Kinematics of Machines

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Module - 9 Lecture - 2

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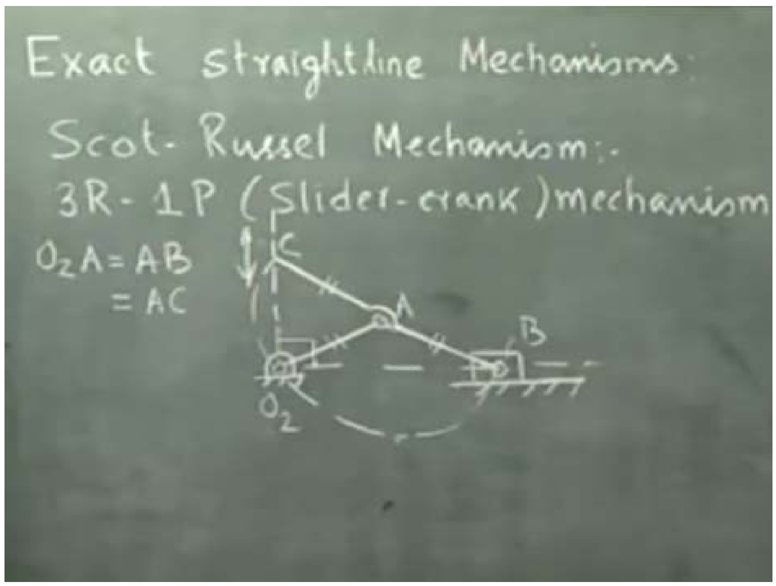

In our last lecture, we have seen a number of four link mechanism of the four R mechanisms, the link dimensions of which are fixed that a particular coupler point in each of this linkages could generate an approximate straight line, which we call approximate straight line mechanism. That means, the coupler curve has an approximate straight line portion, but it is known that by using four links we can never generate an exact straight line. In today's lecture, we will discuss what we call exact straight line mechanism. So far as exact straight line mechanism is concerned, we start our discussion with a mechanism which is known as scott-russel mechanism. This scott-russel mechanism is nothing but a slider crank or a 3R – 1P mechanism. This slider-crank mechanism is of specific dimensions such that, one point of the connecting rod generates an exact straight line perpendicular to the direction of the slider. We may think, what is the necessity of having a prismatic pair and then to try to generate straight line. The thing is that the straight line will be generated perpendicular to the direction of the slider.

This is may be useful, if I want to generate straight line, but I do not have sufficient space to use the prismatic pair. Let us now discuss this scott-russel mechanism. This is as I said is nothing but a slider-crank mechanism, where the crank and the connecting rod are of same dimensions and on the connecting rod, on the extension of the connecting rod I take another point. Let say this, crank check this O_2, crank pin is A, the slider is at B and I take a point C on the extension of that connecting rod that is on the line AB. Such that the length O_2 A AB and AC are all same, we make crank link O_2 A is equal to the connecting rod link AB is also the equal to the link AC, that is the point C is chosen on the extension of the connecting rod such that AB equal to AC, we are started with slider crank for the crank length O_2 A is same as AB. The direction of this prismatic pair is horizontal in this figure, what we can easily show that as this mechanism moves, this points C will generate a vertical straight line. It is obvious because AB equal to AC equal to A O_2 these three points O_2, B and C always lie on A circle whose center is A and the radius is A O_2 because AB equal to AC equal to A O_2. That means, these three points, namely B, O_2 and C will be equidistant from the point A. So we can always draw circle passing through these three points namely C, O_2 and b with center at A.

As this is the semi circle this angle will be always 90 degree. The angle between line O_2 B and O_2 C is always 90 degree because this is the angle on a semi circle. Therefore if the line O_2 B remains horizontal, the line O_2 C will always remain vertical, that is the point C will move along this exact vertical straight line. This is what we call scott-russel mechanism, as we see we are using a 3R – 1P mechanism not a four R linkage. We shall discuss some exact straight line mechanism, using only R pair. Obviously we have to go for number of links, more than four because four R mechanisms can never generate an exact straight line. Exact straight line mechanism using only R pairs are designed on the basis of the geometric principle which is called inversion.

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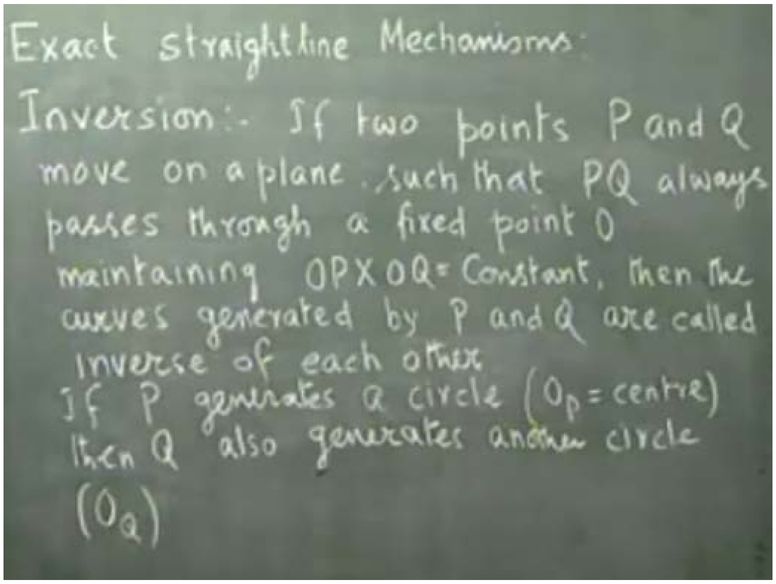

