Kinematics of Machines

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Module - 11 Lecture - 2

We finished our last lecture with the geometrical construction, which expresses the pressure angle for a translating roller follower which is given by this expression.

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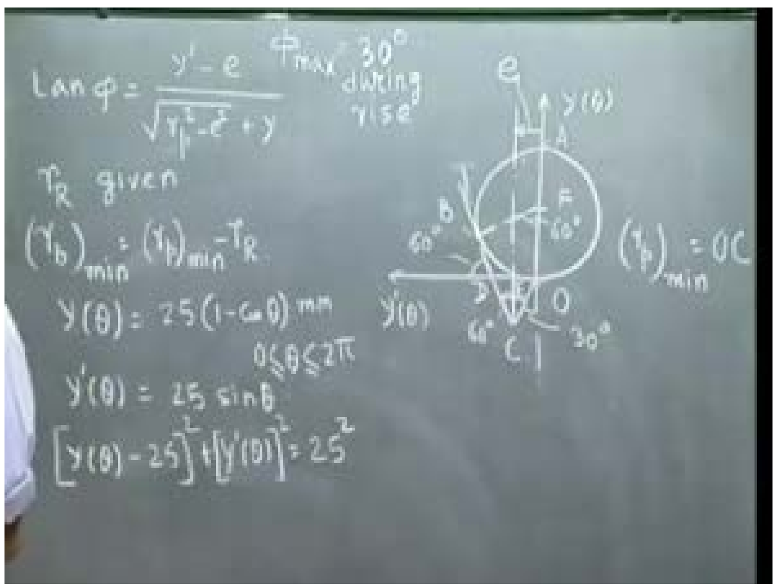

Today, we will take up an example to show from this geometrical construction, we can get the optimum value of this prime circle radius and the offset. Of course, the roller radius r_R is assumed to be given and if I get r_p the minimum value, then r_b the base circle radius minimum value I can get from (r_p) minimum value minus r_R.

Towards this end I take an example say, the follower displacement y theta is given by 25 into 1 minus cosine theta so many millimeters and this expression is valid for all the values of theta that is the entire cam rotation 0 to 2 pi. If this is y theta, then differentiate it once with respect to theta, I get y prime theta which is 25 sine theta.

From these two expressions of y theta and y prime theta it is very easy to see that, y theta minus 25 whole squared plus y prime theta whole squared is 25 square. So, the plot of y theta versus y prime theta will be a circle of radius 25 and centre at y equal to 25 and y prime equal to 0. If we remember x minus a whole squared plus y minus b whole squared, r squared is the equation of a circle of radius r with centre at a and b.

If I plot y theta versus y prime theta, satisfying this equation and that equation is the circle with centre at y equal to 25 and y prime equal to 0 and radius equal to 25. So this is a circle. Let us say, phi $_{max}$ is prescribed as 30 degree during the rise. We are not bothered about the pressure angle during return, during the rise phase phi $_{max}$ must be limited to 30 degree maximum value of phi should not go beyond 30 degree.

In this diagram, as we see from O to this point A, y prime is positive, that is the rise phase and this is the return phase when y prime is negative. During this rise phase, the value of phi should never exceed 30 degree. To keep that maximum value 30 degree, we draw a tangent to this circle such that, the tangent is inclined to the vertical by an angle 30 degree. This line is inclined to the vertical by 30 degree that is this angle is 60 degree and from here, I again draw a line at 30 degree to the vertical and these 2 lines intersect at the point C. Then if I draw a vertical line through C, this gives me the optimum amount of offset and OC gives me a minimum value of r_p, because if we recall the last lecture, with any point we take on this curve then join it with C, then the inclination of that line from the vertical gives the pressure angle.

As we see from this point to all other points, the inclination to the vertical never goes beyond 30 degree. Similarly, in this phase whatever lines I draw the inclination to the vertical never exceeds 30 degree. This is also 30 degree. So, we get this angle as 60 degree. Let me now name these points: Vertical line and the y prime axis this point, let me called E and this point, let me call D and this point of tangency, let me call B and the centre let me call F. As we see, this angle is 120 degree. This angle is 90 degree. This angle is 90 degree. So this angle also is 60 degree. Thus we get.

