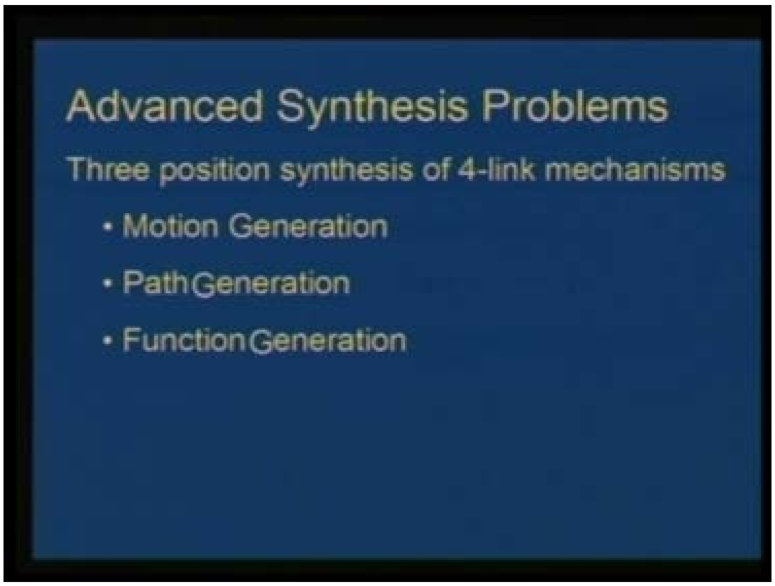
Kinematics of Machines

Prof. A. K. Mallik
Department of Civil Engineering
Indian Institute of Technology, Kanpur

Module - 07 Lecture - 1

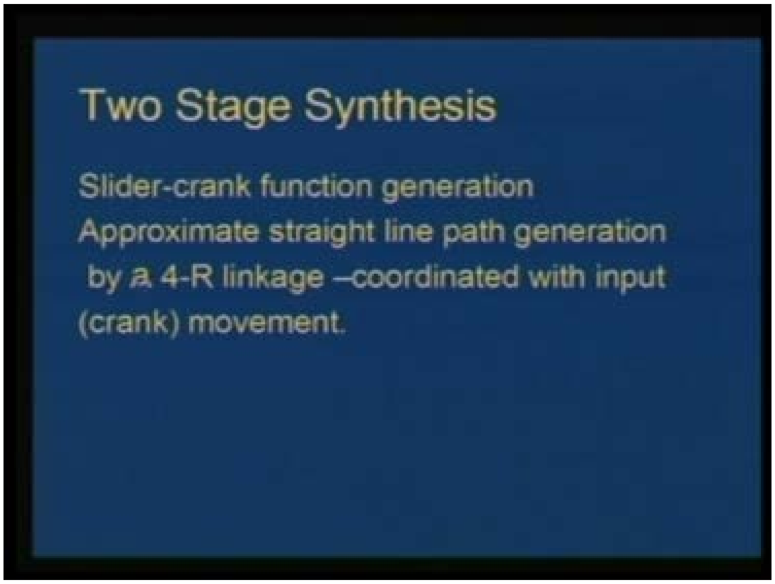
Advanced Synthesis Problems

(Refer Slide Time: 00:25)



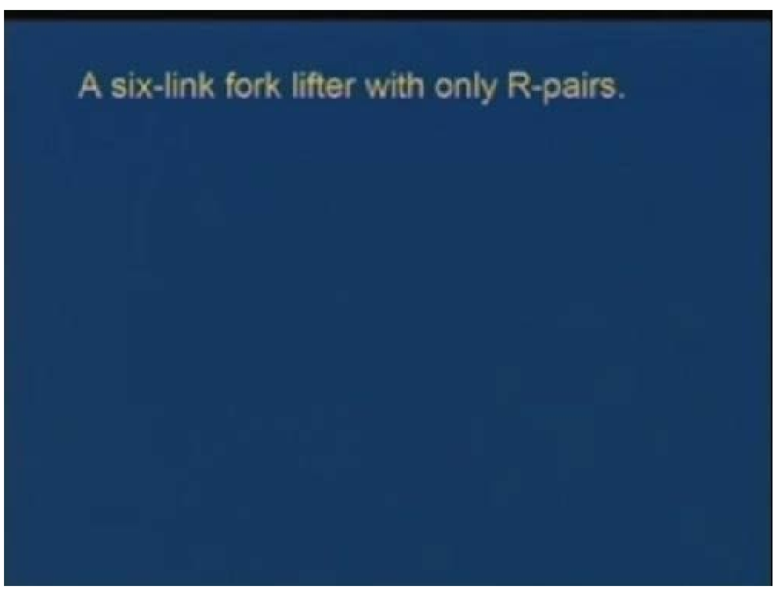
In this module, we will start our discussion with some advanced problems of dimensional synthesis. We have already discussed three position synthesis of 4-link mechanisms namely: motion generation, path generation and function generation.