I would like to remind you that this principle of inversion should not be confused with kinematics inversions, which you discussed earlier on many occasions. What is meant by this principle of inversion? What it says that if two points P and Q move on a plane such that the straight line P Q, that is line joining this point P and Q always passes through fixed point O and maintain relationship point O, maintaining the distance OP into OQ equal to constant, then the curve generated by P and Q as they move are called inverse of each other.

Let us see what is this principle of inversion, I am talking of two points P and Q which are moving a plane, but the straight line PQ at all of this configuration passes through a fixed point O, not only that P and Q move in such way that the distance of OP and OQ, the product of these two distances remains constant. In such a situation, the curve generated by P and Q are called inverse of each other. It can be shown that if the curve generated by P is the circle then the curve generated by Q will also be another circle. That means, circles of inverses of each other, in general if P generates a circle say with OP as center, Q also generates another circle and vice versa, say the center of this circle is OQ.

It can be shown, then these three points fixed point O, the center of the circle generated by P which is OP and the center of the circle generated by Q that is OQ, these three points also will be collinear.

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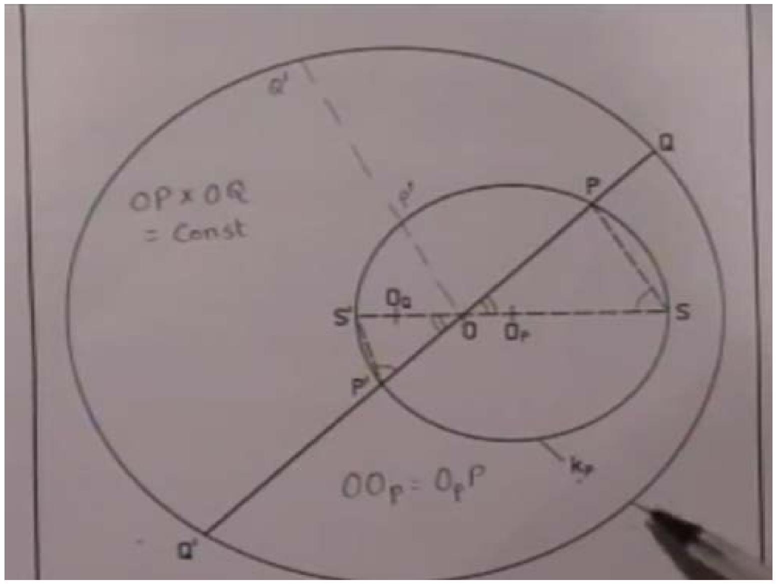

As we just now said, let us say these are the two points P and Q and these two points move in the plane of this papers in such a manner that this straight line P_Q passes this fixed point O, not only that the product of the distances O_P into O_Q remains the constant. As we see here O_P and Q, if P constant here and P comes to say P prime, Q will go to Q prime, such that P prime Q prime again is straight line passing through O, not only that as O_P has reduced, O_Q has increased such that the product OP into OQ remains a constant.

In such a situation, if P generates circle which have call k_p then Q will also generate another circle which I have called k_q. These are the inverses of each other, not only that as we see in this figure the center of the circle k_p is at O_p, whereas the center of the circle k_q is at O_q and this three points namely the fixed point O, O_p and O_q are collinear. This is the inversion in general by circle.

However, if the circle k_p passes through O, that is the distance O O_p is same as OP_p. If the distance OO_p becomes OP_p, in such situation the circle k_p will pass through point O, in that case it will be very easy to show that this circle k_q becomes a straight line that is the radius of this circle becomes infinite. This is what we will prove next that if two point P and Q move in a plane such that the line P Q passes through fixed point O and O_p into O_q remains constant and P generates the circle passing through a fixed point O and then Q will be generating a straight line, that is a circle of infinite radius.

