Kinematics of Machines

Prof. A. K. Mallik

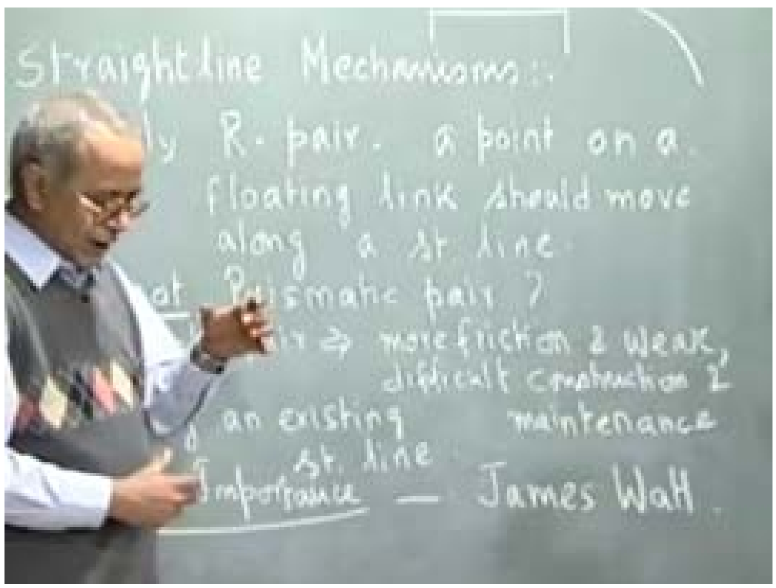
Department of Civil Engineering

Indian Institute of Technology, Kanpur

Module - 9 Lecture - 1

So far we have discussed kinematic analysis and synthesis of planar linkages in general. From today's lecture, we will start discussion only on some special process of mechanism. To start with, let me talk of what is known as straight line mechanism.

(Refer Slide Time: 00:42)



What do we mean by straight line mechanism? By that we mean, we have to come up with a planar linkage consisting only of R pairs, such that a particular point on a floating link, as the mechanism moves, will generate a straight line. So, only R - pair - a point on a floating link. A floating link is the link which is not connected to the ground of the fixed link that is not connected to the frame, like the coupler of the 4R link. The crank and follower are not floating links, it is the coupler which is the floating link. A point on a floating link should move along a straight line.

One may wonder, why take all the trouble to generate a straight line by using only R pair because we know if we have prismatic pair, we can always generate straight lines. Let me say, why not prismatic pair? There are two points to remember why not a prismatic pair: first, prismatic pair implies more friction and wear, difficulty of manufacturing a prismatic pair as compared to R pair, that is difficult construction and maintenance. Moreover, we should also remember that in a prismatic pair we do not generate a straight line, we rather copy an existing straight line. So prismatic pair only means copying an existing straight line. I must emphasize the difference between copying an existing geometric figure and generating a geometric figure.

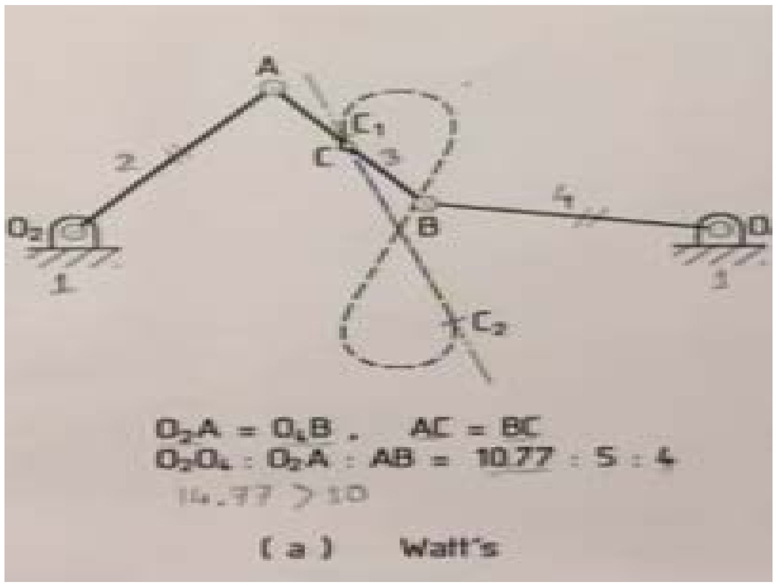
For example, when I draw a circle using either the revolute pair or a compass, we do not copy a straight line rather a point is constrained to move along a circle. It is not that I am taking a circular object then drawing along its periphery, where as in prismatic pair there is always an existing straight line and I am just popping. The third thing to remember is that in the early stages of mechanization, it was very difficult to make a long prismatic pair, that is why the straight line mechanism historically is very important.