(Refer Slide Time: 00:49)



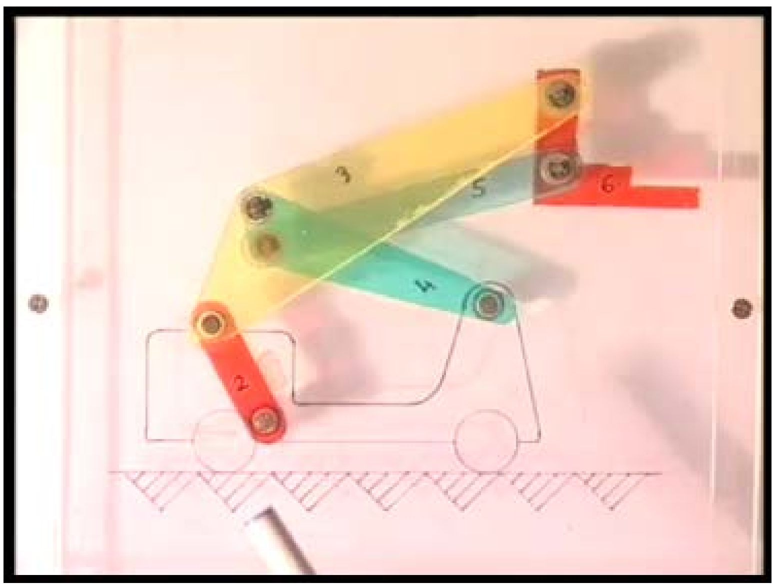
We have already seen that using these three problems and a two stage synthesis technique we could have designed, first a slider-crank function generator and then use that design to generate an approximate straight line path generation of a 4-R linkage coupler curve which could be coordinated with the input, that is the crank movement.

(Refer Slide Time: 01:09)



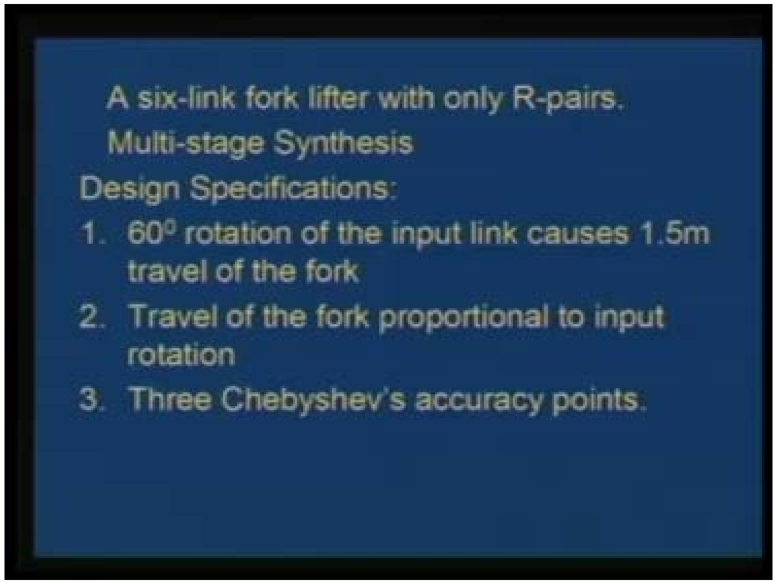
Today we will start our discussion with the design of a six-link fork lifter with only revolute pairs. As we know these fork lifters are used in industries to shift article from one place of the shop floor to another place but such fork lifters normally have two vertical guides or two prismatic pairs. We all know that the prismatic pairs are difficult to maintain and more costly to manufacture. There is more friction also in the prismatic pair whereas, revolute pairs are easy to fabricate and there is virtually no maintenance and because of a rollers that is the revolute pairs with the pin and hole joint there is hardly any friction, because the pin diameters are normally not very large.

(Refer Slide Time: 02:09)



Let me first show you the model of this proposed six-link fork lifter with only R-pairs. This is the model of that proposed six-link mechanism to be used as a fork lifter consisting of only revolute pairs. As we see the body of the truck is the fixed link or link number 1 and there is a link number 2, 3, 4, 5 and 6. As this input link of the crank is moved by a motor what we see that fork that is this link number 6 moves up and down almost vertically, but this vertical movement is without any vertical guide. We will solve and try to design this problem such that these two fixed pivots are located on the body of truck at convenient positions and we have to come up with this linked lengths such that, the rotation of the crank is converted into almost vertical movement of this fork. We will solve this problem by what we call a multi-stage synthesis. Let me now pose the problem with the required design specifications.

(Refer Slide Time: 03:35)



Let us now discuss the details of the design of this six-link fork lifter with only revolute pairs, the model of which we have just now seen. We will achieve this design by a process which we can call as multi-stage synthesis. That means the techniques that we have already learnt for path generation, motion generation and function generation will be applied in a number of stages to get to the final design. The design specifications, for example, in this particular mechanism let us represent it through the following parameters:

