now, when is cylinder size is increasing then swept volume drops, actual volume that enters into the system drops.

So, if you can look at this particular cycle. So, let us see that we are undergoing a compression from p_1 to p_2. So, we can draw this actual cycle that is cycle is abcd. So, free air delivery for cycle would be $\left(V_a - V_d\right)$. Now, if you go to another compressor compression from p_1 to p_3 your FAD per cycle would be $\left(V_a - V_{d'}\right)$ and if you go from p_1 to p_4 then it will be $\left(V_a - V_{d''}\right)$.

So, what we have seen is that, with zero clearance, the volume was swept volume; with increase in the pressure ratio your free air delivery drops. And also because of this pressure ratio your volumetric efficiency also drops. So, because of this region we have to see that a single stage compression is not a viable option. So, we have to go for a multistage compression.

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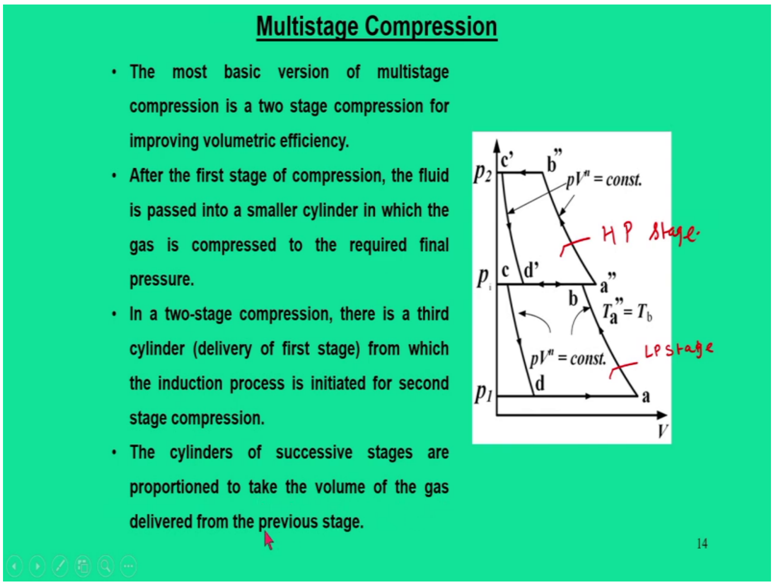

Now, the most basic version of a multistage compression is a two stage compression. So, two stage compression which is shown here that if you want to go for compression process from p_1 to p_2. So, in a first stage compression we can go in a cycle abcd, but during this compression process what happens? Your temperature also increases.

So, the next stage compression has to start at a higher temperature. So, the second stage higher compression starts at point a``b``c`d`. So, we can see this is a LP stage; that means, low pressure stage and this particular you call this as a high pressure stage.

So, in a two stage compressions, we have two stage a low pressure stage and high pressure stage, but what happens here is that, your indicated power also suits of because your temperature goes up. So, for that there is some kind of compromise that you have to do that when we go for a multistage compressions and moreover if your temperature goes up your indicated power also becomes higher. So, we must do some kind of solution for this case.

So, this very basic summary is that after the first stage compression the fluid which passed into this smaller cylinder in which the gas is compressed to the required final pressure. So, in a two stage compression there is a third cylinder from which the induction process is initiated for the second stage compression. So, cylinders of successive stages are proportioned to take the volume of the gas delivered to the previous stage.