In fact, this is one of the first mechanisms which challenge both the ingenuities of both the engineers and the mathematicians - how to produce to straight line without copying an existing straight line. How to generate a straight line using only R pair? The first person to come up with an approximate solution is James Watt. In Watt's engine, as we know, the cylinder and the piston must form a prismatic pair, but during James Watt's time it was not possible to make a piston and a cylinder with close tolerance. There is lot of clearance between the piston and the cylinder. So he wanted to guide this piston along a straight line by using one such straight line mechanism. Watt's mechanism was the first approximate straight line generator or approximate straight line mechanism.

Now, it is known that it is not possible to generate an exact straight line by a 4R-Linkage. No point of a coupler, with any dimension of all the links, can produce a perfect straight line. However, it is possible that a portion of the coupler curve can fairly reproduce an approximate straight line, if the dimensions or the link-lengths are properly chosen that is precisely what James Watt did.

He talked of a Non-Grashof double-rocker of suitable link-lengths, such that the midpoint of the coupler generated an approximate straight line. I will show the figure and explain how it is achieved.

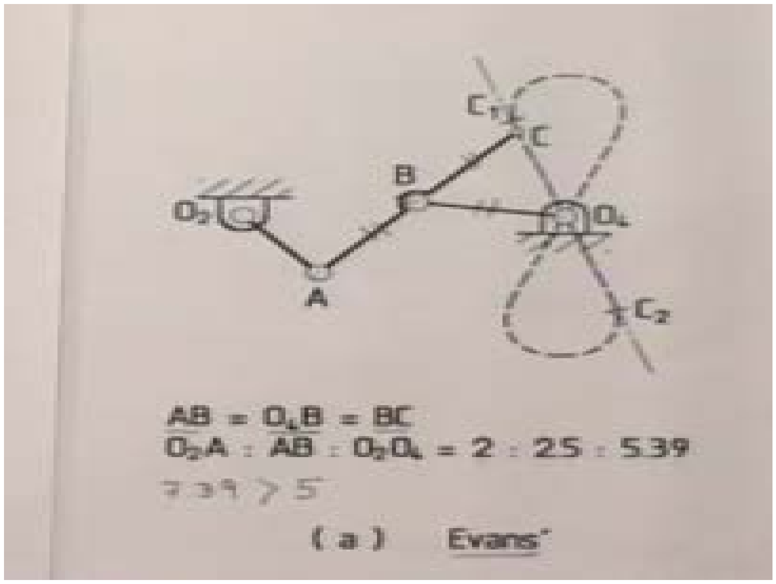
(Refer Slide Time: 07:02)



This figure shows the sketch of what is known as Watt's approximate straight line mechanism as it was first designed by James Watt. As we notice that this is the 4R-Linkage with fixed link, number 1, link number 2 is O_2A, the coupler link number 3 is AB, O_4B is link number 4. In this linkage, the length of O_2A is same as that of O_4B, O_2A is equal to O_4B. C is the midpoint of the coupler AB that is AC is equal to BC. The other link-lengths: O_2O_4 to O_2A to AB are in the ratio of 10.77: 5: 4. These ratios were obviously arrived at by trial and error, such that when this linkage moves, this midpoint C generates a figure of eight. In the figure of eight, there is a portion from C_1 to C_2 which is a very good approximation to a straight line. In fact, the difference between this straight line and this curve is hardly noticeable.

What James Watt did is, he connected his piston rod to this point C and as this linkage moves, the piston goes almost along a straight line. As we notice from these ratios, that l_{max} is 10.77 in some unit and l_{min} is 4. So l_{max} plus l_{min} is 14.77, which is much greater than l prime plus l_2 prime that is O_2A plus O_4B which is 2 times 5 is 10. So, this is a Non-Grashof linkage which means it is a double-rocker.