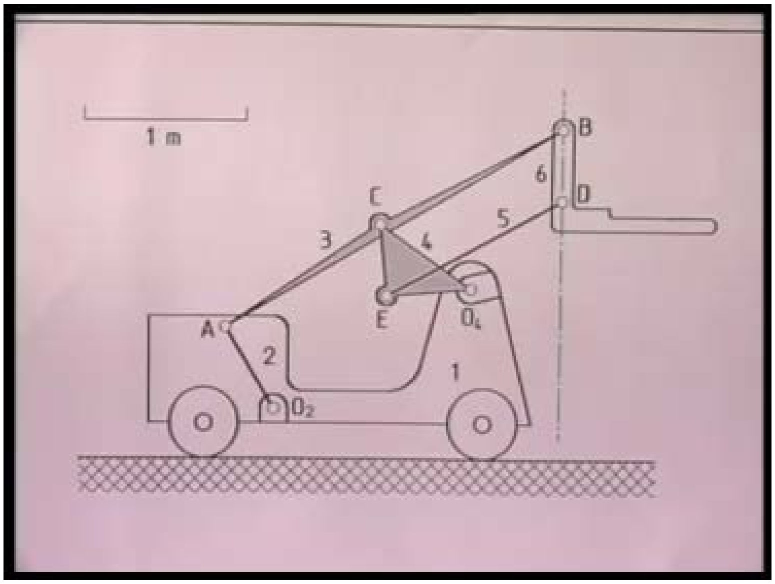
First, the 60 degree rotation of the input link should cause 1.5 meter vertical travel of the fork that is as the input link crank rotates through a 60 degree, the fork should travel almost along a vertical line through a distance of 1.5 meter.

Not only that, the travel of the fork should be proportional to the input rotation. As we know, we cannot have the vertical line exactly vertical at all configurations and we go through what we call a precision point approach.

Then we decided to use three Chebyshev's accuracy points during this entire travel of 1.5 meter.

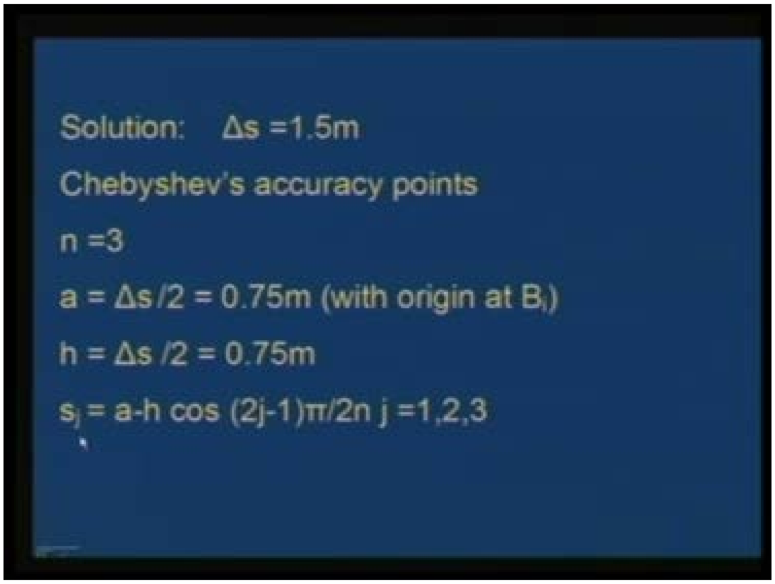
At this stage let me now go back to the sketch of this mechanism and define the other parameters.

(Refer Slide Time: 05:04)



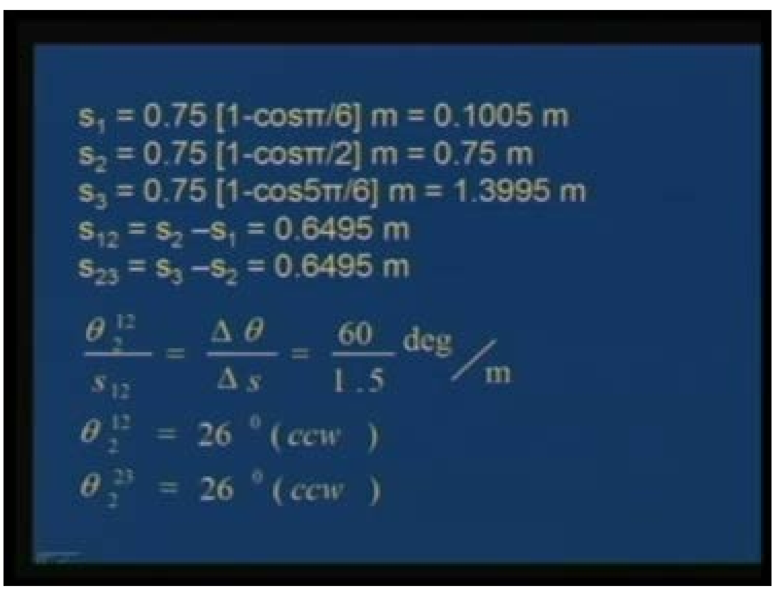
Let me now refer to the proposed sketch of this fork lifter. This two fixed points O_2 and O_4 are to be placed on the body of the truck at convenient locations. These two points B and D belong to the fork and the BD defines the fork completely and we want that 60 degree rotation of this input link should cause 1.5 meter travel along this vertical line. So we start the design assuming a convenient location of O_2 and a vertical line in front of the truck where I want the line BD to move. Let B_i represents the initial point of B and B_f is the final point of B on this vertical line. B_i to BF, this distance is 1.5 meter and the corresponding rotation of this input link should be 60 degree. Now, first step is to determine the three Chebyshev's accuracy points in this range B_i to B_f which we shall call it as B_1, B_2 and B_3. The three Chebyshev's accuracy points in this interval from B_i to B_f can be determined as we know both analytically and graphically. First of all let me determine these three accuracy points B_1, B_2 and B_3 analytically.