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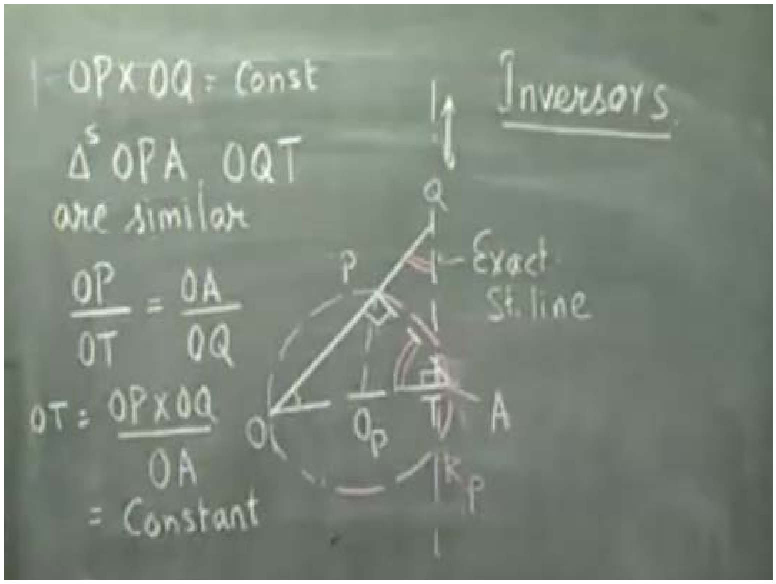

Let us now prove this special case of inversion, that is two points P and Q by passing through the fixed point O, this is the point P and this is the point Q and we have OP into OQ is constant. We are talking about special situation, such that the point P as it moves generates the circle passing through O. As these two points Q and P move, the line P Q always passes through the fixed point but P moves on a circle passing through O. In this situation, we will be able to prove that Q will move along a straight line, which will be perpendicular to the diameter of this circle which I name k_p passing through the point O. To prove this, let us say this is O the center of this circle of k_p let me call it O_p, the other end of the diameter of O let me call it A and if I draw a perpendicular from Q to the diameter O A, that is this angle is 90 degree let me call this foot of this perpendicular as T.

What you see that if we consider two triangles namely OPA, which is the right angle triangle and OQT which is again a right angle triangle, these 2 triangles are similar. Why? Because the angle at P is 90 degree, angle at T is 90 degree this angle is common to both the triangles OPA and OQT which immediately imply that these two triangles as are similar, because two of their angles is same which will mean that this angle is equal to this angle.

If these are two triangles are similar then we can write the ratio of this side will be proportional that means, opposite to this angle I have OP in this triangle, OP divided by opposite to this angle in the bigger triangle OQT, I have OT should be equal to opposite to this right angle is OA and opposite to this right angle I have OQ, because of these two triangles are to similar this relationship between the length of the sides is good OP by OT is OA by OQ. Where I can write, OT is OP into OQ divided by OA, we have already assumed that P and Q are moving in such a way that this distance remains constant and OA is nothing but diameter of the circle K_p that is also constant. This distance OT remains a constant. What is the implication of OT remains constant as the point Q moves? That Q must be moving along this straight line such that the projection of OQ on OA remains constant. Q is generating this exact straight line. This is the special case of that inversion when the circle generated by Q becomes circle of infinite radius that is a straight line.

Linkage is only R pairs having more than four links have been designed to generate exact straight line using this principle and those are called inversors, because we use this principle of this inversion such exact straight line generated are called inversors. We shall discuss two such inversors shortly.

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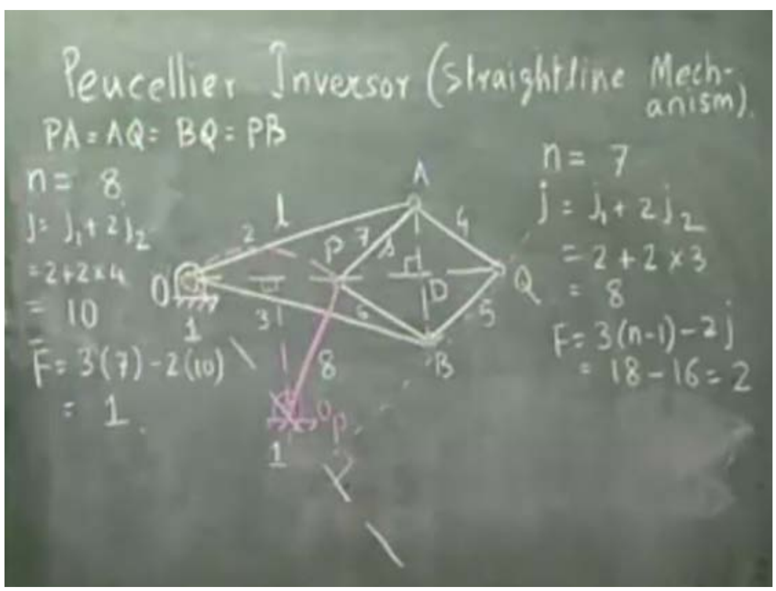

We discuss an inversor which is known as Peucellier inversor or exact straight line mechanism. To follow the Peucellier straight line mechanism, let me start with four equal link connected by revolute pairs making a rhombus. Let me call this point P, this point A, this point Q and this point B. This is the rhombus, that is all link length is equal PA equal to AQ equal to BQ equal to PB. At A and B, we connect two more links of equal length, at this point O attach two equal length OA equal to OB and now make this point O fixed. We have figure with 1, 2, 3, 4, 5, 6, 7 links and 1, 2, 3, 4, 5 the revolute pairs out of which is A, B and O are revolute of pair of higher orders because at each of these three links are connected at A, AP, AQ and AO at B, BP, BQ and BO and at O the fixed link O 1 and the link OA and Ob.