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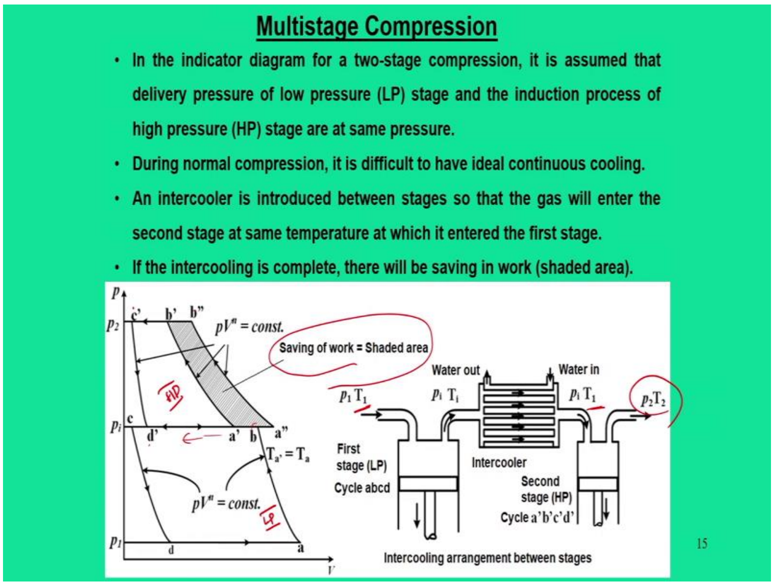

So, typical version of two stage compression goes like this, if you look at this particular p-V diagram, we have seen that the LP stage is the bottom part, high pressure stage is the top part and cycle in the high pressure stage is a``b``c`d`.

But what happens is that the at point b temperature is high and away from the point b, you can see that incurred area will be also higher. So, you will try to push this fellow b towards the left side so that temperature comes down and the power requirement in the HP sides should be less. So, for that the other possible alternative is that we add a intercooler.

So, we have a LP first stage that is low pressures stage for which the cycle is abcd and the second one is the high pressure side HP side. So, the cycle is a`b`c`d` now in between we add a intercooler. So, what does this intercooler do? So, at the beginning stage we have p_1 and T_1 stage. And after the first stage compression the peak pressure becomes p_i side by side your temperature also becomes T_i.

We have also seen that delivery temperature also increases with the pressure ratio, but the problem is that these T_i is a higher temperature and because of this thing; that means, the second stage compression instead of b will start from a``. And since it starts from a`` and if you continue a``b``c`d`, then it will have higher area.

So, what the entire objective is that we have to bring down the temperature of the gas through an inter cooling arrangement. So, what does inter cooling do? We have a circulating water and this circulating water take out heat from this gas with an intention is that with a complete inter cooling the whatever rise in the temperature that happens in the first stage the entire gas is now with a pressure p_i.

But at temperature T_1, T_1 is nothing but the way the gas that has entered. And the second stage again starts with higher pressure, but with a lowest temperature. So, with second stage compression we get the desired pressure p_2 and T_2. So, this is the entire objective so that what intercooler does this intercooler takes out this heat from this gas.

And through this process what benefit we get? So, instead of point b shifting toward a``, point b shift toward a`. So, as a result we can see the hatched area a`a``b``b` is nothing but your saving of work in the high pressure stage. So, this is the advantage of multistage thing and because of this region always multistage compressions is also integrated with an inter cooling arrangement.

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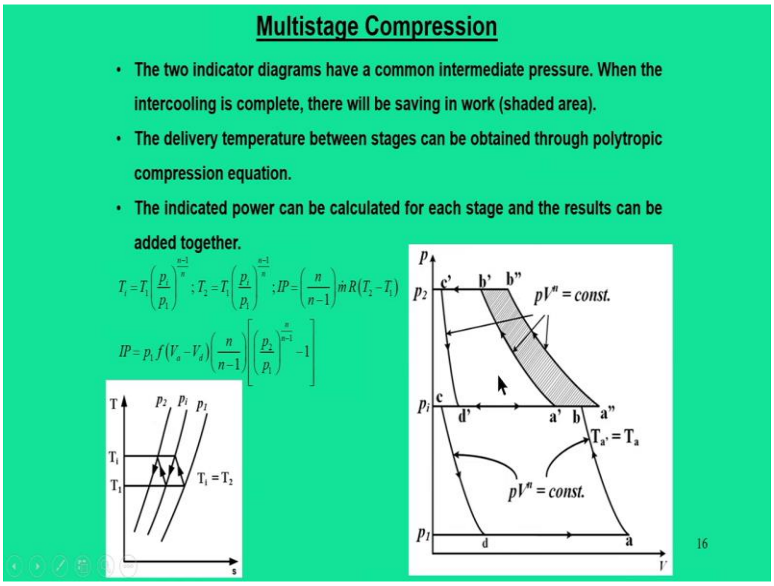