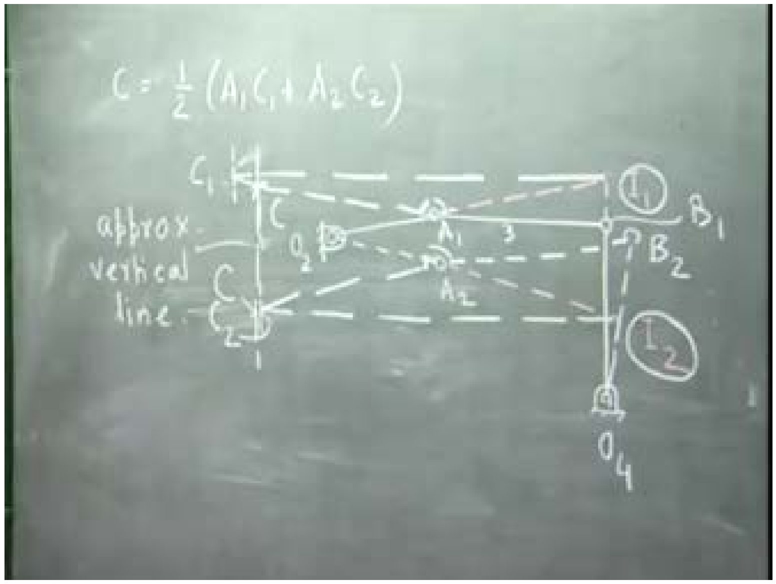
(Refer Slide Time: 09:06)



Let us now look at the case of another approximate straight line mechanism which is known as Evans or Grashof mechanism. In this mechanism, the fixed link is O_2O_4, O_2A is one of the cranks and O_4B is the other link connected to the frame and AB is the coupler. Here the coupler point C is taken on the extension of AB and as this mechanism moves, the point C again generates this figure of eight, in which the portion from C_1 to C_2 is again a very good approximation to a straight line. As we see in this mechanism, O_4B is equal to AB is equal to BC. AB is same as O_4B, the link length is also equal to BC. The other link-lengths are in the following ratios: O_2A: AB: O_2O_4 is 2: 2.5: 5.39.

If we take these link-lengths, then this point C on the extension of the coupler AB will generate an approximate straight line, this is known as Evans' approximate straight line mechanism. Let us see what is the maximum link-length? That is O_2O_4 is 5.39 and the minimum link-length is 2. So l_{min} plus l_{max} is 7.39. AB which is same as O_4B, so l prime plus l_2 prime is 2 times 2.5, which is 5, that means l_{min} plus l_{max} is greater than l prime plus double prime so this is also a Non-Grashof linkage and consequently, this mechanism just like the Watt's mechanism is also a double-rocker. Let me now discuss a methodology, such that, I can determine a suitable coupler point in a 4R link, for example, to generate a vertical straight line.

(Refer Slide Time: 11:31)



What we do is, we start with a 4R-Linkage say O_2, A_1, B_1, O_4. I want that as this linkage moves from this O_2A_1 to say O_2A_2, when the fourth link moves from O_4B_1 to O_4B_2, I want to determine a point on the extension of this coupler AB, such that point C generates an approximate vertical straight line. The methodology is as follows. What we do is we find what the instantaneous center of the coupler is, which is link 3 in the first configuration. As we have seen, we can extend O_2A_1 and O_4B_1, wherever they intersect, that gives me the instantaneous center of the couple at this configuration, which is I_1.

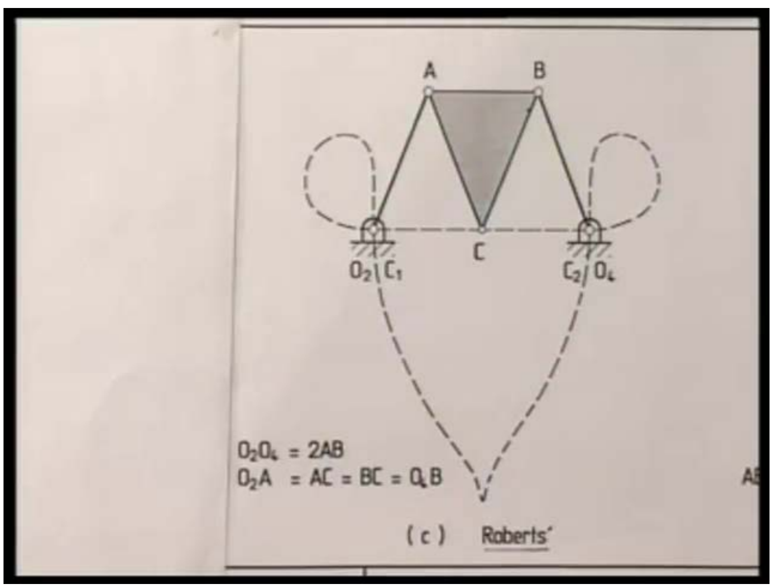
Now, I draw a horizontal line through I_1 and let AB extended meets this horizontal line at C_1 then I know that at this instance, I_1 is the instantaneous center for the coupler 3, so the velocity of the point of C will be perpendicular to I_1C_1, that means C_1 move along this line.