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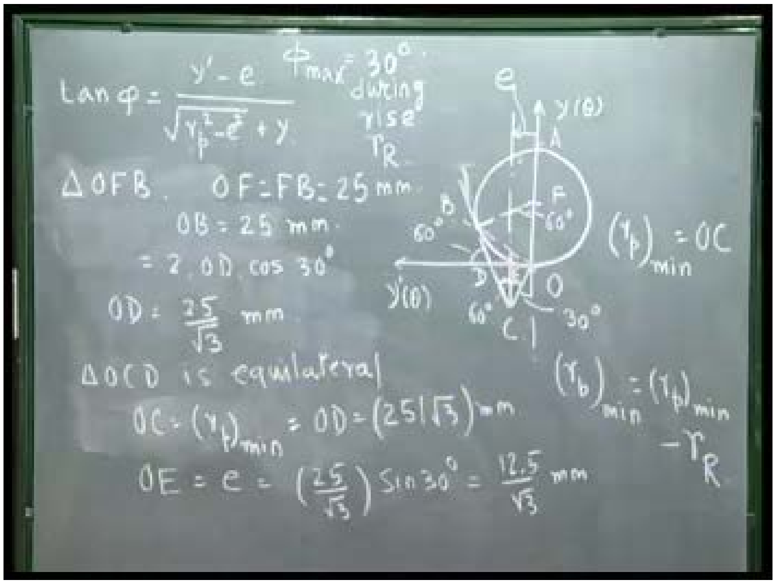

If I draw this line just for the sake of construction OFB, OF is FB. In triangle OFB, I get OF is FB is equal to the radius of this circle which was 25 millimeters and this angle is 60 degree. So this is an equilateral triangle. So I can write OB also 25 millimeters. If I drop a perpendicular from here, which is easy to see that OB is nothing but twice of OD into, this angle is 30 degree, this is 120 these 2 are equal, so 30 degree and 30 degree, so twice OD cosine 30 degree. So that tells me OD is first, that is root 3 by 2, so 25 by root 3 millimeters.

Here we see this is 60 degree, this is 60 degree and this is also 60 degree. So this is again, an equilateral triangle OCD, triangle OCD is equilateral. All sides are equal because, all the angles are 60 degree. So OC, that is (r_p) minimum or optimum turns out to be same as OD, which is 25 by root 3 millimeters. What is the optimum amount of offset? That is OE, OE I can easily see, this is 30 degree this is OC. So this is OC sine 30 degree, so half of that. So, optimum e is 25 by root 3 into sine of 30 degree which is half so we can write 12.5 by root 3 millimeters. We get both (r_p) minimum and e, and once I get (r_p) minimum, I can easily get the base circle radius minimum value which is (r_p) minimum minus the roller radius which has to be specified. This is given and also we have to give that roller radius.

This is the simple diagram verses y theta, versus y prime theta. We get a circle. So everything I could calculate analytically, but if y theta is a complicated function, it changes during rise and then dwell then again return, then dwell, whatever it is. I can always calculate y prime for every value of theta and get a closed curve and do the same construction. Draw a tangent here, and take a line exactly at phi_{max} with the vertical.

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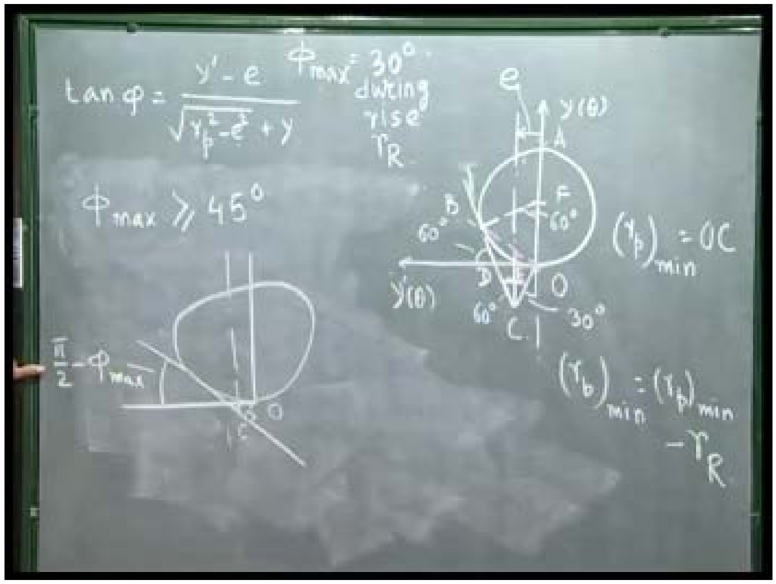

If we talk of a very theoretical situation, that if phi_{max} is more than 45 degree, which normally never is, but suppose if phi_{max} is more than equal to 45 degree then $(r_p)_{minimum}$ calculation would have been a little different. What I would have done is, if this is the closed curve, then I draw a, if phi_{max} is greater than 45 degree, so the tangent I draw at an angle phi_{max} to the vertical, so this angle is phi by 2 minus phi_{max}. Then to get minimum value of OC, that is r_p, I just need to draw a perpendicular to this line. If this is O, this point would have been C, because in this case the pressure angle during this part of the rise and this part of the rise is automatically guaranteed to be less than 45 degree. I have to ensure only that during this phase, phi_{max} does not go beyond 45 degree, because if this angle is phi by 2 minus phi_{max} so this angle is phi_{max} and if this is less than 45 degree, and this is 90 degree. If phi_{max} is more than 45 degree, then this angle will be automatically less than 45 degree.