Let me make the link length, they do not look good. Let me now calculate the degree freedom of this assembly, how many links you have? Link number n is 1, 2, 3, 4, 5, 6, 7 n is 7. How many hinges we have? There are two simple hinges at P and Q so J_1 plus 2 times J_2, J_1 is two plus two times; there are second order hinges at O, because it is connecting three links namely 1, 2 and 3 and one at B, because it is connecting link number 3, 5 and 6 and one at O connecting link number 2, 7 and 4, J_2 is 2. You have 3 into 2 plus 2 is 8. The degree of freedom of this assembly is 3 times (n minus 1) minus 2J which is 3 into n minus one that is 18, minus 2j that is 2 into 8 is 16, degree of freedom is 2.

This assembly has degree of freedom two, what is the implication of that? That I can take this point Q anywhere in this plane, because it has two degrees of freedom I can keep it at any x coordinate and any y coordinator that means this assembly can be deformed such that Q can be taken anyway in this plane. Of course restricted by the link length but it can independently moved in two perpendicular directions that means Q can be taken anywhere in the plane.

Due to these geometric constants, that the figure AP BQ is a rhombus and OA equal to OB what you find that however Q moves, these three points O, P and q must lie on one line. Why? Because it is rhombus the diagonal must intersect normal. This angle must always remain 90 degree, because these are the two diagonals of this numbers BQ and AB are two diagonals of this rhombus AP, BQ. However Q moves this diagonal is always intersects each of the perpendicularly. This angle will be 90 degree. Again if I consider this APB this is the isosceles triangle, because PB equal to PA. Again this line perpendicular must pass through P because this is the isosceles triangle and this perpendicular to the base must be passing to the vertex. The same arguments holds good for this isosceles triangle namely OAB. This perpendicular line to the base of this isosceles triangle AOB this is at mid point I am drawing a perpendicular, must pass through O, this must pass through P which immediately implies that however this figure deforms these three points namely O, P and Q must remain on one line. We are satisfying this conditions that when P and Q move, O, P and Q remain collinear that satisfy one of the conditions needed for inversion.

We should be able to prove that OP into OQ also remains constant. To prove that, let me call this midpoint the intersection of PQ and AB these two diagonal bisect each other perpendicularly at this point, let me call this point D. Let me now try to calculate however this figure moves what happens to this product of these two distances, Op into OQ. OP I can write OD minus PD and OQ I can write OD plus DQ, because the diagonal of a rhombus bisect each other perpendicularly I can write PD is same as DQ. This DQ if write as PD, I get OD to the power of two minus PD to the power of two, because BQ I can write OD plus PD and OD minus PD into OD plus PD will give me OD to the power of two minus PD to the power of two.

This I can write, I can subtract this AD to the power of two from each of this terms, OD to the power of two because this is write angle triangle using Pythagoras theorem taken, I can write OD to the power of two is OS to the power of two minus AD to the power of two I can write this is OA to the power of two minus AD to the power of two. Similarly PD to the power of two again this is the right angle triangle, PD to the power of two, I can write AP to the power of two minus AD to the power of two. This minus AP to the power of two minus AD to the power of two, AD to the power of two AD to the power of two cancel, this gives me OA to the power of two minus AP to the power of two. This product however this figure deforms OP into OQ remain OA to the power of two minus AP to the power of two and now ignore that OA to the power of two this is longer link length is l and AP is this short link length say s, which do not change as the figure deforms, however the point P and Q move, this remains a constant which is the difference of these two link length squares, which is the constant.

In this figure, what you have achieved so far by having this rhombus and these two sides equal is that O, P and Q always remain collinear as P and Q moves where O is the fixed point. Not only that however these two points P and Q move not only they remain collinear passing through O the product that distance is OP into OQ remains l to the power of two minus s to the power of two which is again a constant, our l is this longer link length and s is this shorter link length. We have achieved two conditions required by this inversion, only thing we have to ensure that the locus of P is the circle passing through O that will automatically ensure that Q will generate straight line, perpendicular to the diameter of the circle k_p passing through O. It is very easy to ensure by adding one more link the P passes circle passing through O, to ensure that P passes through a circle passes through O we draw a mid normal to OP perpendicular bisector to OP and join a link let me call OP and join one more link which is O_PP. As we see because OO_P is O_PP as the point P moves, it must move along a circle passing through the point O. Now we have completed what you call Peucellier inversor

Let us see how many links it has, we had seven links to start with and now I have added one more link, which is link number 8, n is 8, J which is J_1 plus 2 times J_2. What we have done? By having this link we have converted this simple hinge which we considered earlier into second order hinge because now three link namely 6, 7 and 8 are connected here. How many simple hinges is left? One at Q and one at O_P, so J_1 is 2. How many J_2? One, two, three and four. J_2 is 4. Two into four, J becomes 10 and degree of freedom of the entire assembly F is three times n minus one that is, seven minus two into J that is, 10, that is 21 minus 20 is 1. We have got an eight link mechanism with single degree of freedom, where the dimension are so chosen that Q will move along a straight line, which is perpendicular to this line OO_P, because P is moving along a circle passing through O and this is the diameter of that circle k_p which we call along the line OO_P. When this eight link mechanism moves, Q will move along a straight line perpendicular to this diameter of k_p. This is what you call Peucellier inverse.