In the second configuration, I again locate the extension of O_2A_2 and the intersection of O_4B_2, which gives me the instantaneous center of the coupler I_2 at the second curve. Now, I draw a horizontal line to I_2 and extend the coupler AB, such that these two lines, this horizontal line through A_2 and the extension of the coupler meet at the point C_2.

So, at this second configuration, the velocity of the point C_2 is perpendicular to I_2C_2 that means the path of C_2 is a vertical straight line. Now obviously, this distance A_1C_1 will be different from the distance A_2C_2. So what we do is we take a point C on the extension of the coupler, say here which is the mean of these two distances, which is the half of A_1C_1 plus A_2C_2. With this point C_2, we see that the path of C will be a fairly, a good straight line. A_2C is more than A_2C_2 and A_1C_1 is more than A_1C. A_2C_2 is less then A_2C and A_1C_1 is more than A_1C, the point is C_1.

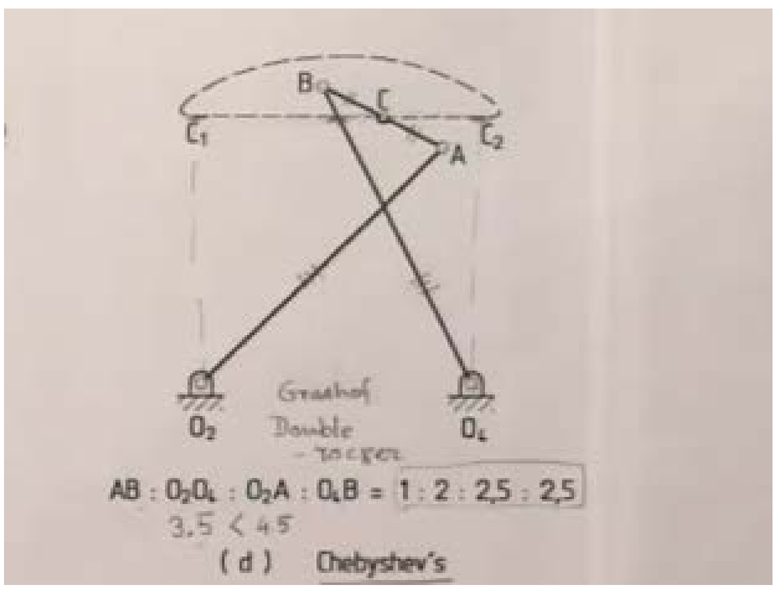
The point C_1 has a fairly good straight line here at the first configuration, the point C_2 has a fairly could straight line at the second configuration. I take a point C in between C_1 and C_2, such that during this entire range it goes almost along a straight line, approximately vertical line. This way we can design an approximate straight line mechanism. It must be emphasized that with the 4R link, with any point on the coupler or any choice of link-length, I can never generate a perfect straight line. For that, we will discuss later that we have to go for linkages with more number of link that is more than four. Before that let me discuss, two other approximate straight line mechanisms which are historically famous, which are designed by a mathematicians, to generate an approximate straight line.

(Refer Slide Time: 15:42)



Let us now look at the sketch of another approximate straight line mechanism which is known as Robert's mechanisms. As we see, in this Robert mechanism is nothing but 4R-linkage. This is fixed link 1, link number 2, this is the coupler link number 3 and this is link 4. In this coupler, we choose a point C, such that when this mechanism moves, the C generates this curve with cusp here (Refer Slide Time: 16:20) and two loops, one here and here. During this movement of the coupler C, it is C_1C_2 from O_2 to O_4, it is fairly coincident with line O_2O_4. The discrepancy cannot be seen by naked eyes, but definitely C is not going along a straight line of O_2O_4, but the discrepancy between the path of C and the line O_2O_4 is negligible. This again a straight line mechanism and the straight line is generated at O_2O_4. For this mechanism the dimension as follows: O_2O_4 is twice of AB and O_2A, AC, BC, O_4B are of equal length, O_4B equal to O_2A which is also same as AC and BC and O_2O_4 is twice of link-length AB. In this Robert mechanism, the thing to note that, it generates a very good approximate straight line, the straight line is coincident with the line of frame O_2O_4.