If this angle is pi by 2 minus phi_{max}, then this angle is phi and if this phi_{max} is more than 45 degree, then this angle is automatically less than 45 degree because this is pi by 2 minus phi_{max}. So during this phase and this phase, pressure angle is ensured to be less than 45 degree. But this is only theoretical because, phi_{max} we never take such a large value. For phi_{max} less than 45 degree, I have to draw a tangent which is inclined to the vertical by phi_{max} and from O, I draw a line which is inclined to the vertical by phi_{max} and wherever they intersect, that determines the point C and OC gives me the minimum value of r_p and the vertical line, if I draw the distance between the y axis and this vertical line gives me the optimum offset. So thus we have finished our discussion to get the basic dimensions for cam profile for translating follower both roller and flat face.

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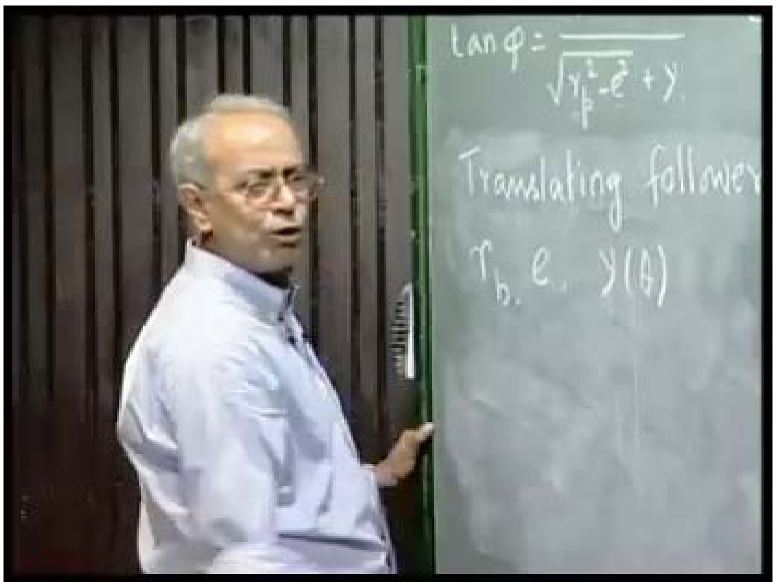

Our next task will be to design or synthesize the cam profile, when we are given or determine r_b and e. To generate a particular follower motion, what should be the cam profile? That is the discussion that we will take up next.

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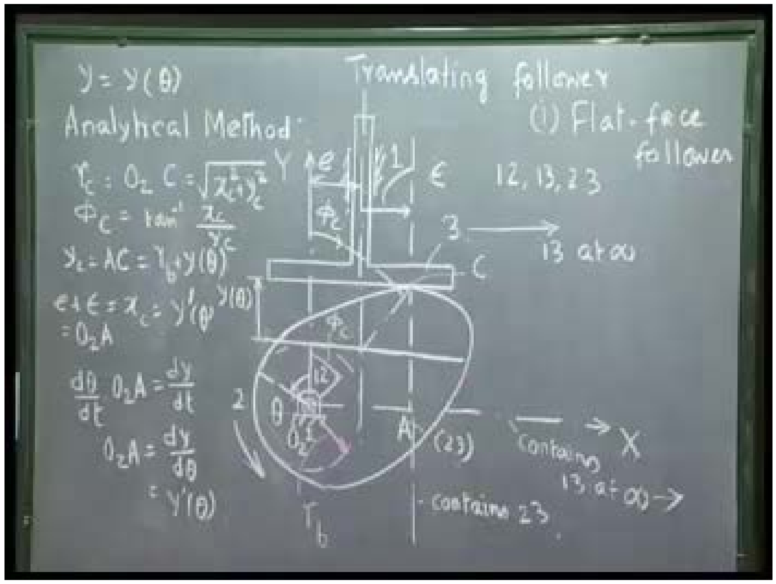

We know that how to size the cam that is, how to choose the value of the base circle radius and the proper amount of offset, the next task is to synthesize the cam profile to generate the desired follower motion given by y equal to y theta. We start with analytical procedure. For the time being, we are restricting our discussion to translating follower and first, we discuss if we have a flat face follower.

This is the flat face follower. Suppose, the cam was rotated by theta, this is the cam with the cam shaft here. This is what we call the offset, cam is rotating counter clockwise. If we draw the base circle of this cam, this is the base circle radius r_b. When the follower touches the base circle that is the lowest position of the follower, this tangent at the top most point of the base circle is the lowest position of the follower.