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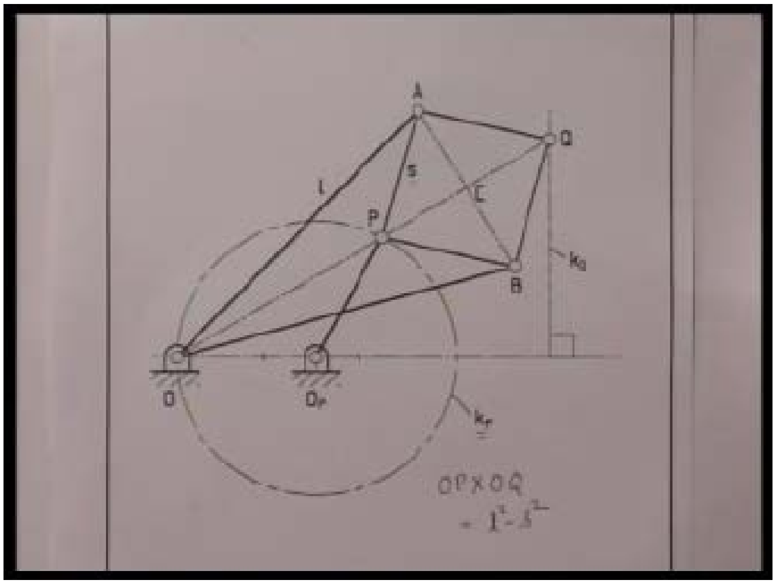

Let me now recapitulate Peucellier straight line mechanism. What we did? We had this A, P, B and Q consisting of four equal link lengths say the link length is denoted by S. We get a figure which is the rhombus. At A and B we connected two equal links of length l, OA equal to OB is equal to the length l. We have seen that in this figure we can ensure that as this figure is deformed, P and Q move but O, P and Q always remain along a straight line, not only that we have also seen when this figure is deformed, OP into OQ remains constant and is equal to l to the power of two minus s to the power of two, where l is this longer link length and s this shorter link length.

In the next step, we added these links O_PP to ensure that P moves along a circle passing through the point O, which I call k_p, O_PP is same as O_PO. This ensures that P moves along a circle, this distance is constant and this is fixed point and because O_PP is same as OO_P. The circle passes to the point O. In such a situation this point Q will generate a straight line, that is it will move along a straight line which is perpendicular to the line OO_P. This angle is 90 degree this is what you call Peucellier inverse, suppose this OP is chosen elsewhere, somewhere here or somewhere there, then this condition still holds good that PQ passes a fixed point O maintaining OP into OQ constant. Only thing that now P generates a circle but that circle does not pass through O and consequently the radius of this circle k_q is not infinity, it will be finite and Q will move along a circle.

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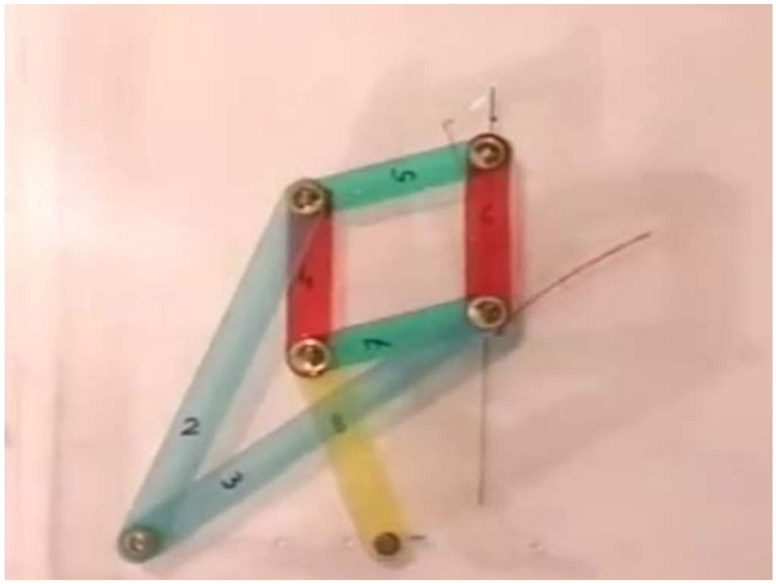

We shall now demonstrate this Peucellier inverse through a module. This is the model of the Peucellier as straight line mechanism, which as eight links the fixed link and four links to make the rhombus, two links to make that isosceles triangle and this is the extra link to ensure that this point passes through this fixed point O. This was our O, P, A, B and this is OPP.