(Refer Slide Time: 17:32)



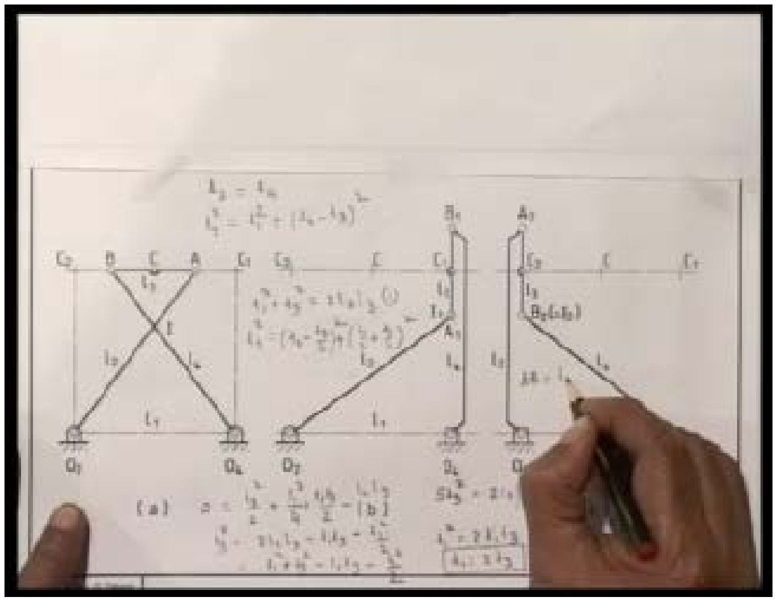
Let us have a look at the stage of another approximate straight line 4R-Linkage which is known as Chebyshev's link. This is again a Non-Grashof double-rocker and as we see, in this linkage O_2ABO_4, the midpoint of the coupler C, generates a fairly good straight line from C_1 to C_2, as this linkage moves. This line C_1 to C_2 is parallel to O_2O_4. Again, I emphasize that the path of C is not an exact straight line, but the discrepancy between the path of C and this straight line C_1C_2 parallel to O_4 is negligible. The entire coupler curve looks like this D-shaped curve with a fairly horizontal portion which is parallel to O_2O_4. In this Chebyshev's linkage, the link-lengths are as follows: C is the midpoint of the coupler, that is, AC is same as BC and the link-length AB: O_2O_4: O_2A which is same as O_4B, that means link 4 and link 2 are again of same length. The ratios are as follows: AB (the coupler length): O_2O_4 (the fixed link-length): O_2A or O_4B, which is same, is in the ratio of 1: 2: 2.5: 2.5. So here let us see, l_{max} is 2.5 and l_{min} is 1 that is l_{min} plus l_{max} is 3.5.

Let us now look at the link-length of this particular linkage. What do we see? AB, the coupler length to the fixed link-length O_2O_4 to the input link say O_2A which is same as the O_4B are in the following ratios: 1: 2: 2.5: 2.5. So that, minimum link length is AB the coupler link which is of unit length and the longest link is O_2A which is of 2.5. So l_{min} plus l_{max} is 3.5, which is less than l prime, say O_2O_4 and l double prime which is (Refer Slide Time: 20.08) 2 plus 2.5 is 4.5. So this is obviously, a Grashof linkage, with a shortest link as the coupler, which means it will be again a double-rocker, but this is a Grashof double-rocker, but the point C generates a fairly good approximate to a straight line.

Next we show, how did we get this link-length? The idea is, that this point C_1 is exactly above O_2 and this point C_2 is exactly above O_4. We determine the link-length of this symmetric linkage, that is O_2A equal to O_4B, such that the velocity of the coupler point C at C_1 is horizontal direction same is true for (Refer Slide Time: 21:20) C_2. When the coupler point comes to C_2, again the velocity of this coupler point will be horizontal. At the midpoint of C_1C_2, that is at the midpoint of O_2O_4, in that configuration again the velocity of this coupler point will be horizontal. If I say, that at these three locations, the velocity is horizontal, that is the coupler curve is horizontal then there is very little deviation between these three points, very little deviation of the coupler curve from this straight line C_1C_2. It is exactly tangential to the coupler curve at C_1, at this midpoint and at C_2. It will be clear when I explain this with help of sketches and get to these relations. This can be derived by the simple configuration that at C_1, C_2 and at the midpoint, when

C occupies the midpoint of C_1C_2, the velocity of the coupler point should be horizontal. This horizontal straight line will be tangential to the coupler curve at these three location and in between there is hardly any difference between the coupler curve and the horizontal straight line.