(Refer Slide Time: 06:40)



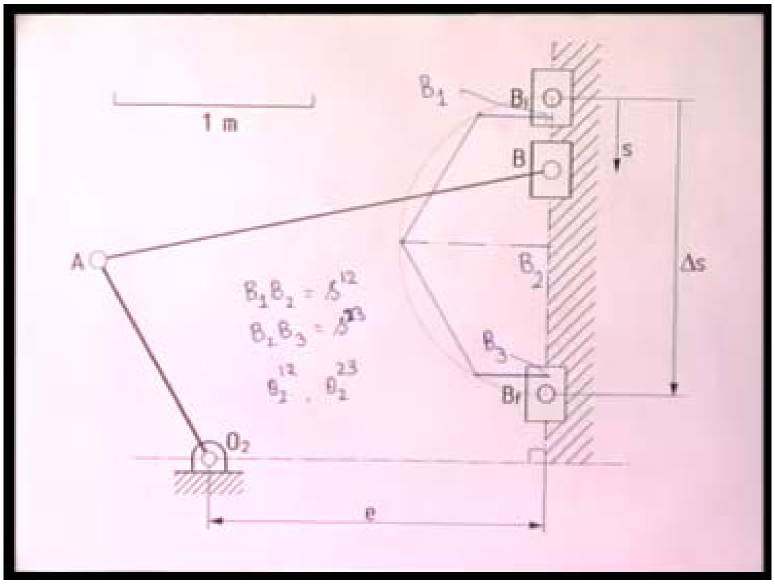
We have been given the delta s that is the total movement is 1.5 meter. We are going for three Chebyshev's accuracy points that is n is equal 3. a, that is the distance of the midpoint from B_i is delta s by 2, which is 0.75 meter with the origin at Bi. The half of the interval h is also delta s by 2, so again h again comes out as 0.75meter.

(Refer Slide Time: 07:22)



Then, the three accuracy points, s_j with j going from 1, 2 and 3 can be obtained from the formula as 0.1005 meter, 0.75 meter and 1.3995 meter.

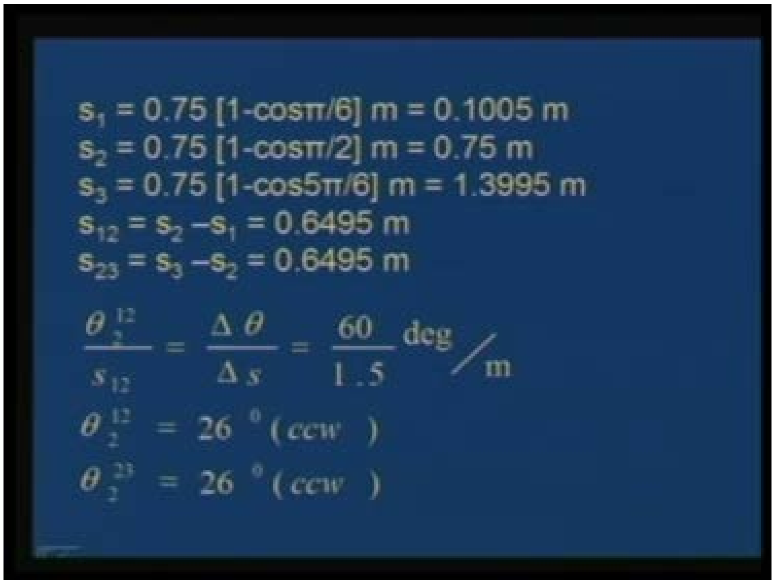
(Refer Slide Time: 07:42)



Let me now obtain the same three points B_1, B_2 and B_3 graphically. To determine the three accuracy points graphically, let me refer to this figure. This is the initial point B_i which I have chosen on this chosen vertical line and this is the point B_f and to this scale B_i to B_f this delta s is 1.5 meter. We have to locate three accuracy points in this range from B_i to B_f, graphically to do that first we draw a semicircle with B_i, B_f as diameter. This is a semicircle with B_i, B_f as diameter, because, n is equal to 3, now I draw a regular hexagon with two of its sides perpendicular to this diameter and inscribed in this circle. The half of this regular hexagon is shown, this hexagon is inscribed within this circle. So, the projection of these three vertices 1, 2 and 3 on to this diameter B_i, B_f locates three accuracy points namely, B_1, B_2 and B_3. B_1B_2 is the travel of the fork we represent by s 1 2. Similarly B_2B_3, the travel of the fork from the second to the third accuracy points is what I call is s 2 3. So s 1 2, s 2 3 is now known I can either measure it from this diagram to this scale or I have already calculated analytically.

The second requirement is that the travel of this fork should be proportional to the rotation of this input link of the crank O_2A. We calculate what is the corresponding movement of this link 2, which I call it as $theta_2$ 1 2 and $theta_2$ 2 3. We have decided the three accuracy points corresponding to the fourth point b namely at B_1, B_2 and B_3 both analytically and graphically.