As we see, as the mechanism is moved we find that this point Q will generate this exact straight line. As this mechanism is moving, the point P is generating a circle which will pass through O, consequently this point Q is moving along this straight line. If I remove this center OP from this particular point, such that this distance is not the same as O_PP, then P will move the circle, but definitely will not pass through O. In that situation if I remove this OO_P and take it to some other location then as we see, this point Q is going along this circular R, of the very large radius, because all the conditions of the inversors is being satisfied. OP into OQ is constant, OPQ always remaining along a straight line, P is a moving along a circle only thing that circle is not passing through the fixed point, because this distance is not equal to this distance. Consequently, Q will also move along a circle.

In fact, we are able to get a mechanism such that, I can draw circular arc of very large radius. In fact, this circle the radius is very large. If I move this point to the other side I could have generated this circle, but only I make OO_P is O_PQ, the radius of the circle becomes infinity. This is the one curvature, this the opposite curvature and this is the no curvature. If I positive curvature, negative curvature and zero curvature which means a straight line. That is a circle of infinite radius. This is what you call a Peucellier straight line mechanism only if I ensure that OO_P is O_PP.

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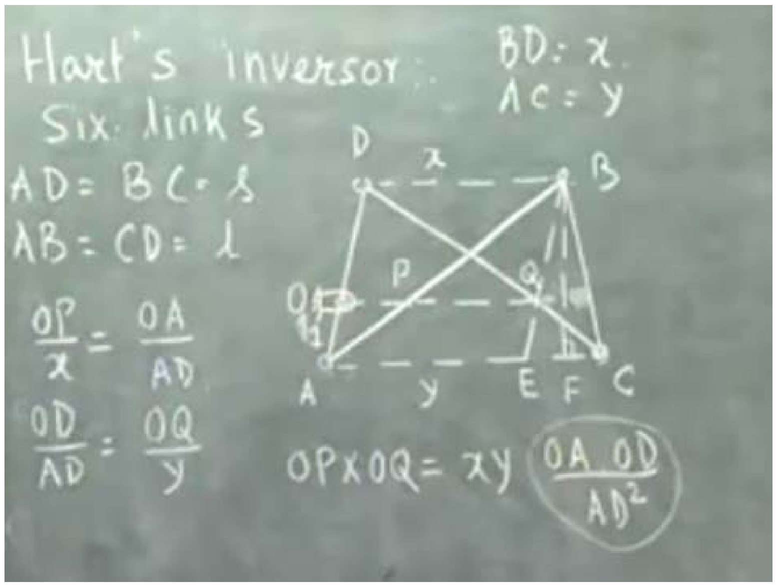

We have just now discussed Peucellier straight line mechanism which consist of eight links and all are pairs. We discuss another inversor which is known as Hart's inversor or Hart's straight line mechanism. As we will see, this will have only six links instead of eight links as we have seen in case of Peucellier mechanism. To explain Hart's inversor let me start with an anti- parallelogram.

Let me call this figure A, B, C and D. In this AD, this link length is equal to BC the other link length, let me call it s and AB equal to CD, which is the longer link length which is called l. We start with anti-parallelogram that means, the opposite sides are of equal length, but it is in the cross configuration. On this link AD let me take a point O and make it fixed, this point let me call O.

If I find the degrees of freedom of this assembly, what we see? We have four link and a fixed link, n is total number of links, n is five and we have one, two, three, four, five simple hinges, J equal to J_1 is also five. The degree of freedom is three into n minus1 that is, three into four minus two into J that is two into five which is two. Again we find, this figure is deformable that means I can take this point B or C or D anywhere in this plane restricted only by the link length, but however this figure is deformed, this BC will always be remaining equal to AD and AB will remain equal to CD. That means the parallelogram will remain always parallelogram however this body deforms. Consequently I can say that because of the anti parallelogram BD will always remain parallel to AC. Through O I draw a line which is parallel to these two lines namely BD or AC. At this line intersects the line AB let me call it P and this point of intersection let me call Q. As we see, the point O is not moving and however the figure deforms, this line will always remain parallel to BD and AC that means these three points namely O, P and Q always remain on one line.

One condition of the inversor is being satisfied that P and Q move, but P Q the line always passes through the point O, not only that we will be able to show that whenever this figure deforms, OP into OQ remain constant. To prove that let me proceed as follows. I draw line through B parallel to AD let me call it E, so A, E, B, D this figure will be parallelogram because BD is parallel to AE and I have drawn BE parallel to AD, which means AD will be also equal to BE because they are opposite sides of a parallelogram and OE will be equal to BD. BC is also equal to AD is also equal to BE, I get a isosceles triangle BEC. I drop a mid-normal from B unto E, C and let me call this point F, let me say BD this distance to x which keeps on changing as the mechanisms moves, as this assembly moves, and AC let me call it y which also changes, BD I write x and AC I write y.