(Refer Slide Time: 22:34)

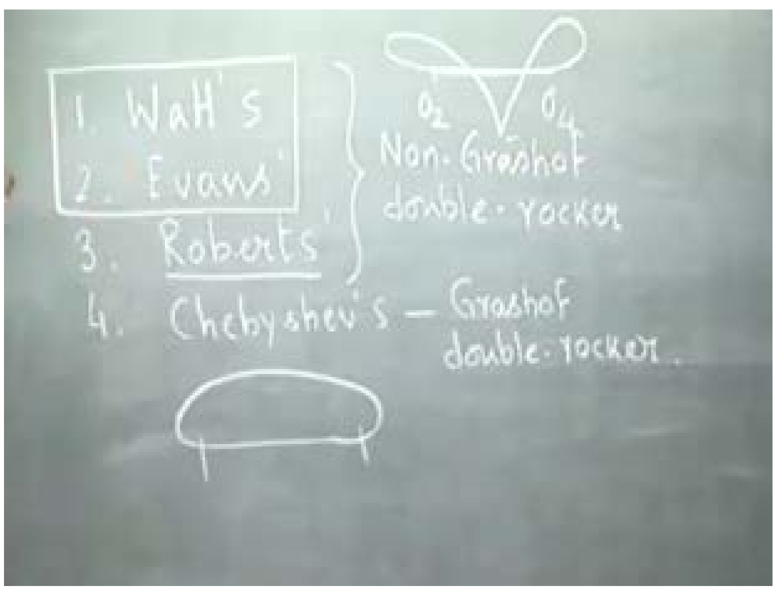


So let me now calculate the required ratios of this l_1, l_2, l_3, and l_4 such that these three configurations are possible. We know, because of this symmetric linkage l_2 is equal to l_4. If I consider this configuration, then I get l_2 square is l_1 square plus l_2 minus l_3 whole square. So I write l_2 square is l_1 square plus this is, l_2: because l_4 is l_2 minus l_3, l_2 minus l_3 whole square. If I expand, what I get from here is: l_1 square plus l_3 square is equal to $2l_2l_3$, this is equation one. From this configuration, what I get, l_2 square is this vertical height which is same as this vertical height. So I can write l_2 square is, l_2 minus l_3 divided by two whole square plus, this horizontal square, l_1 by 2 plus l_3 by 2 whole square.

If I expand this, what we get is, l_2 square cancels, so I get l_3 square by 4 here, l_3 square by 4 here so that is, zero equal to l_3 square by 2 plus l_1 square by 4 plus l_1 l_3 by 2 minus l_2l_3. So we get, l_3 square is, $2l_2l_3$ minus l_1l_3 minus l_1 square by 2. Here, $2l_2l_3$ is l_1 square plus l_3 square, so if I write that here, I get l_1 square plus l_3 square minus l_1l_3 minus l_1 square by 2. So l_3 square cancels, l_1 square by 2 is l_1l_3, that is $2l_1l_3$ which means, l_1 is $2l_3$. From this expression, if I can cancel l_3 square from both side, I am left with l_1 square minus l_1 square by 2 minus l_1l_c. That is, l_1 square is $2l_1l_3$. If I cancel one l_1, what we get? l_1 is $2l^3$,that is the fixed link-length is O_2O_4, is twice of the coupler link AB.

Now using this expression in equation one, what we find, it is l_1 square is $4l_3$ square plus l_3 square is $5l_3$ square is $2l_2l_3$, which means l_2 is 5 by $2l_3$. So over all what we are getting, the coupler - the shortest link-length AB, if I call l_3 then O_2O_4 - the fixed link-length, is standing out to be $2l_3$ and O_2A and O_4B which is l_2, which is equal to l_4 is 2.5 times l_3. These are the three ratios that I showed earlier. So, this is the Chebyshev's approximate straight line mechanism, which generates a very good straight line path in this length C_1 to C_2, then it goes high above and makes a D-shaped curve. So far we have discussed four different 4R-Linkages which are all capable of generating approximate straight line mechanism.

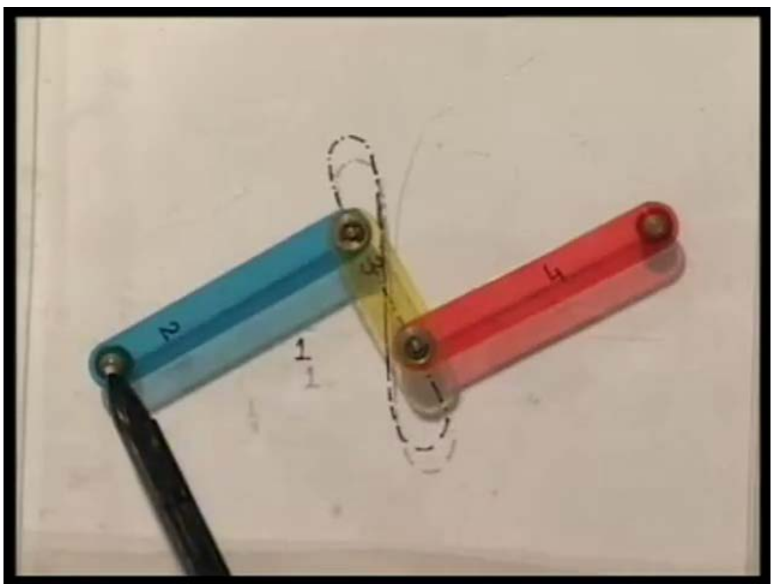
(Refer Slide Time: 28:27)



