Kinematics of Machines

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Module - 9 Lecture - 3

In the first two lecturers of this module, we discussed some special mechanism namely, approximate straight line mechanism and exact straight line mechanism. In today’s lecture, we will discuss another special mechanism, which are used for steering the automobiles and are called ‘steering mechanisms’.

As we know, when an automobile wants to take a turn, the front two wheels have to rotate about the vertical axis.

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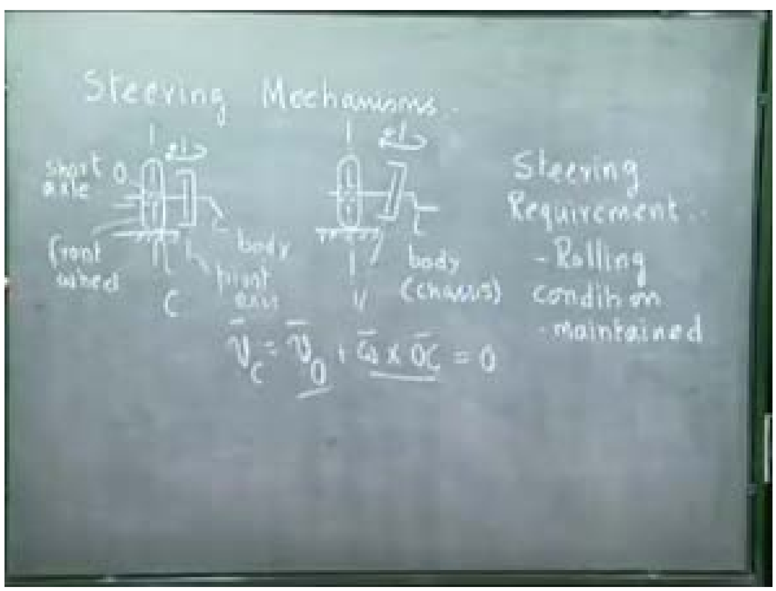

Let us say, this is the front view of the one of the wheels, front wheel’s front view. As we see, the front wheel is normally mounted on a short axle which is pivoted to chassis and the pivot axis can be vertical, this is the body of the automobile and this is the pivot axis. When car wants to take a turn, it is this front wheel, rotates about this pivot axis. There are other types of steering mechanisms, where this pivot axis may not be vertical, it is inclined from the vertical. Again, this is the front view of another front wheel. This is the road surface and this is the short axis about which the front wheel can rotate and during the steering action, when the car wants to take a turn, this wheel will rotate about this pivot axis and this is the body or chassis of the automobile. What you see in this arrangement, while taking a turn, the body of the automobile is lifted. Consequently, when the steering wheel is released, the car automatically comes to its original position and the wheel becomes straight. We shall discuss this type of steering mechanism in our subsequent discussion. First, we should discuss, what is the steering requirement?

When the automobile takes a turn, either a right hand turn or a left hand turn, it is imperative that all the wheels that is the two front wheels and the two back wheels continue to roll. What is the rolling condition? That are the contact point velocity must be zero. It is easy to see that, if I consider this contact point C and it is center of the wheel, if I call 'O', we know velocity of C is velocity of the center of the wheel O plus omega of the wheel cross OC. Now, for V_c to be zero, it is this vector must cancel this vector, that means, these two vectors must lie in the one plane. Now, omega cross OC lies in the plane of the wheel. So, the V_o must lie in the plane of the wheel. This is the basic steering requirement that the velocity of the center of the wheel must lie in the plane of the wheel, such that, when the wheel rotates, the contact point velocity can be zero that is the perfect rolling condition can be maintained.

Steering requirement is rolling condition must be maintained. I hope this is clear that when the car takes a turn, the velocity of the center of the wheel, if it does not lie in the plane of the wheel, then there will be velocity of the contact point, because this lie in the plane of the wheel. Omega is this way, OC is this way, so omega cross OC is in this direction, that is in the plane of the wheel. So, V_o must lie in the plane of the wheel such that V_c can be zero, which is required for perfect rolling conditions. So the front wheels must turn appropriately, such that the center velocity must lie in the plane of the wheel. To maintain this steering requirement, let us see what is the condition needed.

Let us now see the condition that needs to be satisfied, such that the steering requirement is fulfilled.