This is the contact point, when the cam has rotated by an angle theta and consequently, the follower has moved by this distance which is y theta. The contact point let me call C and through this cam shaft O_2, let us take X-axis horizontal and Y-axis vertical. First of all, our objective is to find the polar coordinates of this contact point C. What is the polar coordinate? To find the polar coordinate that is O_2C and say I measure the angle from Y-axis in the clockwise direction that is this angle which we call phi_C. So the polar coordinates of this contact point r_c is O_2C and the other coordinate is phi_C. As we see phi_C is measured from the Y-axis in the clockwise direction.

To find this r_c, we will first find what are X and Y coordinates of the point C. This is the distance which we called eccentricity of the driving effort epsilon and this point let me call A. So y_C is AC and that is easy to see, that AC is nothing but r_b plus y theta.

If we recall yesterdays lecture, this distance O_2A is e plus epsilon which is x_C was shown to be y prime theta. The x co-ordinate which is O_2A. It can be shown very easily that this distance is y prime theta. Suppose, I use that Arnold- Kennedy theorem of 3 centers, this is the fixed link. There is the guide 1, 1, cam is 2 and the flat face follower is 3. So it is a three link mechanism consisting of the fixed link cam, which I have number 2 and this follower which have number 3.

Out of which 12 is that O_2 because there is a revolute pair, the cam can rotate with respect to the fixed link about this axis. So this O_2 is nothing but 1 2, 2 3 is on this common normal or that is on this vertical line. This line contains 2 3 and because 3 is in vertical translation with respect to 1, 1 3 is on the horizontal direction at infinity. So if I draw a horizontal line through O_2 that is X-axis, this contains 1 3 at infinity and now I know these 3 relative instantaneous centers, namely 1 2, 1 3 and 2 3 are collinear by Arnold- Kennedy theorem. So 1 2 is here 1 3 is at infinity, 2 3 is on this line. So this point A is nothing but 2 3. The velocity of this point A, if I consider it to be a point on the cam, that is on body 2, whatever velocity I get, it will be the same velocity at this point for the body 3, that is the follower.

If I consider it to be a point on body 2, the angular velocity of the cam is d theta divided by dt into O_2A in the vertically upward direction and if I consider to be a point on body 3, which is the follower and follower is in perfect translation in the vertical direction. All points of the follower have the same velocity. So this same velocity is of the follower, which is dy divided by dt. So that clearly tells me, O_2A is nothing but dy divided by d theta which is y prime theta. So given y theta, I can find both the x -coordinate of the contact point and the y -coordinate of the contact point, x_C and y_C. y_C is r_b plus y theta, when r_b is known, y theta is known, I can find y_C. If y theta is given, I can find y prime theta and x_C is nothing but y prime theta, r_c is nothing but square root of x_C squared plus y_C squared.

What is phi_C? This angle nothing but, tan inverse x_C by y_C, because this is the angle measured from the vertical. So tan of this is x_C by y_C. This is x_C, this is y_C and this angle I am calling phi. Do not confuse it with the pressure angle. That is why, I am witting phi_C. Phi is the contact point. C is the contact point. This is the polar coordinates of the contact point in the fixed frame of reference in this XY axis, which is fixed in space that is, that belongs to this fixed body 1.

What is the cam profile? Cam profile is nothing but the locus of this contact point, but it has to be expressed in a coordinate system fixed in the cam. So, I fix the coordinates system in the cam. Suppose this line in the cam, I take as my fixed line.

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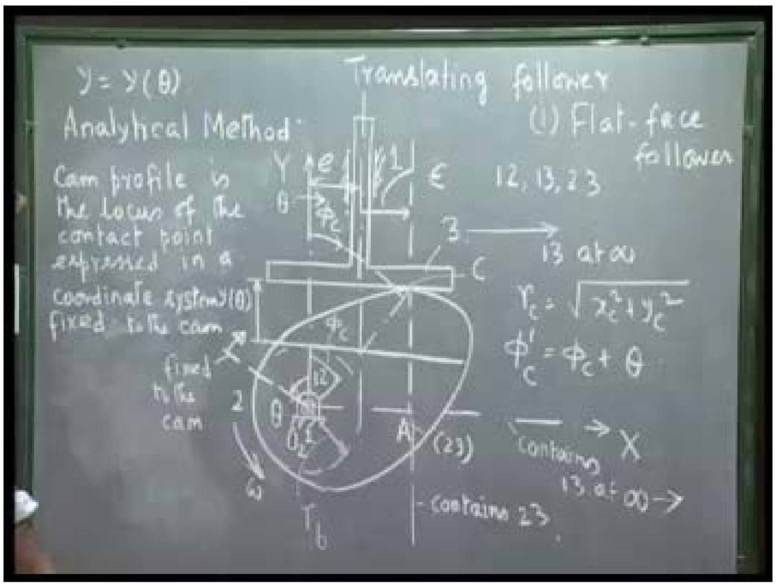

First of all, let me write cam profile, is the locus of the contact point for various values of theta, as the cam rotates, contact point expressed in a coordinate system, fixed to the cam. How do I express this coordinate of the point C, in a cam fixed coordinate system? If I fix the cam, then it is nothing but a kinematic inversion. So the fixed link 1 rotates in the clockwise direction with the same value of omega. If the cam is rotating with a constant angular velocity omega and if I hold the cam fixed, the same kinematics is maintained.