So, if you look at another version of temperature entropy diagram, what we have see? You have pressure p_1 p_2 and side by side we have intermediate temperature T_i is nothing but to this is T_2 and from this isentropic equations we can find out the intermediate temperatures and of course, we can get the indicated power from that expression

$T_i = T_1\left(\frac{p_i}{p_1}\right)^{\frac{n-1}{n}}$; $T_2 = T_1\left(\frac{p_i}{p_1}\right)^{\frac{n-1}{n}}$; $IP = \left(\frac{n}{n-1}\right)\dot{m}\,R\left(T_2 - T_1\right)$ and from this we will say that we will have a saving of work as any this shaded area.

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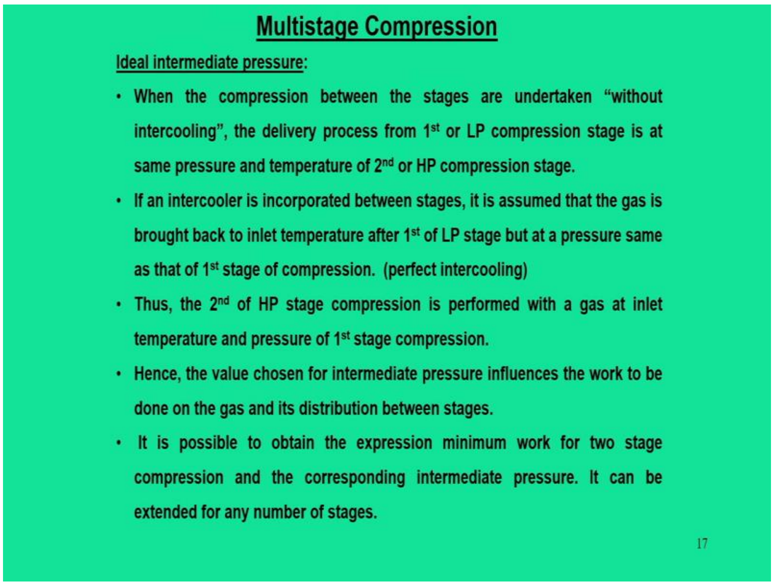

Now, the question is that what is the ideal conditions of this intermediate pressure. Through an inter cooling, So, what happens that when the compression between the stages is undertaken without inter cooling, the delivery process from the first stage or LP compression stage is at the same pressure and temperature of 2nd stage or HP compression stage.

But when an intercooler is incorporated between these stages, it is assumed that the gas is brought back to this inlet temperature after first or LP stage but at the pressure same as that of 1st stage compressions. So, this is a case of perfect inter cooling. Through, this process the 2nd stage compression is performed with a gas at inlet temperature, but pressure of the with the 1st stage compressions.

Now, the value of intermediate pressure has to be chosen in such a way that the indicated power requirement will be the minimum. So, for this case we mathematically it is possible to obtain and expressions for minimum work for a two stage compression systems corresponding to the intermediate pressure. And, also this particular philosophy can be extended for the many number of stages.