They were in order, one is called Watt's linkage or Watt's mechanism, then we discussed Evans' mechanism, third one we discussed was Robert's mechanism and the last one was Chebyshev's mechanism. Out of these, these three were Non-Grashof double-rocker and this one was a Grashof double-rocker. If we remember, these two, with those particular dimensions mentioned earlier, were capable of generating figures of eight, in which a fairly long portion was in approximate straight line. In this Robert's mechanism, we had a cusp, curve of this type where there is a cusp and this fairly approximate straight line portion which was coincident with the locations of the fixed hinge O_2 and O_4 where as in the Chebyshev's mechanism, we had a D-shaped curve, which had a fairly approximate straight line portion, which is parallel to the pivot line O_2O_4 fixed line O_2 O_4.

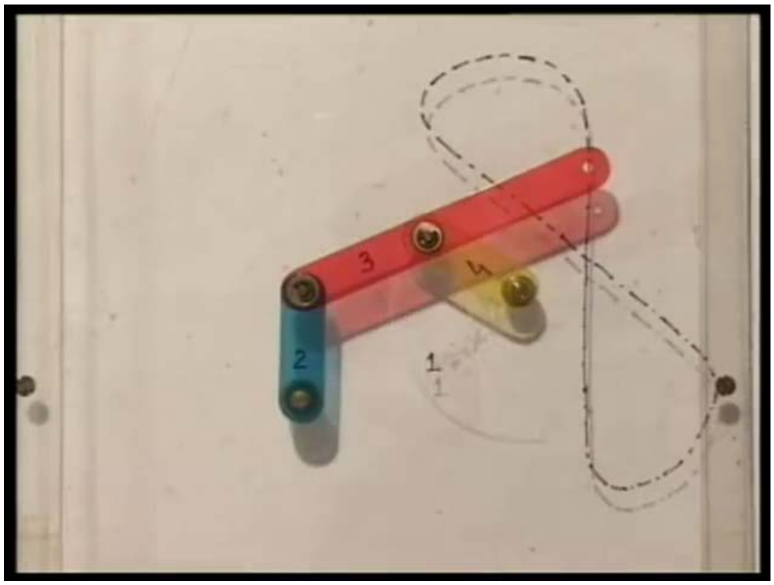
We shall demonstrate these four mechanisms through model you should watch, how good an approximate straight line all these coupler curves are. These are all 4R-Linkages, we consider a particular point on that coupler and the corresponding coupler curve has a fairly good portion of approximate straight line. We will now demonstrate these four mechanisms through models.

(Refer Slide Time: 30:42)



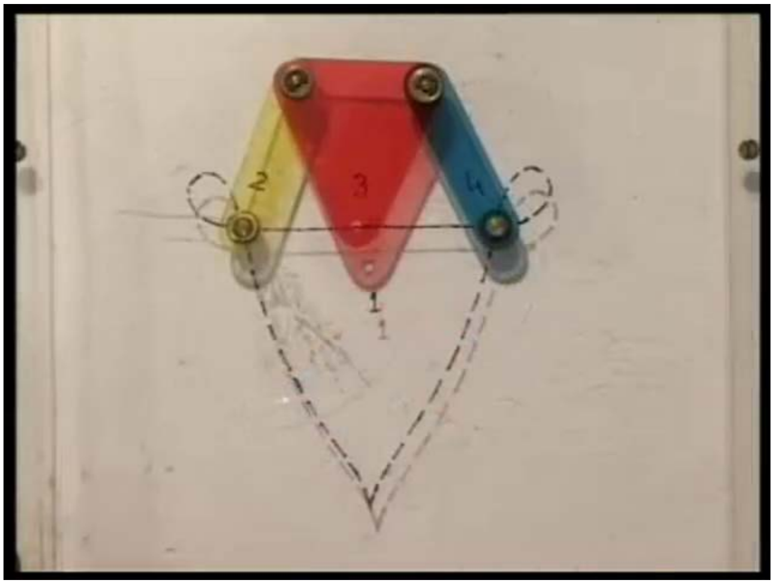
This is the model of the Watt's approximate straight line mechanism. This is the point O_2, this is the point O_4, this is A, this is B, this is the coupler. Let us concentrate on this midpoint of the coupler, that is the coupler curve generated by this midpoint of the coupler. As this mechanism moves, we notice that this mid-point is generating this figure of eight. In this figure of eight, from here to there, there is a fairly good approximation of a straight line. This is the figure of eight that is being generated and from here to here, this portion is very good approximation of a straight line. This is the Watt's approximate straight line mechanism, which as I said earlier, James Watt used this particular point to guide the piston within the cylinder.