If the fixed link is rotated in the opposite direction by the same amount, that is kinematic inversion. This is the relative motion of the cam rotation with respect to the fixed link, in the counter clockwise direction. Fix the cam, that means kinematic inversion implies. The fixed link 1 will rotate in the clockwise direction, with the same angular velocity. This line fixed in space will rotate in the clockwise direction by theta.

What is the angle of this, if I measure it from this line fixed to the cam? This angle is theta and from this y axis which is rotating, so this angle will be always by measured by theta, because if this line has rotated from the fixed line by an angle theta in the counter clockwise direction. If I fix this line, the vertical axis, y axis rotates in the opposite direction, with the same value of theta. If I want to use the cam fixed coordinates system, then I write r_c is same as before. x_C squared plus y_C squared, but I write phi_C prime, which is the angle measured from this line fixed to the cam, for this line O_2C, that is phi_C plus theta and it has to be measured in the clockwise.

We already have the expression of r_c and phi_C. So I get the cam profile, in the polar coordinate system. I plot all these points by measuring from the $O_{2,}$ this distance r_c and measuring the angle of that line O_2C, from this line fixed in the cam, by this theta plus phi_C, where phi_C is nothing but tan inverse x_C by y_C which I wrote earlier.

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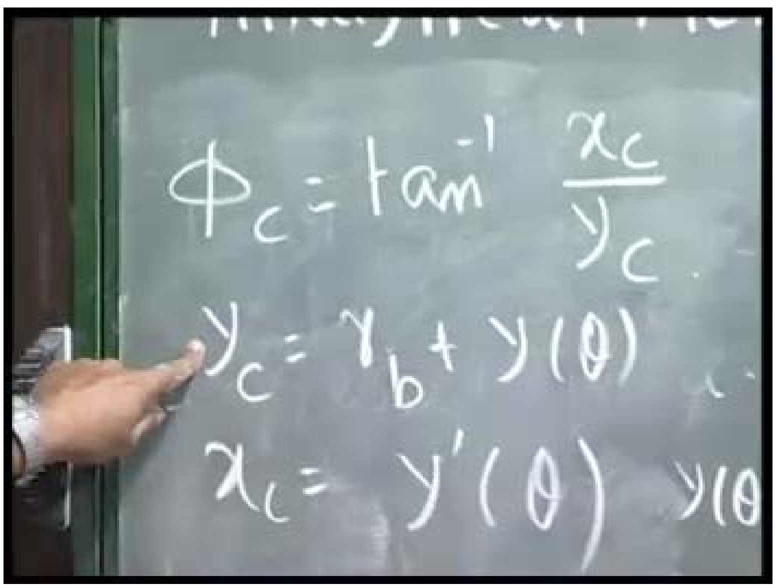

Phi_C is tan inverse x_C by y_C and if we remember y_C is nothing but r_b plus y theta, and x_C is y prime theta. So given r_b and y theta, I get x_C and y_C so I get phi_C and then I can get r_c and phi_C prime and to get the polar coordinate representation of the cam profile with O_2 as the origin. I will continue now this, for the similar exactly same technique for the roller follower.

Just now we have discussed the analytical method of cam profile synthesis for a flat face follower. We obtained the polar coordinates of the contact point and expressed those polar coordinates in a coordinate system, fixed with the cam and that gives the representation of the cam profile. We will extend exactly the same technique for translating roller follower.

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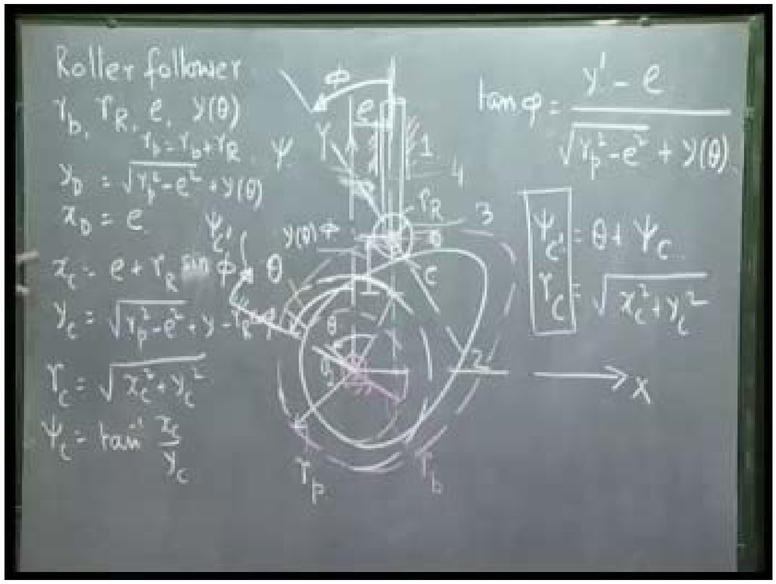