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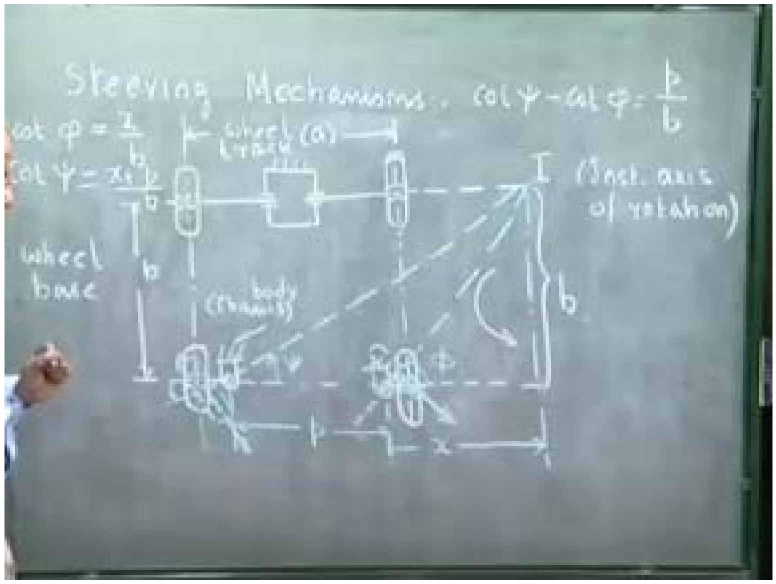

Let us consider the two back wheels. These are the gear wheels. We know these two gear wheels are connected to the differential, which is fixed to the body of the automobile. This differential allows the two gear wheels to rotate at different speed when the car is taking a turn. For example, if the car is taking a turn like this, then this becomes the outer wheel and this becomes the inner wheel and we need that the outer wheel should rotate faster as compared to the inner wheel. This differential speed can be created by this differential, as we shall discuss later on when we discuss gear box.

The distance between these two wheels is called wheel track. Let us say wheel tracking (a). This is the top view of the car and the distance between the gear axle and front axle, this distance let me call b, which is called the wheel base. Similarly, there is another front wheel. So, these are the two gear wheels and these are two front wheels. As we see, the front wheel axles are mounted on these pivots on the body of the automobile. Now, when the car takes a turn in this direction, these two front wheels must rotate. Let us say, this axle rotates about this pivot and this axle rotates about this pivot. These two rotations must be such that, all these axles must intersect at one point. The gear axle and the two front axles, when the car is taking a turn must intersect at one point, let me call it ‘I’, that is, I is the instantaneous axle of rotation for this entire car body as the rigid body. So, this is instantaneous axis of rotation for the car body.

As we see, in this situation, the wheel which is now rotated like this, the velocity of center of the wheel is in this direction that is contained in the plane of the wheel and this wheel which is rotated like this. The velocity of the center of the wheel is perpendicular to this line and again contained in the plane of the wheel, which is the requirement for perfect steering. These two wheels must rotate say, this is phi and this angle is psi. So for perfect steering, this phi and psi should be such that these two front axles and the gear axles pass through one point. This is the steering requirement. So, let me find out what is the condition?

Let us see, if we say that the distance between these two pivots in this direction is p. If I say that distance of this instantaneous axis of rotation from this pivot, say x. So, what we see that this distance is this is the wheel base which I call b. What we can write that, cot phi is x divided by b. Similarly cot psi is x plus p divided by b. If I subtract, we are getting cot psi minus cot phi is p by b. So, for perfect steering requirement the rotation of the two front wheels - the inner wheel by phi which has to rotate more and outer wheel by psi, has to rotate less. This phi and psi should be related by this equation cot psi minus cot phi is p by b, but p is the distance between the two pivots on to which is front short axles are mounted and b is the wheel base.

Now, we will see how we can achieve this condition by using different types of steering mechanism. Now, that we have obtained the steering requirement, that is the condition that must be fulfilled by steering mechanism. Let us discuss a particular steering mechanism known as ‘Davis steering gear’.