Let us see that, APO and ABD these two triangles will be similar, because this line OP will always remain parallel to BD. What we can write, OP by x is same as OA by AD. Considering these two similar triangle namely OAP and ABD because this line is parallel to this line I can easily write OP by x is OA by AD. Let me consider this two triangle namely DOQ and DAC, again in this two triangle OQ is parallel to AC. These two triangles will also be similar. What I can write DO that is OD divided by DA, AD is same as OQ by y, OQ by AC which I call y. Let me cross multiply these two equations then we can see that what we get, OP into OQ is x y into OA into OD divided by AD to the power of two. If I multiply these two equations I get OP into OQ is same as xy into OA by AD into OD by AD which is OA into OD by AD to the power of two

In these expressions as we see when this figure is deformed, the link lengths OA OD or AD never changes, this factor is constant. If I can show that x y remains constant that will imply OP into OQ also remains constant. To maintain the second condition of inversion that as this points P and Q move not only the line P Q passes through O, the distance OP into OQ remains constant, will hold good only if I can show that whenever this figure is deformed this product xy that is BD into AC remains constant.

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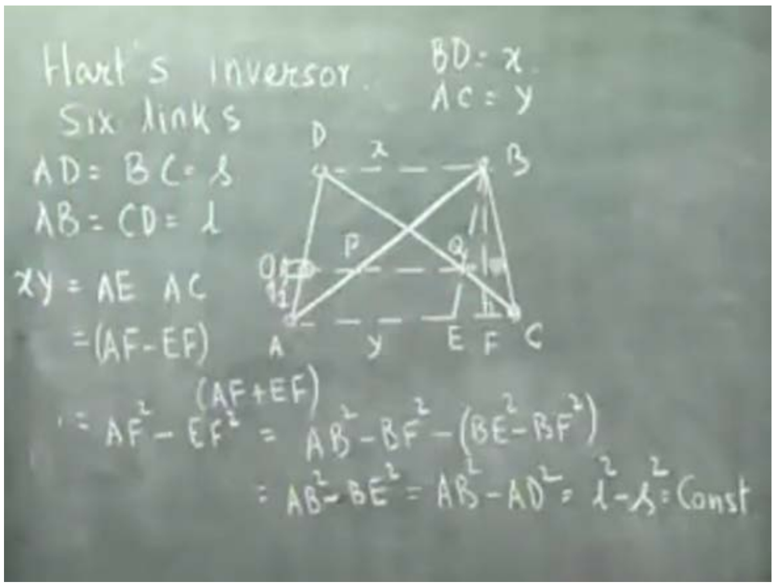

Let show that is simple, we to show the xy remains constant. What is xy? xy is BD which is same as AE, x is AE and what is y? y is AC, xy is AE into AC. This I can write AE is AF minus EF and AC is AF plus CF which is same as EF, because this is an isolation triangle, this is the perpendicular bisector. This is equal to this. AF plus FC that I can write AF plus EF, this we get AF to the power of two minus EF to the power of two. A to the power of two I can write because A, B, F is the right angle triangle, AF to the power of two I can write from Pythagoras theorem AB to the power of two minus BF to the power of two.

Similarly EF to the power of two I can write from this right angle triangle BE to the power of two minus BF to the power of two so BF to the power of two cancels, we get AB to the power of two minus BE to the power of two and BE is same as AD, because this figure is a parallelogram I can write AB to the power of two minus AD to the power of two. AB I call l and AD I call s which are link lengths. However the figure deforms they never change. What we have done so for? We have first seen, OP into OQ is xy into some constant, in terms of the link lengths. If the xy remains constant then OP into OQ will also remain constant and all other point PQ move the line PQ passes through O, this is constant which implies OP into OQ is also constant. Consequently we have satisfied all the conditions needed by inversor. If we ensure that the point P moves along a circle passing through O, then point Q and this link CD will generate a straight line. As in the case of Peucellier mechanism, we add one more length so that the point P is guided along a circle passing through O, which is simple.

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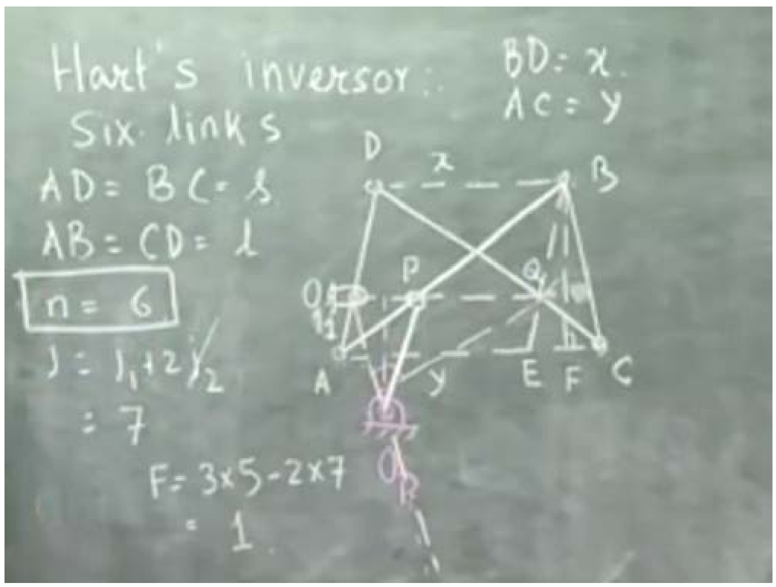