We have been given r_b, r_R, e and the desired motion of the follower y theta and we have to obtain, what is the curve representing the cam surface or the cam profile? We first, draw the cam. This is the cam shaft axis O_2 and this circle is as usual represents the base circle of radius r_b. Suppose, the roller follower is offset that is the roller centre is here. The roller radius is given to us as r_R. This is the guide of the fixed link 1, this is the follower 4, this is the roller 3, this is the cam 2.

We have also seen that I can draw, what we called a pitch curve and this is the prime circle radius r_p. As before, we take X and Y-axis to O_2, measure pi in the clockwise direction. Let me not confuse this phi as the pressure angle. So I use a different symbol here, the polar angle, I call psi. This is the roller centre say C. This contact point, let me call C and the roller centre, I call D. So our job is to find, what is the polar coordinate of this contact point C?

Through C and the roller centre, if I draw a line that is the common normal and this angle we define as the pressure angle phi. This angle is the pressure angle phi. When the roller centre intersects the prime circle, that is the lowest point, this point is the lowest point of the follower and this is the movement of the follower, when the cam has rotated to theta. So this is y theta, and this angle is theta.

Let me first, try to get the x and y-coordinates of this contact point C. For the roller centre, I can write coordinates of y_D and x_D. First let me get to the roller centre. x_D is very simple, that is nothing but the amount of offset. This distance is e. So x_D is e and y_D is this distance plus y theta. This distance as we discussed in our last lecture, is nothing but square root of r_p squared minus e squared. This is a right angle triangle with hypotenuse r_p and this horizontal side e. So the vertical side is square root of r_p squared minus e squared. This is the vertical side plus the roller centre has moved from here to there by amount y theta. So y_D is square root of r_p squared minus e squared plus y theta.

If we get to the coordinate of D, then that of C, I can easily see x_C is e plus r_R sine phi. As this angle is phi and this is $r_{R,}$ so extra horizontal distance is r_R sine phi and y coordinate is below that level D and by r_R cos phi. So y_C is y_D minus r_R cos phi. So square root of r_p squared minus e squared plus y theta minus r_R cosine phi, where phi is the pressure angle and we already got an expression for tan of phi is, y prime minus e divided by square root of r_p squared minus e squared plus y theta. I get the x and y-coordinate of the contact point, in this xy coordinate system, because e is known to us. Given the function y, I can calculate this and I get the value of phi for values of theta. Once I know that phi, I can get x_C which is e plus r_R sine phi.

Similarly, given this quantities y_C, I can get because I know the value of phi from here, I can calculate cos phi. rp is nothing but r_b plus r_R that is known, so I get x_C and y_C. The polar coordinates of this point C, r_c is nothing but square root of x_C squared plus y_C squared and psi_C is nothing but tan inverse x_C by y_C. This is the contact point polar coordinates in the fixed frame of reference, but the cam profile is nothing but the locus of the contact point in the cam fixed system. So psi is measured from this vertical line, but this vertical line which is the line belonging to the fixed link 1, after kinematic inversion, that is if we hold the cam fixed, we will rotate in the clockwise direction. So if this is the configuration at theta, so I draw a line at an angle theta from the vertical and this line, I hold fixed. This is the line fixed to the cam and I hold this fixed. So as a consequence, this y axis rotates in the clockwise direction and that angle will be always theta.

What will be the polar coordinate in this cam fixed system? If the angle is measured from this line, which I called as before psi$_C$ prime, so psi$_C$ prime is nothing but theta plus psi$_C$, and r$_c$ is same, which is square root of x$_C$ squared plus y$_C$ squared. These are the polar coordinates representation of the contact point in a coordinate system, fixed in the cam. This line, which I am holding fixed in the cam, when the cam has rotated by an angle theta, I draw this line at an angle theta counter clockwise direction from the vertical. I measure this psi$_C$ prime, from this line in the clockwise direction. How do I obtain these values? Let me go through it once more.