(Refer Slide Time: 11:45)

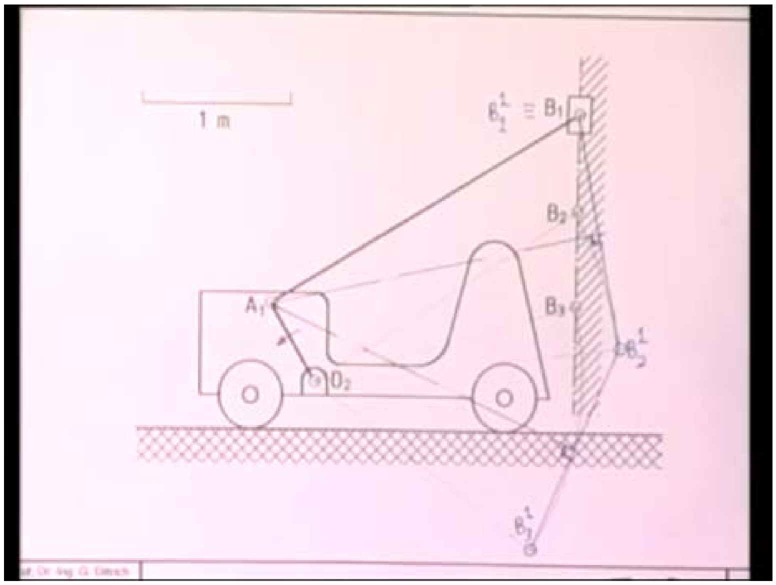


We have got s_{12}, that is s_2 - s_1 is equal to 0.6495 meter analytically and in the drawing we got it to some scale and s_{23} is exactly the same, because the rotation of the crank has to be proportional to the movement of the fork, the relationship is $theta_2$ 1 2 divided by s_{12} is same as delta theta by $delta_s$, where delta theta represents the total travel of the crank and $delta_s$ represents the total travel of the fork. We are trying to design O_2 A, B as a slider crank, where B is the location of the slider and O_2A is the crank. Delta theta is given as 60 degree into $delta_s$ is given as 1.5 meter.

From this relation because s_{12} is known, we can find $theta_2$ 1 2 which turns out to be 26 degree counter-clockwise and $theta_2$ 2 3 again is 26 degree counter-clockwise. In the first stage of the synthesis the problems boils down to designing a slider crank O_2AB such that, the sliding movement at the slider positioned at B is coordinated with the rotation of the crank O_2A according to this relation, that 26 degree counter- clockwise rotation of the crank should produce 0.6495 meter vertical travel of the slider. Further 26 degree rotation in the counter-clock wise direction of the crank will again cause further 0.6495 meter vertical travel of the slider.

Let me now at the first stage design this slider cranks mechanism, which I now explain through the usual graphical method of which we have learnt earlier.

(Refer Slide Time: 13:45 min)



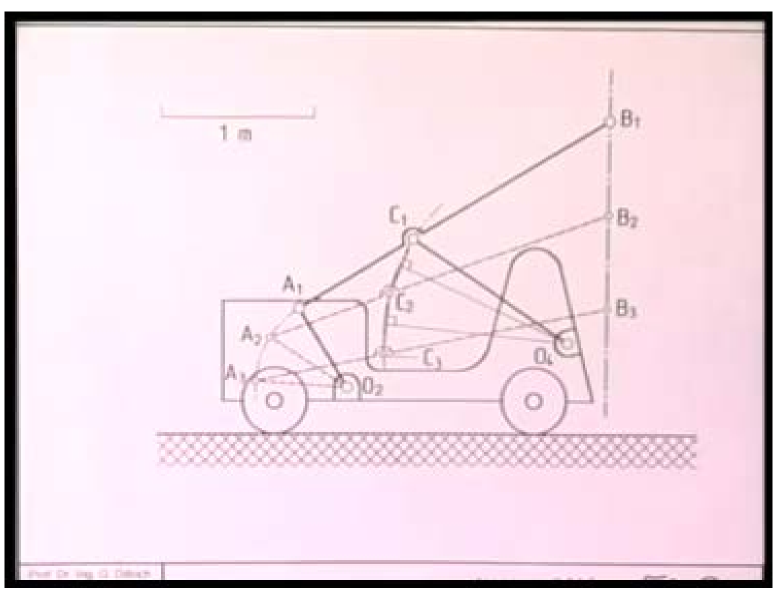
At the first stage of this synthesis problem, let me start B_1, B_2 and B_3, these are the three accuracy points corresponding to the slider movement at B. We choose O_2 arbitrarily and the whole problem right now is a function generation problem as a slider crank such that this downward movement from B_1 to B_2 which is 0.6495 meter should occur during 26 degree rotation of the crank in the counter-clock wise direction. Further 26 degree rotation of the crank will make the point B_2 to come up to B_3. This we have already learnt how to do this, we choose O_2 at a convenient location on the body of the truck, we have chosen this line conveniently at the front of the truck and I have located B_1, B_2 and B_3 according to Chebyshev's risk. We apply the method of inversion, we hold the crank fixed at its first position and the crank pin at A_1 has to be located. Here, we have already drawn the solution but my objective is to locate the required point A_1 such that the slider crank O_2AB acts as a function generator. If we hold the crank fixed, then we know this O_2B_2 has to be rotated through 26 degree but in the clockwise direction. We rotate B_2

with O_2 as center, this O_2B_2 is rotated by 26 degree and this point we call B_2 inverted on the first position or B_2 1.