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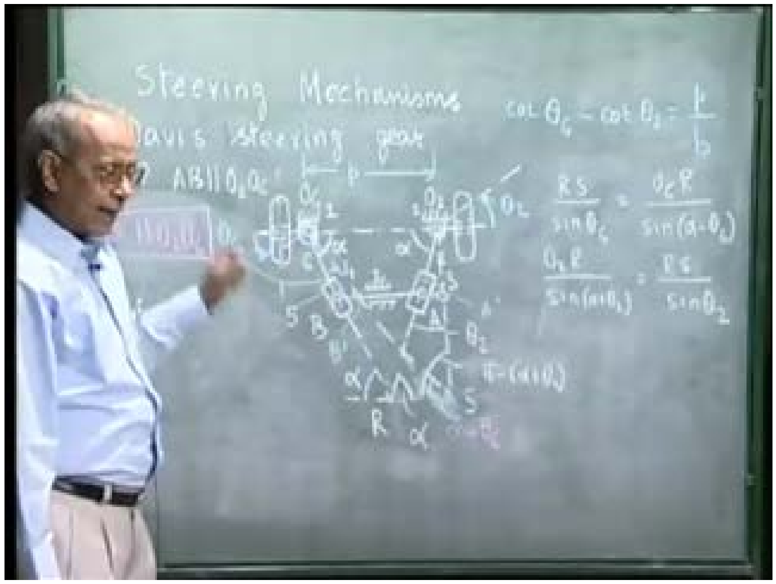

In this Davis steering mechanism, we use a six link mechanism with seven lower pairs. Let me first draw the mechanism. This is the Davis steering mechanism, as we see these are the two front wheels mounted on short axles and the axles are pivoted on the body of the automobile. Let us consider that the car is taking a turn in this direction. As we see, let me call this point O_2, this point A, this point B, and this point O_6. This is the six link mechanism, link number one refers to the body of the automobile. This is link number two and this block in which this link number 2 can slide, this is link number 3, link number 3 is hinged to this link which is link number 4 and this block which is hinged to link number 4 at B is link number 5 and this is link number 6, which is nothing but the axle of this outer link.

Similarly, link number 2 is nothing but the axle of the inner link. These are the axles which can slide in this link. This is another axle which can slide in this link and these two links are connected by link number 4 to revolute pairs. So, over all, we get 6 link mechanism, 1, 2 is the axle of the inner link, 3 is this block in which this link 2 can slide. Link 4 is this rod this link which can slide in the body of the automobile parallel to the direction of O_2O_6. Link 4 is in sliding motion with respect to link number 1, that is, body of the automobile, there is a prismatic pair here. Link number 4 has a revolute pair of the link number 5 and the link number 5 has the prismatic pair in this direction with link number 6, which is the nothing but the axle of the outer link.

Obviously, the mechanism has to be symmetric, such that similar turn can take place either in this direction or in this direction. The car can turn in this way or the car can turn that way. As we see, this mechanism is symmetric, that means this angle is same as this angle. Now, our only requirement is to find the value of alpha, such that the steering requirement is satisfied. Let us say, the rotation of this link 2, how it is steered? It is this rod is moved in this direction. That means, suppose the A comes to this point equal A prime and B comes to exactly by the same distance, here let me call this point B. This rod AB moves horizontally parallel to O_2O_6 from AB to A prime B prime. Consequently, this link 2, rotate by $theta_2$ and if I join O_6 with B prime, link 6 as we rotate by this angle, which is $theta_6$ that is this angle is fixed. So, $theta_2$ is rotation of this axle, $theta_6$ is the rotation of this axle.

As AB goes from AB to A prime B prime, you have to find the value of alpha such that the steering requirement is fulfilled. If we remember, this distance between the pivots is denoted by p and steering requirement was cot of $theta_6$ minus cot of $theta_2$ was p by b. In our previous diagram, $theta_6$ is denoted as the rotation of the outer wheel, which is psi and $theta_2$ as the rotation of the inner wheel, which is denoted by phi and we said cot psi minus cot psi should be p by b. The p is the distance between the pivot and b is the wheel base. Let us say, in the state configuration, these two lines meet here and in this rotated configuration, these two lines meet there. We call this point R and this point S. Extension of link 2 and extension of link 6. In the state configuration, the wheel was straight, meet at R and when the wheel turns, they meet at S. It is very easy to show that this line RS, let me use a different color, this line RS is parallel to O_2O_6. We can easily prove that RS is parallel to O_2O_6. To prove that, we have to remember that this link length four between the two revolute pairs is same, that is A prime B prime is same as AB. BB prime is same as AA prime and AB, that is the length of the rigid link 4 is A prime B prime. We already stated that this prismatic pair was parallel to O_2O_6 that is AB is parallel O_2O_6.