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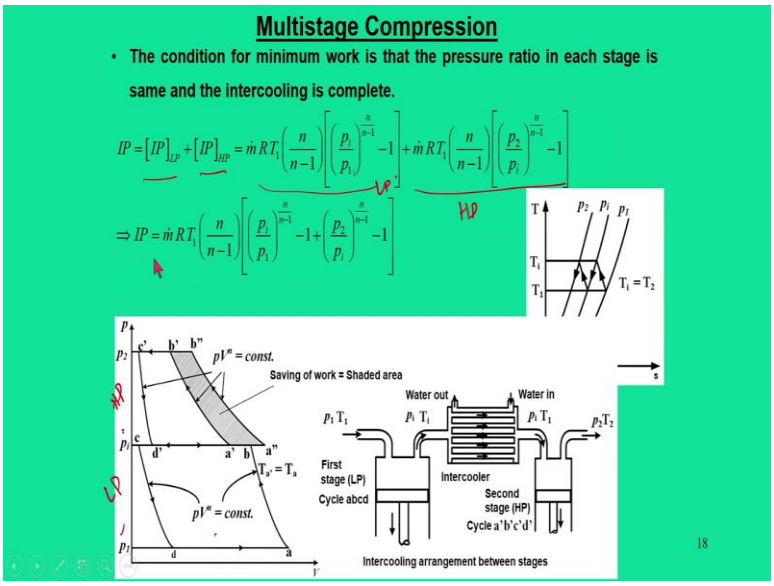

So, the conditions for minimum work is that, the pressure ratio in each stage is same and intercooling is completed. Since there are two stages, the total indicated power we can calculate for the LP stage and for the HP stage.

And individually for low pressure stage the pressure suits from p_i to p_1 and in the HP stage the pressure suits between p_2 and p_i. So, we say this is HP and this is LP and this goes from p_1 to p_i that is LP and p_i to p_2 is your HP stage. And this equation we can simplify in this form.

$$IP = [IP]_{LP} + [IP]_{HP} = \dot{m}RT_1\left(\frac{n}{n-1}\right)\left[\left(\frac{p_i}{p_1}\right)^{\frac{n-1}{n}} - 1\right] + \dot{m}RT_1\left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_i}\right)^{\frac{n-1}{n}} - 1\right]$$

$$\Rightarrow IP = \dot{m}RT_1\left(\frac{n}{n-1}\right)\left[\left(\frac{p_i}{p_1}\right)^{\frac{n-1}{n}} - 1 + \left(\frac{p_2}{p_i}\right)^{\frac{n-1}{n}} - 1\right]$$

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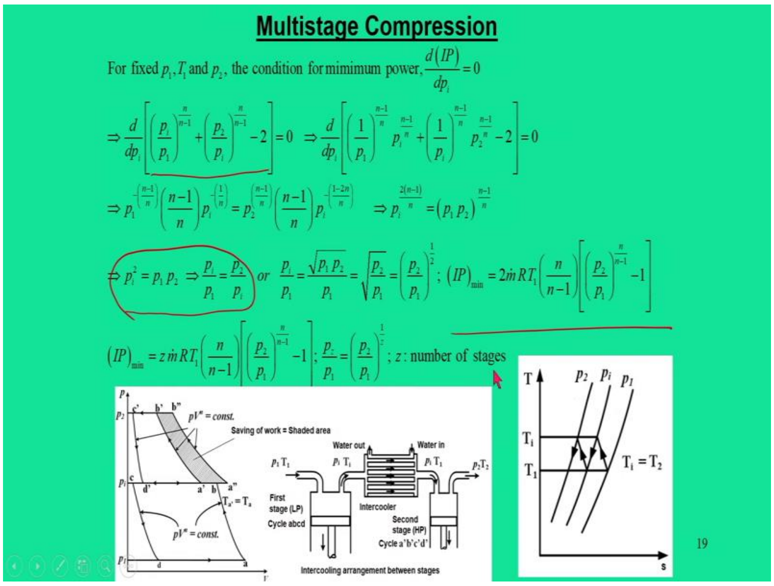

And if you dig into little bit of mathematics into it. So, what we have is that for a minimum for a fixed value of inlet conditions p_1 and T_1 and fix delivery pressure p_2 that is the requirement the condition for minimum power we can write $\frac{d(IP)}{dp_i}=0$.