Because, we are holding the first position fixed, the inverted position of B_3 I can get by rotating O_2B_3 by 52 degree. This line O_2B_3 is rotated through 52 degree and we get the inverted position of B_3 which we call B_3 1. This is B_3 1, this is B_2 1 and this is B_1 which is as same as B_1 1. Then A_1 has to be at the center of the circle passing through B_1 1, B_2 1 and B_3 1. To determine that, we do the usual simple geometric procedure, we draw the mid normal of B_2 1 and B_3 1, which is this line, this is the mid normal of B_2 1, B_3 1 and we also draw the mid normal of B_1 1 and B_2 1. This is the line B_1 1, B_2 1 and we draw the perpendicular bisector of this line and these two lines intersect at A_1. Finally, at the end of the first stage, we have designed a slider -crank mechanism namely $O_2A_1B_1$, where A_1 is the crank pin such that I know the 26 degree counter-clockwise rotation of this crank will cause the slider to come from B_1 to B_2, further 26 degree counter-clockwise rotation of the crank will cause the slider to come from B_2 to B_3.

In the next stage we will get to a four bar linkage such that I can remove the slider and this coupler point B_1 will automatically pass through B_1, B_2 and B_3 when the four bar linkage is moved and that is the second stage of synthesis.

(Refer Slide Time: 18:40)



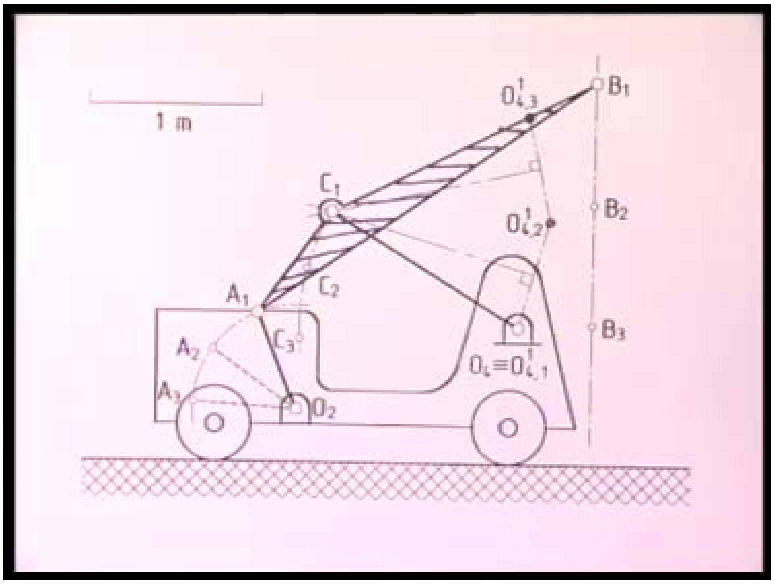
In the first stage we have obtained this O_2, A_1, and B. Now, I removed the slider at B_1 and treat this link AB as the coupler of a four bar link namely, O_2, A, C, O_4 and B is the coupler point of this 4-R link O_2, A, C, O_4. Our objective is how we determine the correct location of O_4 so that when this 4-R linkage is moved, this point B_1 will automatically pass through B_1, B_2 and B_3 and maintaining the coordination with the input movement. This problem we have solved earlier without elaborating.

Let me say, I choose C_1 somewhere in the middle range of this AB not too close to A, not too close to B, but somewhere in this middle region, so I have chosen C_1 When this slider crank goes from O_2, A_1, B_1 to O_2, A_2, B_2, C_1 goes to C_2. I draw A_1 goes to A_2, that is 26 degree, B_1 to B_2 that is 0.6495 meter and I get O_2, A_2, B_2 because this length is already fixed, so from A_2 I measure this length and get to C_2. I locate the point C_1. Similarly, in the third configuration O_2, A_3, B_3 I find the corresponding location of C and I call it as C_3. I can always draw a circle through these three points namely, C_1, C_2, C_3 and the center of this circle is at O_4. The center can be easily determined by drawing the mid normal of C_1C_2 which is this line and mid normal of C_2C_3, which is this line. These two lines intersect at O_4 determining the required location of the fixed hinge O_4. The point to note is that, because I choose C_1 somewhat arbitrarily I have no control over the location of O_4. Here, it happens to be within the boundary of this truck body, but with different choice of C_1 this O_4 could have been very much away from the body of the truck and then we have to do more iteration such that with the proper choice of C_1 only I can get O_4 in a convenient location.

At this stage, that is at the end of the second stage, I have got to a four bar linkage namely, O_2, A, C, O_4 such that when this 4-R-moves this coupler point B_1 has the desired movement and is also coordinated with the input movement.

At the second stage, I will do it little differently, because as I have seen that with an improper choice of C_1, O_4 can go to an inconvenient position. The second method what we will do I will choose that O_4 conveniently at the body of the truck and try to locate the required C_1 on this coupler link AB.

(Refer Slide Time: 22:00)