We draw the mid normal of OP and this point we call OP and here we put another hinge and connect by rigid link, because $O_P P$ is OO_P, P will go along a circle which $O_P P$ is radius and that circle passes through O. The third conditions of straight line inversor also satisfied. Consequently if this mechanism moves, the point Q will generate a straight line which will be perpendicular to the line OO_P. Q will generate a straight line which will be perpendicular to this line OO_P. Let us see how many links we got in the Harts inversor, the fixed link, this four links that consider the first anti-parallelogram and I have added one more link, n is six.

Let me count J is J_1 plus $2J_2$. As we see we have one, two, three, four, five, six and seven and all of them are simple hinges there is no J_2, J_2 is 0 and J_1 is seven, so J is seven the degree of freedom of this is three into five times (n minus one) minus 2 into J is one. Again we get a six link single degree of freedom mechanism consisting of only R pairs and when this linkage moves, this particular point Q and this link CD will generate and exact straight line this is called Hart's inversor.

Again if I don't take OP in a manner such that, O_PP is OO_P then P will generate circle, Q will also generated circle, but as soon as I make this distance equal to this distance such that P moves along a circle passing through O then radius of the circle of this Q becomes infinity which means Q will be moving along a straight line, because straight line is nothing but a circle of infinite radius.

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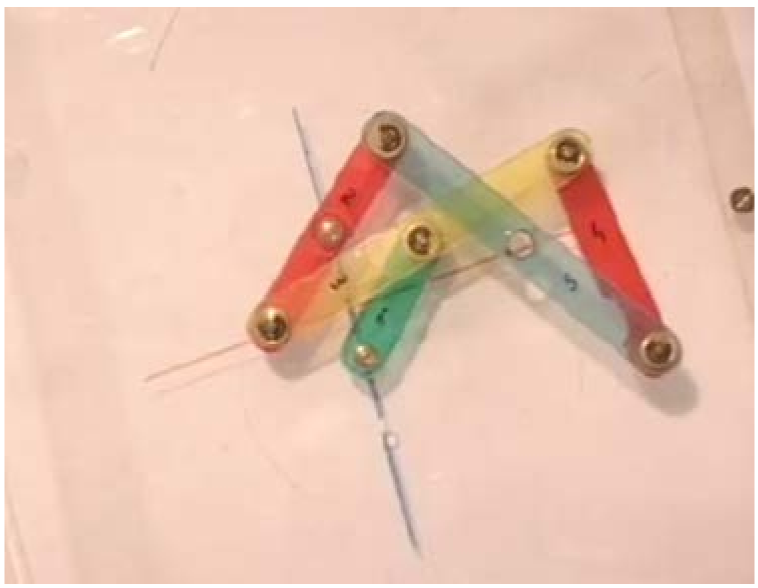

We will demonstrate this Hart's inversor through a model. This is the model of the Hart's inversor which is the six link mechanism. As we seen fixed link, links number two, three, four, five and this is sixth link. These two link length is same and these two link length same constituting an anti-parallelogram, opposite sides of this quadrilateral are same that is in a cross configuration. This is what we call A, B, C, D. We take one point on this link fixed, then think of a straight line which is parallel to this line and this line, because this is the an anti-parallelogram these two lines always remains parallel and I think of a line passing through this six point O, which is parallel to this line and this line.

The intersection of that straight line from to O this link I call peak and this is the intersection of that straight line of this link which I call Q. Now I make O OP is OPP this link length, this point OP I choose on the perpendicular bisector of these two points such that OPP is OPO. Consequently, P will go along a circle with this point as center and this as radius and that circle will pass through this point O. As a result since OP into OQ remains constant which we have proved, EQ passes through this point O, in all configurations that is shown, so Q will generate straight line. As we can see that this point Q will generate this straight line. As just seen this point Q is generating this straight line. This is the Hart's inversor which is the six link chain as compared to a Peucellier inversor which was the eight link mechanism.

Let me now summarize today's lecture, we have discussed today only exact straight line mechanism. We started with scot-Russel mechanism which is simple slider crank mechanism that the 3R-1P linkage where the length of the connecting rod and the length of the crank are equal. In such a situation if we take a point on the extension of the connecting rod suitably, then we found that point on the connecting rod generates an exact straight line perpendicular to the direction of the sliding of the slide. After that, we discussed the principle of inversion I would like to emphasis that, do not confuse this principle of inversion with kinematics inversion. We have explained in this principle of inversion and used that principle to generate two exact straight line mechanism using only RPF. We discussed two of them namely Peucellier straight line mechanism which has eight links and then Hart's straight line mechanism or Hart's inversor which has only six links.