What we can write? That RA by O_2R is same as RB by O_6R is Ab by O_2O_6 because this line is parallel to this, these two triangles are similar triangles, RAb and RO_2O_6 are similar triangles, because AB is parallel to O_2O_6. So, we can write RA by O_2R is same as RB by O_6R is AB by O_2O_6. Now, A prime B prime is again parallel to O_2O_6. So, for these two triangles, S A prime B prime and SO_2O_6, we can have similar relationship, that is SA prime divided by O_2S is SB prime by O_6S is A prime B prime by O_2O_6. These two white triangles are similar and this triangle and this blue triangle are similar, because this line is parallel to this line. Now, AB is same as A prime B prime. So, these are also same. So, we can write RA by O_2R is same as SRA by O_2R is same as SA prime by O_2S, RA by O_2R is AB by O_2O_6 and SA prime by O_2 S is A prime B prime by O_2O_6. AB is same as A prime B prime, so this is equal to this. Now, this RA by O_2R is same as S A prime by O_2S, then, I can say this line A prime is parallel to RS which means RA is also parallel to O_2O_6. So we have just proved that this line RS is parallel to O_2O_6. To determine the required value of alpha, such that the steering condition is fulfilled, let us note that because RS is parallel to O_2 O_6, this angle is also alpha.

Similarly, this angle is also alpha. Now, this is the angle $theta_6$, this angle is alpha. So, this angle is alpha minus $theta_6$. This angle is alpha and this angle is $theta_2$ so this angle is pi minus alpha plus $theta_2$. Now, we use the triangle sin law for this triangle namely, O_6RS, we can write RS by sin of this angle which is $theta_6$. RS by sin of $theta_6$ is same as O_6R divided by sin of this angle, which is alpha minus $theta_6$. So, this is O_6R sin of alpha minus $theta_6$. This angle is alpha and this angle is $theta_6$, so this angle is alpha minus $theta_6$. Now, in this triangle, we apply the sin law RS by sin of $theta_6$ is O_6R divided by sin of this angle, which is alpha minus theta. Again, we apply the triangle sin law for O_2RS, this triangle. So, O_2R divided by sin of this angle, which is O_2R by sin of pi minus alpha plus $theta_2$ is same as alpha plus $theta_2$ is same as RS divided by sin of this angle, which is $theta_2$. So, we apply the sin law for triangle O_6RS and O_2RS to get these two relations.

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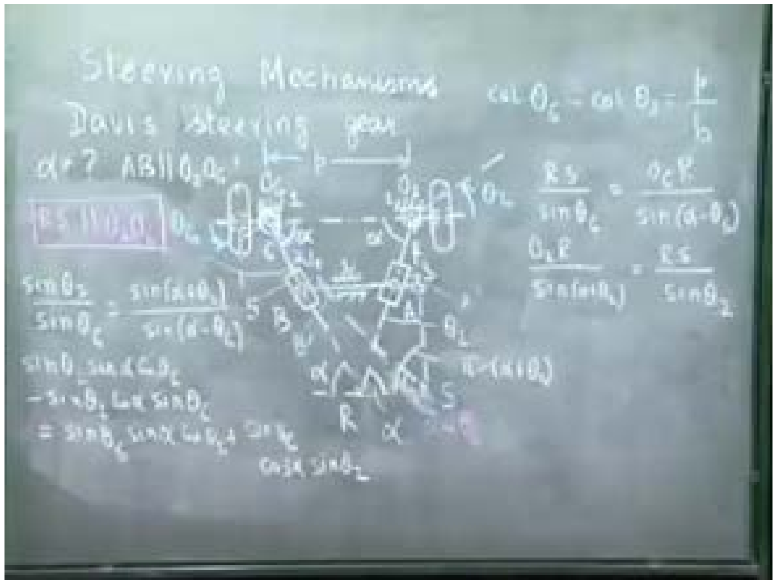

Now, from here, if I divide this equation by this equation, we get sin θ_2 divided by sin θ_6 is same as sin alpha plus θ_2 divided by sin alpha minus θ_6. We just divide this equation by this and this equation by this to get these relations. If we expand, we get sin θ_2 sin alpha cos θ_6 minus sin θ_2 six cos alpha sin θ_6 is same as sin θ_6 sin alpha cos θ_2 plus sin θ_6 cos alpha sin θ_2. Just multiplying this with this, this with this, we get this relation.