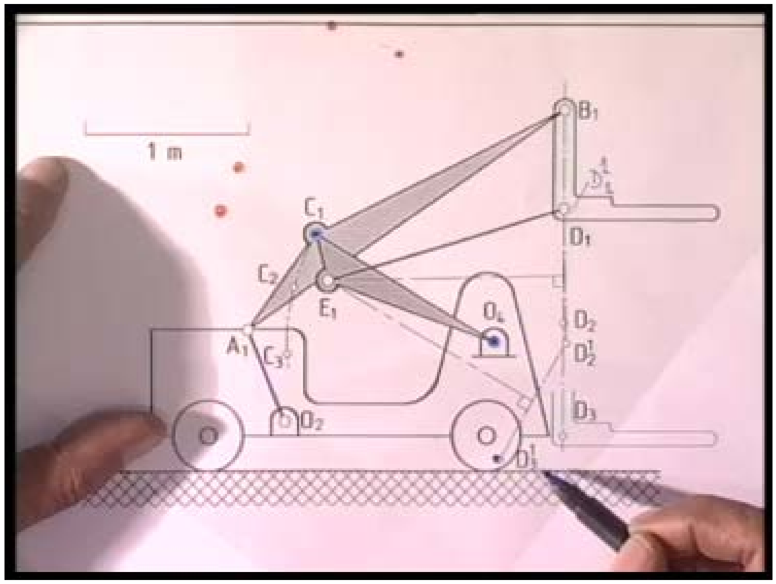
In this alternative procedure we say that O_2, A_1, B_1, $O_2A_2B_2$ and O_3, A_3, B_3 are already known to us. Instead of choosing C_1, I rather choose O_4 conveniently on the body of the truck say at this location which we call O_4. To determine the point C_1 on the coupler link, I make a kinematic inversion holding the coupler fixed at its first configuration and obtain the inverted position of O_4 as we know, because this C belongs to the coupler and C, O_4 does not change. So, if we hold the coupler fixed at its first location, this point O_4 will move on a circle. To obtain the inverted position of O_4 we follow the usual procedure, we mark A_2, B_2 and O_4. These are the relative positions of A_2, B_2 and O_4 because the link AB is not moving when I make a kinematic inversion, I will take this A_2 and B_2 to coincide with A_1 and B_1 respectively so I move. If we remember this point represents B_2, this point represents A_2 and this cross represents O_4 and they define the relative position of A, B and O_4 at the second configuration. Kinematic inversion means this relative position should not change, though I am not allowing AB to move from A_1B_1. I move this tracing paper A_2 coincide with A_1, B_2 coincide with B_1 and wherever O_4 goes that point I mark by piercing on this tracing paper and mark this point as $O_{4,2}$ 1, because this is the second position of O_4 inverted on to the first position.

Following the same logic, I also obtain the inverted position of the third configuration that is on the tracing paper I mark A_3, B_3 and O_4. These are the relative positions, but inverting on the first position A_3, B_3 coincides with A_1 and B_1 respectively and wherever O_4 goes it has come here that point I call as $O_{4,3}$ 1. So, this way we have obtained the three inverted position of O_4 which is as same as $O_{4,1}$ 1, $O_{4,2}$ 1 and $O_{4,3}$ 1. If I draw a circle passing through these three inverted position, then the center of the circle will be C_1. As usual I draw the perpendicular bisector of this line and the perpendicular bisector of this line, these two bisectors meet at the point C_1 determining this revolute pair on the coupler link at C_1. Now I have got a different four bar linkage namely O_2, A_1, C_1, O_4 with B_1 as the coupler point which will pass through B_1, B_2, B_3 maintaining the coordinated movement with the input link.

Here, the location of O_4 is convenient, but as we see the coupler has become not a very lean member but a heavier member, that is triangular in shape. Previously the coupler was almost a straight link and now this coupler has become triangular in shape, but we have controlled over the fixed hinge O_4 that I can choose conveniently on the body of the truck. At the end of this second stage either by following the previous procedure or all this alternative procedure I can get through this four bar linkage with the coupler point movement coordinated with the input link.

In the third stage, I will convert this to a six-link mechanism such that B, D which is not seen here B, D will be my fork of the lifter.

(Refer Slide Time: 27:00)



So at the end of the second stage of the synthesis, we have determined A_1, $C_{1.}$ We had already chosen O_2, O_4 and this vertical line on which I had B_1, B_2 and B_3. If this fork is hinged at only one point this will be swinging like a pendulum. To guide this fork vertically I choose another convenient point D_1, this is chosen arbitrarily. B is at B_1, D is at D_1. When at the second configuration, when B goes to B_2, because this distance remains same I can locate D_2 and similarly D_3 corresponding to the third position on the same vertical line. The only task is to determine this revolute pair on this link at E_1 such that D_1 passes through D_1, D_2 and $D_{3.}$ To determine this E_1, I make again a kinematic inversion keeping this link fixed, that is the follower link of this O_2, A, C, $O_{4.}$ This four bar linkage, the follower link I hold fixed at its first position and determine the inverted positions of D_1 generating the same relative movement, but holding this link four fixed.