I get the coordinate x$_C$, y$_C$ because the pressure angle phi, I can always get once y, y prime, e and r$_p$, these quantities are known. So for various values of theta, I can get x$_C$ and y$_C$. So for various values of theta, I can get r$_{c,}$ and for various values of theta, I get first psi$_C$ and then add theta to get psi$_C$ prime. If I go on plotting these values r and psi, r is the distance from O$_2$ and psi is measured from this line, which has been drawn at an angle theta from the vertical line. Then plot all those points and the locus of all these points will give me the cam profile. This is how we get the analytical expression for the cam profile for a translating roller follower.

So far we have discussed in great details, the determination of the basic dimensions and the cam profile synthesis with reference to translating follower. So far the oscillating follower is concerned, determination of basic dimension is a little more involved.

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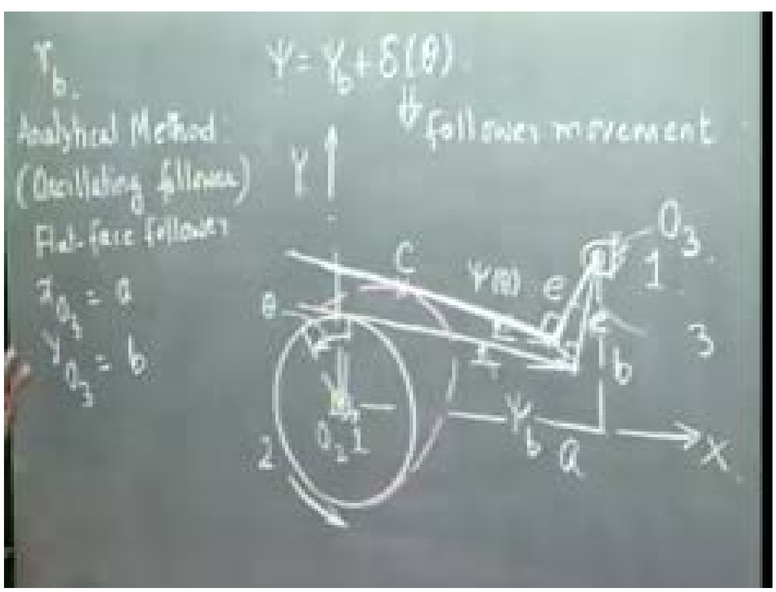

We will assume, that the basic dimensions like base circle radius is determined. It is already given to us and the desired follower motion is prescribed and how to synthesize the cam profile or how to determine the cam profile by analytical method? So we continue our discussion with analytical method: but with reference to oscillating follower. Here again, I will restrict our discussion, only to a flat face follower. Exactly same methodology can be applied to roller follower, but the algebra involved is a little different, but the methodology is just the same. Flat face follower is equally applicable for a roller follower.

First let me discuss, how do we describe or how do we express the follower motion incase of an oscillating follower? As we know, the size of the cam is described by this base circle area. Let us say this is the base circle radius of a cam, this is as usual the cam shaft axis. When the oscillating flat face follower is in contact with the base circle that is the extreme position from where the follower motion is measured or the rise of the follower starts. Let us assume that the follower face which is in contact with the base circle radius is represented by this line and this follower is hinged at this point. It is the oscillating follower. So it is hinged at one point. There is a revolute pair with the fixed link and the whole follower will oscillate about that point. So I draw a perpendicular to this follower face. This is 90 degree. I represent the follower by these two straight lines the follower face and these gives me another kinematic dimension which is necessary. Here as we know the fixed link is 1 and cam which is number 2. This is the base circle of the cam and this is the follower which is link 3. So let me call this revolute pair O_3.

Due to the shape of the cam, as the cam rotates this point was here the follower was at this position and now follower will be, this is the current position and if I drop a perpendicular to this line this dimension remains e. This is moving in a circle. So this rigid body rotates in the clockwise direction as the cam rotates in the anticlockwise direction. This is the basic motion geometry of the flat face follower which is oscillating and as we see, this distance changes because the contact point on the follower face moves.

How do I express this particular follower motion? Suppose, I draw a horizontal line, this is the lowest position of the follower which I express this angle by psi_b and when the cam has a follower rotated in the counterclockwise direction, this I called psi has a function of theta. So psi is which is a function of theta is psi_b plus some angle which is a function of theta. It is this delta theta which represents the follower movement. Instead of a trace point, I am talking of this follower movement in terms of this angle directly. That is more convenient. We should also note that, we have already got another kinematic dimension namely this e which is perpendicular drop from O_3 to this follower face which remains same, because this is the same rigid body. This angle is always 90 degree.