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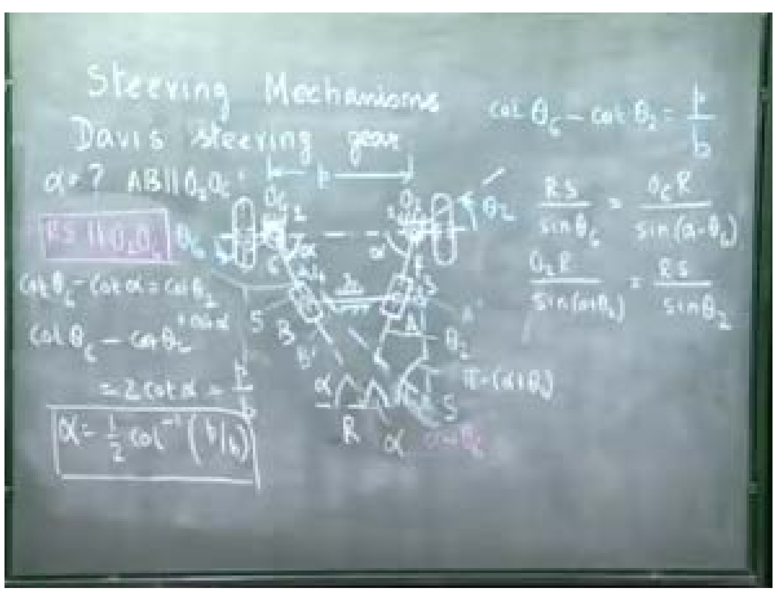

Now, we can divide all these four terms by sin $theta_2$ sin alpha sin $theta_6$, so that I get, Cot of $theta_6$, sin $theta_2$ sin alpha cancels, divided by sin $theta_6$ I get cot. Again, from here, I get minus cot alpha. This sin cancels and this gives me cot alpha. Here, I get equal to cot $theta_2$, then here I get plus cot alpha. So that gives me, cot $theta_6$ minus cot $theta_2$ is two cot alpha. For perfect steering condition to be maintained, I know cot $theta_6$ minus cot $theta_2$ should be p by b, while p is the distance O_2 O_6 between these two pivots and b is the wheel base. So, this is equal to p by b. So, alpha is half of cot inverse p by b.

Let me explain this Davis steering gear once more. This is the six link mechanism, where link 1 refers to the body or chassis of the automobile. Link 2 is inner wheel axle, we consider a turn like this and with this front wheel becomes the inner wheel and link 6 is the short axle of the outer wheel, that is link number 6. Link 3 has a prismatic pair between 2 and 3. Link 5 and 6 has a prismatic pair between them. Link 4 has two revolute pair at its end and a prismatic pair with link 1 and the direction of this prismatic pair is parallel to O_2O_6.

The mechanism is symmetric, because we have to take identical left turn and right turn. Only, design parameter is to find out this angle alpha and we found out that alpha should be half cot inverse p by b, where p is distance between O_2, O_6 and b is the wheel base. Then, this steering action will be maintained.

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So this is a six link mechanism, where you have, R and a P pair, then again an R pair, then again a P pair then again an R pair, then again P pair and an R pair, so it has seven lower pairs, alternately R-P-R-P-R-P-R. Though this Davis steering gear can maintain the perfect steering condition for all radius of turning, it is cumbersome to make because there are so many prismatic pairs and it is difficult to maintain.

So, Davis steering gear is a six link mechanism with one, two, three prismatic pair and four revolute pairs. Due to difficultly of this prismatic pairs, it is decided to forego this advantage of maintaining the steering condition for all turning radius. We can go for a simple mechanism, which will maintain the steering conditions for certain values of turning radius and will not be exactly satisfied for all other turning radius. There will be a little tendency of slight slip or the transverse motion of the wheel at the point of contact.

Let me now summarize, what we have covered today. We started discussion of steering mechanisms which are used in automobile. The front wheels of the automobile are mounted are short axles, which are pivoted to the body of the automobile. For perfect steering condition, that is the wheel continued to roll even when the car is taking a turn, we need that the velocity of the wheel center must lie in the plane of the wheel, so that the contact point of the wheel has zero velocity maintaining pure rolling. This requires that the two front wheels must be rotated by different angles, depending on the turning radius. The condition we obtained was in terms of the distance between the pivots of these two short axles and the wheel base. We showed that is six link mechanism, using alternately R and P pair, seven of these 4- R, 3 - P pairs; which we call Davis steering gear can maintain this perfect steering condition for all turning radius, but there is difficultly of this complicated mechanism, both for manufacturing, it is costly and also for maintenance because of three prismatic pairs.