(Refer Slide Time: 30:42)



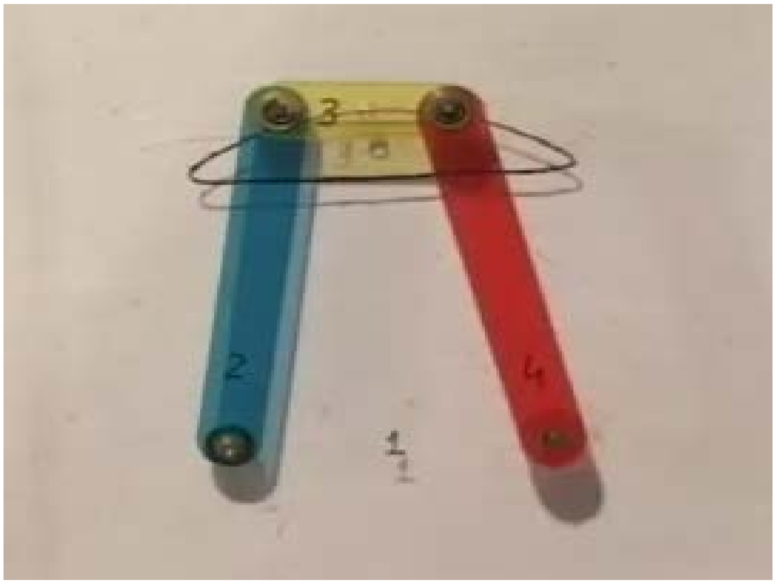
This is the model of the Evans' or Grashof's 4-link mechanism, which can generate an approximate straight line. This is the point O_2, A, B, O_4. Coupler point C is taken on the extension of the line AB. If the proportion of the link-length, as we said earlier, as we see as this mechanism moves, this coupler point generate this figure of eight in which from here to there, this portion is a very good approximation of a straight line. From here, as we notice that both the Watt's mechanism seen earlier and this Evans' mechanism a double-rocker, neither link 2 nor link 4 can rotate completely. As we see, that this rocker is crossing the line of frame, which implies, that it must be Non-Grashof double-rocker. This is the line of frame and link number 4 is coming below this so this a Non-Grashof double-rocker just as in the case of a Watt's linkage.

(Refer Slide Time: 33:21)



This is the model of the Robert's approximate straight line mechanism. This is O_2, A, B, O_4 and this is the coupler, with this coupler point as the C. As this mechanism moves, we should notice, that this coupler point C on link 3, is generating almost the straight line O_2O_4, with the very close approximation. Here, there is a cast and there are two loops at O_2 and O_4. This is again a Non-Grashof type double-rocker because both the rockers are crossing the line of frame, but they cannot rotate completely. Both of these are rockers but they are rocking above and below the line of frame. This is a Non-Grashof double-rocker with a very good approximate straight line generation between O_2 and O_4, this coupler point C.

(Refer Slide Time: 34:29)



This is the model of the Chebyshev's 4-link mechanism which can generate an approximate straight line by this midpoint of the coupler. O_2, A, B, O_4 and this is the point C, which is the mid point of the coupler AB. As we calculated the link-length ratios, this O_2O_4 is twice of AB, the twice of couple link and this link length and this link length are equal and is equal to 2.5 times coupler link length - coupler length AB. If this is unity, then this is 2, then these two lengths are 2.5.

If I move this we should notice, the curve generated by this coupler point. As we see that, this portion of the coupler point, is a very good straight line, very good approximation of the straight line. This is one configuration that we consider where O_2A and AB are vertical. This is C_1, this is again O_4B and AB are vertical and this is C_2 - the coupler point. This is the symmetric configuration, which we call C, O_2ABO_4 with this pointer C. So this is the Chebyshev's, which is again a double-rocker, but as we see the rocking is all away up the line of frame so it is a Grashof double-rocker, unlike the previous three.