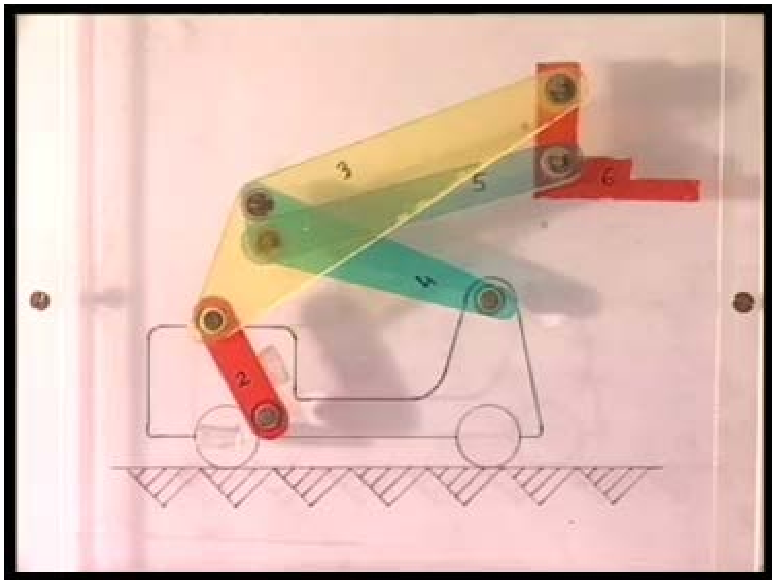
At the second stage of our synthesis we have determined this linkage O_2, A_1, C_1, O_4 out of which O_2 and O_4 had been chosen conveniently on the body of the truck. The line of travel of the fork was also conveniently chosen in front of the truck and we have come up to this design O_2, A_1, C_1, B1 and O_4. As we see this fork, if it is hinged only at one point it will swing like a pendulum. To guide the fork along this vertical path, I will choose another point which I call it as D_1 which I choose conveniently on the fork and now this point D_1 with a revolute pair has to be connected to this link O_4, which is O_4, C_1 such that D_1 goes through D_2 and D_3. This distance BD remains the same, so from B_2 I can locate D_2 on this vertical line and D_3 also on this vertical line. I have got D_1, D_2 and D_3. To locate the position of E_1, that is this revolute pair on link number 4, again I apply the method of inversion holding this link that is link number four O_4C fixed at its first configuration and determine where is D_2 1 and where is D_3 1. To do this we follow the same method of kinematic inversion.

This is D_2, this is O_4 and this is C_2. So, I mark O_4, C_2 and D_2. Now, I am holding this link O_4, C fixed at its first position. So, What do I do? I make C_2 coincide with C_1 and O_4 does not move and wherever D_1 goes that becomes my D_2 1. Let me repeat, to get the inverted position corresponding to the second configuration inverted on the first position, I mark O_4, C_2 and D_2. Since O_4, C is not moving from the first configuration, I make O_4C_2 coinciding with O_4, C_1, that is C_2 coincides with C_1, O_4 coincides with O_4 and wherever this D_2 moves that I call it as D_2 1.

Similarly, to obtain D_3 1, I mark O_4, C_3 and D_3. Holding the link 4 fixed at its first position, I make O_4 at O_4, C_3 at C_1 and wherever D_3 moves that I call it as D_3 1. Since link 4 is fixed at its first position, this link length is of constant length so E_1 can be located at the center of the circle passing through D_1 which is same as D_1 1, D_2 1 and D_3 1. Again following the usual technique, I find the perpendicular bisector of these two inverted position D_2 1 and D_3 1 and the perpendicular bisector of D_1 1 and D_2 1 which is given by this line. These two lines intersect at E_1 determining the location of this revolute pair on link 4 and I connect E_1 and D_1 by a rigid link.

If we do not allow this link to move then obviously D_1 goes on a circle with E_1 as center. So, E_1 is determined at the center of the circle passing through D_1 1, D_2 1 and D_3 1. At this stage I have finished the design of a six-link mechanism which gives me a fork lifter, where the fork goes almost vertically, at least it passes through B_1, B_2, B_3 and the point B passes through D_1, D_2, D_3 lying on this vertical line. Of course there will be deviation a little bit as it goes from B_1 to B_2 from this vertical configuration, but the deviation is not much and it acts as a very good fork lifter particularly the load is not heavy, where the movement of the fork has been coordinated with the rotation of the crank and the fork goes almost vertically and we end up getting a fork lifter without any vertical guide.

(Refer Slide Time: 33:37)



Let me now summarize this multi- stage synthesis of guideless fork lifter as we have completed in today's lecture. We have chosen first O_2 this is a fixed pivot at a convenient location of the body of the truck. We have also chosen a vertical line in front of the truck which is convenient for the movement of the fork. First, we design O_2, A, B as a slider crank such that the slider movement at B is coordinated with the input motion of the crank. 60 degree counter-clock wise rotation of the crank has caused 1.5 meter vertical travel of this point B using three Chebyshev's accuracy points in this interval.

Next, we chose O_4 conveniently on the body of the truck. Then remove the slider and got coupler point here at C such that O_2, A, C, O_4 this 4-R-link has a coupler point B which has an approximate straight line path and the movement along that straight line path is coordinated with the motion of the input crank. In the third stage, we have chosen this point D again conveniently on the body of the fork such that a second point is guided along the same vertical path. After choosing this point D, we found the inverted positions of the D corresponding to the second and third configuration holding this link, the link number 4 fixed at its first configuration and the center of the circle passing through these three inverted positions of D namely, D_1 1, D_2 1 and D_3 1 I found the center of that circle at this point which I call it as E.

Finally, we get this six-link mechanism where I have a fork lifter without any vertical guide. The techniques that we have learnt for function generation, motion generation and path generation can be synthesized together to have a multi-stage synthesis problem for even more complicated problem.

In today's lecture, we have designed a six-link fork lifter using only revolute pairs by multi- stage synthesis processes which are basically the combination of the three position synthesis for function motion and path generation of four-link mechanisms which we had discussed earlier. In our next lecture, we will try to design another six-link mechanism for a rail-less garage door. A garage door which will be opening inside the garage rather than coming out of the garage and in the open position it will become horizontal to the roof of the garage using again only revolute pairs and not using any prismatic pair.