$$\Rightarrow \frac{d}{dp_i}\left[\left(\frac{p_i}{p_1}\right)^{\frac{n-1}{n}}+\left(\frac{p_2}{p_i}\right)^{\frac{n-1}{n}}-2\right]=0 \quad \Rightarrow \frac{d}{dp_i}\left[\left(\frac{1}{p_1}\right)^{\frac{n-1}{n}} p_i^{\frac{n-1}{n}}+\left(\frac{1}{p_i}\right)^{\frac{n-1}{n}} p_2^{\frac{n-1}{n}}-2\right]=0$$

$$\Rightarrow p_1^{-\left(\frac{n-1}{n}\right)}\left(\frac{n-1}{n}\right)p_i^{-\left(\frac{1}{n}\right)} = p_2^{\left(\frac{n-1}{n}\right)}\left(\frac{n-1}{n}\right)p_i^{-\left(\frac{1-2n}{n}\right)} \qquad \Rightarrow p_i^{\frac{2(n-1)}{n}} = \left(p_1\, p_2\right)^{\frac{n-1}{n}} \Rightarrow p_i^2 = p_1\, p_2$$

So, we land off having an very simple expressions $\Rightarrow p_i^2 = p_1\, p_2$. So, it is a geometric mean. So, intermediate pressure is nothing but the geometric mean of p_1 and p_2 and there are other ways of rewriting that same expression in the form of $\Rightarrow \frac{p_i}{p_1}=\frac{p_2}{p_i} \quad or \quad \frac{p_i}{p_1}=\frac{\sqrt{p_1\, p_2}}{p_1}=\sqrt{\frac{p_2}{p_1}}=\left(\frac{p_2}{p_1}\right)^{\frac{1}{2}}$.So, accordingly if you do that we can find that minimum indicated power for a perfect inter cooling arrangement will be $\left(IP\right)_{\min} = 2\dot{m}RT_1\left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{2n}}-1\right]$. Now if there are z number of stages then we can write this particular expression $\frac{p_z}{p_1}=\left(\frac{p_2}{p_1}\right)^{\frac{1}{z}}$. So, accordingly we can rewrite this particular expression as $(IP)_{\min}=z\dot{m}RT_1\left(\frac{n}{n-1}\right)\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{zn}}-1\right]$ where z is the number of stages.

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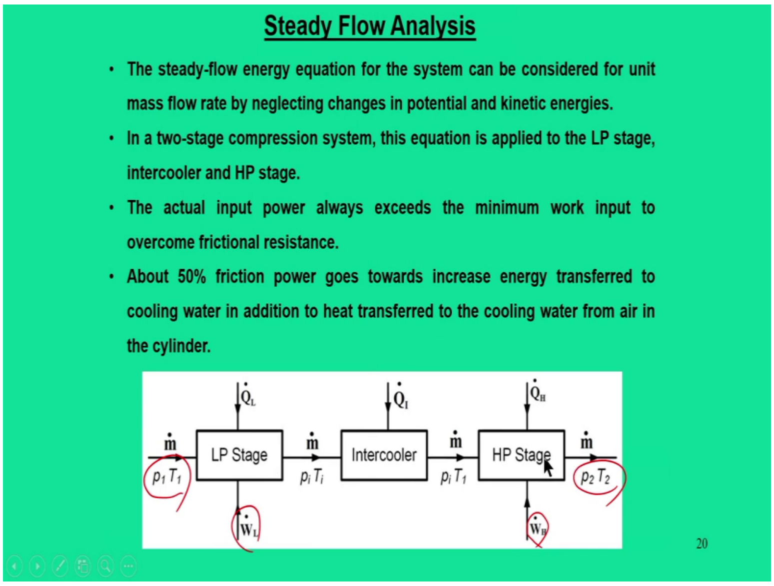

The next part of our analysis is a steady flow analysis and which we are going to deal with for a two stage compression systems. So, after dealing with the multi stage compression let us see that if we can do some kind of analysis with main intention is that we have seen that always this compressor is integrated with thermal circuits to take out the heat as much as possible with an main intention is that we have to resemble this compression process close to isothermal nature to minimize the work. So, what happens you can see that W_L is the work which is gets added for a given mass flow rate of air at condition p_1 T_1 in the LP stage heat and work is getting added.