The relative position of O_2 and O_3, these two revolute pair is also two important kinematic dimensions. Suppose I draw by X-axis here and Y-axis here. So the x and y- coordinate of O_3 are two more important dimensions, I call it a and b, x-coordinate of O_3 is a, y-coordinate of O_3 is b, where O_3 is the hinge point for the oscillating follower and O_2 is the cam shaft, e is another kinematic dimension which has been obtained by dropping a perpendicular from O_3 to the follower face and then the follower is treated as this rigid body consisting of two straight lines at right angle. This is the lowest position when the follower was in contact with the base circle that is, when this point was here. Now the cam has rotated through an angle theta. As the cam has rotated through an angle theta, the follower has rotated through an angle delta theta and the angle that this follower face makes with the X-axis or horizontal line that I write as psi.

At theta equal to 0, the value of psi was psi_b, when delta theta is also 0. At theta equal to 0, delta theta is 0, and psi is equal to psi_b. So delta theta is the follower movement which is the function of theta that is how it prescribed. Our objective will be to find the polar coordinates of this contact point C which will define the cam profile, when I express this coordinate in the cam fixed coordinate system. That is basically the method. But to do this, we need to do a little bit of elaborate geometry as we will see just now.

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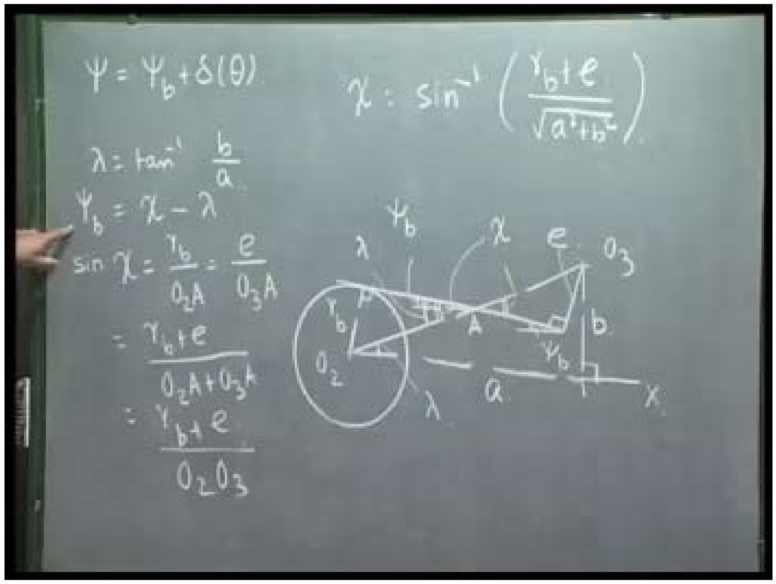

For oscillating flat face follower, the angle that the follower face makes with the horizontal was expressed as psi_b plus delta theta. So before we try to get the contact point coordinates, first let me get the expression for this angle psi_b, which the follower face makes at its extreme position when it is contact with the base circle. So let us say this is the base circle with O_2 as center and the follower face is tangent to the base circle. This is the point O_3 where the follower is hinged and this angle is 90 degree. The angle that this follower face makes with the horizontal, that we call psi_b.

If I draw a line which is horizontal, this angle is psi_b. Now the line O_2O_3 this is a horizontal line this distance we call a and this distance we call b. This is 90 degree. This is the X-axis. This angle is clearly seen to be tan inverse b by a and this angle is nothing but this angle. This angle is tan inverse lambda is the angle which this line is making with the horizontal. So this line is the horizontal line. So this angle is same as this angle. This is lambda.

Lambda is tan inverse b by a and psi_b that is this angle I can get, if I find this angle and then subtract lambda from here. So to find this angle, let me join this line this is at 90 degree. This is at 90 degree and this angle is same as this. This angle if we give a name say chi, then psi_b is seem to be chi minus lambda and to get chi I can easily get it, if I write this as the base circle radius r_b this dimension is e and this point, let me name A, the point of intersection of the flat face at the lowest position with O_2AO_3. So this angle is equal to this angle. This angle is 90 degree. This angle is 90 degree. So from here, I can write sine of chi is r_b by O_2A and from here I can write sine of the same angle chi is e by O_3A. So this is same as e by O_3A.

Using this relation, I can write, this is also equal to r_b + e by O_2A+O_3A which is same as r_b+e by O_2O_3 and O_2O_3 is nothing but square root of a squared plus b squared, because this angle is 90 degree. So chi I get sine inverse r_b plus e divided by square root of a squared plus b squared and psi_b is nothing but chi minus lambda where lambda is this angle. The angle O_2O_3 makes with X axis that is tan inverse b by a. So once these geometric dimensions are given like a, b, e and r_b, I am in a position to find the value of psi_b.

In our next lecture I will take off from this point and I will assume this value of psi_b which we can obtain given the values of a, r_b, e, b all that. Then I will try to get the coordinates of the contact point between the flat face and the cam profile and that will give me the cam profile in polar coordinates as we did in case of translating follower.