And finally, from the LP stage when the gas goes out same mass flow, but at different pressure p_i and T_i then it goes to an intercooler, for intercooler there is no addition of work rather there is a saving of work, there is no change in the pressure. So, it is becomes p_i, but temperature drops to T_1 again.

So, T_1 is nothing but the temperature at which is enters into the LP stage and this is the one amount of work that has to be heat has to be rejected, similarly for HP stage we have also work transfer as well as heat transfer and ultimately from p_1 and T_1, as a user will get p_2 T_2 conditions.

But here this is a work transfer which is given as input, but in all the cases what we can see, always there is a heat rejection. Of course, here heat is added into systems, but in reality the heat is getting rejected from the LP stage or intercooler or HP stage. Now, with this analysis we are going to quantify what are this heat losses that comes out from this LP stage intercooler and Q_H.

So, this is the entire motive of this steady flow analysis. So, in a two stage compression we can apply this equations for intercooler and HP stage and the actual power input always exceeds the minimum work input due to frictional resistance. So, you can say that about 50 percent of friction power goes towards the increase in the energy transfer to the cooling water.

So, this particular has to be a realistic estimate. So, although we say that your work requirement, but we also have to quantify the amount of heat that has to be taken out because the power requirement will be 50 percent higher than that mainly due to the friction power and temperature rise.

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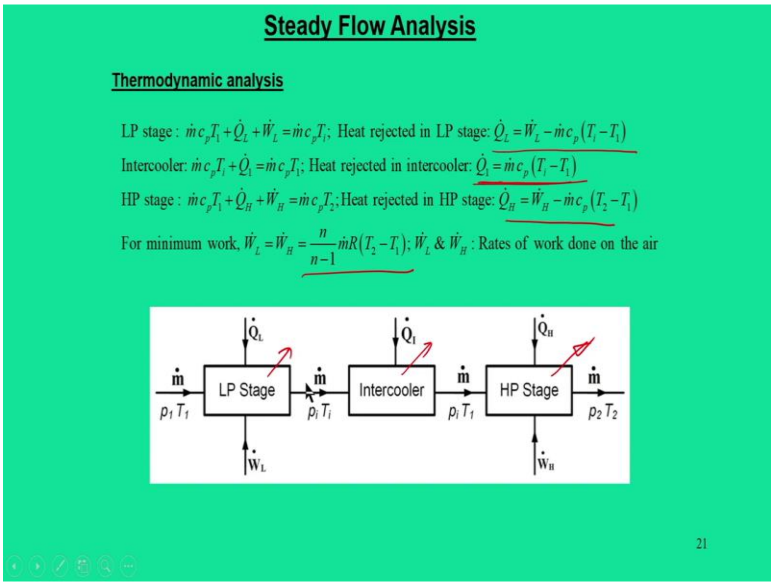

So, for that a simple steady flow analysis equations for LP stage intercooler and HP stage carried out here, for LP stage we can see the energy balance equations $\dot{m}c_pT_1 + \dot{Q}_L + \dot{W}_L = \dot{m}c_pT_i$. And ultimately we find the heat rejected from the LP stage is $\dot{Q}_L = \dot{W}_L - \dot{m}c_p\left(T_i - T_1\right)$.

And for the intercooler there is Q_I that is the heat rejected from this intercooler that goes out $\dot{Q}_I = \dot{m}c_p\left(T_i - T_1\right)$ and similarly for HP stage we say heat rejected $\dot{Q}_H = \dot{W}_H - \dot{m}c_p\left(T_2 - T_1\right)$. While deriving the expressions we have used heat as input, but actually heat is getting rejected. So, we have written this that heat is rejected from the HP stage.

And when there is a minimum work, the low pressure stage and high pressure stage work transfer are equal that is $\dot{W}_L = \dot{W}_H = \frac{n}{n-1}\dot{m}R\left(T_2 - T_1\right)$. So, this is a very simple thermodynamic analysis through steady flow equations. So, with this we conclude this reciprocating compressors. So, whatever we have covered today, I will just try to solve a simple problem based on the our analysis or discussion today.

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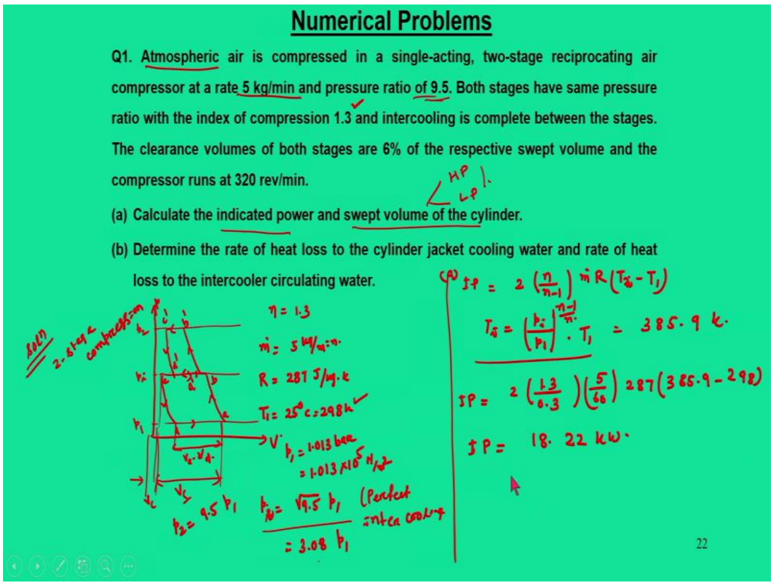

The problem for discussion today is a 2 stage compression. So, if you come across such a problem first thing that we must do is you have to draw this thermodynamic p-V diagram for a two stage compression systems.

So, this diagram has basically three parts one is pressure p_1 at which gas enters pressure p_2 the at which gas is delivered and there is a intermediate pressure pi and through clearance volume you can draw this first stage compression systems we say it is a-b-c-d and so, V_a - V_d is your volume induced into this cylinder.

So, we have clearance volume, we have stroke volume. Since it is a inter cooling things. So, it has to start at some lower temperature. So, it is a` and the compression goes a`-b`-c`-d`. So, our main target we have to find out the indicated power and swept volume into the cylinder. Swept volume into the cylinder there are two parts one is high pressure other is low pressure you have to calculate it separately.

And this is to be happened for a given mass flow rate that is 5 kg/min and the pressure ratio is 9.5 and the gas is at atmospheric conditions. So, there are some basic assumptions that we have to take first. So, index of compression we are assuming as 1.3, m is given as 5 kg/min and R for air is 287 J/kg-K.

T_1 we assumed to be 25°C or 298K, p_1 we say it is 1.013 bar that is 1.013 x 10^5 N/m^2 and p_i is $\sqrt{9.5}p_1 = 3.08p_1$ that is intermediate pressure with a perfect inter cooling.

And $p_2 = 9.5p_1$.First thing that we are going to calculate is indicated power. So, indicated power is $IP = 2\left(\frac{n}{n-1}\right)\dot{m}R\left(T_i - T_1\right)$ as it is a two stage.

So, $T_i = T_1\left(\frac{p_i}{p_1}\right)^{\frac{n-1}{n}} = 385.9K$. Now, $IP = 2\left(\frac{1.3}{0.3}\right)\frac{5}{60}287\left(385.9 - 298\right) = 18.22\text{kW}$

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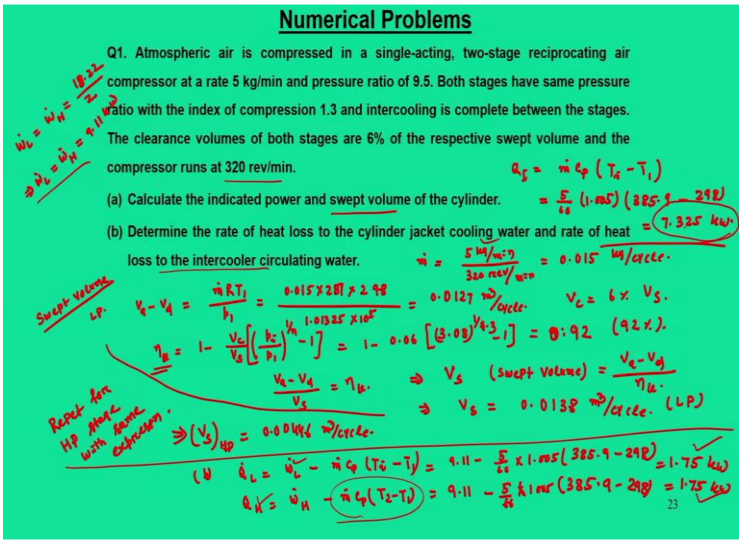

So, the swept volume has two parts, first is LP cylinder and for this LP cylinder we can write this $V_a - V_d = \frac{\dot{m}RT_1}{p_1}$ and here as the compressor is running at 320 rpm, so, $\dot{m} = \frac{5\,\text{kg}/\text{min}}{320\,\text{rev}/\text{min}} = 0.015\,\text{kg}/\text{cycle}$.

So, we can write this induced volume $\frac{0.015 \times 287 \times 298}{101325} = 0.0127\,\text{m}^3/\text{cycle}$. So, we can now calculate this volumetric efficiency as $\eta_v = 1 - \left(\frac{V_c}{V_s}\right)\left[\left(\frac{p_2}{p_1}\right)^{1/n} - 1\right]$, so here $V_c = 6\% V_s$

So, $\eta_v = 1 - 0.06\left[(3.08)^{1/1.3} - 1\right] = 0.92$ or 92%. So, by definition of this volumetric efficiency we can write $\eta_v = \frac{V_a - V_d}{V_s} \Rightarrow V_s = \frac{V_a - V_d}{\eta_v} = 0.0138\,\text{m}^3/\text{cycle}$. So, this is for LP.

So, repeat for HP stage with same expression and this will imply V_s for HP stage will be equal to 0.00446 m^3/cycle and third part is the heat loss to the cooling water. So, there are three parts we can write $\dot{Q}_L = \dot{W}_L - \dot{m}c_p\left(T_i - T_1\right)$.

So, and this indicated power already we found that $\dot{W}_L = \dot{W}_H = \frac{18.22}{2} = 9.11\text{kW}$.

So $\dot{Q}_L = 9.11 - \frac{5}{60} \times 1.005(385.9 - 298) = 1.75\text{kW}$.

Then we can find $\dot{Q}_H = \dot{W}_H - \dot{m}c_p(T_2 - T_1) = 9.11 - \frac{5}{60} \times 1.005(385.9 - 298) = 1.75\text{kW}$

Now what remains is that, what heat loss in the intercooler? Now for this intercooler the difference between these two is nothing but the intercooler heat. So, we can write $\dot{Q}_I = \dot{m}c_p(T_i - T_1) = \frac{5}{60} \times 1.005(385.9 - 298) = 7.325\text{kW}$

So, the answer that goes that heat loss to the cooling water jacket is for LP stage is 1.75 kilowatt, for high HP stage is 1.75 kilowatt, but the intercooler takes about 7.325 kilowatt and because of this region we will have a saving in the work in the high pressure stage.

With this I conclude this module that is last module reciprocating air compressors and of course, with this also I conclude this total lecture topic on the applied thermodynamics. With this I hope you will learners will get benefit out of this course.

Thank you for your attentions. That is all for this course on Applied Thermodynamics.